Estimating the Confidence Intervals for the Coefficients of Variation Delta-Gamma Distributions

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Abstract: - A common application of the coefficient of variation (CV), which is the ratio of the population standard deviation to the population mean, is frequently used to assess quality control and economic processes, among others. The fiducial quantity approach, Bayesian confidence intervals (CIs) using the Jeffreys, uniform, or normal-gamma-beta (NGB) priors, and highest posterior density (HPD) intervals using the Jeffreys, uniform, or NGB priors were used to provide estimators for the CI for the ratio of CV of two delta-gamma distributions. An evaluation of their performance in terms of average length and coverage probability was carried out using Monte Carlo simulations. The results of this study indicate that the HPD using the Jeffreys prior and fiducial quantity methods are the best for estimating the CI for the ratio of the CV of two delta-gamma distributions. Rainfall data from Mae Hong Son province in Thailand was used to illustrate their practicability when analyzing real-life processes.

Key-Words: - Delta-gamma distribution, Highest posterior density, Jeffreys prior, Normal-Gamma-Beta prior, Simulation, Uniform prior, Fiducial quantities.

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1 Introduction

Thailand's rainy season lasts from mid-May to mid-October, with the southwest monsoon bringing an abundance of annual rainfall. For the majority of the country, the greatest rainfall occurs from August to September whereas January and December are exceptionally dry. Thus, rainfall data in Thailand and other nations typically includes zero readings at certain times of the year, which should be considered when researching rainfall. Aitchison presented a model for situations in which there are zero observations by assigning a probability of δ that the dataset contains zero observations and $1 - \delta$ as the residual probability for the positive observations, [1]. The delta-lognormal distribution first suggested by Aitchison and Brown, includes a random variable with a lognormal distribution for the positive observations and a binomial distribution for the number of zero observations, [2].

The CIs for the parameters of the delta-gamma distribution and other related distributions have been determined by numerous researchers using various methods for statistical inference. For instance, Muralidharan and Kale created the CIs for the mean of the mixed distribution and a modified gamma distribution that includes a singularity at zero, [3]. Lecomte et al. suggested applying the compound Poisson-gamma and delta-gamma distributions to handle zero-inflated continuous data inside the variable sampling volume regime, [4].

The population CV can be defined as the ratio of the population mean to the population standard deviation, [5]. Among other fields, biology, economics, and quality control all frequently use the CV, [6]. CIs have been provided by many researchers for the CV of different distributions. For instance, Methods for estimating the CI for the ratio of the CVs within two gamma distributions were proposed in [7]. In [8], it is proposed various methods to estimate the CI for the ratio of the CVs in two inverse gamma distributions.

To compare CVs in two populations, one appropriate method is to use the ratio of the CVs of the two populations of rainfall data that contain zero observations. This ratio can be represented by utilizing the two delta-gamma distributions. So far, no publications have been forthcoming on estimating the CIs for the ratio of the CVs within delta-gamma distributions. In the examination, we used the fiducial quantity (FQ) and six Bayesian-based approaches to estimate the CI for this scenario. The Bayesian methods are Bayesian CIs based on the Jeffreys (B.Jef), uniform (B.Uni), or NGB (B.NGB) priors and three HPD based on the Jeffreys (H.Jef), uniform

(H.Uni), or NGB (H.NGB) priors. Furthermore, we demonstrate their practicability for real-life situations by applying them to analyze rainfall data from Mae Hong Son province in Thailand.

2 Preliminary

The delta-gamma distribution function can be specified as follows:

$$G\left(x_{ij};\delta_i,\alpha_i,\beta_i\right)=\begin{Bmatrix} \delta_i & ; & x_{ij}=0, \\ \delta_i+\left(1-\delta_i\right)F\left(x_{ij};\alpha_i,\beta_i\right) & ; & x_{ij}>0. \end{Bmatrix} \tag{1}$$

The gamma cumulative distribution function can be indicated as $F\left(x_{ij};\alpha_i,\beta_i\right)$.

Moreover, $\alpha_i\beta_i$ and $\alpha_i\beta_i^2$ are the respective means and variances of gamma (α_i,β_i) distribution with shape parameter (α_i) and scale parameter (β_i). For the zero and positive observations, identified by $n_{i(0)}$ and $n_{i,(1)}$ respectively, where $n_i=n_{i,(0)}+n_{i,(1)}$, the positive observations, follow a gamma distribution and the zero observations follow a binomial distribution.

The maximum likelihood estimators (MLEs) of δ_i α_i,, and β_i are respectively described as:

$$\hat{\delta}_i=n_{i,(0)}/n_i\,,\ \hat{\alpha}_i=\frac{1}{2\left(\log\bar{X}_{ij}-\sum_{i=1}^{n_i}\log X_{ij}/n_i\right)},\text{ and}$$

$$\hat{\beta}_i=\bar{X}_{ij}/\hat{\alpha}_i,.$$

where $\bar{X}_{ij}$ represents the sample mean of X_{ij} [11].

Following [1], the population mean and variance of X_{ij} are defined as follows:

$$E(X_{ij})=(1-\delta_i)\cdot(\alpha_i\beta_i) \tag{2}$$

and

$$Var(X_{ij})=(1-\delta_i)\cdot(\alpha_i\beta_i^2)+\delta_i(1-\delta_i)\cdot(\alpha_i\beta_i)^2 \tag{3}$$

Consequently, the CV of X_{ij} can be expressed as

$$CV\left(X_{ij}\right)=\nu_i=\sqrt{\frac{1+\delta_i\alpha_i}{\left(1-\delta_i\right)\alpha_i}}. \tag{4}$$

The ratio of their CVs is given by

$$\varphi=\frac{\nu_1}{\nu_2}. \tag{5}$$

The methods to construct the CI for φ are provided in the next subsections.

2.1 The Fiducial Quantity Method

Krishnamoorthy and Wang derived an FQ based on cube root-transformed samples, [9]. Let $X_{ij}=\left(X_{i1},X_{i2},\ldots,X_{in_i}\right);i=1,2$ and $j=1,2,\ldots,n_i$ be a sample from a delta-gamma distribution with shape parameter a_i and scale parameter b_i. For $Y_{ij}=X_{ij}^{\frac{1}{3}};i=1,2$ and $j=1,2,\ldots,n_i$, then Y_{ij} is approximately normally distributed with means μ_i and variances σ_i^2 provided by

$$\mu_i=(b_i a_i^{\frac{1}{3}})\left(1-\frac{1}{9a_i}\right)\text{ and }\sigma_i^2=\frac{b_i^{\frac{2}{3}}}{9a_i^{\frac{1}{3}}}. \tag{6}$$

