Adaptive Fuzzy Control of a Four-Wheeled Mobile Robot Subject to Wheel Slip

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Abstract: - In this paper, the adaptive fuzzy tracking control of a four-wheeled mobile robot subject to wheels slip is considered. We proposed an adaptive scheme in that fuzzy logic approximators are used to approximate the unknown system functions in designing the adaptive tracking control of a mobile robot. Fuzzy systems are expressed as a series expansion of basis functions, to adaptively compensate for the mobile robot nonlinearities. The proposed control system works online, parameter adaptation is realized in every discrete step of the control process, and a preliminary learning phase of fuzzy system parameters is not required. The stability of the algorithm is established in the Lyapunov sense, with tracking errors converging to a neighborhood of zero. Simulation results illustrate the effectiveness of the approach.

Key-Words: **-** Fuzzy system, Lyapunov stability, mobile robot, tracking control, wheels slip.

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1 Introduction

Application of modern methods of realization of motion of wheeled mobile robots, in which a fundamental role is played by artificial intelligence methods, belongs to priority research direction in the field of modern technologies of autonomous robots. Despite significant advances in the field of autonomous robotics, still, many problems remain unsolved. Most difficulties are associated with a description of the natural work environment of an autonomous robot. Usually, the knowledge about the environment is, in general, incomplete, uncertain, and approximate. To this field belong, for example, the problems concerning the inclusion of the phenomena of mobile robot wheel slips into control algorithms. Recently, a lot of attention is devoted to the problems of modeling and control of wheeled mobile robots taking into account wheel slips [3], [5], [6], [7], [8], [9], [11], [12], [13], [18], [23], [24], [29], which follows from possibility of using those objects in practical applications, characterized, for instance, by irregular surfaces and various parameters of wheels contact with the ground. In the conventional control theory, most of the control problems are usually solved by mathematical tools based on the system models. Fuzzy controllers are assumed to work in situations where the plant parameters and structures have some uncertainties or unknown variations. As we know, based on the universal approximation theorem, [26], [27], where fuzzy logic systems have been shown to be capable of uniformly approximating any well-defined nonlinear function to any degree of accuracy, many important adaptive fuzzy control schemes have been developed to directly incorporate the expert information systematically and various stable performance criteria are guaranteed by theoretical analyses, [20], [21], [22], [28]. Based on the established fuzzy system properties, various adaptive fuzzy control schemes have been systematically developed, by which the stability of the closed-loop system can be guaranteed by theoretical analyses, [22], [27]. Among these approaches, the adaptive tracking control method with a radial basis function fuzzy system, [17], is proposed for nonlinear systems to adaptively compensate the nonlinearities of the systems, [4]. The indirect and direct adaptive control schemes using fuzzy systems for nonlinear systems have also been shown in [19], to provide design algorithms for stable controllers. In addition, control systems based on a fuzzy control scheme are augmented with variable structure control, [27], [29], to ensure global stability and robustness to disturbances.

In this paper, the intelligent stable adaptive fuzzy control system for the position and heading of a four-wheeled mobile robot with the inclusion of longitudinal and lateral slips is proposed, in which fuzzy systems are used for compensation of nonlinearities and variable operating conditions of a mobile robot.

The structure of the paper is as follows. In section 2 basic kinematic relationships are discussed, and generalized velocities required for realization of the desired robot motion, understood as kinematic controller, are determined using the backstepping method. Dynamic equations of motion of a four-wheeled mobile robot taking into account wheel slips are given in section 3. Section 4 concerns the description of the adopted structure of an adaptive fuzzy system for compensation of robot nonlinearities. In section 5 synthesis of tracking control of mobile robots is conducted and stability analysis of the control algorithm is carried out based on Lyapunov's theory. In section 6 obtained results of simulations of the introduced solution are presented. Conclusions are given in section 7.

2 Kinematic Controller for WMR

The object analyzed in the present article is a four-wheeled mobile robot. A diagram of its kinematic structure is shown in Fig. 1, [24], [25].

