Dynamic-chance-constrained-based Fuzzy Programming Approach for Optimizing Wastewater Facultative Ponds for Multi-period Case

K. KARTONO[1], S. SUTRISNO[1,*], S. SUNARSIH[1], W. WIDOWATI[1],
TOSPORN ARREERAS[2], MUHAMMAD SYUKUR[2]
[1]Department of Mathematics, Diponegoro University,
Tembalang 50275 Semarang,
INDONESIA

[2]School of Management, Mae Fah Luang University,
Chiang Rai 57100,
THAILAND

[*]*Corresponding Author*

Abstract: - In this article, a novel optimization model that was specifically designed as a dynamic-chance-constrained fuzzy uncertain programming framework is introduced. This model serves the purpose of optimizing the efficiency of facultative ponds utilized in domestic wastewater treatment. The primary focus of this study was maximizing the amount of the wastewater treated in the facility subject to quality requirements via the assessment of wastewater quality through the measurement of Biological Oxygen Demand (BOD). The model's development was grounded in a real-world scenario, where decision-makers encountered uncertainties in various parameters, such as the rate of BOD degradation and the incoming wastewater load, both characterized by fuzzy membership functions. In light of this uncertainty, the decision-maker aimed to maximize the wastewater treatment capacity while maintaining a suitable safety margin for both objective and constraint functions, employing policies founded on probability and chance. A case study was carried out at the Bantul domestic wastewater treatment plant, situated in Yogyakarta, Indonesia. The study successfully identified optimal decisions regarding wastewater flow rates and processing times. As a result, it can be concluded that the proposed model effectively resolved the problem at hand, making it a valuable tool for decision-makers in similar contexts.

Key-Words: - Biological oxygen demand, chance-constrained optimization, domestic wastewater, dynamic fuzzy programming, dynamic optimization, facultative ponds, wastewater treatment

Received: May 17, 2022. Revised: August 21, 2023. Accepted: September 22, 2023. Available online: November 16, 2023.

Nomenclature

Decision variables on the observation day *j*:

$Q_0(j)$: The rate of the wastewater volume inflow at the inlet (m^3)

$Q_i^e(j)$: The rate of the wastewater volume at the facultative pond *i* (m^3)

$t(j)$: Average detention time (day)

Fuzzy parameters:

L_0 : The daily rate of the waste load at the facility inlet (kg)

k : The daily BOD degradation rate

Semi-decision variables:

$L_i(j)$: Waste load at the inlet of the facultative pond *i* (kg) on the observation day *j*

$L_i^e(j)$: Waste load processed in the facultative pond *i* (kg) on the observation day *j*

Crisp or deterministic parameters:

$C_i(j)$: The BOD concentration at pond *i* (mg/L) estimated on the observation day *j*

$E_i(j)$: The BOD degradation efficiency index at pond *i* (in percentage) estimated on the observation day *j*

$E_i^r(j)$: Target or reference point for the BOD degradation efficiency index at pond *i* estimated on the observation day *j*

BM : Wastewater quality standard

$p_i, i = 1,2$: Percentage of waste load processed in pond *i*

1 Introduction

Before disposal, wastewater needs to undergo a stabilization process to uphold water and environmental sustainability objectives. However, the availability of wastewater treatment plants remains limited, especially in developing countries, necessitating the optimization of their performance to handle wastewater to the fullest extent possible.

In most wastewater treatment plants, microorganisms such as algae, bacteria, and zooplankton are commonly harnessed to reduce pollutant concentration, [1]. Among the parameters employed to assess the quality of treated water, the focus in this study was on Biological Oxygen Demand (BOD). The types of wastewater typically encountered are of domestic origin, originating from households, hotels, and general industries. Facultative ponds are chosen for use in wastewater treatment plants due to their straightforwardness in degrading pollutants in domestic wastewater until they meet specified concentration standards, often measured through BOD levels, [2].

Mathematical optimization models have been integrated into wastewater treatment management to enhance the capacity and efficiency of facultative ponds. Numerous models have been devised for wastewater management, each tailored to address specific challenges faced by decision-makers, see e.g., [3], [4], [5]. These models vary in complexity and purpose, encompassing simple models like the one proposed in, [6], to manage pollutant concentrations based on quantitative prototypes. Other models cater to distinct scenarios, including linear models with deterministic parameters, [7], models focusing on adsorbents for wastewater treatment analysis, [8], quantitative models for sewage treatment, [9], and models assessing the construction costs of wastewater plants, [10].

