Investigation on Wear due to Corrosion of the Earth Electrode and the Rate of Increase in Earth Resistance

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Abstract: This paper investigates the rate of increase in earth resistance due to corrosion of the earth electrode. When the earth electrode corrodes, the predetermined value cannot be maintained due to the earth resistance increasing. As there has been no study of the relationship between corrosion of the earth electrode and earth resistance, it is difficult to predict the rate of increase in earth resistance due to electrode corrosion. In this paper, we have derived a calculation formula for determining changes in electrode shape that takes both electrolytic corrosion and natural corrosion into account, and this makes it possible to estimate the lifetime of the earth electrode. Furthermore, by using this calculation formula, it is possible to predict the rate of increase in earthing resistance value when electrolytic corrosion and natural corrosion progress simultaneously.

Key-Words: - earth resistance, earth resistivity, hemispherical electrode, rod-shaped electrode, electrode shape, electrolytic corrosion, natural corrosion, formula.

Received: November 15, 2025. Revised: January 11, 2026. Accepted: February 12, 2026. Published: May 29, 2026.

1 Introduction

It is known that when an electrode is buried in soil, the electrode wears out due to natural corrosion, [1]. It is also known that when DC current flows from an electrode into the soil, the electrode is consumed by corrosion due to electrolysis (electrolytic corrosion), [2].

However, until now, there has been no study of a calculation formula for electrode corrosion that takes both electrolytic corrosion and natural corrosion into account. Furthermore, no study has investigated the increase in earthing resistance when electrolytic corrosion and natural corrosion are taken into account at the same time. For this reason, the only way to determine the wear status of the earth electrode was to dig up the soil and visually check it.

In addition, the only way to predict an increase in the earth resistance of the earth electrode is to periodically measure the earth resistance, [3], [4], [5], [6].

Therefore, in this study, we have developed a calculation formula for determining changes in electrode shape by simultaneously considering electrolytic corrosion and natural corrosion for hemispherical electrodes and rod-shaped electrodes, making it possible to estimate the lifetime of earthing electrodes.

2 Regarding the Increase in Earthing Resistance due to Corrosion of the Earthing Electrode

Corrosion of the earth electrode is one of the typical cases where earth resistance increases. Since the earth electrode is made of metal, corrosion occurs. The earth electrode material is usually iron and may be coated with copper. Once an earth electrode is buried, its condition cannot be easily observed. In many cases, the disappearance of the earth electrode is only noticed after a problem occurs. Therefore, it is important to estimate the lifetime of the earth electrode. Electrode corrosion can normally be divided into natural corrosion and electrolytic corrosion. Water and oxygen behavior are important factors in natural corrosion in soil. Water in soil dissolves minerals and organic matter and acts as an aqueous solution of acids, salts, and alkalis, but the component concentrations and oxygen concentrations of this aqueous solution vary depending on the soil type. Also, depending on the surface condition of the metal in contact with each metal, an anode part and a cathode part are formed on the metal surface, creating an electric circuit of anode → electrolyte (soil) → cathode, and the metal turns into ions at the part that becomes the anode. Earth resistance is the resistance from the earth electrode to the depths of the earth. The earth resistivity of the location can be measured using Wenner's four-electrode method, [7]. The earth resistance can be calculated from the shape of the electrode and the resistivity of the soil (earth resistivity). Resistivity is affected by water content and salinity in the soil. Resistivity is an index that expresses the ease with which current flows in soil. In other words, the lower the resistivity, the easier the current will flow, which is an indicator of soil corrosiveness. The resistivity of soil is about 10 to 1000 Ω·m, and in ordinary soil it is several 100 Ω·m. Table 1 lists the relationship between natural corrosion rate and resistivity in the case of iron, [8].

Table 1. Relationship between natural corrosion rate and resistivity in the case of iron

corrosive	Natural corrosion rate (mm/year)	Resistivity (Ω・m)
Extremely large	≧0.125	0～10
Large	0.04~0.125	10~50
Middle	0.01~0.04	50~100
Small	0.0025~0.01	100~1000
Extremely small	≦0.0025	≧1000

Source: [8]

On the other hand, electrolytic corrosion is electrolysis of an earth electrode due to stray current from a DC（Direct Current） electric railway. Electric railway is mostly DC, and the rails are used as a return path for train current. As shown in Figure 1, stray currents (leakage currents) from the rail flow to the ground due to insufficient insulation between the rail and the ground. DC electric railways require a large amount of current, with a voltage of 1500 V and a 15-car train having a maximum starting current of 3000A. For this reason, even a small percentage of leakage current causes problems, [2]. Since stray current flows through a path with low electrical resistance under the ground, the current tends to flow into and out of the earth electrodes, and it is known that the earth electrodes through which the current flows out are severely corroded by electrolytic corrosion. The current value varies over time, but the current flowing through the lead wire of the earth electrode can be measured using a clamp-type ammeter that utilizes the Hall effect. The time-averaged value of this current can be used for calculating the stray current described below. However, the effect of the current needs to be estimated within a certain range based on the obtained average value.