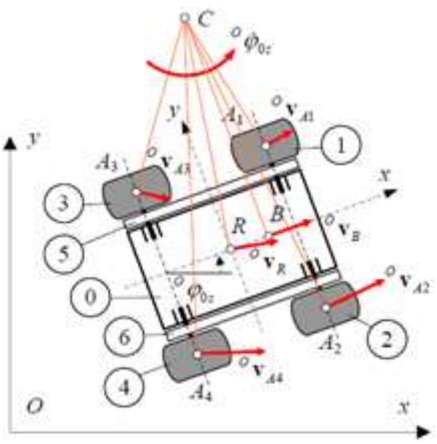

Fig. 1: Model of the analyzed robot

In the model, the following basic robot assemblies can be distinguished: 0 – mobile platform (body with additional control and measurement frame attached to it), 1-4 – wheels, 5-6 – toothed belts (caterpillars). In the analyzed robot, the front wheels are coupled with the back wheels by means of the toothed belts. The following symbols are adopted for *i*-th wheel: A_i – geometric centre, r_i – radius, θ_i – wheel spin angle. Mobile platform spin angle is denoted ${}^{o}\dot{\varphi}_{oz}$. It is assumed that the motion of the mobile robot occurs in the *Oxy* plane (as shown in Fig. 1). Position and orientation of the mobile platform are described by generalized coordinates vector:

$$ {}^{o}\mathbf{q} = \left[{}^{o}x_R,\ {}^{o}y_R,\ {}^{o}\varphi_{oz} \right]^T \tag{1}$$

where: ${}^{o}x_R$, ${}^{o}y_R$ – coordinates of the point *R* of the mobile platform, $\varphi_z = {}^{o}\varphi_{oz}$ – the spin angle of the mobile platform with respect to *z*-axis of stationary coordinate system $\{O\}$. Generalized velocities vector $\dot{\mathbf{q}}$ can be determined based on the value of the velocity of motion of the point *R* of the robot along the direction of the *x*-axis of the $\{R\}$ system connected with the robot, that is v_R, and angular velocity of spin of the mobile platform, that is $\dot{\varphi}$, based on the kinematic equations of motion in the form:

$$ \dot{\mathbf{q}} = \begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\varphi}_z \end{bmatrix} = \begin{bmatrix} \cos(\varphi_z) & 0 \\ \sin(\varphi_z) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_R \\ \dot{\varphi} \end{bmatrix}. \tag{2}$$

The above equation is valid if the robot moves on horizontal ground. In the control of the position and heading of the robot, one assumes that the motion of the robot is realized based on the desired vector of its position and heading, which has the form:

$$ \mathbf{q}_d = \left[x_{Rd},\ x_{Rd},\ \varphi_d \right]^T, \tag{3}$$

where: x_{Rd}, y_{Rd} – desired coordinates of the characteristic point *R* of the robot in the $\{O\}$ coordinate system in (m), $\varphi_d = {}^{o}\varphi_{ozd}$ – the desired spin angle of the mobile platform with respect to *z*-axis of $\{O\}$ coordinate system in (rad). To define the problem of tracking control, based on the relationship (2) let us define desired parameters of motion of the point *R* in the form of the equation:

$$ \dot{\mathbf{q}}_d = \begin{bmatrix} \dot{x}_{Rd} \\ \dot{y}_{Rd} \\ \dot{\varphi}_d \end{bmatrix} = \begin{bmatrix} \cos(\varphi_d) & 0 \\ \sin(\varphi_d) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_{Rd} \\ \omega_d \end{bmatrix} \tag{4}$$

