Mathematical and Computer Modeling of a Dynamic System for Effectively Combating Disinformation

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Abstract: - The work investigated a mathematical and computer model of a dynamic system for effectively combating disinformation. In the compartmental model of false information dissemination in society, there are groups of citizens: - At risk, prone to the perception of misinformation; Adepts - those who accepted false information and with Immunity - those who rejected false information from the very beginning or future adepts. A barrier level for the number of adherents will be introduced as a measure of the information security of society. As a result of a computer experiment, the possibility of an uncontrolled growth in the number of adherents was identified, threatening the safety of society as a whole.

Key-Words: - Dynamic system, mathematical and computer model, computer experiment, Internet Technologies, disinformation, controllability, optimal control task.

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1 Introduction

The article discusses mathematical and computer modeling of social processes. In particular, one of the types of Information Warfare - the fight against disinformation. In the conditions of the universal Internet, developed social networks, etc. information dissemination has become publicly available. This opportunity is often negatively used by some forces to spread false information. The goals of disseminating false information can be different, including imposing on the part of society the “needed” point of view and thereby determining the desired model of implementation. We can recall the scandal related to the US elections when several false accounts created by external forces were discovered on social media, through which information was disseminated. In the consequences, this fact was called “an attempt to influence the US elections”. In the European Union, disinformation has begun to be fought in an organized manner. In 2015, by decision of the Heads of State and Government of the Council of Europe, a special group “Strategic Communication with the East” was established - EastStratCom Task Force, to counter the disinformation campaign aimed at discrediting the EU's Eastern Neighbourhood Policy. In less than ten years, the group's budget has grown from one to eleven million Euros. On the group’s official website, you can find information about how many articles containing misinformation were reviewed and what responses were prepared and sent out. Logically, a campaign against disinformation can be planned and conducted more effectively if there is a clear idea and quantitative characteristics of the false information spread. Naturally, the study of disseminating false information using mathematical and computer modeling, conducting computer experiments along with other methods of studying the task, make it possible to effectively describe the process under study and plan measures against disinformation campaigns. When constructing a mathematical model for spreading false information and combating it, we will use the compartmental approach proposed in the works, [1], [2], [3], [4], [5], [6], [7], [8], [9], [10].

2 Problem Formulation

Let us assume that in a society with a constant number N of people, information flows are distributed at each moment of time $t \in [0,T]$ by two operators. The second operator distributes false information in the amount of $y_5(t)$, the first operator distributes anti-false information, in contrast to the second operator, in the amount of $y_4(t)$. These two operators, spreading the corresponding information in fact, in figurative terms, lead the battle for the minds of members of society. As a result of this, the following groups are formed in society: - Y_1. Risk group with a number $y_1(t)$, that has not yet decided which operator to follow; - Y_2 group of Adherents of false information with the number $y_2(t)$, who became followers of the second operator; - Y_3 Immunity group with the number $y_3(t)$, rejected false information, and thereby perceived the first operator.

We will assume that people who find themselves in the Immunity group no longer leave it. If we manage to determine the dynamics of the transition of people from one group to another, we can thereby establish the degree of influence of false and anti-false information on society. Structurally, the transition of people from one group to another under the influence of false and anti-false information can be depicted as follows, in Figure 1.

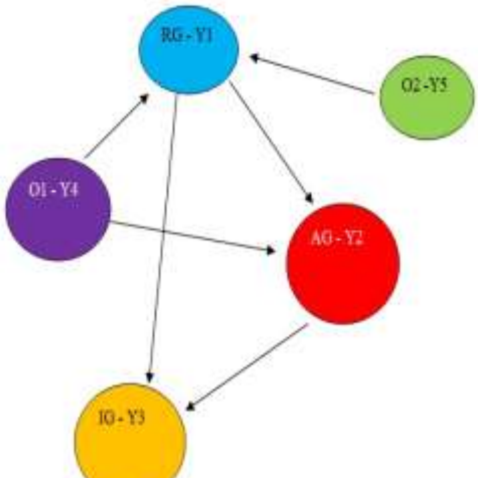

Fig. 1: Structure of transitions from one group to another

As can be seen from Figure 1, operator 2 acts only on the Risk group, and operator 1, in addition to the Risk group, also acts on the group of Adepts. Over time, the size of the Risk group decreases and approaches zero, and this occurs as a result of the mutual action of operators 1 and 2. Naturally, if the size of the group of adherents is small, then false information does not significantly affect society, and it can adequately perceive and respond to challenges that may arise for society. Operator 1 acts to reduce the number of Adepts by spreading anti-false information. The more intensively anti-false information is spread, the smaller the number of Adepts is. However, the creation and dissemination of one unit of anti-false information requires certain financial and other resources. It is quite possible that, given certain limited resources, Operator 1 will not be able to reduce the size of the group of Adepts. Therefore, the task arises of determining the required amount of resources for operator 1 to reduce the number of Adepts and at the same time effectively use these resources. Those. The problem of optimal control of combating false information arises.