The respective FQs of μ_i and σ_i^2 are

$$Q_{\mu_i}=\bar{x}_{ij}+\frac{Z_i\sqrt{n_i-1}}{\chi^2_{n_i-1}}\cdot\frac{s_i}{\sqrt{n_i}}\text{ and }Q_{\sigma_i^2}=\frac{(n_i-1)s_i^2}{\chi^2_{n_i-1}}, \tag{7}$$

where $\bar{x}_{ij}$ and s_i are the observed values of $\bar{X}_{ij}$ and S_i, respectively; $Z_i \sim N\left(0,1\right)$; and $\chi^2_{n_i-1}$ is an independent random variable from a Chi-squared distribution. The FQs for the shape parameter are

$$Q_{\alpha_i}=\frac{1}{9}\left\{\left(1+0.5\frac{Q_{\mu i}^{\ 2}}{Q_{\sigma_i^2}}\right)+\left[\left(1+0.5\frac{Q_{\mu i}^{\ 2}}{Q_{\sigma_i^2}}\right)^2-1\right]^{\frac{1}{2}}\right\}. \tag{8}$$

Then, the FQs for δ_i are as follows, [10]

$$Q_{\delta_i}:\ \frac{1}{2}\text{Beta}(n_{i,(1)},n_{i,(0)}+1)+\frac{1}{2}\text{Beta}(n_{i,(1)}+1,n_{i,(0)}). \tag{9}$$

Therefore, the FQs for ν_i of a delta-gamma distribution have the following specifications:

$$Q_{\nu_i}=\sqrt{\frac{1+Q_{\delta_i}Q_{\alpha_i}}{(1-Q_{\delta_i})Q_{\alpha_i}}}. \tag{10}$$

Now, the FQs for φ is given by

$$Q_\varphi=\frac{Q_{\nu_1}}{Q_{\nu_2}}. \tag{11}$$

Subsequently, the equal-tailed $100(1-\tau)\%$ FQ interval for φ can be derived as

$$CI_{FQ}=\left[Q_\varphi(\tau/2),Q_\varphi(1-\tau/2)\right], \tag{12}$$

where $Q_\varphi(\tau/2)$ and $Q_\varphi(1-\tau/2)$ are the $(\tau/2)100$th and $(1-\tau/2)100$th percentiles of the distribution of Q_φ, respectively.

The CI for φ can be obtained by executing Algorithm 1.

Algorithm 1.
1. Utilizing $X_{ij} \sim \Delta(\delta_i, \alpha_i, \beta_i)$, compute $\bar{x}_{ij}$ and s_i^2 of the cube root-transformed samples.
2. Generate $Z_i \sim N(0,1)$ and $\chi^2_{n_i-1}$.
3. Generate $\text{Beta}(n_{i,(1)}, n_{i,(0)}+1)$ and $\text{Beta}(n_{i,(1)}+1, n_{i,(0)})$.
4. Calculate Q_{μ_i} and $Q_{\sigma_i^2}$ using Equation (7).
5. Calculate Q_{α_i} and Q_{δ_i} using Equations (8) and (9).
6. Calculate Q_{v_i} and Q_φ using Equations (10) and (11).
7. Steps 2-6 are repeated 5,000 times to obtain the Q_φ.
8. Calculate the 95% CIs for φ using Equation (12).
9. To acquire the average lengths (ALs) and coverage probabilities (CPs), steps 1 through 8 are repeated 10,000 times.

2.2 The Bayesian Methods

The posterior distribution is used to determine confidence intervals (CIs) based on the Bayesian technique for the parameter of interest. [11], whereas posterior distributions are used to create the HPD intervals using a Bayesian approach. The parameter values with the highest posterior density make up the HPD, whereas the narrowest interval that can be discovered for a parameter of interest with probability $100(1-\tau)\%$ is the HPD interval. [5], [12]. The HPD was first provided by [13].

Let $p(\omega | y)$ be a posterior density function. A region R in the parameter space of ω is called a HPD region of content $(1-\tau)$ if

(i) $Pr(\omega \in R | y) = 1-\tau$,

(ii) For $\omega_1 \in R$ and $\omega_2 \notin R, p(\omega_1 R | y) \geq p(\omega_2 R | y)$.

2.1.1 Jeffreys Prior

Jeffreys presented a prior created from the square root of the Fisher information matrix characterized as $p(\theta) \propto \sqrt{|I(\theta)|}$, [14]. The Jeffreys prior for δ of a binomial distribution is characterised as $p(\delta_i) \propto (\delta_i)^{-\frac{1}{2}}(1-\delta_i)^{\frac{1}{2}}$, from which the marginal posterior distribution of δ_i is obtainable in the manner described below:

$$\delta_{i(jef)} | x_{ij} \sim Beta\left(n_{i,(0)} + \frac{1}{2}, n_{i,(1)} + \frac{3}{2}\right). \tag{13}$$

Subsequently, the Jeffreys prior for σ_i^2 in a lognormal distribution is $p(\sigma_i^2) \propto \sigma_i^{-2}$. Hence, the respective marginal posterior distributions of σ_i^2 are as follows:

$$\sigma^2_{i(jef)} | x_{ij} \sim IG\left(\frac{n_{i,(1)}}{2}, \frac{\sum_{i=1}^{n_i}(x_{ij}-\mu_i)^2}{2}\right). \tag{14}$$

Likewise, the following are the marginal posterior distributions of μ_i:

$$\mu_{i(jef)} | \sigma_i^2, x \sim N(\bar{x}_{ij}, \sigma^2_{i(jef)} / n_{i,(1)}). \tag{15}$$

Additionally, we may use $\mu_{i(jef)} | \sigma_i^2, x$ and $\sigma^2_{i(jef)} | x_{ij}$ to get the gamma distribution's mean and variance in the following ways:

$$M_{i(B.Jef)} = \left\{\frac{\mu_{i(jef)}}{2} + \sqrt{\left(\frac{\mu_{i(jef)}}{2}\right)^2 + \sigma^2_{i(jef)}}\right\}^3, \tag{16}$$

$$V_{i(B.Jef)} = \left\{\frac{\mu_{i(jef)} + \sqrt{\mu^2_{i(jef)} + 4\sigma^2_{i(jef)}}}{2(9^{-1/4})(\sigma^2_{i(jef)})^{-1/4}}\right\}^4. \tag{17}$$

Then

$$v_{i(B.Jef)} = \frac{\sqrt{V_{i(B.Jef)}}}{M_{i(B.Jef)}}. \tag{18}$$

So that

$$\varphi_{B.Jef} = \frac{v_{1(B.Jef)}}{v_{2(B.Jef)}}. \tag{19}$$

Based on the B.Jef and H.Jef methods, the CI and HPD intervals for φ of a delta-gamma distribution are defined as

$$CI_{B.Jef} = [\hat{\varphi}_{B.Jef}(\tau/2), \hat{\varphi}_{B.Jef}(1-\tau/2)]. \tag{20}$$

2.2.2 Uniform Prior

The prior probability is a constant function, [15], that certainly sets a prior for all possible values, [16]. The uniform prior for δ_i of a binomial distribution is $p(\delta_i) \propto 1$, [12], from which the marginal posterior distribution of δ_i can be obtained as follows:

$$\delta_{i(unif)} \mid x_{ij} : \; Beta(n_{i,(0)}+1, n_{i,(1)}+1). \quad (21)$$

Kalkur and Rao indicated the uniform prior for σ_i^2 is $\sigma_i^2 \propto 1$, [17].

