A New Functional Observer Design of Delayed Systems in Time and Frequency Domains

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Abstract: - This paper presents new time and frequency domain designs of a functional unknown input observer for a class of linear delayed systems fed by unknown input in both dynamic and output equations. In addition a comparative analysis between time domain, frequency domain based on time domain results and direct frequency domain is provided. The main interest here is to propose an observer that estimates a functional state and an unknown input vector. The time domain procedure design is based on Lyapunov-Krasovskii stability theory after giving the existence condition of such observers. The optimal gain implemented in the functional observer with internal delay design is obtained in terms of Linear Matrix Inequalities (LMIs). The frequency procedure design is derived from time domain results by applying the factorization approach where we define some useful Matrix Fraction Descriptions (MFDs). The efficiency of the proposed approach is shown by a numerical example.

Key-Words: - Functional Observer, Delay, Unknown Input, Time Domain, Frequency Domain, LMI,

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1 Introduction

It is obvious that the design of observers for linear systems has been a topic of recurrent researches since unknown inputs have been considered, [1], [2].

Moreover, linear delay systems have been a recurring subject of much interest researches.Indeed, the importance of such systems stems from their ability to describe systems with which standard state space representations are not applicable. Therefore, the observers' design for linear delay systems is of considerable interest, [2], [3], [4], [5].

Not least, delay modeling has been extensively studied as it influences the stability dynamic and other performances of the systems, [6], [7], [8]. This situation becomes obvious when dealing with communication networks, economic systems and chemical processes, [6], [7], [9], [10].

In the frequency domain, there is less studies in the state estimation relative to that of the time domain, [4], [11], [12], [13], [14], although it is the basis of most of the analyzes carried out in control systems and implementation procedures, [4].

Motivated by these facts, a new time and a new frequency domains technics have set up the main methods of designing functional observers for time delaying linear systems with unknown inputs that have to be presented.

The time domain procedure is based on Lyapunov stability theory, where we give the existence conditions of such observers to get a gain implemented in the design using (LMI). The design of the procedure in the frequency domain is derived from the time domain results where we propose some appropriate (MFDs). The application of the factorization approach helps to give a polynomial description of the proposed function observer.

The paper outlines are as follows. The previous section has already been part of the related work. Section 3 will present the problem formulation that we propose to solve. Section 4 will give a time domain solution for the observer design problem. A LMI approach is then applied to optimize the gain implemented in the observer. The fifth section will present the frequency domain representation of the unknown input functional observer in terms of polynomial matrices. This representation should be based on time domain solution. In section 6, we will sum up the time and the frequency domain design steps of the functional observer. Section 7, will illustrate a numerical example of our approach and then section 8 will conclude the whole work.

2 Related Work

Several studies have been interested on the estimation problem for a class of linear delayed systems. So observers have been developed with respect to stability performances. Then unmeasured state vector components are reconstructed according to a convergent dynamic.

In this scope, we propose a new estimator scheme for linear delayed systems, [15], [16], [17]. Relevant results have been, also, shown for the estimation of a fault vector present in both state and output descriptions. As presented by [18], [19], frequency method based on temporal contribution has been highlighted in this work.

Furthermore, polynomial approaches are operated to design the frequency scheme of the proposed observer. Compared to [4], [18], [19], precision and ultra observer dynamic reaction is proved especially when estimating the unknown input vector. In fact, the fault vector is detected and copied from the conditioned output measurement. In addition, proposed approaches are verified through a servomotor example used in a temperature measurement installation.

3 Problem Formulation

Considering the following continuous-time linear system.

$$\begin{aligned}\dot{x}(t) &= Ax(t) + A_d x(t-d) + Bu(t) \\ &\quad + B_d u(t-d) + R_1 v(t) && (1)\\ y(t) &= Cx(t) + R_2 v(t) && (2)\\ z(t) &= Lx(t) && (3)\end{aligned}$$

Where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$, $v(t) \in \mathbb{R}^{m_v}$ and $z(t) \in \mathbb{R}^{m_z}$are respectively the state, the input, the output, the unknown inputs and the functional state vectors.
A, A_d, B, B_d, R_1, R_2, L and C are the known matrices of appropriate dimensions.
$d \in \mathbb{R}_+$ is the state and the input delay.
We assume that:

$$\psi = \begin{pmatrix} x(t) \\ v(t) \end{pmatrix} \tag{4}$$

Then, system of equations (1), (2), and (3) can be written as :

$$\begin{aligned}E\dot{\psi}(t) &= \bar{A}\psi(t) + \bar{A}_d\psi(t-d) \\ &\quad + \bar{B}u(t) + \bar{B}_d u(t-d) && (5)\\ \bar{y}(t) &= \bar{C}\psi(t) && (6)\\ \bar{z}(t) &= \bar{L}\psi(t) && (7)\end{aligned}$$

Where

$$E = \begin{pmatrix} I_{n\times n} & 0_{n\times m_v} \\ 0_{m_v\times n} & 0_{m_v\times m_v}\end{pmatrix}, \bar{A} = \begin{pmatrix} A & R_1 \\ 0_{m_v\times n} & 0_{m_v\times m_v}\end{pmatrix},$$

$$\bar{B} = \begin{pmatrix} B \\ 0_{m_v\times m}\end{pmatrix}, \bar{C} = \begin{pmatrix} C & R_2\end{pmatrix}, \bar{B}_d = \begin{pmatrix} B_d \\ 0_{m_v\times m}\end{pmatrix},$$

$$\bar{A}_d = \begin{pmatrix} A_d & 0_{n\times m_v} \\ 0_{m_v\times n} & 0_{m_v\times m_v}\end{pmatrix}, \bar{L} = \begin{pmatrix} L & 0_{m_v\times m_z} \\ 0_{m_z\times m_v} & I_{m_z}\end{pmatrix} \tag{8}$$

In the sequel, we suppose that :

Hypothesis 1 [18], [20]

1. $rank\,(E) = r \leq (n+m_v)$
2. $rank \begin{bmatrix} E \\ \bar{C}\end{bmatrix} = n + m_v$

4 Time Domain Design

Under assumption 1 in hypothesis 1, there exists a no singular matrix :

$$S = \begin{pmatrix} a & b \\ c & d\end{pmatrix} \tag{9}$$

Such as:

$$aE + b\bar{C} = \bar{L} \tag{10}$$

$$cE + d\bar{C} = 0 \tag{11}$$

with $a \in \mathbb{R}^{(m_v+m_z)\times(n+m_v)}$, $b \in \mathbb{R}^{(m_v+m_z)\times(p+m_v)}$, $c \in \mathbb{R}^{p\times(n+m_v)}$, $d \in \mathbb{R}^{p\times(p+m_v)}$
The functional observer of system of equations (5), and (7), take the form of :

$$\begin{aligned}\dot{z}(t) &= Fz(t) + F_d z(t-d) + Hu(t) \\ &\quad + H_d u(t-d) + L_1\bar{y}(t) + L_2\bar{y}(t-d) && (12)\\ \bar{\hat{z}}(t) &= z(t) + b\bar{y}(t) + E_2 d\bar{y}(t) && (13)\end{aligned}$$

$\dot{z}(t)$ is the state vector of the observer and $\bar{\hat{z}}(t)$ is the functional state estimate.
The matrices F, F_d, H, H_d, L_1, L_2 and E_2 will be determined using the LMI approach.