Beyond wastewater treatment optimization, additional models serve various objectives, such as sewage management, [10], [11], energy analysis, [12], effluent and sludge analysis, [13], microplastics removal, [14], and (bio)energy generation from wastewater, [15], [16], [17], [18], [19], [20]. However, none of these models have been formulated in a chance-constrained framework, enabling decision-makers to impose chance-based constraints on uncertainty-containing parameters.

To address this gap, a new model was developed to optimize the performance of facultative ponds in wastewater treatment, accommodating uncertain parameters like pollutant concentration at the inlet, and in a dynamic manner over time. This allows decision-makers to introduce additional chance-based constraints to the model, such as the probability of violating uncertain constraints under predefined values. These uncertainties are treated as fuzzy parameters with membership functions determined based on the observations of the decision-maker. Among the parameters investigated in this study, BOD levels were monitored, using the Bantul facultative ponds in Yogyakarta, Indonesia as a case study to develop the model and compute optimal decisions based on the proposed framework.

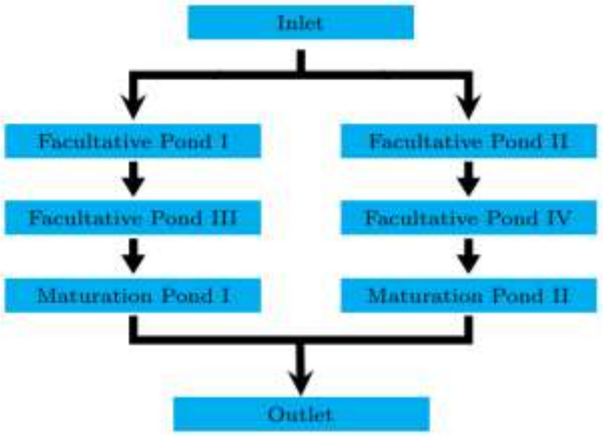


Fig. 1: The layout view of the Sewon wastewater treatment plant

2 Mathematical Model

2.1 Problem Setting and Assumptions

This study focuses on the degradation of Biological Oxygen Demand (BOD) in facultative ponds, specifically designed within the layout of the Bantul wastewater treatment plant situated in Yogyakarta, Indonesia. The facility processes domestic wastewater originating from households, industries, offices, and hotels through a multi-step treatment process, as illustrated in Figure 1. The primary objective is to maximize the processing capacity of all facultative ponds to ensure that the BOD concentration meets the quality standard, considering certain uncertain parameters. Furthermore, the problem is solved dynamically in terms of observation time periods, meaning that the model should be able to provide optimal decisions for multiple periods of implementation in one calculation. To be precise and to provide a clearer understanding, the specifications and assumptions used in this research are elaborated as follows.

The parameter for assessing wastewater quality was BOD. Data related to BOD were collected from wastewater samples at specific grid points within each facultative pond. The BOD degradation rate was treated as a fuzzy parameter, with the decision-maker developing its membership function. Furthermore, An index value was employed to regulate the BOD degradation process, as described in the mathematical model.

The source of domestic wastewater entering the facility was exclusively from the Yogyakarta province. The uncertain inflow waste load was monitored at the inlet per day, incorporating fuzzy uncertainty. The decision-maker established the membership function for the inflow waste load based on observations and historical data. Secondary historical data and observations informed the formulation of this membership function.

Optimizations were conducted over days, and they covered multiple days of observations in one model and one calculation. Moreover, all fuzzy parameters were assumed to have discrete membership functions.

The methodology adopted in this study can be summarized as follows: Initially, the decision-maker constructs membership functions for the fuzzy parameters and assesses the likelihood of not violating the lower bounds of the chance-based constraints within the constraint functions. Subsequently, the objective function, representing the wastewater inflow rate, is formulated. Additionally, the BOD efficiency index control term is defined as the quadratic difference between a reference point and the actual efficiency index. The reference point is determined based on the decision-maker's intuition and experience with managing the facultative ponds' performance. Furthermore, constraint functions are formulated and expressed in a mathematical model, taking into account the structure of the wastewater treatment facility and the necessary conditions that must be met.

The formulated mathematical model is then solved using a computer, with the model being translated into a programming language using LINGO 19.0 and subsequently solved with the embedded solver in the software. Chance-constrained-based programming is employed to calculate the optimal decision, as detailed in, [21]. Finally, the generated solution is applied to the wastewater treatment facility.