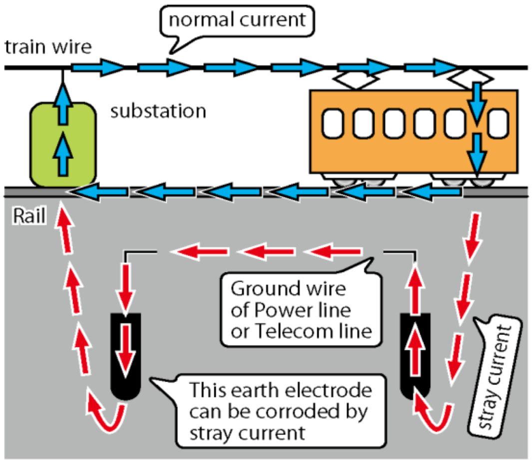

Fig. 1: Electrolytic corrosion due to stray current.
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3 Derivation and Solutions of the Calculation Formula for Electrode Corrosion

Here, we will derive a formula for calculating electrode wear when considering electrolytic corrosion and natural corrosion at the same time for hemispherical electrodes and rod-shaped electrodes.

As shown in Figure 2, the hemispherical electrode is assumed to have a hemispherical shape with a radius r, and the direct current I flows uniformly from the surface of the hemispherical electrode to the Earth. In addition, as shown in Figure 3, the shape of the rod-shaped electrode is a rod with radius r and length L, and the tip is semispherical with radius r, so that the direct current I flows uniformly from the surface of the rod-shaped electrode to the Earth. At this time, in both the case of a hemispherical electrode and the case of a rod-shaped electrode, the equation representing the wear of the electrode due to corrosion is equation (1), [9].

Here, the first term on the right side of equation (1) represents electrolytic corrosion due to direct current, and the second term on the right side represents wear of the electrode due to natural corrosion.

$$-\mathrm{d}r = \frac{iM\mathrm{d}t}{zFd} + V_0\mathrm{d}t \tag{1}$$

dr: decrease in electrode radius r at time dt
I: total current flowing out of the electrode
i: Current density flowing out from the electrode
M: Atomic weight of electrode metal
z: Valence of electrode metal
d: specific gravity

F: Faraday constant
V_0: Natural corrosion rate of electrode metal in soil

3.1 Corrosion Calculation Formula for Hemispherical Electrodes

In the case of a hemispherical electrode, the current density *i* flowing from the electrode surface to the Earth is calculated by dividing the total current *I* by the surface area of the hemispherical electrode and is expressed by equation (2).

$$i = \frac{I}{2\pi r^2} \tag{2}$$

Substituting equation (2) into equation (1) and rearranging this results in equation (3).

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\left(\frac{A}{r^2} + V_0\right) \tag{3}$$

where, $A = \frac{MI}{2\pi zFd}$

when the differential equation of (3) is solved under the initial condition that when time $t = 0$ s, the radius *r* of the earth electrode is r_0, the relationship between time *t* and the electrode radius *r* at that time can be expressed as (4).

Although this differential equation contains some approximations, we have obtained an analytical solution as follows:

$$t = \frac{r_0 - r}{V_0} - \frac{1}{V_0}\left(\sqrt{\frac{A}{V_0}}\arctan\sqrt{\frac{V_0}{A}}r_0 - \sqrt{\frac{A}{V_0}}\arctan\sqrt{\frac{V_0}{A}}r\right) \tag{4}$$

In equation (3), the first term on the right-hand side represents the effect of electrolytic corrosion, while the second term represents the effect of natural corrosion. The solution in the absence of electrolytic corrosion is clearly a linear line, which appears in the first term of equation (4). However, we are interested in regions where the influence of electrolytic corrosion is significant.