4.1 Condition of Synthesis of the Unknown Input Observer

Using equations (7), and (13) the estimation error is given as follows.

$$\begin{aligned}e_z(t) &= \bar{z}(t) - \bar{\hat{z}}(t) \\ &= [a + E_2 c]E\psi(t) - z(t) \\ &= GE\psi(t) - z(t) && (14)\end{aligned}$$

With

$$G = a + E_2 c \tag{15}$$

Having given the continuous-time linear system of equations (1), (2), and (3)and the functional observer of equations (12), and (13) we aim at designing the observer matrices F, F_d, H, H_d, L_1, L_2 and E_2 where $\bar{\hat{z}}(t)$ asymptotically converges to $\bar{z}(t)$, so :

$$\lim_{t\to+\infty} e(t) = 0 \tag{16}$$

That's why, we suggest the following theorem :

Theorem 1 The functional observer of equations (12), and (13) is an unknown input observer for continuous-time delayed systems of equations (1), (2), and (3) if and only if the following equations are satisfied :

i. $\dot{e}_z(t) = Fe_z(t) + F_d e_z(t-d)$ is asymptotically stable.

ii. $G\bar{A} - FGE - L_1\bar{C} = 0$

iii. $G\bar{A}_d - F_dGE - L_2\bar{C} = 0$

iv. $G\bar{B} - H = 0$

v. $G\bar{B}_d - H_d = 0$

Proof 1 The derivative of equation (14) is given as follows :

$$\dot{e}_z(t) = GE\dot{\psi}(t) - \dot{z}(t) \tag{17}$$

Using equations (5), and (12), equation (17) becomes:

$$\begin{aligned}\dot{e}_z(t) = Fe_z(t) &+ F_d e_z(t-d) + [G\bar{A} - FGE \\ &- L_1\bar{C}]\psi(t) + [G\bar{A}_d - F_dGE \\ &- L_2\bar{C}]\psi(t-d) + [G\bar{B} - H]u(t) \\ &+ [G\bar{B}_d - H_d]u(t-d)\end{aligned} \tag{18}$$

In order to fit equation (16), we impose a dynamic of the estimation error. Based on equation (18), conditions ii) to v) become obvious.

4.2 Determination of Observer Matrices

This design procedure is based on Lyapunov-Krasovskii stability theory using LMIs tests. The functional observer equations (12), and (13) estimates a functional state and an unknown input vectors.
Replacing equation (15) in condition ii)-iv) and using equations (10), and (11), we obtain :

$$a\bar{A} = FaE + J\bar{C} - E_2c\bar{A} \tag{19}$$

Likewise

$$a\bar{A}_d = F_daE + J_d\bar{C} - E_2c\bar{A}_d \tag{20}$$

With

$$J = L_1 - FE_2d \tag{21}$$

And

$$J_d = L_2 - F_dE_2d \tag{22}$$

Equations (19), (20), (21), and (22) can be written as the following matrix form :

$$X\Sigma = \theta \tag{23}$$

With

$$X = [F \quad F_d \quad J \quad J_d \quad -E_2] \tag{24}$$

$$\Sigma = \begin{pmatrix} aE & 0 \\ 0 & aE \\ \bar{C} & 0 \\ 0 & \bar{C} \\ c\bar{A} & c\bar{A}_d \end{pmatrix} \tag{25}$$

$$\theta = \begin{bmatrix} a\bar{A} & a\bar{A}_d \end{bmatrix} \tag{26}$$

Note that a general solution of equation (23) exists if the condition in equation (27) satisfied :

$$rang \begin{bmatrix} \Sigma \\ \theta \end{bmatrix} = rang(\Sigma) \tag{27}$$

If we respect condition of the equation (27) we will find :

$$X = \theta\Sigma^+ - Z(I - \Sigma\Sigma^+) \tag{28}$$

Then Σ^+ is the pseudo-inverse of the matrix Σ and Z is an unknown matrix of appropriate dimension, which will be determined by the LMI approach.
The unknown matrix F is as follows :

$$F = X \begin{pmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = A_{11} - ZB_{11} \tag{29}$$

After replacing the expression of the equation (28) in equation (29), we have obtained the following equation :

$$A_{11} = \theta\Sigma^+ \begin{pmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, B_{11} = (I - \Sigma\Sigma^+) \begin{pmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{30}$$

Similarity, we obtain the matrix F_d:

$$F_d = X \begin{pmatrix} 0 \\ I \\ 0 \\ 0 \\ 0 \end{pmatrix} = A_{22} - ZB_{22} \tag{31}$$

with

$$A_{22} = \theta\Sigma^+ \begin{pmatrix} 0 \\ I \\ 0 \\ 0 \\ 0 \end{pmatrix}, B_{22} = (I - \Sigma\Sigma^+) \begin{pmatrix} 0 \\ I \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{32}$$

The same case for the matrix J in order to reach :

$$J = X \begin{pmatrix} 0 \\ 0 \\ I \\ 0 \\ 0 \end{pmatrix} = A_{33} - ZB_{33} \quad (33)$$

We consider that:

$$A_{33} = \theta\Sigma^+ \begin{pmatrix} 0 \\ 0 \\ I \\ 0 \\ 0 \end{pmatrix}, B_{33} = (I - \Sigma\Sigma^+) \begin{pmatrix} 0 \\ 0 \\ I \\ 0 \\ 0 \end{pmatrix} \quad (34)$$

Similarity, we obtain the matrix J_d

$$J_d = X \begin{pmatrix} 0 \\ 0 \\ 0 \\ I \\ 0 \end{pmatrix} = A_{44} - ZB_{44} \quad (35)$$

With

$$A_{44} = \theta\Sigma^+ \begin{pmatrix} 0 \\ 0 \\ 0 \\ I \\ 0 \end{pmatrix}, B_{44} = (I - \Sigma\Sigma^+) \begin{pmatrix} 0 \\ 0 \\ 0 \\ I \\ 0 \end{pmatrix} \quad (36)$$

At the end E_2 is defined by :

$$-E_2 = A_{55} - ZB_{55} \quad (37)$$

With

$$A_{55} = \theta\Sigma^+ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ I \end{pmatrix}, B_{55} = (I - \Sigma\Sigma^+) \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ I \end{pmatrix} \quad (38)$$

In order to determine arbitrary matrix Z based on the Lyapunov-Krasovskii stability theory, we have proposed the following theorem :

Theorem 2 The functional observer in the form of equations (12), and (13) is an unknown input functional observer for the system of equations (1), (2), and (3) if there exist matrices $P = P^T > 0$, $Q = Q^T > 0$ and Y satisfying the following (LMI) :

$$\begin{pmatrix} Q_1 & PA_{22} - YB_{22} \\ A_{22}^T P - B_{22}^T Y^T & -Q \end{pmatrix} < 0 \quad (39)$$

Where

$$Q_1 = A_{11}^T P - B_{11}^T Y^T + PA_{11} - YB_{11} + Q \quad (40)$$

The gain Z is given by

$$Z = P^{-1}Y \quad (41)$$

Proof 2 Theorem 2 is based on the use of the Lyapunov functional which is presented as follows :

$$v(e_z, t) = e_z^T(t)Pe_z(t) + \int_{t-d}^{t} e_z^T(k)Qe_z(k)dk \quad (42)$$

Where $P = P^T > 0, Q = Q^T > 0$
Using condition i) of theorem 1, the differentiating of equation (42) is defined by :

$$\begin{aligned} \dot{v}(e_z, t) &= e_z^T(t)[F^T P + PF + Q]e_z(t) \\ &\quad + e_z^T(t-d)F_d^T Pe_z(t) + e_z^T(t)PF_d e_z(t-d) \\ &\quad - e_z^T(t-d)Qe_z(t-d) \end{aligned} \quad (43)$$

This can be written as :

$$\dot{v}(e_z, t) = \delta^T(t) \begin{bmatrix} F^T P + PF + Q & PF_d \\ F_d^T P & -Q \end{bmatrix} \delta(t) \quad (44)$$

Where: δ(t)=$[e_z(t) \quad e_z(t-d)]$

From equation (43) $\dot{v}(e_z, t) < 0$ if and only if :

$$\begin{bmatrix} F^T P + PF + Q & PF_d \\ F_d^T P & -Q \end{bmatrix} < 0 \quad (45)$$

All the matrices of the observer become well determined, if the matrix Z is solved.