Consequently, the following is the marginal posterior distribution of σ_i^2:

$$\sigma^2_{i(unif)} \mid x_{ij} : \; IG\left(\frac{n_{i,(1)}-2}{2}, \frac{\sum_{i=1}^{n_i}(x_{ij}-\mu_i)^2}{2}\right). \quad (22)$$

Likewise, the respective marginal posterior distribution of μ_i are as follows:

$$\mu_{i(unif)} \mid \sigma_i^2, x_{ij} : \; N(\bar{x}_{ij}, \sigma^2_{i(unif)} / n_{i,(1)}). \quad (23)$$

Additionally, we may use $\mu_{i(unif)} \mid \sigma_i^2, x_{ij}$ and $\sigma^2_{i(unif)} \mid x_{ij}$ to calculate the gamma distribution's mean and variance in the following ways:

$$M_{i(B.Uni)} = \left\{\frac{\mu_{i(unif)}}{2} + \sqrt{\left(\frac{\mu_{i(unif)}}{2}\right)^2 + \sigma^2_{i(unif)}}\right\}^3, \quad (24)$$

$$V_{i(B.Uni)} = \left\{\frac{\mu_{i(unif)} + \sqrt{\mu^2_{i(unif)} + 4\sigma^2_{i(unif)}}}{2(9^{-1/4})(\sigma^2_{i(unif)})^{-1/4}}\right\}^4, \quad (25)$$

Then

$$\nu_{i(B.Uni)} = \frac{\sqrt{V_{i(B.Uni)}}}{M_{i(B.Uni)}}. \quad (26)$$

So that

$$\varphi_{B.Uni} = \frac{\nu_{1(B.Uni)}}{\nu_{2(B.Uni)}}. \quad (27)$$

Based on the B.Uni and H.Uni methods, the CI and HPD intervals for φ of a delta-gamma distribution are defined as

$$CI_{B.Uni} = [\varphi_{B.Uni}(\tau/2), \varphi_{B.Uni}(1-\tau/2). \quad (28)$$

2.2.3 Normal-Gamma-Beta Prior

Maneerat and Niwitpong recommended employing the H.NGB interval to calculate the common mean for several delta-lognormal distributions, which worked well on small-to-large sample sizes, [18]. Let $Y_i = \log W_i; i = 1,2$ be a random variable of a normal distribution with mean μ_i and precision λ_i where $W_i : \; LN(\mu_i, \lambda_i)$ and $\lambda_i = \sigma_i^{-2}$. The H.NGB for $\xi_i = (\mu_i, \lambda_i, \delta_i)$ is indicated as $p(\xi_i) \propto \lambda_i^{-1}\left[(1-\delta_i)\delta_i\right]^{-1/2}$, where μ_i, λ_i has the normal-gamma distribution, and δ_i has the beta distribution. Hence, the marginal posterior distributions of δ_i, σ_i^2, and μ_i are as follows:

$$\delta_{i(NGB)} \mid x_{ij} : \; Beta\left(n_{i,(0)} + \frac{1}{2}, n_{i,(1)} + \frac{1}{2}\right), \quad (29)$$

$$\sigma^2_{i(NGB)} \mid x_{ij} : \; IG\left(\frac{n_{i,(1)}-1}{2}, \frac{\sum_{i=1}^{n_{i,(1)}}(x_{ij}-\mu_i)^2}{2}\right), \quad (30)$$

$$\mu_{i,NGB} \mid x_{ij} : \; t_{2(n_{i,(1)}-1)}\left(\bar{x}_{ij}, \frac{\sum_{i=1}^{n_i}(x_{ij}-\bar{x}_{ij})^2}{n_{i,(1)}(n_{i,(1)}-1)}\right). \quad (31)$$

Additionally, we may use $\mu_{i(NGB)} \mid x_{ij}$ and $\sigma^2_{i(NGB)} \mid x_{ij}$ to calculate the gamma distribution's mean and variance in the following ways:

$$M_{i(B.NGB)} = \left\{\frac{\mu_{i(NGB)}}{2} + \sqrt{\left(\frac{\mu_{i(NGB)}}{2}\right)^2 + \sigma^2_{i(NGB)}}\right\}^3, \quad (32)$$

$$V_{i(B.NGB)} = \left\{\frac{\mu_{i(NGB)} + \sqrt{\mu^2_{i(NGB)} + 4\sigma^2_{i(NGB)}}}{2(9^{-1/4})(\sigma^2_{i(NGB)})^{-1/4}}\right\}^4. \quad (33)$$

Then

$$\nu_{i(B.NGB)} = \frac{\sqrt{V_{i(B.NGB)}}}{M_{i(B.NGB)}}. \quad (34)$$

So that

$$\varphi_{B.NGB} = \frac{\nu_{1(B.NGB)}}{\nu_{2(B.NGB)}}. \quad (35)$$

Based on the B.NGB and H.NGB methods, the CI and HPD intervals for φ of a delta-gamma distribution are defined

as $CI_{B.NGB} = [\varphi_{B.NGB}(\tau/2), \varphi_{B.NGB}(1-\tau/2)]$. (36)

Algorithm 2.

1. Utilizing $X_{ij} : \; \Delta(\delta_i, \alpha_i, \beta_i)$, compute $\bar{x}_{ij}$ and s_i^2 of the cube root-transformed samples.
2. Generate $\delta_i \mid x_{ij}$ using Equations (13), (21) and (29).
3. Generate $\sigma_i^2 \mid x_{ij}$ using Equations (14), (22), and (30).
4. Generate $\mu_i \mid \sigma_i^2, x_{ij}$ using Equations (15), (23) and (31).

5. Calculate the mean and variance using Equations (16), (17), (24), (25), (32) and (33).

6. Calculate v_i and φ using Equations (18), (19), (26), (27), (34), and (35).

7. Calculate the 95% CIs and HPD for φ using Equations (20), (28) and (36).

8. Steps 1-7 are repeated 10,000 times to obtain the CPs and ALs.

3 The Monte Carlo Simulation Study

The performances of the CI estimators for the ratio of CVs of two delta-gamma distributions constructed with FQ, B.Jef, H.Jef, B.Uni, H.Uni, B.NGB, and H.NGB were compared in terms of their CPs and ALs, with the most effective one for a given situation providing the CP close to or above 0.95 and the shortest AL. There were 10,000 repetitions (M) used in the Monte Carlo simulation and 5,000 replicates (m) for FQ with a nominal confidence level of 0.95 employing R statistics software (version 4.1.0). The data were generated for $X_{ij} \sim \Delta(\delta_i, \alpha_i, \beta_i); i = 1,2$ and $j = 1,2,...,n_i$. We chose (15,15), (25,25), (50,50), or (100,100) for equal sample sizes $(n_1 = n_2)$ and (15,25), (25,50), or (50,100) for unequal sample sizes $(n_1 \neq n_2)$. The probabilities of zeros (δ_1, δ_2) were set as (0.2,0.2), (0.4,0.4), or (0.6,0.6), the shape parameters (α_1, α_2) as (0.05,0.05), (0.05,0.06), (0.06,0.05) or (0.06,0.06), and the rate parameter (β_1, β_2) as (2,2).