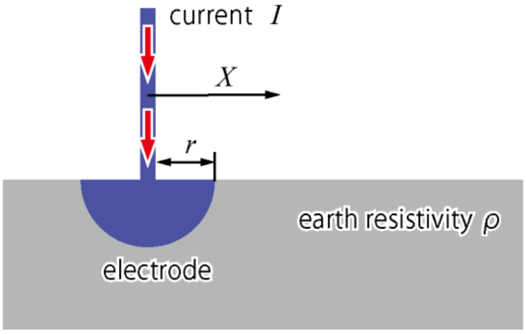

Fig. 2: Hemispherical electrode shape
Source: created by the authors

3.2 Corrosion Calculation Formula for Rod-Shaped Electrodes

In the case of a rod-shaped electrode, the current density *i* flowing from the electrode surface to the Earth is calculated by dividing the total current *I* by the surface area of the rod-shaped electrode (the sum of the side area and the surface area of the tip), which is (5).

$$i = \frac{I}{2\pi(r^2 + rL)} \tag{5}$$

Substituting (5) for (1) and reorganizing it gives formula (6)

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\left(\frac{A}{r^2 + rL} + V_0\right) \tag{6}$$

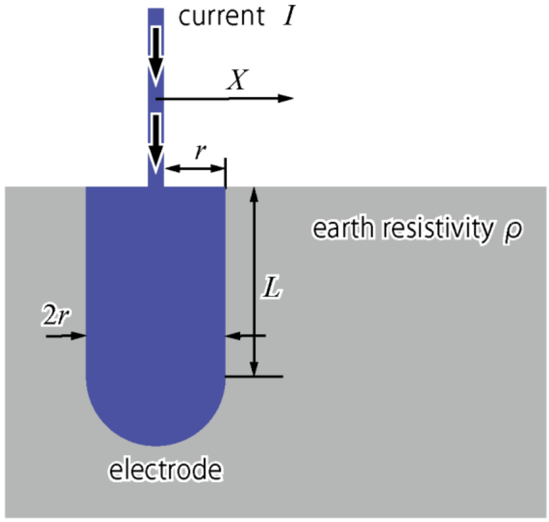

Fig. 3: Shape of rod electrode
Source: created by the authors

In addition, the equation obtained by differentiating this (5) with respect to time *t* is shown in (6).

When the differential equation of (6) is solved under the initial condition that when time $t = 0$ s, the radius *r* of the electrode is r_0, the relationship between time *t* and the electrode radius *r* at that time can be expressed as (7). We also obtained an analytical solution as follows:

$$t = \frac{r_0 - r}{V_0} + \frac{A}{V_0^2 \alpha}\left(\ln\left(\frac{\sqrt{r_0 + L/2 + \alpha}}{\sqrt{r_0 + L/2 - \alpha}}\right) - \ln\left(\frac{\sqrt{r + L/2 + \alpha}}{\sqrt{r + L/2 - \alpha}}\right)\right)$$

$$: \left[\frac{A}{V_0} < \left(\frac{L}{2}\right)^2\right] \quad \text{(7-1)}$$

$$t = \frac{r_0 - r}{V_0} - \frac{A}{V_0^2 \beta}\left(\arctan\left(\frac{r_0 + L/2}{\beta}\right) - \arctan\left(\frac{r + L/2}{\beta}\right)\right)$$

$$:\left[\left(\frac{L}{2}\right)^2 < \frac{A}{V_0}\right] \quad (7\text{-}2)$$

$$t = \frac{r_0 - r}{V_0} + \frac{A}{V_0^2}\left(\frac{1}{r_0 + L/2} - \frac{1}{r + L/2}\right)$$
$$:\left[\frac{A}{V_0} = \left(\frac{L}{2}\right)^2\right] \quad (7\text{-}3)$$

here

$$\alpha = \sqrt{\left(\frac{L}{2}\right)^2 - \frac{A}{V_0}}$$
$$\beta = \sqrt{\frac{A}{V_0} - \left(\frac{L}{2}\right)^2}$$

The first term on the right-hand side of equation (6) represents the effect of electrolytic corrosion, and the second term represents the effect of natural corrosion. In this case as well, the solution when there is no effect of electrolytic corrosion matches the first terms of equations (7-1) to (7-3). When the stray current is increased, it is observed that as the current value increases, the form of the solution undergoes continuous phase transitions in the order of linear function, equation (7-1), (7-3), and (7-2).

4 Calculation Formula for the Earth Resistance Increase Rate

4.1 Calculation Formula for the Earth Resistance Increase Rate of a Hemispherical Electrode

Focusing on hemispherical electrodes, we are investigating the increase in earth resistance when the electrodes are worn out due to electrolytic corrosion and natural corrosion. The earth resistance R of a hemispherical electrode of radius r is expressed as (8), [10].

$$R = \frac{\rho}{2\pi r} \quad (8)$$

Here, ρ is the earth resistivity.