5 Frequency Domain Design

5.1 Frequency Domain : Method 1

In this section we propose an easier technique of designing an unknown input functional observer described in the frequency domain by applying the left co-prime factorization of the transfer matrix. The observer transfer matrix is presented by the following theorem :

Theorem 3 The observer frequency description given by equations (12), and (13) for system of equations (1), (2) and (3) is defined by :

$$\bar{\hat{z}} = T_1 u(s) + T_2 u(s) + T_3 \bar{y}(s) + T_4 \bar{y}(s) \quad (46)$$

Where

$$T_1 = N_1^{-1}(s)M_1(s) = (sI - F_x(s))^{-1}H \quad (47)$$

$$T_2 = N_2^{-1}(s)M_2(s) = (sI - F_x(s))^{-1}H_d e^{ds} \quad (48)$$

$$\begin{aligned} T_3 = N_3^{-1}(s)M_3(s) &= (sI - F_x(s))^{-1}L_1 \\ &\quad + D \end{aligned} \quad (49)$$

$$T_4 = N_4^{-1}(s)M_4(s) = (sI - F_x(s))^{-1}L_2 e^{ds} \quad (50)$$

With

$$F_x(s) = F + F_d e^{ds} \quad (51)$$
$$D = b + E_2 d \quad (52)$$

After using the left co-prime factorization and based on [7], [18], we can deduce all matrices implemented in this design which are shown by :

$$N_1(s) = -(sI - F_x(s) + P_1)^{-1} + I \quad (53)$$
$$M_1(s) = (sI - F_x(s) + P_1)^{-1} H \quad (54)$$
$$N_2(s) = (sI - F_x(s) - P_2)^{-1} P_2 + I \quad (55)$$
$$M_2(s) = (sI - F_x(s) - P_2)^{-1} H_d e^{-ds} \quad (56)$$
$$N_3(s) = -(sI - F_x(s) + P_3)^{-1} + I \quad (57)$$
$$M_3(s) = (sI - F_x(s) + P_3)^{-1}(L_1 - P_3 D)D \quad (58)$$
$$N_4(s) = (sI - F_x(s) - P_4)^{-1} P_4 + I \quad (59)$$
$$M_4(s) = (sI - F_x(s) - P_4)^{-1} L_2 e^{-ds} \quad (60)$$

Note that P_1, P_2, P_3 and P_4 are respectively matrices of appropriate dimensions such as : $det(sI - F_x(s) + P_1)$, $det(sI - F_x(s) - P_2)$ $det(sI - F_x(s) + P_3)$ and $det(sI - F_x(s) - P_4)$ which are stable.

Proof 3 When applying the Laplace transformation on system of equation (12) and considering equation (51), we can write here :

$$\begin{aligned} z(s) &= (sI - F_x(s))^{-1} H u(s) + (sI - F_x(s))^{-1} \\ &\quad H_d e^{-ds} u(s) + (sI - F_x(s))^{-1} L_1 \bar{y}(s) \\ &\quad + (sI - F_x(s))^{-1} L_2 e^{-ds} \bar{y}(s) \end{aligned} \quad (61)$$

Using equation (61) and taking into consideration equations (51), and (52), the Laplace transformation of equation (13) will be :

$$\begin{aligned} \bar{\hat{z}}(s) &= (sI - F_x(s))^{-1} H u(s) + (sI - F_x(s))^{-1} \\ &\quad H_d e^{-ds} u(s) + [(sI - F_x(s))^{-1} L_1 + D]\bar{y}(s) \\ &\quad + (sI - F_x(s))^{-1} L_2 e^{-ds} \bar{y}(s) \end{aligned} \quad (62)$$

So the frequency description of the proposed observer can be justified.

5.2 Frequency Domain : Method 2

In this part, system of equations (1), (2), and (3) can be written as follows :

$$\begin{aligned} \dot{\psi}(t) &= \bar{A}\psi(t) + \bar{A}_d \psi(t-d) + \bar{B}u(t) \\ &\quad + \bar{B}_d u(t-d) + \bar{F}_m(t) \end{aligned} \quad (63)$$
$$\bar{y}(t) = \bar{C}\psi(t) \quad (64)$$
$$\bar{z}(t) = \bar{L}\psi(t) \quad (65)$$

Where

$$\bar{A} = \begin{pmatrix} A & R_1 \\ 0_{m_v \times n} & 0_{m_v \times m_v} \end{pmatrix}, \bar{A}_d = \begin{pmatrix} A_d & 0_{n \times m_v} \\ 0_{m_v \times n} & 0_{m_v \times m_v} \end{pmatrix},$$

$$\bar{F} = \begin{pmatrix} 0_{m_v \times n} \\ G \end{pmatrix}, \bar{L} = \begin{pmatrix} L & 0_{m_v \times m_z} \\ 0_{m_z, \times m_v} & I_{m_z} \end{pmatrix}, \quad (66)$$

$$\bar{B} = \begin{pmatrix} B \\ 0_{m_v \times m} \end{pmatrix}, \bar{B}_d = \begin{pmatrix} B_d \\ 0_{m_v \times m} \end{pmatrix}, \bar{C} = \begin{pmatrix} C & R_2 \end{pmatrix}$$

With

$$\dot{v}(t) = -v(t) + Gm(t) \quad (67)$$

G is an unknown matrix and $m(t)$ is an unknown function. In this section, we propose to raise a functional observer for delayed linear system with unknown input mentioned in the frequency domain by applying the polynomial approach, [22], [23]. The proposed functional observer has to take the following scheme :

$$\bar{\hat{z}}(s) = G_u(s)u(s) + G_{\bar{y}(s)}\bar{y}(s) \quad (68)$$

After applying the Laplace transformation in equations (63), and (64) we have reached:

$$\bar{y}(s) = G_u(s)u(s) + G_m(s)m(s) \quad (69)$$

with

$$G_u(s) = \bar{C}(sI - A_1)^{-1} B_1 \quad (70)$$
$$G_m(s) = \bar{C}(sI - A_1)^{-1} \bar{F} \quad (71)$$

and

$$A_1 = \bar{A} + \bar{A}_d e^{-ds} \quad (72)$$
$$B_1 = \bar{B} + \bar{B}_d e^{-ds} \quad (73)$$