where: v_{Rd}, ω_d – respectively desired linear velocity of the characteristic point *R* of the robot in (m/s) and desired angular velocity of its mobile platform in (rad/s), in the stationary coordinate system $\{O\}$. In the problem of tracking control, one should determine the vector of control of position and heading of the robot $\mathbf{u}_s = [v_s, \omega_s]^T$, such that $\mathbf{q} \to \mathbf{q}_d$ for $t \to \infty$. The errors of the robot's position and heading in the coordinate system associated with the robot $\{R\}$ and in the stationary system $\{O\}$ can be determined from the relationship:

$$\mathbf{q}_e = \begin{bmatrix} e_F \\ e_L \\ e_O \end{bmatrix} = \begin{bmatrix} \cos(\varphi_z) & \sin(\varphi_z) & 0 \\ -\sin(\varphi_z) & \cos({}^R\varphi_z) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{Rd} - x_R \\ y_{Rd} - y_R \\ \varphi_{zd} - \varphi_z \end{bmatrix}$$

$$\mathbf{e} = \begin{bmatrix} e_x \\ e_y \\ e_\varphi \end{bmatrix} = \begin{bmatrix} x_{Rd} - x_R \\ y_{Rd} - y_R \\ \varphi_{zd} - \varphi_z \end{bmatrix} \tag{5}$$

where e_F, e_L, e_O are respectively longitudinal position error in (m), lateral position error in (m), and heading error in [rad]. Generalized velocities required for the desired motion of the robot can be determined using various methods. A popular method used for this purpose is the so-called backstepping method, [1], [2], [7], [15]. According to it, the vector of desired generalized velocities of motion of the robot's mobile platform expressed in the robot's coordinate system $\{R\}$ can be determined based on the following relationship:

$$\mathbf{u}_d = \begin{bmatrix} v_s \\ \omega_s \end{bmatrix} = \begin{bmatrix} k_F e_F + v_{Rd} \cos(e_o) \\ \omega_d + k_L e_L v_{Rd} + k_o v_{Rd} \sin(e_o) \end{bmatrix} \tag{6}$$

where: v_s, ω_s – desired velocities of robot motion expressed in the coordinate system $\{R\}$, that is, the linear velocity of characteristic point R in (m/s) and angular velocity of the mobile platform in (rad/s), k_F (s^{-1}), k_L (rad/m^2), k_o (rad/m) – chosen positive parameters.

3 Dynamic Model of a WMR Subject to Wheel Slip

In Fig. 2 a schematic diagram of the analyzed robot with marked reaction forces acting on the robot in the wheel-ground plane of contact is presented, [24].

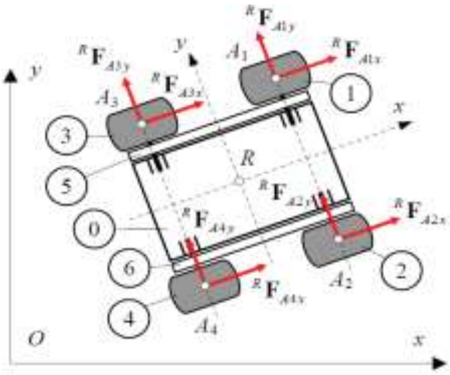

Fig. 2: Diagram of reaction forces acting on the robot in the wheel-ground contact plane

In the description of the motion of the four-wheeled robot, it is assumed that the tire-ground coefficient of adhesion changes according to the Kiencke model and values of longitudinal slip ratios λ_3 and λ_4 depend respectively on angular velocities of driven wheels $\dot{\theta}_3$ and $\dot{\theta}_4$. Additionally, equality of driving torques for passive and active wheels is assumed, that is, $\tau_1 = \tau_3$ and $\tau_2 = \tau_4$. After taking into account the above assumptions, dynamic equations of motion for the hybrid chassis system, i.e. with wheels and toothed belts, are written as [24]:

$$\begin{bmatrix} 2a_1 & 0 \\ 0 & 2a_1 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_3 \\ \ddot{\theta}_4 \end{bmatrix} + \begin{bmatrix} 2a_2 \operatorname{sgn}(\dot{\theta}_3) + 4\lambda_p \lambda_3 / (\lambda_p^2 + \lambda_3^2)(a_3 + a_4 a_{Rx} - a_5 a_{Ry}) + \\ + 2(a_6 + a_7 a_{Rx} - a_8 a_{Ry}) \operatorname{sgn}(\dot{\theta}_3) \\ 2a_2 \operatorname{sgn}(\dot{\theta}_4) + 4\lambda_p \lambda_4 / (\lambda_p^2 + \lambda_4^2)(a_3 + a_4 a_{Rx} + a_5 a_{Ry}) + \\ + 2(a_6 + a_7 a_{Rx} + a_8 a_{Ry}) \operatorname{sgn}(\dot{\theta}_4) \end{bmatrix} = \begin{bmatrix} \tau_3 \\ \tau_4 \end{bmatrix}, \tag{7}$$

where $\lambda_p, a_{Rx}, a_{Ry}$ are respectively: a constant associated with a model of wheel-ground adhesion, projections of acceleration of characteristic point R of the robot in the coordinate system associated with the robot $\{R\}$. In turn, constants a_i that occur in equation (7) result from geometry, masses, and distribution of masses of the analyzed robot and were determined in the work, [24]. From the kinematic relationships of the analyzed model of the mobile robot, one can determine angular velocities of driven wheels as functions of control signals that realize the desired trajectory of the robot's motion, according to the following relationship:

$$\begin{bmatrix}\dot{\theta}_3\\ \dot{\theta}_4\end{bmatrix}=\frac{1}{r}\begin{bmatrix}1 & -W/2\\ 1 & W/2\end{bmatrix}\mathbf{u}_R, \qquad (8)$$

where the control signals' vector has the form:

$$\mathbf{u}_R=\begin{bmatrix}v_R & \omega\end{bmatrix}^T. \qquad (9)$$

After introducing equation (8) into dynamic equations of motion of a mobile robot (7), one obtains:

$$\mathbf{M}\dot{\mathbf{u}}_R+\mathbf{F}_R(\mathbf{u}_R)+\boldsymbol{\tau}_z=\boldsymbol{\tau},$$
$$\mathbf{M}=\frac{1}{r}\begin{bmatrix}2a_1 & -a_1W\\ 2a_1 & a_1W\end{bmatrix}, \qquad (10)$$

where: $\mathbf{M}$ is a constant positive-definite inertia matrix, $\mathbf{F}_R(\mathbf{u}_R)\in\mathbf{R}^{2x1}$ is a vector describing robot nonlinearities, $\boldsymbol{\tau}_z\in\mathbf{R}^{2x1}$ denotes bounded unknown disturbances which include, for example, motion phenomena not taken into account in the description, $\boldsymbol{\tau}\in\mathbf{R}^{2x1}$ is control signals' vector identical with torques of robot driving wheels 3 and 4. The dimensionality of equation (10) results from the assumption of active torques of wheels 3 and 4 and passive torques of wheels 1 and 2 being respectively equal.

4 Fuzzy Systems, Fuzzy Basis Function Expansion and Function Approximation

Problems of control of wheeled mobile robots with the inclusion of wheels' slips are complex and their solution requires the application of complex methods. Because of the lack of a systematic approach to analysis and synthesis of control of nonlinear systems so far, the adaptive fuzzy systems became an attractive tool used in the theory of nonlinear systems. Fig. 3 shows an adaptive fuzzy system. An adaptive fuzzy system is defined as a fuzzy system equipped with a learning algorithm, where the fuzzy system is constructed from a set of fuzzy IF-THEN rules using fuzzy logic principles and the learning algorithm adjusts the parameters of the fuzzy system based on the training information, [14], [16], [17], [20], [26], [28]. Adaptive fuzzy systems can be viewed as fuzzy logic systems whose rules are automatically generated through a training process. In this section, we will give the mathematical formulas of fuzzy systems and fuzzy basis functions.