Assuming that the initial radius of the earthing electrode without corrosion is r_0, the initial value R_0 of the earthing resistance at this time is given by the following equation by setting $r = r_0$ in (8).

$$R_0 = \frac{\rho}{2\pi r_0} \quad (9)$$

By normalizing (8) with (9), the rate of change in earth resistance can be found. That is:

$$\frac{R}{R_0} = \frac{r_0}{r} \quad (10)$$

From (8), the rate of change in earth resistance R due to the decrease in electrode radius r due to corrosion is expressed as (11) by differentiating (8) with r.

$$\frac{\mathrm{d}R}{\mathrm{d}r} = -\frac{\rho}{2\pi r^2} \quad (11)$$

From (11), it can be seen that the rate of increase in earth resistance R is inversely proportional to the square of the electrode radius r. Further, the rate of change of the earth's resistance R with respect to time t is expressed as (12-1) by differentiating (8) with respect to time t.

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{\partial R}{\partial r} \cdot \frac{\partial r}{\partial t} \quad (12\text{-}1)$$

By substituting (1) and (11) into the right-hand side of (12-1), we obtained (12-2).

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{\rho}{2\pi r^2}\left(\frac{A}{r^2} + V_0\right) \quad (12\text{-}2)$$

From (12-2), the rate of change of earthing resistance R over time t is the sum of two items: the part that is inversely proportional to the fourth power of the radius r due to electrolytic corrosion, and the part that is inversely proportional to the square of the radius r due to natural corrosion.

4.2 Calculation Formula for the Earthing Resistance Increase Rate of the Rod-Shaped Electrode

Focusing on rod-shaped electrodes, we are investigating the increase in earthing resistance when the electrodes are worn out due to electrolytic corrosion and natural corrosion. The earthing resistance R of a rod-shaped electrode with radius r and length L is expressed as (13), [11].

$$R = \frac{\rho}{2\pi L}\left(\ln\left(\frac{4L}{r}\right) - 1\right) \quad (13)$$

Here, ρ is the earth's resistivity.

Assuming that the initial radius of the earth electrode without corrosion is r_0, the initial value R_0 of the earth resistance at this time is as follows by setting $r = r_0$ in (13).

$$R_0 = \frac{\rho}{2\pi L}\left(\ln\left(\frac{4L}{r_0}\right) - 1\right) \quad (14)$$

By normalizing (13) with (14), the rate of change in earth resistance can be found. That is:

$$\frac{R}{R_0} = \frac{\ln\left(\frac{4L}{r} - 1\right)}{\ln\left(\frac{4L}{r_0} - 1\right)} \quad (15)$$

From (13), the rate of change in earth resistance R due to the decrease in electrode radius r due to corrosion is expressed as (16) by differentiating (13) with r.

$$\frac{dR}{dr} = \frac{\rho}{2\pi L}\left(-\frac{1}{r}\right) \quad (16)$$

The rate of change of the earth resistance R with respect to time t is expressed as (17-1) by differentiating (16) with respect to time t.

$$\frac{dR}{dt} = \frac{\partial R}{\partial r}\cdot\frac{\partial r}{\partial t} \quad (17\text{-}1)$$

By substituting (6) and (16) into the right-hand side of (17-1), we obtained (17-2)

$$\frac{dR}{dt} = \frac{\rho}{2\pi Lr}\left(\frac{A}{r^2+rL} + V_0\right) \quad (17\text{-}2)$$

From (17-2), it can be seen that the rate of change in earth resistance R over time t is the sum of two items: the part due to electrolytic corrosion and the part due to natural corrosion.