The right co-prime factorization of $G_u(s)$ can be shown as [4], [13], [25] :

$$G_u(s) = N_u(s) M_u^{-1}(s) \quad (74)$$

Where $N_u(s)$ and $M_u(s)$ are matrices $\in R_\infty$such as :

$$M_u(s) = [A_k, B_1, K, I] \quad (75)$$
$$N_u(s) = [A_k, B_1, \bar{C}, 0] \quad (76)$$
$$A_k = A_1 + B_1 K \quad (77)$$

K has been chosen in condition that $det(sI - A_1 - B_1 K)$ is stable.
Let's define $\beta(s)$, which is called pseudo-state as follows :

$$u(s) = M_u(s)\beta(s) \quad (78)$$

Replacing respectively $G_u(s)$ and $u(s)$ by their expressions of equations (74), and (78) in equation (69), we reach the following from :

$$\begin{aligned}\bar{y}(s) &= N_u(s)M_u^{-1}(s)M_u(s)\beta(s) + G_m(s)m(s) \\ &= N_u(s)\beta(s) + G_m(s)m(s) \qquad (79)\end{aligned}$$

After exploring equations (70), and (74), and supposing that $\bar{C} = I$, we can have :

$$(sI - A_1^{-1})B_1 = (sI - A_K)^{-1}B_1M_u^{-1}(s) \qquad (80)$$

In the sequel, we can deduce :

$$\begin{aligned}\psi(s) &= (sI - A_1)^{-1}(B_1u(s) + \bar{F}m(s)) \\ &= (sI - A_K)^{-1}B_1M_u^{-1}(s)B_1u(s) \\ &+ (sI - A_1)^{-1}\bar{F}m(s) \qquad (81)\end{aligned}$$

If we replace $u(s)$ by its given equation, in equation (81), we will have :

$$\begin{aligned}\psi(s) &= (sI - A_K)^{-1}B_1\beta(s) \\ &+ (sI - A_1)^{-1}\bar{F}m(s) \qquad (82)\end{aligned}$$

Therefore the functional observer $\bar{z} = \bar{L}\psi(s)$ can be performed as follows :

$$\bar{L}\psi(s) = L_\beta(s) + L_m(s)m(s) \qquad (83)$$

with

$$L_\beta(s) = \bar{L}(sI - A_K)^{-1}B_1 \qquad (84)$$

$$L_m(s) = \bar{L}(sI - A_1)^{-1}\bar{F} \qquad (85)$$

The proposed observer gives an estimation of both the functional state and the unknown input at the same time in equation (65) as follows :

$$\lim_{t\to\infty}(\bar{z}(t) - \bar{\hat{z}}(t)) = 0 \qquad (86)$$

Here, we propose the following theorems :

Theorem 4 The system of equation (68) is a functional observer for system of equation (1) if and only if :

$$K_u(s)M_u(s) + K_{\bar{y}(s)}N_u(s) = L(s) \qquad (87)$$

Proof 4 For $m = 0$, after replacing $u(s)$ and $\bar{y}(s)$ by their expressions in equation (68), we have found :

$$\begin{aligned}\bar{\hat{z}}(s) &= K_u(s)u(s) + K_{\bar{y}(s)}\bar{y}(s) \\ &= K_u(s)M_u(s)\beta(s) + K_{\bar{y}(s)}N_u(s)\beta(s) \\ &= (K_u(s)M_u(s) + K_{\bar{y}(s)}N_u(s))\beta(s) \\ &= L(s)\beta(s) \qquad (88)\end{aligned}$$

By the identification, $L(s) = K_u(s)M_u(s) + K_{\bar{y}(s)}N_u(s)$ theorem 4 is then held.

Theorem 5 the system of equation (68) is a linear functional observer for system of equations (1), (2), and (3) if and only if :

$$K_u(s) = L(s)Y_u(s) - Q_1\hat{N}_u(s) \qquad (89)$$

$$K_{\bar{y}(s)} = L(s)X_u(s) - Q_1\hat{M}_u(s) \qquad (90)$$

$$Q_1(s) \in R_\infty \qquad (91)$$

With $\hat{N}_u(s)$ and $\hat{M}_u(s)$ is the left co-prime factorization of $G_u(s)$:

$$\hat{N}_u(s) = \bar{C}(sI - A_L)^{-1}B_1 \qquad (92)$$

$$\hat{M}_u(s) = -\bar{C}(sI - A_L)^{-1}L_1 \qquad (93)$$

$$A_L = A_1 - L_1\bar{C} \qquad (94)$$

L_1 has to be chosen in condition that $det(sI - A_1 + L_1\bar{C})$ is stable.
$X(s)$ and $Y(s)$ are matrices which satisfy the Bezout identity as in [4], [13], [21], [23] :

$$X(s) = -K(sI - A_L)^{-1}L_1 \qquad (95)$$

$$Y(s) = -K(sI - A_L)^{-1}B_1 + I \qquad (96)$$

And

$$X(s)N_u(s) + Y(s)M_u(s) = I \qquad (97)$$

Proof 5 $\hat{N}_u(s)$ and $\hat{M}_u(s)$ are the left co-prime factorization of $G_u(s)$. $N_u(s)$ and $M_u(s)$ are the right co-prime factorization of $G_u(s)$. We can then deduce the following equation :

$$N_u(s)M_u^{-1}(s) - \hat{M}_u^{-1}(s)\hat{N}_u(s) = 0 \qquad (98)$$

After using equations (95), and (96) and choosing $Q_1(s) \in RH_\infty$, we will find :

$$\begin{aligned}L(s) &= L(s)[X_u(s)N_u(s) + Y_u(s)M_u(s)] \\ &+ Q_1(s)[\hat{M}_u(s)N_u(s) - \hat{N}_u(s)M_u(s)] \\ &= L(s)X_u(s)N_u(s) + L(s)Y_u(s)M_u(s) \\ &+ Q_1(s)\hat{M}_u(s)N_u(s) - Q_1(s)\hat{N}_u(s)M_u(s) \\ &= [L(s)X_u(s) + Q_1(s)\hat{M}_u(s)]N_u(s) \\ &+ [L(s)Y_u(s) - Q_1(s)\hat{N}_u(s)]M_u(s) \qquad (99)\end{aligned}$$

Thus, according to theorem 5, we notice that :

$$L(s) = K_u(s)M_u(s) + K_{\bar{y}(s)}N_u(s) \qquad (100)$$

6 Algorithm for Functional Observer Design

6.1 Time Domain Algorithm

1. Check that conditions 1 and 2 of hypothesis are satisfied.

2. Calculate S using equations (10), and (11).
3. Calculate Σ and θ using equations (25), and (26).
4. Deduce the values of matrices A_{11}, B_{11}, A_{22} and B_{22} from equations (31), and (32).
5. Verify if the condition of equation (28) is satisfied. Then the resolution of the proposed LMI of equations (39), and (40), gives the gain matrix Z.
6. Compute F and F_d using equations (29) and (31).
7. Calculate J, J_d and E_2 respectively from equations (33), (35) and (37).
8. Cet L_1 and L_2 using equations (21) and (22).
9. Get H and H_d using iv) and v) from Theorem 1.

6.2 Frequency Domain Algorithm : Method 1

1. Determine matrices P_1, P_2, P_3, and P_4 in a way that $det(sI - F_x(s) + P_1)$, $det(sI - F_x(s) - P_2)$, $det(sI - F_x(s) + P_3)$ and $det(sI - F_x(s) - P_4)$ are stable.
2. Calculate $N_i(s)$ and $M_i(s)$, for $i \in 1, 2, 3, 4$ using equations (53), (54), (55), (56), (57), (58), (59), and (60).
3. Deduce T_1, T_2, T_3 and T_4 from equations (47), (48), (49), and (50).
4. Synthesize the estimated state vectors in the frequency domain using equation (46).