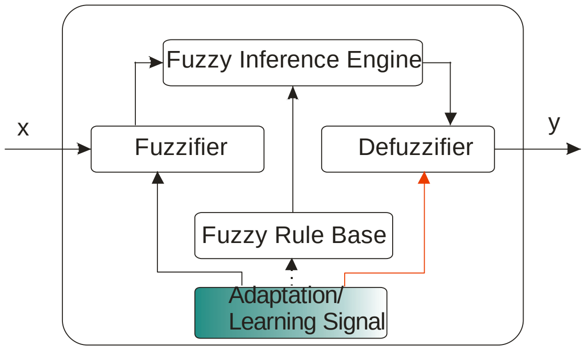

Fig. 3: Adaptive fuzzy system

Without loss of generality, we assume that fuzzy systems are MISO systems $f:U\in\Re^n\rightarrow V\in\Re$, where $U=U_1\times U_2\times\ldots\times U_n\in\Re^n$ is the input space and $V\subset R$ is the output space. Consider a fuzzy logic system (FLS) with rules in the following form

$$R^j: \text{IF } x_1 \text{is } A_1^j \text{ and} \ldots \text{and } x_n \text{ is } A_n^j \text{ THEN } y \text{ is } \gamma_j,$$
$$j=1,2,\ldots,N \qquad (11)$$

Where A_i^j are fuzzy sets defined by their respective membership functions $\mu_{A_i^j}(x_i)$, $i=1,2\ldots,n$ and $\gamma_j\in\Re$ are singleton rule consequents. When a product and operator and a product implication method are used together with the center of gravity defuzzification method, this leads to a fuzzy logic system with the following form

$$y=f(\mathbf{x})=\frac{\sum_{j=1}^{N}\left(\prod_{i=1}^{n}\mu_{A_i^j}(x_i)\right)\gamma_j}{\sum_{j=1}^{N}\prod_{i=1}^{n}\mu_{A_i^j}(x_i)}. \qquad (12)$$

Which coincides with the Takagi-Sugeno model, [22]. When all the parameters of the FLS in (12) are considered free, methods such as back-propagation learning can be applied. The idea introduced in [14], [20], [26], is to fix the premise parameters of the FLS such that the resulting fuzzy system is equivalent to a linear combination of nonlinear functions called fuzzy basis functions.

Definition 4.1, [26], defines fuzzy basis functions (FBF) as

$$p_j=\frac{\prod_{i=1}^{n}\mu_{A_i^j}(x_i)}{\sum_{j=1}^{N}\prod_{i=1}^{n}\mu_{A_i^j}(x_i)}, j=1,2,\ldots,N. \qquad (13)$$

Now the fuzzy system (12) is equivalent to a linear combination of an FBFs

$$f(\mathbf{x}) = \sum_{j=1}^{N} p_j(\mathbf{x})\gamma_j. \qquad (14)$$

In order to develop learning algorithms for these fuzzy systems, we need to specify the functional form of the fuzzy membership function for a fuzzy set A_i^j. The membership function can be any continuous bounded function, e.g., the Gaussian membership function

$$A(x; c, \delta) = \exp\left[\frac{-(x-c)^2}{\delta_w^2}\right], \qquad (15)$$

and Fig. 4 shows an example of FBFs in one-dimensional premise space.

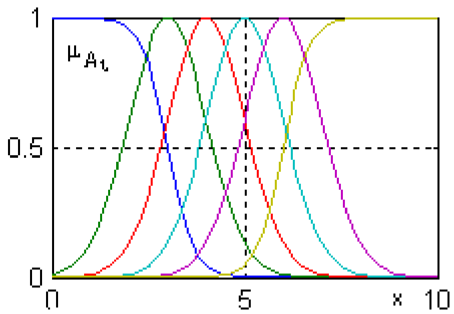

Fig. 4:. Membership functions

It has been shown, [10], [21], [27], that FLS possess the universal approximate property. That is, for any given continuous function $g(\mathbf{x})$ on a compact set U and any given real number $\varepsilon > 0$, there exists a fuzzy system $f(\mathbf{x})$ in the form (14) such that

$$\sup_{\mathbf{x}\in\mathbf{U}} |g(\mathbf{x}) - f(\mathbf{x})| < \varepsilon. \qquad (16)$$