5 Calculation Examples

5.1 Case 1

A calculation example will be shown below, targeting a rod-shaped electrode. The material of the earthing rod is iron, the shape is radius $r_0 = 7.0$ mm, length L = 1400 mm, natural corrosion rate of iron V_0 is V_0 = 0.02 mm/year (middle rate listed in Table 1), atomic weight of iron M = 55.85 g/mol, valence z = 2, specific gravity d = 7.87 g/cm^3, and Faraday constant F = 96500 C/mol. These physical constants, such as atomic weight of iron, valence z, specific gravity d, and Faraday constant F can be referenced, for example, using the National library of medicine, [12]. Even though the earth electrode may be coated with copper, its thickness is about 0.01mm, so we ignored it. Soil is not uniform in the depth direction. As standard rod electrodes are driven into the ground from the surface using a hammer, electrode lengths of 0.5 to 1.4 meters are normally used. Therefore, even when using the maximum length of 1.4 meters, the soil to this depth can be considered to have an almost uniform resistivity. Even if the earth's resistivity is not uniform, as shown in equation (15), the rate of change in earth resistance is normalized, so it has no effect for corrosion simulation. If the electrodes corrode and iron ions dissolve, they diffuse due to the concentration gradient and disappear, so they do not affect the resistivity distribution in that soil. Using (7), Figure 4 shows the relationship between time t and radius r of the earthing rod when current I is used as a parameter.

The colored areas in the figure represent the range that can be obtained when V_0 is varied from 0.01 to 0.04 mm/year to estimate the uncertainty of the natural corrosion rate. In Equation (6), when I = 1 mA, the first term is about 0.019 mm/year at $r = r_0$, which is nearly the same magnitude as the second term, so the effect of changes in V_0 is significant. However, it can be confirmed that this effect decreases as the current increases.

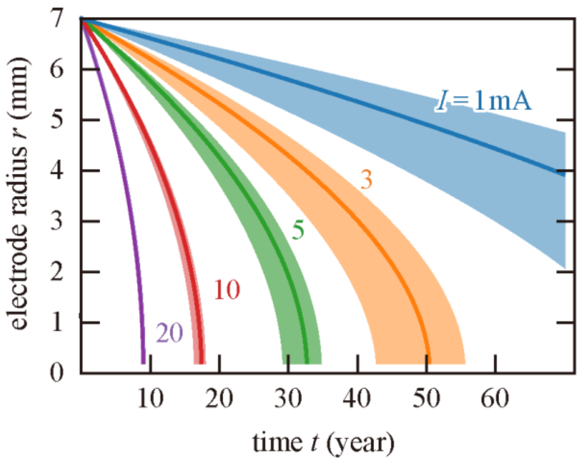

Fig. 4: Relationship between time t and radius r of earth rod
Source: created by the authors

Figure 5 shows the relationship between the time t for the whole electrode to disappear and the current I.

Using (7) and (15), Figure 6 shows the relationship between time t and normalized earth resistance R/R_0 when current I is used as a parameter.

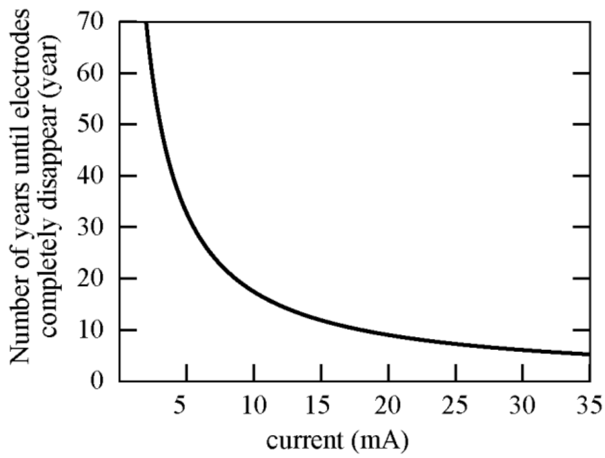

Fig. 5: Relationship between time t for the whole electrode to disappear and current I
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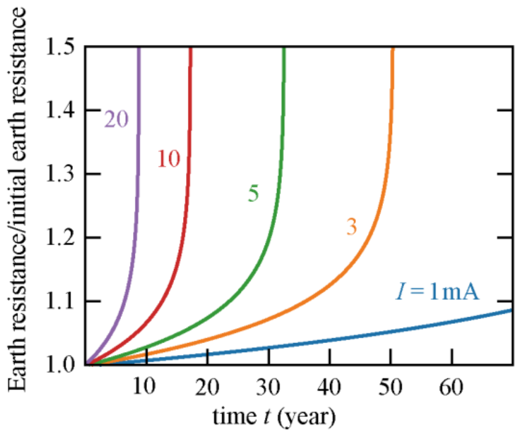

Fig. 6: Relationship between time t and normalized earth resistance R/R_0
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5.2 Case 2

Here is another calculation example.

The material of the earthing rod is iron, its shape is radius r = 3.5 mm, length L = 700 mm. The other conditions are the same as in Case 1.

Figure 7 shows the relationship between the time t and radius r of the earthing rod.