3 Problem Solution

From the Risk group (Y_1) as a result of the influence of false information ($y_5(t)$), its members can leave it and go to the Adept group (Y_2), and under the influence of anti-false information ($y_4(t)$) to the Immunity group (Y_3). Interpersonal relationships of members of Risk and Adept groups also affect the transition from Risk to Adept group. The interpersonal relationships of members of Risk and Immunity groups affect the transition from Risk to Immunity group. Note that the number of members of the Risk group does not increase, the function $y_1(t)$ does not grow. Thus, you can write in mathematical relations the rate of change in the number of Risk groups:

$$\frac{dy_1(t)}{dt} = -\lambda(t)y_4(t)y_1(t) - \kappa(t)y_5(t)y_1(t) - \\ -\alpha_1(t)y_1(t)y_2(t) - \alpha_2(t)y_1(t)y_3(t),$$

where $\lambda(t)$, $\kappa(t)$, $\alpha_1(t)$, $\alpha_2(t)$ are non-negative variable coefficients. Note that in the resulting ratio, given a specific example, the number of members of the Risk, Adept, and Immunity groups can be determined by sociologists using the appropriate methodology. In general, to derive the relationship between the transition of people from one group to another requires joint research by scientists from different specialties. For example, not all information disseminated by operator 2 may be false; this may serve the purpose of increasing the "reliability" of operator 2's information. Therefore, it is necessary to isolate false information from the entire flow of information from operator 2, in fact, in real time. Expert Systems can probably cope with this task. To determine the value of $\lambda(t)$, $\kappa(t)$, $\alpha_1(t)$, $\alpha_2(t)$ and other coefficients in real time, it is preferable to use, in addition to Expert Systems, other capabilities of the artificial intelligence system.

Similarly, we can approach other groups, taking into account the principle of balance and the fact that from the Adept group the transition to the Immunity group can occur spontaneously. As a result, we will get a dynamic system for combating disinformation in society:

$$\begin{cases} \dfrac{dy_1(t)}{dt} = -\lambda(t)y_4(t)y_1(t) - \kappa(t)y_5(t)y_1(t) - \\ -\alpha_1(t)y_1(t)y_2(t) - \alpha_2(t)y_1(t)y_3(t), \\ \\ \dfrac{dy_2(t)}{dt} = \alpha_1(t)y_1(t)y_2(t) + \kappa(t)y_5(t)y_1(t) - \\ -\lambda_1(t)y_4(t)y_2(t) - \gamma(t)y_2(t) - \\ -\beta_1(t)y_2(t)y_3(t), \\ \\ \dfrac{dy_3(t)}{dt} = \gamma(t)y_2(t) + \alpha_2(t)y_1(t)y_3(t) + \\ +\beta_1(t)y_2(t)y_3(t) + \lambda_1(t)y_4(t)y_2(t) + \\ +\lambda(t)y_4(t)y_1(t), \\ \\ \dfrac{dy_4(t)}{dt} = \omega_1(t)y_2(t)\left(1 - {y_4(t)}/{M_1}\right), \\ \dfrac{dy_5(t)}{dt} = \omega_2(t)y_1(t)\left(1 - {y_5(t)}/{M_2}\right). \end{cases} \tag{1}$$

where, all the system coefficients (1) are positive and variable. The parameters M_1 and M_2 correspond to the levels of those Internet Technologies with the help of which information flows by operators are distributed accordingly.

Suppose that at an initial point in time, the numbers of all groups and the volume of operator flows are known, i.e.

$$\begin{cases} y_1(0) = y_{10}, \quad y_2(0) = y_{20}, \quad y_3(0) = y_{30}, \\ y_4(0) = y_{40} \quad y_5(0) = y_{50}. \end{cases} \tag{2}$$

Thus, a mathematical model was built for the spread of disinformation and the fight against it in the form of the Cauchy Problem (1), (2). Where (2) is the initial conditions for the dynamic system (1).