6.3 Frequency Domain Algorithm : Method 2

1. Based on MFD's, we can calculate L_1 and K as $det(sI - A_1 + L_1\bar{C})$ and $det(sI - A_1 + B_1K)$ are stable.
2. Get $\hat{N}_u$ and $\hat{M}_u$ using equations (92), and (93).
3. Get N_u and M_u using equations (75), and (76).
4. Calculate $X_u(s)$ and $Y_u(s)$ using equations (97), and (98).
5. Determine the configuration matrix Q_1

7 Numerical Example

In order to point the automated sensor temperature effectively and accurately to the frontal area of the human body, it is sufficient to regulate the angular position of the servomotor according to the angle α as shown in Figure 1:

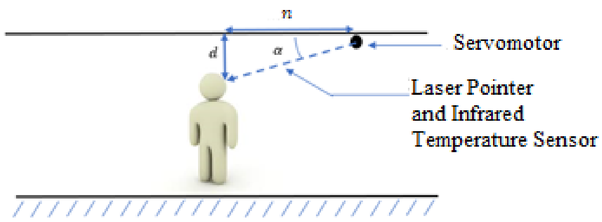

Fig. 1: Measuring principle

d as the distance that separates the highest point of the visitor from the ceiling of the tunnel as follows:

$$d = H_T - T_h$$

and H_T as the height of the tunnel and T_h as the height of the visitor (Human).
Having Known that n is the distance that separates the visitor from the projection of the servomotor on the mass, we obtain :

$$n = arctg(\frac{d}{n})$$

The DC servomotor is assimilated to a second-order system following a physical description using differential equations of its mechanical and electrical feed back, [25], [26].
With

$$H(s) = \frac{M_\delta}{JR_n s^2 + (BR_n + M_\delta M_n)s}$$

Let $u(t) \in \mathbb{R}^q$ and $\theta(t) \in \mathbb{R}^p$ be respectively the known input vector and the output which measures the angular position of a point on the servomotor shaft.
We note by $w(t)$ the speed of rotation of the servomotor as follows:

$$w(s) = s\theta(s)$$

Knowing that :

$$H(s) = \frac{\theta(s)}{U(s)}$$

and by developing the equation $H(s)$, we have :

$$JR_n sw(s) + Z_n w(s) = M_\delta U(s)$$

Add to it :

$$Z_n = BR_n + M_\delta M_n = 24$$

so

$$sw(s) = -\frac{Z_n}{JR_n}w(s) + \frac{M_\delta}{JR_n}U(s)$$

After an adequate modeling, we have obtained the model of the equations (1), (2), and (3).
With :

$$A = \begin{pmatrix} 0 & 1 \\ 0 & \frac{-Z_n}{JR_n} \end{pmatrix}, B = \begin{pmatrix} 0 \\ \frac{M_\delta}{JR_n} \end{pmatrix}$$

B: Coefficient of viscosity-friction.
R_n: Armature resistance which is equal to 4.
M_δ: Torque constant which is equal to 2.
M_n: Back electromotive force (f.e.m) constant.
J: Inertial constant which is equal to 1.
Let's consider system of equations (1), (2), and (3) where :

$$E = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, A = \begin{pmatrix} 0 & 1 \\ 0 & -6 \end{pmatrix}, L = \begin{pmatrix} 0 & 1 \end{pmatrix},$$

$$B = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}, B_d = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 \end{pmatrix},$$

$$R_1 = \begin{pmatrix} 1.5 \\ 5 \end{pmatrix}, R_2 = (1), A_d = \begin{pmatrix} 0 & 0 \\ -1 & -2.3 \end{pmatrix}$$

7.1 Functional Unknown Input Observer : Time Domain Algorithm

We suggest that the known and the unknown input signals are presented respectively by Figure 2 and Figure 3.

According to the condition given by equations (10), and (11), we obtain :

$$a = \begin{pmatrix} 0 & 1 & 3 \\ -1 & 0 & -2 \end{pmatrix}, b = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$
$$c = \begin{pmatrix} 0 & 0 & 2 \end{pmatrix}, d = (0)$$

The LMI resolution presents the observation matrices as follows :

$$F = \begin{pmatrix} -6 & 5 \\ -1 & -1.5 \end{pmatrix}, F_d = \begin{pmatrix} -2.3 & 1 \\ 0 & 0 \end{pmatrix}$$
$$J = \begin{pmatrix} 5 \\ -1.5 \end{pmatrix}, J_d = 10^{-16} \begin{pmatrix} 0.2395 \\ 0 \end{pmatrix}$$
$$H = \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}, H_d = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, E_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
$$L_1 = \begin{pmatrix} 5 \\ -1.5 \end{pmatrix}, L_2 = 10^{-16} \begin{pmatrix} 0.2395 \\ 0 \end{pmatrix}$$

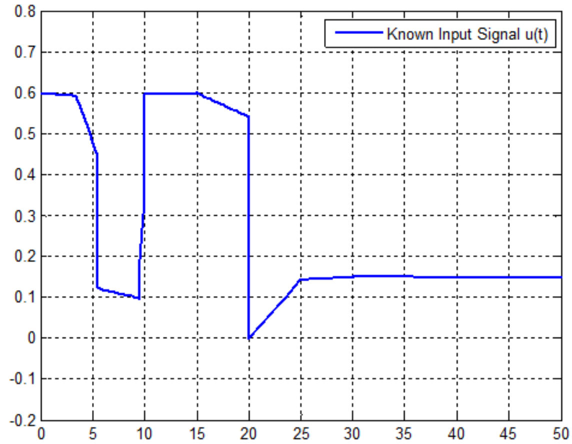

Fig. 2: Known Input Signal $u(t)$

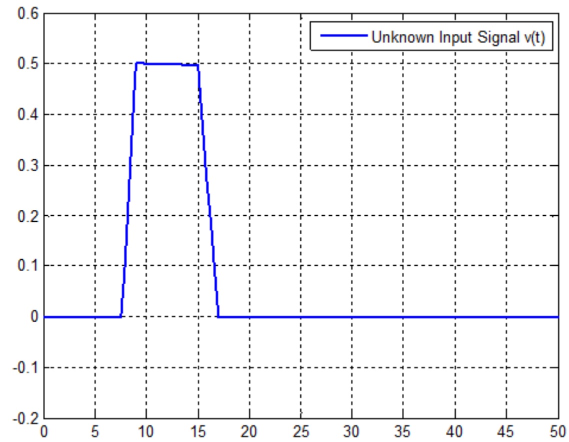

Fig. 3: Unknown Input Signal $v(t)$

Figure 4 and Figure 5 show, respectively, a comparison between the real and the estimated components of the functional state and the unknown input vector.