Because the electrode has become shorter and thinner, the effect of electrolytic corrosion is significant, and even at I = 1 mA, the influence on the change of V_0 is small. Also, a phase transition of the solution occurs between I = 4 mA and I = 5 mA, but no dramatic change is observed in the shape of the graph.

Figure 8 shows the relationship between the time t for the whole electrode to disappear and the current I.

Using (7) and (15), Figure 9 shows the relationship between time t and normalized earth resistance R/R_0 when current I is used as a parameter.

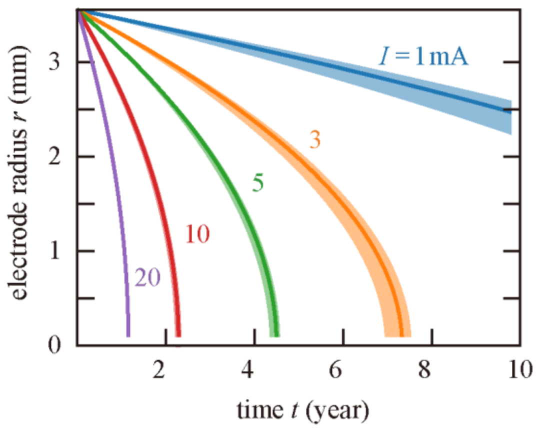

Fig. 7: Relationship between time t and radius r of the earth rod
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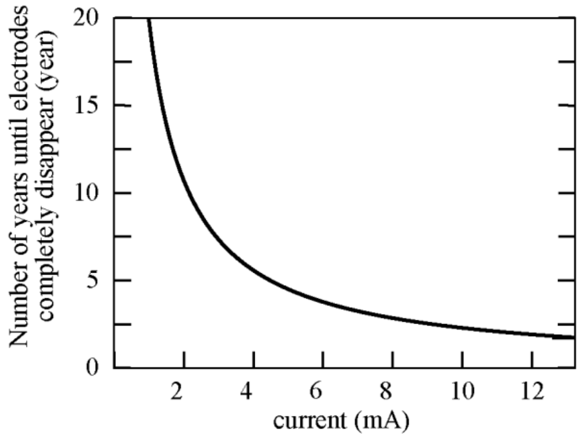

Fig. 8: Relationship between time t for the whole electrode to disappear and current I
Source: created by the authors

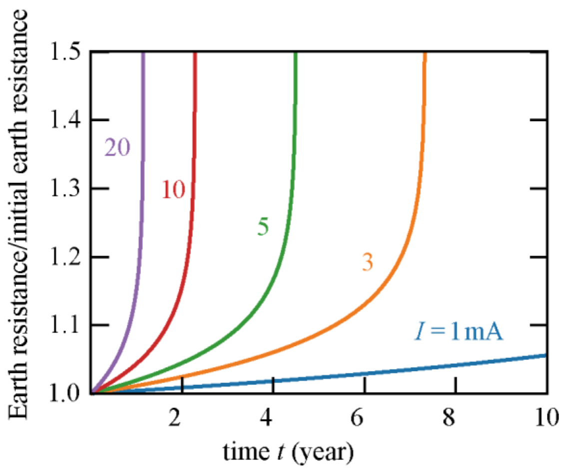

Fig. 9: Relationship between time t and normalized earth resistance R/R_0
Source: created by the authors

6 Conclusions

We have clarified a calculation that allows us to estimate the wear of the earthing electrode by considering both electrolytic corrosion and natural corrosion at the same time.

Until now, the only way to determine the corrosion status of earthing electrodes buried underground was to dig up the soil and visually check. Furthermore, the only way to predict an increase in earth resistance is to periodically measure the earth resistance.

However, by using the proposed calculation formula, it was possible to predict the wear of the earth electrode and the increase in earth resistance when electrolytic corrosion and natural corrosion progress simultaneously.

In the future, we would like to carry out experimental investigations using a large pool to simulate natural conditions. When a small container is used, the electric field between the earth electrode and the opposing electrode becomes specific according to the shape. In other words, even in the case of rod-shaped electrodes, if a large pool is used, the opposing electrode is placed far enough away, allowing the natural conditions to be simulated.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- Motom Sato, Chihiro Nishimoto and Naoki Kusumoto calculated the equations.
- Motom Sato, Chihiro Nishimoto and Hitoshi Kijima formulated the differential equation to be able to determine the degree of corrosion of metal earth rods.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

No funding was received for this study.

Conflicts of Interest

The authors declare no conflicts of interest relevant to the content of this article.