The significance of functions $y_2(t)$ determines how much misinformation has penetrated society. If the goal is set that during what is important voting, society should either be completely freed from misinformation, or the "carriers" of disinformation should not exceed or equal five percent of the number of voters. Usually, five percent is the barrier in some European countries that political actors must overcome in elections to enter the legislature. If the elections are scheduled for time T , then by this time the number of adherents of operator 2 - $y_2(T)$ should be less than five percent of voters. Let the number of voters coincide with the size of society, then the following must be fulfilled:

$$y_2(T) < N/20. \tag{3}$$

Thus, the task arises - the dynamic system should be transferred from state (2) to state (3), so that a society, mainly free from disinformation, would make a choice. To transfer the system from state (2) to (3) in operation, [11], it is proposed to consider the flow volume of the first operator as a control parameter and set the task of optimal control of the fight against disinformation:

$$J(y_4(t)) = J(\omega_1(t), M_1) = \int_0^T \phi(t) y_4(t)dt \to inf. \tag{4}$$

where $\phi(t)$ the price of spreading one unit of anti-false information at a particular point in time $t \in [0,T]$. Since the flow of anti-false information is determined by the levels of Internet Technologies M_1 and the parameter $\omega_1(t)$, we actually have two control parameters. Thus, the task of optimally controlling the fight against disinformation - (4), (1) - (3) makes the following sense. Operator 1 must produce such an amount of anti-false information that will satisfy the system (1), the boundary values (2), (3) and the price for its creation will be minimal.

3.1 Stationary Solutions - Singular Points

Let

$\vec{y}(t) = (y_1(t), y_2(t), y_3(t), y_4(t), y_5(t))^T$,

$\vec{F}(\vec{y}(t)) = \left(f_1(\vec{y}(t)), f_2(\vec{y}(t)), f_3(\vec{y}(t)), f_4(\vec{y}(t)), f_5(\vec{y}(t))\right)^T$

then the autonomous system (1) can be represented in the following form-

$$\frac{d\vec{y}(t)}{dt} = \vec{F}(\vec{y}(t)) \tag{5}$$

Let the coefficients in (1) be constant and equal - $\lambda = 0{,}015$; $\lambda_1 = 0{,}011$; $\kappa = 0{,}009$; $\alpha_1 = 0{,}013$; $\alpha_2 = 0{,}014$; $\beta_1 = 0{,}013$; $\gamma = 0{,}0092$ $\omega_1 = 0{,}018$; $\omega_2 = 0{,}017$; $M_1 = 45$; $M_2 = 60$. For these coefficient values, we find stationary solutions of system (1) or (5), i.e. let's solve a system of nonlinear equations:

$$\vec{F}(\vec{y}) = \vec{0}. \qquad (6)$$

Note that system (6) has a trivial solution $\vec{y} = (0,0,0,0,0)^T$. To find other stationary solutions, we will use the **MatLab** application package, namely the **fsolve** function. To do this, we will create two **m-files**:

adsys2.m

```
function F=adsys2(y)
l=0.015; l1=0.011; k=0.009; a1=0.031; a2=0.014;
b=0.013;g=0.0092;
o1=0.018;o2=0.017; m1=45; m2=60;
F = [-l*y(1).*y(4)-k*y(1).*y(5)-a1*y(1).*y(2)-a2*y(1).*y(3)
    a1*y(1).*y(2)+k*y(1).*y(5)-l1*y(2).*y(4)-g*y(2)-b*y(2).*y(3)
g*y(2)+a2*y(1).*y(3)+b*y(2).*y(3)+l1*y(2).*y(4)+l*y(1).*y(4)
    o1*y(2).*(1-y(4)/m1)
    o2*y(1).*(1-y(5)/m2)]
end
```

znles.m

```
y0 = [9; 13; 11; 10;  8];
[y,fval] = fsolve(@adsys2,y0,optimset('Display','off','TolFun',1e-4))
```

As a result of a computer experiment, when the system initially approaches zero, y0 = [9; 13; 11; 10; 8], we obtain a stationary solution:

$$\vec{y}^{\,*} = (0;0;9.6806;4.8259;20.6643)^T \qquad (7)$$

For each stationary solution, its nature should be determined. To do this, you need to find the eigenvalues of the Jacobian matrix of system (5):

$$\frac{D(f_1, f_2, f_3, f_4, f_5)}{D(y_1, y_2, y_3 y_4, y_5)} = \left(\frac{\partial f_i}{\partial y_j}\right), \; i, j = 1,2,3,4,5; \qquad (8)$$