The performance of the different techniques for estimating the nominal 95% CIs for the ratio of the CV of two delta-gamma distributions are shown in Table 1, Table 2 and Figure 1, Figure 2, Figure 3 and Figure 4 in Appendix. The simulation results are reported in Table 1 and Table 2, and the CPs and ALs from Table 1 and Table 2 are compiled in Figure 1, Figure 2, Figure 3 and Figure 4 in Appendix.

4 Application of the Methods to Real-World Data Situations

The Upper Northern Region Irrigation Hydrology Center in Mae Hong Son province, Thailand, provided monthly rainfall data that were utilized to compare the CI estimators' performances.

Initially, we employed four models: Cauchy, Normal, Gamma, and Lognormal to find the best fitting distribution for the positive rainfall data using the Akaike information criterion (AIC). AIC is defined as AIC = −2 lnL+ 2k where L is the likelihood function and k is the number of parameters and n be the number of recorded measurements. From the results in Table 3 (Appendix) ; it can be seen that Data fitting to a gamma distribution produced the lowest AIC values, so it was deemed to be the most appropriate.

4.1 The CI for the Ratio of the CVs with Equal Sample Sizes

The monthly rainfall data from Mueang district, Mae Hong Son province, in February from 1987 to 2022 and December from 1987 to 2022 were used as the datasets. The summary statistics in February were $\bar{x}_1 = 14.1461$, $n_1 = 36$, $n_{1,(1)} = 13$, $n_{1,(0)} = 23$ and the MLEs for $\alpha_1, \delta_1, \beta_1$, and v_1 were $\hat{\alpha}_1 = 1.0676$, $\hat{\delta}_1 = 0.64$, $\hat{\beta}_1 = 13.2501$, and $\hat{v}_1 = 2.0888$, respectively. The summary statistics in the December dataset were $\bar{x}_2 = 26.10$, $n_2 = 36$, $n_{2,(1)} = 17$, $n_{2,(0)} = 19$ and the MLEs for $\alpha_2, \delta_2, \beta_2$, and v_2 were $\hat{\alpha}_2 = 0.6115$, $\hat{\delta}_2 = 0.52$, $\hat{\beta}_2 = 42.6811$, and $\hat{v}_2 = 0.9760$, respectively. The 95% CIs estimates for φ are shown in Table 4 (Appendix).

From the simulation study results for $n_1, n_2 = 25$ and $\delta_1, \delta_2 = 0.6$, although all of the techniques achieved CPs close to 0.95, H.Jef obtained the shorter AL. Therefore, H.Jef is the most effective technique for creating the CI for the ratio of CVs of rainfall datasets from the Mueang district in Mae Hong Son province for February from 1987 to 2022 and December from 1987 to 2022.

4.2 The CI for the Ratio of the CVs with Unequal Sample Sizes

The monthly rainfall data from Mueang district, Mae Hong Son province, for January from 2000 to 2022 and November from 1992 to 2022 were used as the datasets. The summary statistics in the January dataset from January were $\bar{x}_1 = 20.48$, $n_1 = 23$, $n_{1,(1)} = 15$, $n_{1,(0)} = 8$ and the MLEs for $\alpha_1, \delta_1, \beta_1$, and v_1 were $\hat{\alpha}_1 = 0.9641$, $\hat{\delta}_1 = 0.35$, $\hat{\beta}_1 = 21.2435$, and $\hat{v}_1 = 1.4573$, respectively. The summary statistics in November dataset were $\bar{x}_2 = 45.8519$, $n_2 = 30$, $n_{2,(1)} = 27$, $n_{2,(0)} = 3$ and the MLEs for $\alpha_2, \delta_2, \beta_2$, and v_2 were $\hat{\alpha}_2 = 1.2111$, $\hat{\delta}_2 = 0.1$, $\hat{\beta}_2 =$

37.8569, and $\hat{\nu}_2 = 1.0141$, respectively. The 95% CIs estimates for φ are showed in Table 5 (Appendix).

From the simulation study results for $n_1 = 15$, $n_2 = 25$, and $\delta_1, \delta_2 = 0.2$, the FQ, H.Jef, H.Uni, B.NGB, and H.NGB techniques achieved CPs close to 0.95 but H.Jef obtained the shorter AL. Therefore, H.Jef is the most effective technique for creating the CI for the ratio of CVs of rainfall data from the Mueang district in Mae Hong Son province for January from 2000 to 2022 and November from 1992 to 2022.

5 Conclusions

We produced estimators for the CI for the ratio of the CVs of two delta-gamma distributions by utilizing the FQ, B.Jef, H.Jef, B.Uni, H.Uni, B.NGB, and H.NGB techniques. To assess their CPs and ALs, a Monte Carlo simulation was run. Following that, monthly rainfall data from Thailand's Mae Hong Son province were used to test the proposed approaches. The findings indicate that the H.Jef and FQ methods are the best for estimating the CI for the ratio of the CVs of two delta-gamma distributions.