2.2 Chance-constrained-based Fuzzy Optimization Model

In this optimization challenge, the following two primary objectives were considered: 1) maximizing the influx of wastewater and 2) minimizing the quadratic expression representing the disparity between the BOD degradation efficiency index and the reference value stipulated by the decision-maker. The setup of these two goals led to the formulation of the following optimization problem subject to constraints functions that were formulated following the specifications and assumptions of the problem described in the previous section (explanations for each will follow afterward):

$$\min Z = -\sum_{j=1}^{J} Q_0(j) + \sum_{j=1}^{J}\sum_{i=1}^{4}[E_i(j) - E_i^r(j)]^2 \qquad (1)$$

subject to, $\forall j = 1,2,\dots,J$:

$$L_i^e(j) = \frac{\left(\mathrm{Q}_i^e(j) \times C_i(j)\right)}{1000}, \forall i = 1,2,3,4; \qquad (2)$$

$$E_i(j) \times C_i(j) \le BM, i = 1,2,3,4; \qquad (3)$$

$$L_1(j) + L_2(j) = L_0; \qquad (4)$$

$$Cr\left\{L_3^e(j) = (1 - p_1(j)) \times L_1(j)\right\} \ge \gamma_1; \qquad (5)$$

$$Cr\left\{L_4^e(j) = (1 - p_2(j)) \times L_2(j)\right\} \ge \gamma_2; \qquad (6)$$

$$Cr\{L_1(j) + L_2(j) = L_0\} \ge \beta_i, i = 1,2,3,4; \qquad (7)$$

$$Q_1^e(j) + Q_2^e(j) = Q_0(j); \qquad (8)$$

$$Q_3^e(j) = 0.5 \times Q_1^e(j) \text{ and } Q_4^e(j) = 0.5 \times Q_2^e(j); \qquad (9)$$

$$Cr\left\{E_i(j) = \frac{k \times t(j)}{1 + k \times t(j)}\right\} \ge \alpha_i, i = 1,2,3,4. \qquad (10)$$

In the preceding minimization problem, we consider the value of $-Q_0$ the objective function since the original problem aims to maximize it. The constraint function (2) denotes that the waste load is determined by both the inflow rate and organic matter while (3)ensuring that the BOD concentration remains below an upper threshold. Additionally, equality (4) signifies that the total waste load entering ponds I and II equals that at the inlet. Inequalities (5) (6) govern the waste load transfer from pond I to pond III and from pond II to pond IV, respectively, where $\gamma_i, i = 1,2$ they represent the confidence levels used for pond I and II provided as an appropriate safety margin by the manager/decision-maker for the corresponding constraint functions to hold. The probabilistic inequality (7) is employed to restrict the treated wastewater load in facultative pond *i*, ensuring it does not exceed its pre-treatment value.

Concerning Figure 1, the combined inflow rate into ponds I and II equals that of the inlet, while the inflow rates into ponds III and IV are half that of ponds I and II, denoted by inequalities (8) and (9) respectively. Formula (10) outlines the computation of the required biological oxygen degradation efficiency index, where *k* represents the BOD degradation rate for the detention time spans one day, and where $\alpha_i, i = 1,2,3,4$ *to* represent the confidence levels used for pond *i* provided as an appropriate safety margin by the manager/decision-maker for the corresponding constraint functions to hold.

Combining both objective functions and all constraint functions yields a probabilistic optimization problem. To address this problem, the chance-constraint programming algorithm is required to compute the optimal decision. Furthermore, it's worth noting that all constraint functions are closed and bounded, ensuring that this optimization problem always possesses an optimal solution as long as its feasible region is not empty.