The Jacobian matrix (8) for a stationary point (solution) (7) has the form

$$j = \begin{pmatrix} -0.3939 & 0 & 0 & 0 & 0 \\ 0.1860 & -0.1881 & 0 & 0 & 0 \\ 0.2079 & 0.1881 & 0 & 0 & 0 \\ 0 & 0.0161 & 0 & 0 & 0 \\ 0.0111 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad (9)$$

Now let's calculate the eigenvalues of the Jacobian (9) in MatLab using **eig**(j) . We get L= [0, 0, 0, -0.1881, -0.3939] real eigenvalues. In this case, the Jacobian determinant is equal to zero **det**(j)=0.

Since there are zeros among the eigenvalues, the first Lyapunov methods for stability are not applicable. In our case, there is a critical singular point and for its further study we should look for the Lyapunov function, and apply Lyapunov's theorems on stability, asymptotic stability and Chetaev's theorem on instability. However, we will refrain from doing so at this stage. What is interesting for us is not so much asymptotic stability, i.e. does the system transition to a stationary position as time tends to infinity, and the behavior of the system over a finite period, is it possible to transfer it to the required position during this time? Because, over a finite period of time, events may unfold in such a way and the system will find itself in such a state that it can cause upheavals in society.

3.2 Controllability Problem

We examine the task of optimal control of the fight against disinformation (4), (1) - (3) for controllability. I.e. find out whether there is such a function that satisfies (2) to (3). We use the **MatLab** Application Package for this, in the system (1) the coefficients will be considered constant. Let us have the following boundary conditions: $T = 15$, $y_1(0) = 400$, $y_2(0) = 50$, $y_3(0) = 10$, $y_4(0) = 1$, $y_5(0) = 17$, $y_2(15) \le 460/20$. Let the coefficients of system (1) be previously determined values. With such coefficients of the autonomous system (1), the conditions for the existence and uniqueness of a solution to the Cauchy Problem(1), (2) in the class of smooth functions are satisfied.

To solve the Cauchy problems (1), (2) with the values indicated above, we use the ode15s solver, the program code is compiled - Listing

```
[X,Y]=ode15s(@adsys,[0 15],[400 50 10 1 17]);
plot(X,Y,'linewidth',2)
legend('y1','y2','y3','y4','y5')
function dydx=adsys(x,y)
l=0.01; l1=0.015; k=0.01; a1=0.03; a2=0.017;
b=0.015;g=0.001; o1=0.019; o2=0.79; m1=40;
m2=55;
```

```
dydx=[-l*y(1)*y(4)-k*y(1)*y(5)-a1*y(1)*y(2)-
a2*y(1)*y(3)
     a1*y(1)*y(2)+k*y(1)*y(5)-l1*y(2)*y(4)-
g*y(2)-b*y(2)*y(3)

g*y(2)+a2*y(1)*y(3)+b*y(2)*y(3)+l1*y(2)*y(4)+l*
y(1)*y(4)
 o1*y(2)*(1-y(4)/m1)
 o2*y(1)*(1-y(5)/m2)]
end
```

A computer experiment shows, for the above data, that operator 1, even with little effort, manages to neutralize the influence of false information by the end of the time interval - the number of adherents is zero, although it is enough that the number of adherents is less than five percent of the population, in Figure 2.

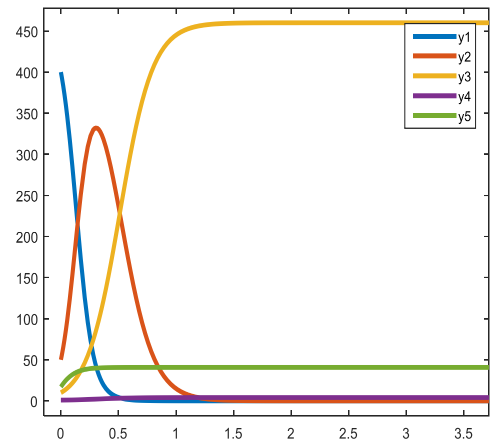

Fig. 2: Manageability of the model for combating disinformation

A similar result is achieved for other values of system parameters - information security is ensured by the end of the interval. However, information security is nevertheless vulnerable, but not at the end of the interval, but at the beginning. If we pay attention to the value of the number of adherents at the beginning of the interval, it turns out that, for example, in our case, its maximum value is 322 at time 0.3, which is 80,5% of the society population, in Figure 3.