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References:
[1] J. Aitchison, "On the Distribution of a Positive Random Variable Having a Discrete Probability Mass at the Origin," Journal of the American Statistical Association, vol. 50, no. 271, pp. 901-908, 1955. DOI: 10.1080/01621459.1955.10501976
[2] J. Aitchison and J.A.C. Brown, The Lognormal Distribution: With Special Reference to Its Uses in Economics London, Cambridge University Press: UK, 1963.
[3] K. Muralidharan and B.K. Kale, "Modified Gamma Distributions with Singularity at Zero," Communications in Statistics-Simulation and Computation, vol. 31, no. 1, pp. 143-158, 2002.
[4] J.B. Lecomte, H.P. Benot, S. Ancelet, M.P. Etienne, L. Bel, and E. Parent, "Compound Poisson-Gamma vs. Delta-Gamma to Handle Zero-Inflated Continuous Data Under a Variable Sampling Volume," Methods in Ecology and Evolution, vol. 4, no. 12, pp. 1159-1166, 2013. DOI: 10.1111/2041-210X.12122
[5] G. Casella and R.L. Berger, Statistical Inference, Cengage Learning: Boston, 2001.
[6] T. Holgersson, P. Karlsson, and R. Mansoor, "Estimating Mean-Standard Deviation Ratios of Financial Data," Journal of Applied Statistics, vol. 39, no. 3, pp. 657-671, 2013.
[7] P. Sangnawakij, S.A. Niwitpong, and S. Niwitpong, "Confidence Intervals for the Ratio of Coefficients of Variation of the Gamma Distributions," In Lecture Notes in Computer Science, pp. 193-203, 2015. DOI: 10.1007/978-3-319-25135-6_19
[8] T. Kaewprasert, S.A. Niwitpong, and S. Niwitpong, "Confidence Interval for the Ratio of the Coefficient of Variation of Inverse Gamma Distributions," Applied Science and Engineering Progres, vol. 16, no. 1, pp. 1-11, 2022. DOI: 10.14416/j.asep.2021.12.002
[9] K. Krishnamoorthy and X. Wang, "Fiducial Confidence Limits and Prediction Limits for a Gamma Distribution: Censored and Uncensored Cases," Environmetrics, vol. 27, no. 8, pp. 479-493, 2016. DOI: 10.1002/env.2408
[10] X. Li, X. Zhou, and L. Tian, "Interval Estimation for the Mean of Lognormal Data with Excess Zeros," Statistics Probability Letters, vol. 83, no. 11, pp. 2447-2453, 2013.
[11] A. Gelman, J.B. Carlin, H.S. Stern, D.B. Dunson, A. Vehtari, and D.B. Rubin, Bayesian Data Analysis, Chapman and Hall/CRC: United States, 2013. DOI: 10.1201/b16018
[12] W.M. Bolstad and J.M. Curran, Introduction to Bayesian Statistics, Wiley: Hoboken, 2016.
[13] G.E.P. Box and G.C. Tiao, Bayesian Inference in Statistical Analysis, Wiley Classics: New York, 1973.
[14] H. Jeffreys, Theory of Probability, Oxford University Press: UK, 1961.
[15] J.V. Stone, Bayes' Rule: A Tutorial Introduction to Bayesian Analysis, Sebtel Press: Shiffield, 2013. DOI: 10.13140/2.1.1371.6801
[16] J.X. O'Reilly and R.B. Mars, "Bayesian Models in Cognitive Neuroscience: A Tutorial," An Introduction to Model-Based Cognitive Neuroscience, pp. 179-197, 2015.
[17] T.A. Kalkur and A. Rao, "Bayes Estimator for Coefficient of Variation and Inverse

Coefficient of Variation for the Normal Distribution," International Journal of Statistics and Systems, vol. 12, no. 4, pp. 721-732, 2017.

[18] P. Maneerat and S.A. Niwitpong, "Estimating the Average Daily Rainfall in Thailand Using Confidence Intervals for the Common Mean of Several Delta-Lognormal Distributions," PeerJ, vol. 9, no. e10758, 1-28, 2021. DOI: 10.7717/peerj.10758

APPENDIX

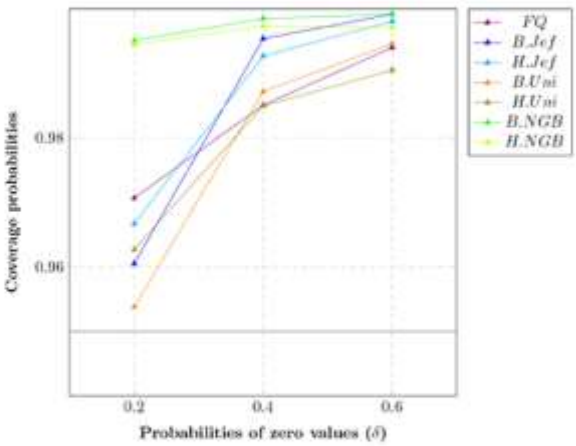

Fig. 1: Charts of lines of the CPs provided by the techniques for the CI of the ratio of the CVs for various probabilities of zero values ($n_1 = n_2$)

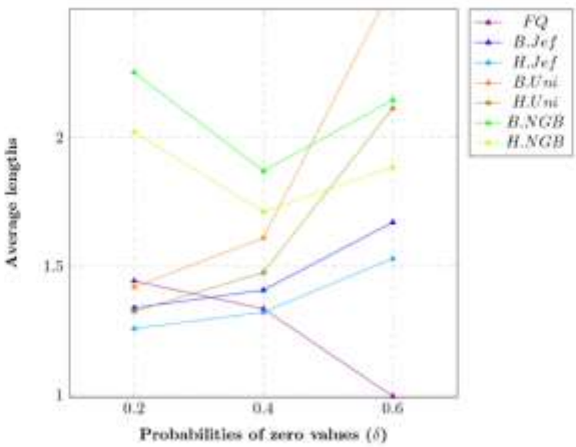

Fig. 2: Charts of lines of the ALs provided by the techniques for the CI of the ratio of the CVs for various probabilities of zero values ($n_1 = n_2$)

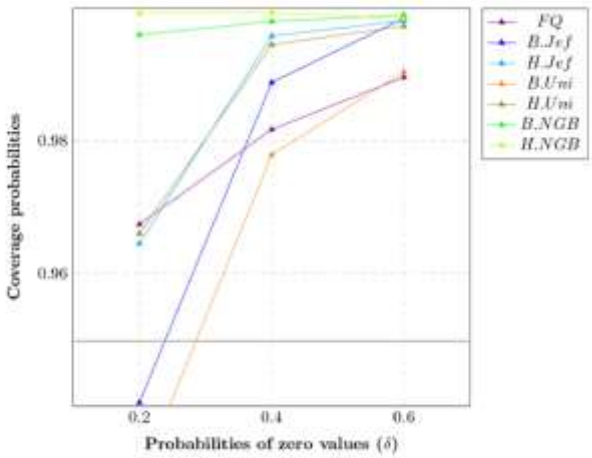

Fig. 3: Charts of lines of the CPs provided by the techniques for the CI of the ratio of the CVs for various probabilities of zero values ($n_1 \neq n_2$)

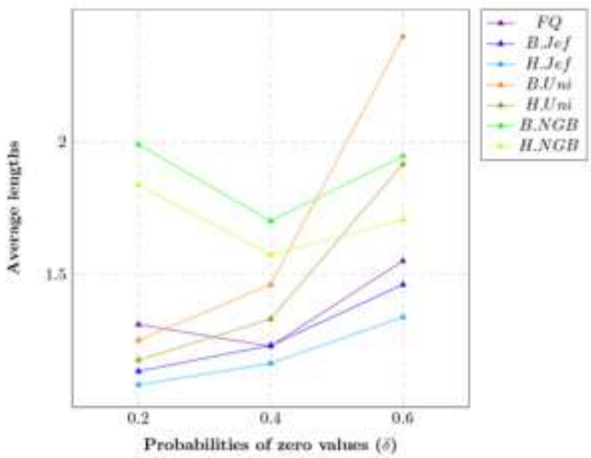

Fig. 4: Charts of lines of the ALs provided by the techniques for the CI of the ratio of the CVs for various probabilities of zero values ($n_1 \neq n_2$)