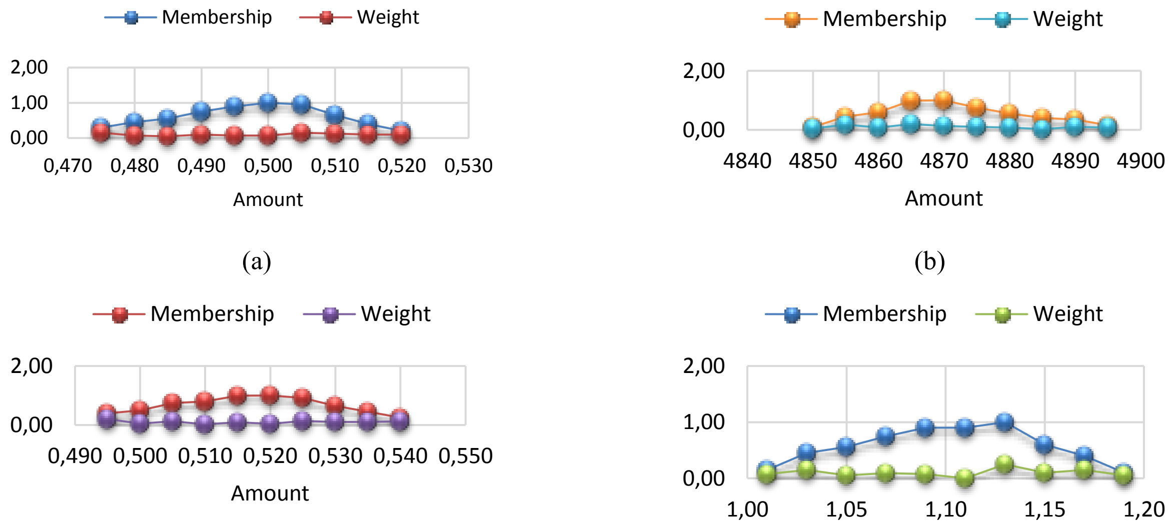
The chance-constrained optimization problem (1) was solved by using the chance-constrained programming method introduced in, [22]. Furthermore, to calculate the optimal decision, the uncertain programming method based on the deterministic equivalent approach provided in, [21], was utilized. To calculate the expectation of fuzzy numbers with discrete membership functions, the fuzzy number theory in, [22], was utilized.

3 Case Study

The case study was carried out at the Bantul wastewater treatment facility, and Figure 1 illustrates the treatment process flow. The subsequent subsection presents the parameters and outcomes of the chance-constrained fuzzy programming model.

3.1 Parameter Setting

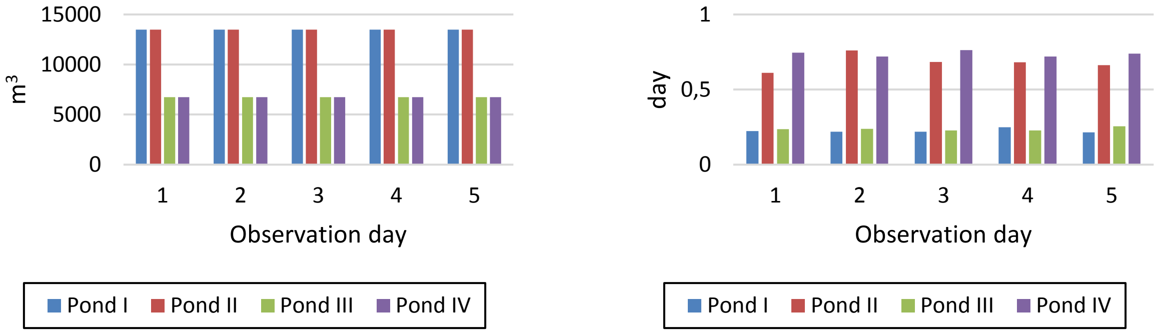
The membership functions for the fuzzy parameters were generated randomly, centered around the mean of the data provided in, [7]. Figure 2 displays both their membership values and weights. In compliance with the Yogyakarta Province's local government policy, the BOD concentration in treated wastewater should not surpass 50 mg/L. Simultaneously, the decision-maker aimed for an efficiency index of 0.5 for each pond. The calculations were executed using the LINGO 19.0 optimization software, employing the generalized reduced gradient algorithm as outlined in references, [21], [22].



(a) (b)

(c) (d)

Fig. 2: Graphs of the membership functions of the fuzzy parameters (a) waste load for ponds I and II (b) waste load for ponds III and IV (c) BOD degradation rate for ponds I and II (d) BOD degradation rate for ponds III and IV



(a) (b)

Fig. 3: The optimal decisions (a) $Q_i^e(j)$: The rate of the wastewater volume at the facultative pond i (m^3) on the observation day j (b) $t(j)$: Average detention time (day) on observation day j

3.2 Results and Discussion

Figure 3 displays the optimal outcome derived from the proposed model. Meanwhile, the optimal inflow rate at the inlet stands at 26950 m^3 per day. The optimized allocation for wastewater treatment across the facultative ponds consists of 13475 m^3/day for both Ponds I and II, along with 6737.5 m^3/day for Ponds III and IV. According to this calculation, the anticipated BOD concentration post-treatment is 50 mg/L, and it is not imperative for the detention time to span the entire day. Should the decision-maker opt for a full-day detention time, the expected BOD concentration would be lower than 50 mg/L.