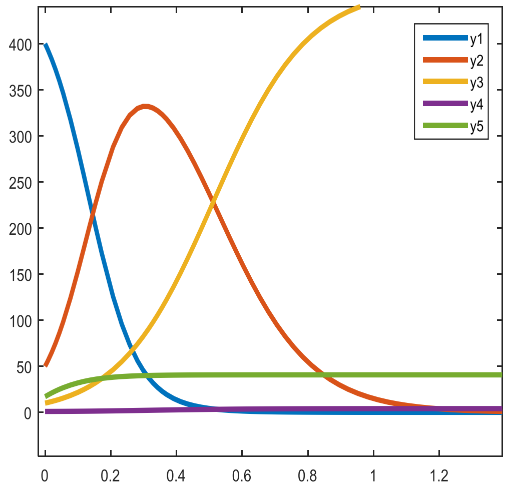

Fig. 3: Information security vulnerability

At the same time, and over a sufficiently long period, the number of adherents is more than 50% of the public. For the information security of society, it is not sufficient to reduce the number of disinformation adherents only at a certain point in time, for example, before any voting. Since at some points, the influence of disinformation on societies can be so strong that society as a whole can legitimize revolutionary, early parliamentary, presidential elections, etc. Therefore, in the optimal control problem of combating disinformation, adjustments should be made and inequality in (3) should be satisfied over the entire period, [11].

$$y_2(t) \le 0{,}05 \times N \text{ , for } \forall t \in [0;T]. \quad (10)$$

The issue of controllability in the optimal control problem (4), (1), (2), and (5) is studied using a computer experiment. By changing the control parameters - $\omega_1(t)$ and M_1, increasing from by an order of magnitude, it is not possible to achieve the fulfillment of condition (10) throughout the entire section $[0;T]$, i.e. there are a sufficient number of adherents of false information in society. The question arises of attracting new control parameters, these could be: $\beta_1(t)$ - the intensity of interpersonal contacts between the Immunity and Adept groups; y_{30} - initial conditions for the Immunity group; y_{40} - initial conditions for the dissemination of anti-false information by the first operator. By varying the values of these new control parameters, it is possible to significantly reduce the maximum value of the number of adherents, for example for $\beta_1(t) = 0.23$, $y_{30} = 40$, $y_{40} = 20$ - [X,Y]=ode15s(@adsys,[0 15], [400 50 40 20 17]), which is clearly seen in Figure 4.

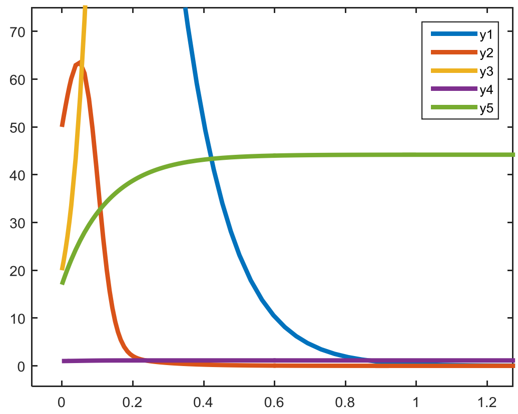

Fig. 4: Result of control new parameters

The maximum number of adherents is 63,48 - reached at the beginning of the time interval and then not for long - soon the number of adherents sharply decreases and becomes less than 24, which is no more than five percent of the population of society. At the same time, the number of adherents at the beginning of the time interval is growing, and for this not to happen, the inequality must be fulfilled $\left.\frac{dy_2(t)}{dt}\right|_{t=0} < 0$, $\alpha_1(0)y_{10}y_{20} + \kappa(0)y_{50}y_{10} - -\lambda_1(0)y_{40}y_{20} - \gamma(0)y_{20} - \beta_1(0)y_{20}y_{30} < 0.$

Let us rewrite the last inequality assuming that,

$$\alpha_{10}y_{10} + \kappa_0 y_{50}y_{10} / y_{20} < \lambda_{10}y_{40} + \gamma_0 + \beta_{10}y_{30}. \quad (11)$$

In inequality (11) there are at least two control parameters - , and with their help - by selecting the appropriate values, it is possible to achieve the fulfillment of condition (11). And this means that the number of adherents from the very beginning will decrease.