Table 1. The CPs and the ALs of the different techniques for estimating the nominal 95% two-sided CI for φ ($n_1 = n_2$)

n_1, n_2	δ_1, δ_2	α_1, α_2	CPs (ALs)						
			FQ	B.Jef	H.Jef	B.Uni	H.Uni	B.NGB	H.NGB
15,15	0.2,0.2	0.04,0.04	**0.9707**	**0.9605**	**0.9667**	**0.9538**	**0.9627**	**0.9951**	**0.9945**
			(1.4425)	(1.3376)	(*1.2569*)	(1.4186)	(1.3244)	(2.2530)	(2.0220)
		0.05,0.05	**0.9677**	**0.9554**	**0.9653**	0.9461	**0.9584**	**0.9960**	**0.9958**
			(1.3944)	(1.2003)	(*1.1374*)	(1.3217)	(1.2400)	(2.1218)	(1.9129)
		0.06,0.06	**0.9649**	**0.9538**	**0.9622**	0.9456	**0.9577**	**0.9948**	**0.9929**
			(1.3384)	(1.0984)	(*1.0471*)	(1.2464)	(1.1725)	(1.9911)	(1.8044)
	0.4,0.4	0.04,0.04	**0.9851**	**0.9954**	**0.9927**	**0.9872**	**0.9850**	**0.9985**	**0.9973**
			(1.3342)	(1.4070)	(*1.3211*)	(1.6104)	(1.4756)	(1.8695)	(1.7111)
		0.05,0.05	**0.9833**	**0.9927**	**0.9927**	**0.9851**	**0.9853**	**0.9977**	**0.9965**
			(1.3373)	(1.3354)	(*1.2569*)	(1.5878)	(1.4513)	(1.8376)	(1.6825)
		0.06,0.06	**0.9859**	**0.9939**	**0.9945**	**0.9878**	**0.9863**	**0.9986**	**0.9981**
			(1.3259)	(1.2748)	(*1.2035*)	(1.5582)	(1.4238)	(1.7969)	(1.6460)
	0.6,0.6	0.04,0.04	**0.9940**	**0.9992**	**0.9981**	**0.9946**	**0.9905**	**0.9992**	**0.9972**
			(*0.9942*)	(1.6701)	(1.5299)	(2.5930)	(2.1140)	(2.1456)	(1.8853)
		0.05,0.05	**0.9959**	**0.9993**	**0.9991**	**0.9961**	**0.9927**	**0.9991**	**0.9988**
			(1.7455)	(1.6467)	(*1.5066*)	(2.7124)	(2.1831)	(2.1908)	(1.9151)
		0.06,0.06	**0.9949**	**0.9997**	**0.9985**	**0.9956**	**0.9899**	**0.9994**	**0.9976**
			(1.7861)	(1.6212)	(*1.4821*)	(2.7826)	(2.2238)	(2.2108)	(1.9253)
25,25	0.2,0.2	0.04,0.04	**0.9688**	0.9273	0.9358	0.9240	0.9345	**0.9976**	**0.9982**
			(*1.0913*)	(0.8949)	(0.8677)	(0.9215)	(0.8921)	(1.7492)	(1.6234)
		0.05,0.05	**0.9665**	0.9273	0.9345	0.9238	0.9314	**0.9972**	**0.9971**
			(*1.0313*)	(0.8046)	(0.7838)	(0.8428)	(0.8189)	(1.5958)	(1.4897)
		0.06,0.06	**0.9630**	0.9255	0.9328	0.9229	0.9309	**0.9958**	**0.9960**
			(*0.9725*)	(0.7484)	(0.7312)	(0.7946)	(0.7737)	(1.4561)	(1.3664)
	0.4,0.4	0.04,0.04	**0.9857**	**0.9885**	**0.9901**	**0.9836**	**0.9848**	**0.9994**	**0.9992**
			(0.9662)	(0.9784)	(*0.9458*)	(1.0214)	(0.9833)	(1.4278)	(1.3504)
		0.05,0.05	**0.9799**	**0.9836**	**0.9854**	**0.9772**	**0.9809**	**0.9993**	**0.9988**
			(0.9496)	(0.9285)	(*0.8994*)	(0.9844)	(0.9489)	(1.3682)	(1.2963)
		0.06,0.06	**0.9810**	**0.9852**	**0.9878**	**0.9800**	**0.9838**	**0.9991**	**0.9989**
			(0.9274)	(0.8899)	(*0.8636*)	(0.9547)	(0.9210)	(1.3090)	(1.2430)
	0.6,0.6	0.04,0.04	**0.9922**	**0.9985**	**0.9983**	**0.9938**	**0.9952**	**0.9997**	**0.9987**
			(*1.0493*)	(1.1545)	(1.1031)	(1.2878)	(1.2098)	(1.4506)	(1.3636)
		0.05,0.05	**0.9913**	**0.9979**	**0.9975**	**0.9953**	**0.9942**	**0.9995**	**0.9993**
			(*1.0616*)	(1.1243)	(1.0750)	(1.2872)	(1.2062)	(1.4412)	(1.3540)
		0.06,0.06	**0.9804**	**0.9852**	**0.9866**	**0.9782**	**0.9817**	**0.9992**	**0.9988**
			(0.9255)	(0.8878)	(*0.8616*)	(0.9538)	(0.9200)	(1.3032)	(1.2377)
50,50	0.2,0.2	0.04,0.04	**0.9701**	0.9007	0.9050	0.9003	0.9042	**0.9992**	**0.9993**
			(*0.7754*)	(0.5754)	(0.5671)	(0.5829)	(0.5743)	(1.2727)	(1.2147)