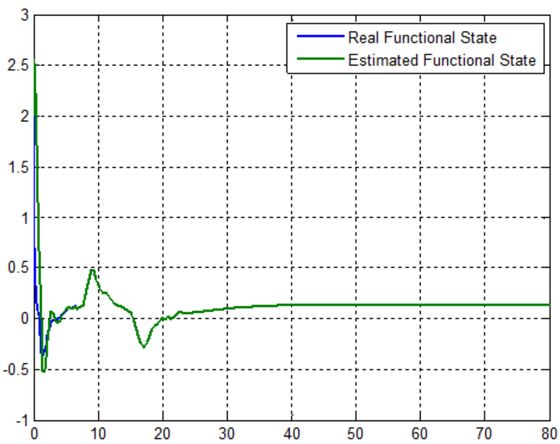

Fig. 4: Real and Estimated Functional State

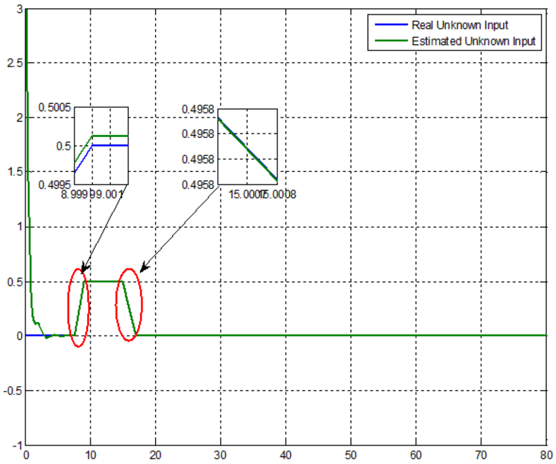

Fig. 5: Real and Estimated Unknown Input

However, according to Figure 4, we can notice that unknown inputs affect estimation dynamic and create transitory phases but the proposed observer retrieves real component in a short time $t \in [0.5, 1]$.

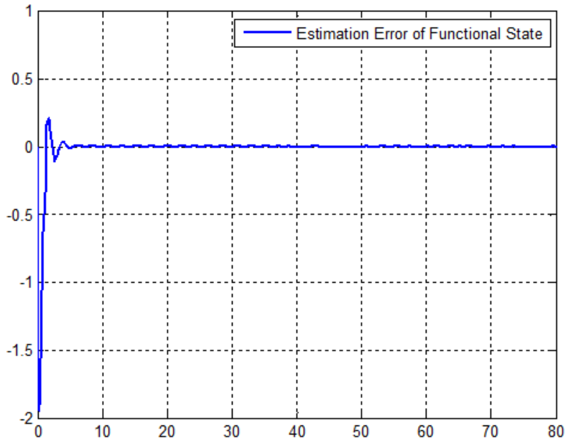

Fig. 6: Estimation Error of Functional State

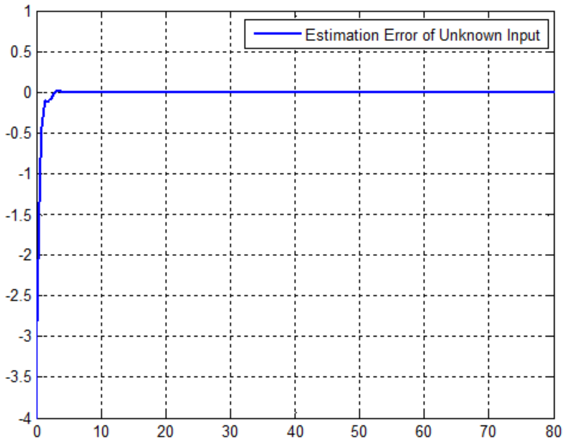

Fig. 7: Estimation Error of Unknown Input

The estimation error is given in Figure 6 and Figure 7. We can deduce a convergence of the error dynamic which proves the efficiency of the proposed approach.

7.2 Functional Unknown Input Observer : Frequency Domain Algorithm : Method 1

After using the left co-prime factorizations, matrices of the frequency domain description of the observer for linear system of equations (1), (2), and (3) are as follows:

$$P_1 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, P_2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}, P_4 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Then

$$N_1 = \frac{1}{\sigma_1} \begin{pmatrix} N_1^{11} & N_1^{12} \\ N_1^{21} & N_1^{22} \end{pmatrix}, M_1 = \frac{1}{\sigma_1} \begin{pmatrix} M_1^{11} \\ M_1^{21} \end{pmatrix}$$

$$N_2 = \frac{1}{\sigma_{21}} \begin{pmatrix} N_2^{11} & N_2^{12} \\ N_2^{21} & N_2^{22} \end{pmatrix}, M_2 = \frac{-1}{\sigma_{22}} \begin{pmatrix} M_2^{11} \\ M_2^{21} \end{pmatrix}$$

$$N_3 = \frac{1}{\sigma_3} \begin{pmatrix} N_3^{11} & N_3^{12} \\ N_3^{21} & N_3^{22} \end{pmatrix}, M_3 = \frac{1}{\sigma_3} \begin{pmatrix} M_3^{11} & M_3^{12} \\ M_3^{21} & M_3^{22} \end{pmatrix}$$

$$N_4 = \frac{1}{\sigma_{41}} \begin{pmatrix} N_4^{11} & N_4^{12} \\ N_4^{21} & N_4^{22} \end{pmatrix}, M_4 = \frac{1}{\sigma_{42}} \begin{pmatrix} M_4^{11} \\ M_4^{21} \end{pmatrix}$$

By keeping the same command and unknown input signals.

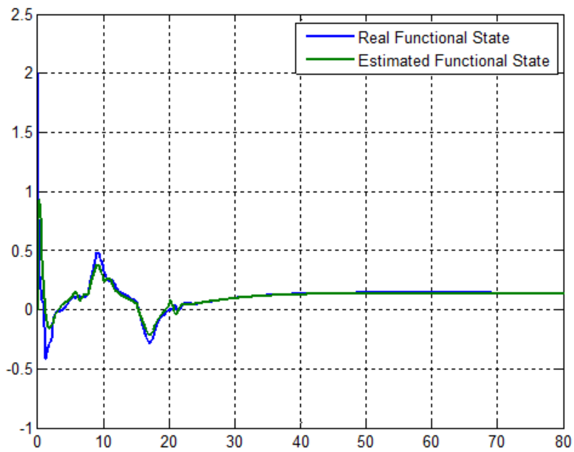

Fig. 8: Real and Estimated Functional State

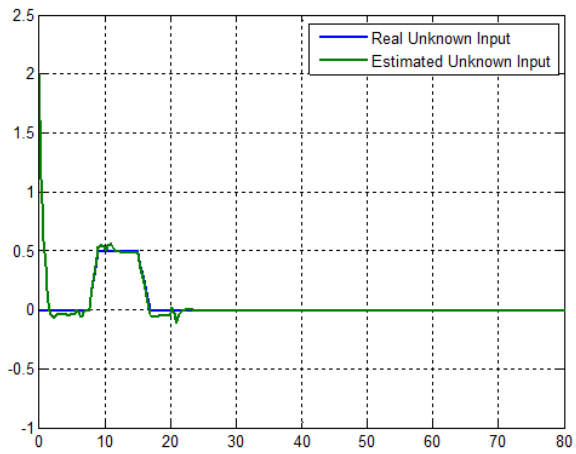

Fig. 9: Real and Estimated Unknown Input

Figure 8 and Figure 9 show, respectively, a comparison between the real and the estimated components of the functional state and the unknown input vector.