4 Conclusion

For the information security of society, it is not sufficient to reduce the number of disinformation adherents only at a certain point in time, for example, before any vote. As it was shown, at some points, the influence of disinformation on societies can be so strong that society as a whole can legitimize revolutionary, early parliamentary, presidential elections, etc. Therefore, in the proposed, [11], adjustments should be made to the task of optimal control of the fight against disinformation, specifically, the number of adherents should be controlled throughout the entire period. To effectively combat disinformation, it is necessary to increase the number of control parameters and the target control functionality and its minimization has the form:

$$J(\omega_1(t), M_1, \beta_1, y_{30}, y_{40}) = \int_0^T \phi(t) y_4(t)dt \to inf. \quad (12)$$

$$\phi(t) \in C; \qquad M_1, \beta_{10}, y_{30}, y_{40} \in R$$

Thus, the problem of optimal control of effective combat against disinformation will take the form (12), (1), (2), (10). If in the initial condition (2) there is inequality $y_{20} > 0.5N$, then condition (10) is not satisfied at the beginning of the time interval, which means the vulnerability of society in terms of Information Security. Mathematical and computer modeling of the effective fight against disinformation allows us to conclude that permanent control of the number of adherents and information flows of operator 2 makes it possible to ensure the stability of society in Information Warfare.

The creation of an Automated System for Effectively Combating Disinformation should occur based on generalized principles and its application will be possible in different countries and different areas of activity. However, it can be assumed that it is especially relevant for my country. Georgia recently received the status of a candidate country for joining the European Union with the expectation that it will implement nine recommendations of the European Commission in the future, among which the first is to "fight disinformation and foreign information manipulation and interference against the EU and its values".

References:

[1] A.P. Petrov, O.G. Proncheva, Modeling Position Selection by Individuals during Informational Warfare with a Two-Component Agenda. *Math Models Comput Simul* 12, 2020, pp. 154–163

[2] I.D. Kolesin, E.M. Zhitkova, Modeling Sociopsychological Processes of the Formation of the Population's Position on Vaccine Prevention in the Context of Information Propaganda. *Math Models Comput Simul* 15 (Suppl 1), 2023, pp. 12–17.

[3] O. Nakonechnyi, A. Pashko, A. Sverstuk and I. Shevchuk, Statistical Simulation of the External Influence of the Information Spreading of the Population Models, *2020 IEEE 2nd International Conference on System*

Analysis & Intelligent Computing (SAIC), Kyiv, Ukraine, 2020, pp. 1-4,.

[4] T. Chilachava, Ts. Gvinjilia, Nonlinear Mathematical Model of Interference of Fundamental and Applied Researches, *International Journal of Systems Science and Applied Mathematics*, 2(6): 2017, pp. 110-115.

[5] Inna Samuilik, Felix Sadyrbaev, Mathematical Modelling of Leukemia Treatment, *WSEAS Transactions on Computers*, vol. 20, 2021, pp. 274-281, https://doi.org/10.37394/23205.2021.20.30.

[6] T. Chilachava, Mathematical Model of Transformation of Two-party Elections to Three-party Elections, *GESJ: Computer Science and Telecommunications*, No.2(52), 2017, pp. 21-29.

[7] G. Pochkhua, L. Sulava, Political Conflicts with the Participation of International Investment Institutions, *Proceedings XXII of Tskhum-Abkhazian Academy of Sciences*, Tbilisi, 2022, pp. 4-22.

[8] T. Chilachava, G. Pochkhua, Research of a Three-Dimensional Dynamic System Describing the Process of Assimilation, *Lecture Notes of TICMI*, Vol. 22, 2021, pp. 63–72.

[9] R. Pandey, M. Pandey and A. Nazarov, Detection of Propaganda in Information Warfare using Deep Learning, *2022 4th International Conference on Advances in Computing, Communication Control and Networking (ICAC3N)*, Greater Noida, India, 2022, pp. 2485-2489.

[10] O. Nakonechnyi, A. Pashko, O. Kapustian, Zinko T, Shevchuk I. Statistical Simulation of the Information Warfare. *In2019 IEEE International Conference on Advanced Trends in Information Theory (ATIT)*, IEEE, 2019, pp. 75-80.

[11] N. Kereselidze, Mathematical and Computer RAI Models of False Information Dissemination, *WSEAS Transactions on Computer Research*, Vol.7, No.9, 2019, pp. 56-63.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

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The author has no conflicts of interest to declare.