Table 1. *continued.*

n_1,n_2	δ_1,δ_2	α_1,α_2	CPs (ALs)						
			FQ	B.Jef	H.Jef	B.Uni	H.Uni	B.NGB	H.NGB
		0.05,0.05	**0.9627**	0.8895	0.8948	0.8872	0.8913	**0.9968**	**0.9971**
			(*0.7201*)	(0.5239)	(0.5171)	(0.5337)	(0.5266)	(1.1243)	(1.0790)
		0.06,0.06	**0.9598**	0.8948	0.8931	0.8943	0.8949	**0.9955**	**0.9955**
			(*0.6650*)	(0.4882)	(0.4825)	(0.4994)	(0.4933)	(0.9931)	(0.9577)
	0.4,0.4	0.04,0.04	**0.9831**	**0.9770**	**0.9768**	**0.9730**	**0.9740**	**0.9995**	**0.9998**
			(0.6788)	(0.6408)	(*0.6300*)	(0.6491)	(0.6379)	(1.0479)	(1.0128)
		0.05,0.05	**0.9794**	**0.9737**	**0.9741**	**0.9703**	**0.9703**	**0.9994**	**0.9993**
			(0.6569)	(0.6110)	(*0.6014*)	(0.6218)	(0.6116)	(0.9731)	(0.9431)
		0.06,0.06	**0.9787**	**0.9754**	**0.9765**	**0.9725**	**0.9734**	**0.9986**	**0.9989**
			(0.6307)	(0.5877)	(*0.5790*)	(0.6007)	(0.5913)	(0.9054)	(0.8796)
	0.6,0.6	0.04,0.04	**0.9916**	**0.9960**	**0.9948**	**0.9934**	**0.9930**	**0.9999**	**0.9998**
			(*0.6987*)	(0.7463)	(0.7304)	(0.7651)	(0.7478)	(1.0328)	(0.9985)
		0.05,0.05	**0.9882**	**0.9946**	**0.9944**	**0.9906**	**0.9926**	**0.9999**	**0.9997**
			(*0.6917*)	(0.7268)	(0.7119)	(0.7504)	(0.7337)	(0.9925)	(0.9609)
		0.06,0.06	**0.9895**	**0.9955**	**0.9949**	**0.9930**	**0.9927**	**0.9998**	**0.9996**
			(*0.6799*)	(0.7120)	(0.6978)	(0.7390)	(0.7227)	(0.9530)	(0.9240)
100,100	0.2,0.2	0.04,0.04	**0.9694**	0.8822	0.8806	0.8839	0.8827	**0.9995**	**0.9995**
			(*0.5520*)	(0.3915)	(0.3880)	(0.3939)	(0.3904)	(0.9206)	(0.8944)
		0.05,0.05	**0.9656**	0.8740	0.8718	0.8726	0.8717	**0.9983**	**0.9985**
			(*0.5064*)	(0.3569)	(0.3540)	(0.3597)	(0.3567)	(0.7881)	(0.7693)
		0.06,0.06	**0.9587**	0.8707	0.8694	0.8721	0.8688	**0.9963**	**0.9971**
			(*0.4619*)	(0.3321)	(0.3296)	(0.3355)	(0.3329)	(0.6765)	(0.6629)
	0.4,0.4	0.04,0.04	**0.9851**	**0.9695**	**0.9705**	**0.9686**	**0.9691**	**0.9999**	**0.9999**
			(0.4854)	(0.4403)	(*0.4359*)	(0.4423)	(0.4379)	(0.7673)	(0.7509)
		0.05,0.05	**0.9816**	**0.9673**	**0.9679**	**0.9661**	**0.9672**	**0.9994**	**0.9992**
			(0.4658)	(0.4199)	(*0.4159*)	(0.4224)	(0.4183)	(0.6994)	(0.6862)
		0.06,0.06	**0.9803**	**0.9693**	**0.9699**	**0.9687**	**0.9684**	**0.9990**	**0.9991**
			(0.4426)	(0.4042)	(*0.4005*)	(0.4072)	(0.4034)	(0.6359)	(0.6251)
	0.6,0.6	0.04,0.04	**0.9909**	**0.9927**	**0.9922**	**0.9916**	**0.9900**	**0.9999**	**0.9998**
			(*0.4941*)	(0.5074)	(0.5012)	(0.5115)	(0.5052)	(0.7568)	(0.7411)
		0.05,0.05	**0.9884**	**0.9913**	**0.9900**	**0.9889**	**0.9889**	**0.9997**	**0.9998**
			(*0.4849*)	(0.4954)	(0.4895)	(0.5001)	(0.4942)	(0.7115)	(0.6978)
		0.06,0.06	**0.9894**	**0.9932**	**0.9921**	**0.9910**	**0.9899**	**0.9997**	**0.9998**
			(*0.4726*)	(0.4864)	(0.4807)	(0.4916)	(0.4859)	(0.6721)	(0.6602)

Table 2. The CPs and the ALs of the different techniques for estimating the nominal 95% two-sided CI for φ ($n_1 \neq n_2$)

n_1, n_2	δ_1, δ_2	α_1, α_2	CPs (ALs)						
			FQ	B.Jef	H.Jef	B.Uni	H.Uni	B.NGB	H.NGB
15,25	0.2,0.2	0.04,0.04	**0.9674**	0.9405	**0.9645**	0.9288	**0.9660**	**0.9960**	**0.9993**
			(1.3107)	(1.1344)	(*1.0835*)	(1.2516)	(1.1762)	(1.9902)	(1.8374)
		0.05,0.05	**0.9628**	0.9350	**0.9584**	0.9243	**0.9601**	**0.9945**	**0.9987**
			(1.2418)	(1.0178)	(*0.9732*)	(1.1579)	(1.0855)	(1.8367)	(1.7028)
		0.06,0.06	**0.9582**	0.9325	**0.9570**	0.9203	**0.9589**	**0.9925**	**0.9980**
			(1.1907)	(0.9500)	(*0.9075*)	(1.1098)	(1.0369)	(1.7087)	(1.5895)
	0.4,0.4	0.04,0.04	**0.9817**	**0.9888**	**0.9958**	**0.9779**	**0.9945**	**0.9980**	**0.9994**
			(1.2286)	(1.2319)	(*1.1644*)	(1.4611)	(1.3313)	(1.7007)	(1.5738)
		0.05,0.05	**0.9808**	**0.9882**	**0.9954**	**0.9757**	**0.9942**	**0.9979**	**0.9992**
			(1.2233)	(1.1702)	(*1.1047*)	(1.4355)	(1.2991)	(1.6635)	(1.5376)
		0.06,0.06	**0.9821**	**0.9878**	**0.9952**	**0.9777**	**0.9943**	**0.9978**	**0.9994**
			(1.2086)	(1.1223)	(*1.0579*)	(1.4205)	(1.2767)	(1.6130)	(1.4894)
	0.6,0.6	0.04,0.04	**0.9896**	**0.9985**	**0.9981**	**0.9904**	**0.9973**	**0.9990**	**0.9984**
			(1.5507)	(1.4614)	(*1.3389*)	(2.3973)	(1.9161)	(1.9469)	(1.7062)
		0.05,0.05	**0.9907**	**0.9982**	**0.9994**	**0.9910**	**0.9978**	**0.9988**	**0.9990**
			(13.9172)	(4.3773)	(*3.3945*)	(61.8114)	(20.9648)	(11.1420)	(7.1361)
		0.06,0.06	**0.9919**	**0.9990**	**0.9993**	**0.9915**	**0.9978**	**0.9992**	**0.9993**
			(1.6515)	(1.4261)	(*1.2965*)	(2.6117)	(2.0281)	(2.0214)	(1.7451)
25,50	0.2,0.2	0.04,0.04	**0.9635**	0.9069	0.9222	0.8999	0.9215	**0.9944**	**0.9989**
			(*0.9494*)	(0.7537)	(0.7342)	(0.7905)	(0.7671)	(1.4994)	(1.4314)
		0.05,0.05	**0.9609**	0.9039	0.9214	0.8931	0.9198	**0.9919**	**0.9984**
			(*0.8858*)	(0.6849)	(0.6667)	(0.7286)	(0.7057)	(1.3358)	(1.2815)
		0.06,0.06	**0.9574**	0.9018	0.9173	0.8958	0.9192	**0.9907**	**0.9985**
			(*0.8250*)	(0.6386)	(0.6212)	(0.6865)	(0.6641)	(1.1938)	(1.1513)
	0.4,0.4	0.04,0.04	**0.9788**	**0.9764**	**0.9882**	**0.9656**	**0.9862**	**0.9974**	**0.9997**
			(0.8556)	(0.8362)	(*0.8107*)	(0.8906)	(0.8566)	(1.2629)	(1.2124)
		0.05,0.05	**0.9751**	**0.9754**	**0.9860**	**0.9649**	**0.9844**	**0.9963**	**0.9999**
			(0.8318)	(0.7939)	(*0.7692*)	(0.8567)	(0.8222)	(1.1896)	(1.1438)
		0.06,0.06	**0.9771**	**0.9799**	**0.9887**	**0.9672**	**0.9871**	**0.9967**	**0.9996**
			(0.8050)	(0.7630)	(*0.7390*)	(0.8312)	(0.7966)	(1.1226)	(1.0810)
	0.6,0.6	0.04,0.04	**0.9886**	**0.9950**	**0.9990**	**0.9888**	**0.9991**	**0.9990**	**0.9999**
			(0.9379)	(0.9913)	(*0.9476*)	(1.1353)	(1.0568)	(1.2998)	(1.2281)
		0.05,0.05	**0.9879**	**0.9958**	**0.9988**	**0.9889**	**0.9982**	**0.9990**	**0.9999**
			(0.9371)	(0.9617)	(*0.9180*)	(1.1246)	(1.0414)	(1.2760)	(1.2040)
		0.06,0.06	**0.9893**	**0.9970**	**0.9993**	**0.9911**	**0.9985**	**0.9992**	**0.9999**
			(0.9368)	(0.9443)	(*0.8997*)	(1.1295)	(1.0399)	(1.2523)	(1.1795)
50,100	0.2,0.2	0.04,0.04	**0.9666**	0.8846	0.8909	0.8817	0.8905	**0.9973**	**0.9991**
			(*0.6665*)	(0.4914)	(0.4841)	(0.5013)	(0.4937)	(1.0882)	(1.0572)