It is worth noting that the efficiency index values varied among the four ponds due to parameter fluctuations, but their average remained at 31%. This implies that overall performance in the facultative ponds needs enhancement, primarily through sludge removal. Notably, Pond II exhibited the highest efficiency index value and should be maintained, while Pond I, with the lowest value, requires improved treatment, such as the addition of an aerator.

From the results, several managerial insights emerge regarding the management of facultative ponds, those are explained as follows. First, decision-makers are likely to consider varying confidence levels for each constraint function in the mathematical model, allowing adjustments based on their experience and intuition. Second, some parameters had unknown actual values during computation, indicating decisions were made under uncertainty. This suggests that achieved goals may differ from the mathematical model's expected values. Therefore, when dealing with multiple probability values, actual results can be either better or worse. Third, it is possible to perform multiple optimizations with different parameter values, such as various membership functions, until the decision-maker gains sufficient confidence to execute a decision. However, computational time should be considered when the decision-maker has the time and expects improved results.

4 Conclusion

In this study, a novel dynamic-chance-constrained fuzzy programming model has been introduced, aimed at improving the efficiency of facultative wastewater stabilization ponds with multiple time periods of observation. An empirical investigation was conducted at the Bantul wastewater treatment plant, and the outcomes indicated the effective optimization of the facility through the proposed method.

Looking ahead, there are several forthcoming challenges. These encompass the development of more intricate models to address complex scenarios, including the management of pollutant degradation processes within maturation ponds. Additionally, exploring the impact of sludge analysis on pond performance is an intriguing avenue for future research.

References:

[1] D. Kang and K. Kim, “Real Wastewater Treatment Using a Moving Bed and Wastewater-Borne Algal–Bacterial Consortia with a Short Hydraulic Retention Time,” *Processes*, vol. 9, no. 1, 2021, doi: 10.3390/pr9010116.

[2] R. Vagheei, “Upgrading of waste stabilization ponds using a low-cost small-scale fine bubble diffused aeration system,” *Water Science and Technology*, vol. 84, no. 10–11, pp. 3104–3121, Aug. 2021, doi: 10.2166/wst.2021.330.

[3] Q. Zhang, Z. Li, and W. Huang, “Simulation-based interval chance-constrained quadratic programming model for water quality management: A case study of the central Grand River in Ontario, Canada,” *Environ Res*, vol. 192, 2021, doi: 10.1016/j.envres.2020.110206.

[4] S. Mahajan, S. K. Gupta, I. Ahmad, and S. Al-Homidan, “Using concave optimization methods for inexact quadratic programming problems with an application to waste management,” *J Inequal Appl*, vol. 2021, no. 1, 2021, doi: 10.1186/s13660-021-02588-w.

[5] Q. Zhang and Z. Li, “Data-driven interval credibility constrained quadratic programming model for water quality management under uncertainty,” *J Environ Manage*, vol. 293, 2021, doi: 10.1016/j.jenvman.2021.112791.

[6] B. sheng Huang *et al.*, “Quantitative study of degradation coefficient of pollutant against the flow velocity,” *Journal of Hydrodynamics*, vol. 29, no. 1, pp. 118–123, 2017, doi: 10.1016/S1001-6058(16)60723-0.

[7] Sunarsih, Widowati, Kartono, and Sutrisno, “Mathematical Analysis for the Optimization of Wastewater Treatment Systems in Facultative Pond Indicator Organic Matter,” *E3S Web of Conferences*, vol. 31, no. 05008, pp. 1–3, 2018.

[8] A. Gopakumar, R. Narayan, S. A. Nagath, N. P, R. Mohammed. S, and S. Chandran. S, “Waste Water Treatment Using Economically Viable Natural Adsorbent Materials,” *Mater Today Proc*, vol. 5, no. 9, pp. 17699–17703, 2018, doi: 10.1016/j.matpr.2018.06.091.

[9] B. K. Kogo, E. K. Biamah, and P. K. Langat, “Optimized Design of a Hybrid Biological Sewage Treatment System for Domestic Wastewater Supply,” *Journal of Geoscience and Environment Protection*, vol. 05, no. 05, pp. 14–29, 2017, doi: 10.4236/gep.2017.55002.

[10] D. O. Olukanni and J. J. Ducoste, “Optimization of waste stabilization pond design for developing nations using computational fluid dynamics,” *Ecol Eng*, vol. 37, no. 11, pp. 1878–1888, 2011, doi: 10.1016/j.ecoleng.2011.06.003.