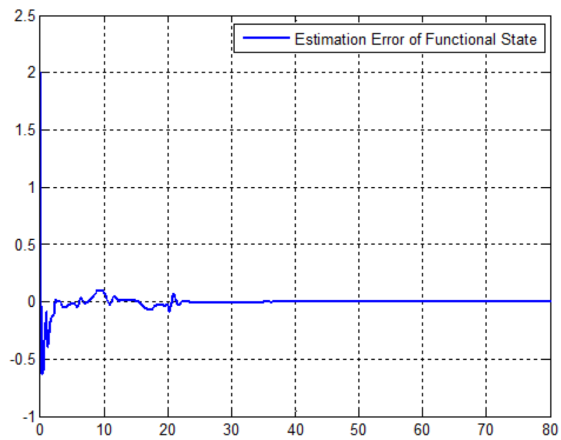

Fig. 10: Estimation Error of Functional State

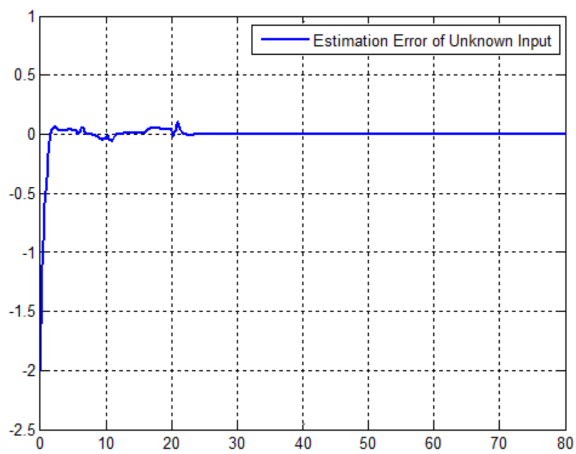

Fig. 11: Estimation Error of Unknown Input

The estimation error is given in Figure 10 and Figure 11.We deduce another convergence of our functional observer in the frequency domain design which is self-evident at the level of the efficiency of the proposed approach.

7.3 Functional Unknown Input Observer : Frequency Domain Algorithm : Method 2

L_1 and K are chosen so that $det(sI - A_1 + L_1\bar{C})$ and $det(sI - A_1 + B_1 K)$ are stable, this results in the following equations:

$$L_1 = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}, K = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$$

Using Pade approximation we reach, [4], [24]:

$$e^{-ds} = e^{-s} = \frac{-s+2}{s+2}$$

$$A_1 = A_K = \begin{pmatrix} 0 & 1 & 1.5 \\ \frac{s-2}{s+2} & \frac{2.3s-4.6}{s+2} - 6 & 5 \\ 0 & 0 & 0 \end{pmatrix}$$

The functional observer transfer function for system equations (1), (2), and (3) is given in the below formulas when taking into account the proposed polynomial approach defined previously, as:

$$\hat{M}_u(s) = \hat{M}_{u11}, \hat{N}_u(s) = \hat{N}_{u11}, M_u(s) = 1$$

$$N_u(s) = N_{u11}, X_u(s) = 0, Y_u(s) = 0$$

$$Q_1(s) = \begin{pmatrix} \sigma_{Q_1} \\ \sigma_{Q_1} \end{pmatrix}$$

$$L_\beta = \begin{pmatrix} L_{\beta_1} \\ 0 \end{pmatrix}, \quad L(s) = \begin{pmatrix} L_1(s) \\ 0 \end{pmatrix}$$

$$K_u = \frac{1}{\sigma_K} \begin{pmatrix} k_{u1} \\ k_{u2} \end{pmatrix}, \quad k_{\bar{y}} = \begin{pmatrix} k_{\bar{y}1} \\ k_{\bar{y}2} \end{pmatrix}$$

The latter command and the unknown input signals have to be kept. Figure 12 and Figure 13 show, undoubtedly, a comparison between the real and the estimated components of the functional state and the unknown input vector.
However, according to Figure 12, we easily notice

that unknown inputs affect estimation dynamic and create transitory phases but the proposed observer retrieves real component in a short time $t \in [2, 10]$ and $t \in [18, 25]$.

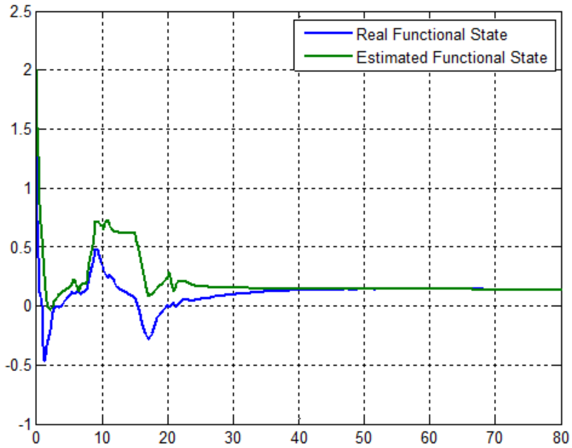

Fig. 12: Real and Estimated Functional State

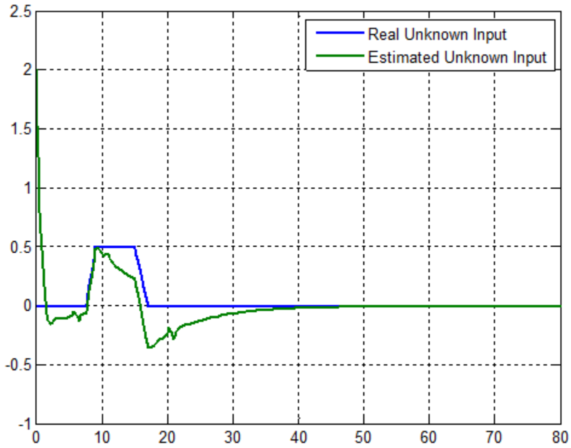

Fig. 13: Real and Estimated Unknown Input

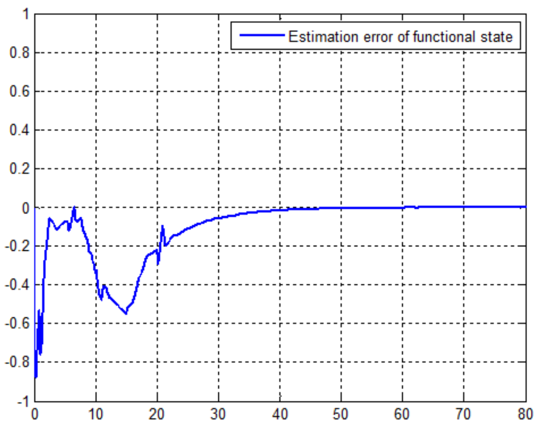

Fig. 14: Estimation error of Functional State vector

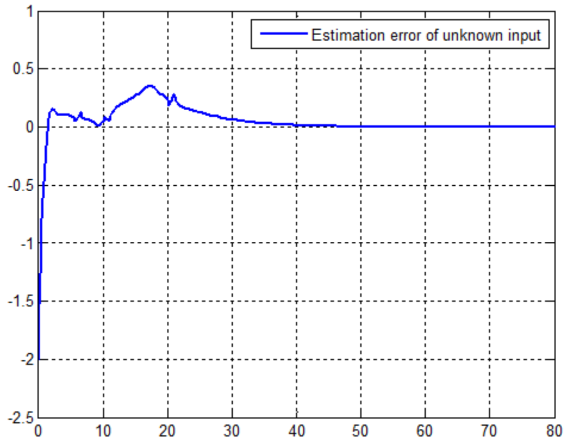

Fig. 15: Estimation error of Unknown Input Vector

The estimation errors is given in Figure 14 and Figure 15.