Table 2. *continued.*

n_1,n_2	δ_1,δ_2	α_1,α_2	CPs (ALs)						
			FQ	B.Jef	H.Jef	B.Uni	H.Uni	B.NGB	H.NGB
		0.05,0.05	**0.9612**	0.8844	0.8907	0.8802	0.8883	**0.9939**	**0.9967**
			(*0.6155*)	(0.4501)	(0.4435)	(0.4614)	(0.4543)	(0.9396)	(0.9186)
		0.06,0.06	**0.9553**	0.8866	0.8888	0.8825	0.8889	**0.9915**	**0.9959**
			(*0.5640*)	(0.4195)	(0.4134)	(0.4316)	(0.4249)	(0.8153)	(0.8012)
	0.4,0.4	0.04,0.04	**0.9803**	**0.9707**	**0.9762**	**0.9652**	**0.9727**	**0.9987**	**0.9995**
			(0.5936)	(0.5519)	(*0.5431*)	(0.5630)	(0.5536)	(0.9203)	(0.8989)
		0.05,0.05	**0.9751**	**0.9653**	**0.9722**	**0.9592**	**0.9691**	**0.9969**	**0.9988**
			(0.5703)	(0.5267)	(*0.5183*)	(0.5394)	(0.5302)	(0.8426)	(0.8253)
		0.06,0.06	**0.9774**	**0.9744**	**0.9767**	**0.9669**	**0.9742**	**0.9974**	**0.9990**
			(0.5431)	(0.5059)	(*0.4978*)	(0.5198)	(0.5108)	(0.7713)	(0.7575)
	0.6,0.6	0.04,0.04	**0.9872**	**0.9912**	**0.9952**	**0.9869**	**0.9935**	**0.9993**	**0.9999**
			(*0.6144*)	(0.6424)	(0.6291)	(0.6644)	(0.6490)	(0.9169)	(0.8930)
		0.05,0.05	**0.9872**	**0.9926**	**0.9952**	**0.9878**	**0.9947**	**0.9986**	**0.9998**
			(*0.6031*)	(0.6248)	(0.6117)	(0.6492)	(0.6338)	(0.8698)	(0.8480)
		0.06,0.06	**0.9873**	**0.9936**	**0.9962**	**0.9901**	**0.9952**	**0.9991**	**0.9996**
			(*0.5918*)	(0.6142)	(0.6010)	(0.6421)	(0.6262)	(0.8296)	(0.8095)

∗ *Italicize the shortest AL and bold the CPs greater than the nominal confidence level of 0.95.*

Table 3. AIC results to check the distributions of the rainfall datasets.

Rainfall Station	Cauchy	Normal	Lognormal	Gamma
Mueang (February)	104.4263	105.8415	101.4244	**98.6124**
Mueang (December)	164.8593	166.1968	151.0278	**147.6980**
Mueang (January)	134.2037	136.2560	124.8226	**124.4980**
Mueang (November)	269.6906	266.3669	273.2349	**263.1739**

Table 4. The 95% CIs for the ratio of CV of rainfall data in Mueang district, Mae Hong Son province ($n_1 = n_2$)

Methods	CI for φ		Length of intervals
	Lower bound	Upper bound	
FQ	0.4551	1.6011	1.1460
B.Jef	0.4779	1.6134	1.1356
H.Jef	0.4297	1.5260	**1.0963**
B.Uni	0.4162	1.6564	1.2402
H.Uni	0.3693	1.5682	1.1989
B.NGB	0.4439	1.6068	1.1629
H.NGB	0.4057	1.5104	1.1047

Table 5. The 95% CIs for the ratio of CV of rainfall data in Mueang district, Mae Hong Son province ($n_1 \neq n_2$)

Methods	CI for φ		Length of intervals
	Lower bound	Upper bound	
FQ	0.0278	0.5704	0.5426
B.Jef	0.0175	0.5525	0.5350
H.Jef	0.0000	0.5058	**0.5058**
B.Uni	0.0223	0.5921	0.5698
H.Uni	0.0009	0.5370	0.5361
B.NGB	0.0233	0.5669	0.5436
H.NGB	0.0003	0.5168	0.5165

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

Wansiri Khooriphan: performed the experiments, analyzed the data, authored or reviewed drafts of the paper; Sa-Aat Niwitpong: concived and designed the experiments, approved the final draft; Suparat Niwitpong: contributed analysis tools, prepared tables.

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Conflict of Interest

The authors have no conflicts of interest to declare.