[11] D. Recio-Garrido, Y. Kleiner, A. Colombo, and B. Tartakovsky, “Dynamic model of a municipal wastewater stabilization pond in the Arctic,” *Water Res*, vol. 144, pp. 444–453, 2018, doi: 10.1016/j.watres.2018.07.052.

[12] S. Borzooei *et al.*, “Optimization of the wastewater treatment plant: From energy saving to environmental impact mitigation,” *Science of The Total Environment*, vol. 691, pp. 1182–1189, 2019, doi: 10.1016/j.scitotenv.2019.07.241.

[13] M. Benito, C. Menacho, P. Chueca, M. P. Ormad, and P. Goñi, “Seeking the reuse of effluents and sludge from conventional wastewater treatment plants: Analysis of the presence of intestinal protozoa and nematode eggs,” *J Environ Manage*, vol. 261, p. 110268, 2020, doi: https://doi.org/10.1016/j.jenvman.2020.110268

[14] H. J. Kwon, H. Hidayaturrahman, S. G. Peera, and T. G. Lee, “Elimination of Microplastics at Different Stages in Wastewater Treatment Plants,” *Water (Basel)*, vol. 14, no. 15, 2022, doi: 10.3390/w14152404.

[15] T. Tobin, R. Gustafson, R. Bura, and H. L. Gough, “Integration of wastewater treatment into process design of lignocellulosic biorefineries for improved economic viability,” *Biotechnology for Biofuels 2020 13:1*, vol. 13, no. 1, pp. 1–16, Feb. 2020, doi: 10.1186/S13068-020-1657-7.

[16] A. Tawfik, H. Niaz, K. Qadeer, M. A. Qyyum, J. J. Liu, and M. Lee, “Valorization of algal cells for biomass and bioenergy production from wastewater: Sustainable strategies, challenges, and techno-economic limitations,” *Renewable and Sustainable Energy Reviews*, vol. 157, no. 112024, pp. 1–15, Apr. 2022, doi: 10.1016/J.RSER.2021.112024.

[17] N. J. Koffi and S. Okabe, “High electrical energy harvesting performance of an integrated microbial fuel cell and low voltage booster-rectifier system treating domestic wastewater,” *Bioresour Technol*, vol. 359, no. 127455, pp. 1–9, Sep. 2022, doi: 10.1016/J.BIORTECH.2022.127455.

[18] C. J. A. Caligan, M. M. S. Garcia, J. L. Mitra, and J. L. G. San Juan, “Multi-objective optimization for a wastewater treatment plant

and sludge-to-energy network," *J Clean Prod*, vol. 368, no. 133047, pp. 1–14, Sep. 2022, doi: 10.1016/J.JCLEPRO.2022.133047.

[19] B. Zhang *et al.*, "Recent Advances in the Bioconversion of Waste Straw Biomass with Steam Explosion Technique: A Comprehensive Review," *Processes*, vol. 10, no. 10, 2022, doi: 10.3390/pr10101959.

[20] A. Shahid *et al.*, "Bioenergy potential of the residual microalgal biomass produced in city wastewater assessed through pyrolysis, kinetics and thermodynamics study to design algal biorefinery," *Bioresour Technol*, vol. 289, p. 121701, 2019, doi: https://doi.org/10.1016/j.biortech.2019.121701

[21] LINGO: *The Modeling Language and Optimizer*. Illinois: Lindo Systems, Inc., 2020.

[22] B. Liu, *Uncertainty Theory*. in Springer Uncertainty Research. Berlin, Heidelberg: Springer Berlin Heidelberg, 2015. doi: 10.1007/978-3-662-44354-5.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- K. Kartono managed and supervised the research activities.
- K. Kartono, S. Sutrisno, S. Sunarsih, W. Widowati, Tosporn Arreeras, Muhammad Syukur modelled and verified the programming.
- S Sutrisno carried out the computational simulation.
- K. Kartono, S. Sutrisno, S. Sunarsih, W. Widowati prepared the draft of the manuscript.
- Tosporn Arreeras, Muhammad Syukur validated the results, reviewed, and edited the manuscript.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

This work is funded by the Faculty of Science and Mathematics, Universitas Diponegoro via Riset Utama 2023 Research Grant contract no. 22/UN7.F8/PP/II/2023.

Conflict of Interest

The authors have no conflicts of interest to declare.

Creative Commons Attribution License 4.0 (Attribution 4.0 International, CC BY 4.0)