A new convergence has been deduced, it deals with error dynamic which proves the proposed approach's suitability.

7.4 Comparison Between the two Methods in Frequency Domain and the Method in Time Domain

The following figures (Figure 16, Figure 17) show, respectively, a comparison between the estimated error in time domain,frequency domain based on time domain and direct frequency domain of the functional state and the unknown input vector.

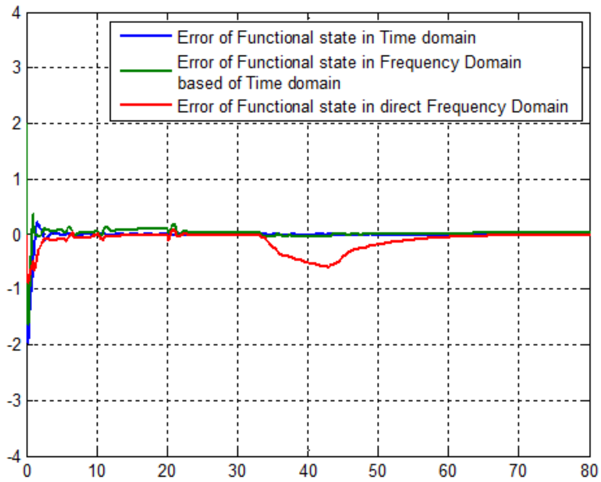

Fig. 16: Comparison between the two Functional status errors

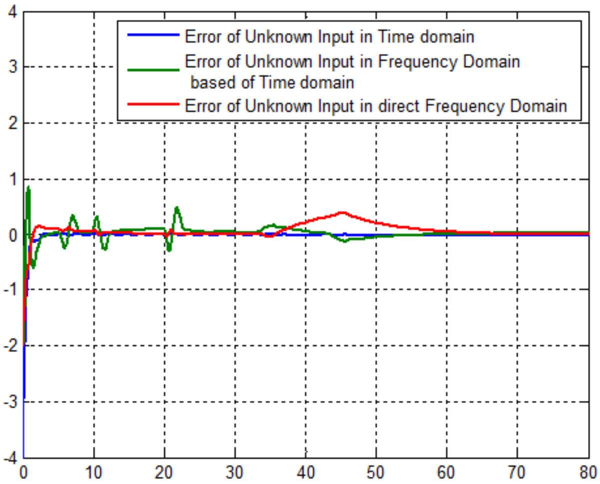

Fig. 17: Comparison between the two Unknown Inputs errors

We notice quicker dynamics in time domain. Then, it is clearer that in frequency domain based on temporal during the permanent phase, the error is better than in direct frequency domain which is quite slower. Where as the observer implementation is more obvious and feasible when using frequency domain design described only by the system input and output and fraction functions.

8 Conclusion

In this paper, we have presented a functional observer design for linear system with constant time delay and unknown input in time and frequency domains. Based on the application of the factorization approach, the frequency domain algorithm is derived from the time domain results. In addition, stable dynamic of the Time Domain approach is ensured by using the Lyapunov-Krasoveskii stability theory and LMI tools. Finally, we have used a numerical example to justify the efficiency of the proposed method through comparative analysis between the time domain , direct frequency domain and the classical one.

Appendix A: Frequency Domain Algorithm : Method 1

$$
\begin{aligned}
\sigma_1 &= 20s^3 + 144s^2 + 583s + 738 \\
N_1^{11} &= -(s+2)(2s+3) * 10 + \sigma_1 \\
N_1^{12} &= -(s+3) * 80, N_1^{21} = (s+2) * 20 \\
N_1^{22} &= -(10s^2 + 57s + 166) * 2 + \sigma_1 \\
M_1^{11} &= (s+2)(2s+3) * 5, M_1^{21} = (s+2) * 10 \\
\sigma_{21} &= 20s^3 + 104s^2 + 439s + 426 \\
\sigma_{22} &= 20s^4 + 144s^3 + 647s^2 + 1304s + 852 \\
N_2^{11} &= (s+2)(2s+1) * 10 + \sigma_{21} \\
N_2^{12} &= 20s^2 + 150s + 300, N_2^{21} = -(s+2) * 20 \\
N_2^{22} &= 20s^3 + 124s^2 + 513s + 678 \\
M_2^{11} &= (20s^3 + 110s^2 - 600) \\
M_2^{21} &= 20s^3 + 34s^2 + 104s - 504 \\
\sigma_3 &= 20s^3 + 204s^2 + 857s + 1230 \\
N_3^{11} &= -(s+2)(2s+5) * 10 + \sigma_3 \\
N_3^{12} &= -(3s+10) * 20 \\
N_3^{21} &= (s+2) * 20 \\
N_3^{22} &= -(10s^2 + 77s + 206) * 2 + \sigma_3 \\
M_3^{11} &= 80s^2 + 210s - 100 \\
M_3^{21} &= 20s^3 + 154s^2 + 392s + 40 \\
\sigma_{41} &= 20s^3 + 104s^2 + 419s + 386 \\
\sigma_{42} &= (s+2)(20s^3 + 104s^2 + 419s + 386) \\
N_4^{11} &= (s+2)(2s+1) * 10 + \sigma_{41} \\
N_4^{12} &= (s+3) * 80 \\
N_4^{21} &= -(s+2) * 20 \\
N_4^{22} &= (10s^2 + 47s + 146) * 2 + \sigma_{41} \\
M_4^{11} &= -(s+2)(2.395 * 10^{-17}s - 4.79 * 10^{-17}) \\
&\quad * (2s+1) * 10 \\
M_4^{21} &= (s+2)(2.395 * 10^{-17}s - 4.79 * 10^{-17}) * 20
\end{aligned}
$$

Appendix B: Frequency Domain Algorithm : Method 2

$$
\begin{aligned}
\hat{M}_{u11} &= \frac{s(10s^3 + 57s^2 + 156s + 20)}{10s^4 + 97s^3 + 444s^2 + 1186s + 1476} \\
\hat{N}_{u11} &= \frac{\hat{N}_{u111}}{\hat{N}_{u112}} \\
\hat{N}_{u111} &= s(10s^3 + 32s^2 + 72s - 272) \\
\hat{N}_{u112} &= (10s^5 + 117s^4 + 638s^3 + 2074s^2 \\
&\quad + 3848s + 2952) \\
N_{u11} &= \frac{10s^3 + 32s^2 + 72s - 272}{10s^4 + 77s^3 + 270s^2 + 332s + 40} \\
\sigma_{Q_1} &= \frac{10s^4 + 97s^3 + 444s^2 + 1186s + 1476}{10s^2 + 77s^3 + +40} \\
L_{\beta_1} &= \frac{-5s^3 + 30s^2 + 20s + 40}{10s^4 + 77s^3 + 270s^2 + 332 + 40}
\end{aligned}
$$

$$\sigma_k = 10s^5 + 97s^4 + 424s^3 + 872s^2 + 704s + 80$$
$$K_{u1} = -15s^4 - 12s^3 + 8s^2 + 352s + 80$$
$$k_{u2} = -s(10s^3 + 32s^2 + 72s - 272)$$
$$L_1(s) = \frac{-5s^3 + 30s^2 + 20s + 40}{10s^4 + 77s^3 + 270s^2 + 332s + 40}$$

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