Therefore, the fuzzy system (14) is qualified to estimate the unknown non-linear function $g(\mathbf{x})$. In fact, there exist, ideal control representatives P, centroids $\mathbf{c}$ and widths δ_w, so that the non-linear function can be represented as

$$g(\mathbf{x}) = \mathbf{\Gamma}^T \mathbf{P}(\mathbf{x}) + \varepsilon, \qquad (17)$$

with the estimation error bounded by $\|\varepsilon\| < \varepsilon_m$. Then an estimate of $g(\mathbf{x})$ can be given by

$$\hat{g}(\mathbf{x}) = \hat{\mathbf{\Gamma}}^T \mathbf{P}(\mathbf{x}), \qquad (18)$$

where $\hat{\mathbf{\Gamma}}$ is an estimate of ideal values provided by a learning algorithm. It is shown in [26], [27], that Gaussian basis functions do have the best approximation property. This is the main reason we choose the Gaussian function as the membership function. In this work, an FBF can be generated based on a numerical input-output pair.

5 Adaptive fuzzy Control Algorithm and Stability

In the present section, the synthesis of control of position and heading of a wheeled mobile robot using the control structure of nonlinear systems will be conducted, which takes into account compensation for robot nonlinearities realized by means of the FBFs linear with respect to parameters described in section 4. The task of this control will be the reduction of the actual control vector (9) to the control vector resulting from the analysis of kinematics (6). To this end, let us define the velocity tracking error:

$$\mathbf{s} = \mathbf{u}_d - \mathbf{u}_R. \qquad (19)$$

After differentiating relationship (19) and inserting it into (7), one obtains dynamic equations of motion written as a function of the velocity error:

$$\mathbf{M}\dot{\mathbf{s}} = \mathbf{f}(\mathbf{x}) + \boldsymbol{\tau}_Z - \boldsymbol{\tau}, \qquad (20)$$

where $\boldsymbol{\tau}_Z$ represents bounded disturbances so that $\|\boldsymbol{\tau}_Z\| \le Z$ and nonlinear function has the form:

$$\mathbf{f}(\mathbf{x}) = \mathbf{M}\dot{\mathbf{u}}_d + \mathbf{F}(\mathbf{u}_R). \qquad (21)$$

Vector $\mathbf{x}$ allowing determination of the value of the nonlinear function can be defined as:

$$\mathbf{x} = \left[\dot{\mathbf{u}}_d^T, \mathbf{u}_R^T\right]^T, \qquad (22)$$

and it should be available for measurement. The function $\mathbf{f}(\mathbf{x})$ involves all parameters of the analyzed wheeled mobile robot such as masses, mass moments of inertia, coefficients of motion resistance, and description of the slip phenomenon. Quantities of this kind usually can be described only in an approximate way. Because the function $\mathbf{f}(\mathbf{x})$ is described approximately, if one adopts the law of control with the inclusion of this approximation in the form:

$$\boldsymbol{\tau} = \hat{\mathbf{f}}(\mathbf{x}) + \mathbf{k}_{\mathbf{p}}\mathbf{s} - \boldsymbol{\delta}, \qquad (23)$$

where $\hat{\mathbf{f}}(\mathbf{x})$ is an output of a fuzzy system, $\mathrm{k_p}$ is a positive-definite diagonal matrix, and $\boldsymbol{\delta}$ is a control signal robust to non-modeled phenomena and other disturbances, then the description of a closed system one may express as:

$$\mathbf{M}\dot{\mathbf{s}} = -\mathbf{k}_{\mathbf{p}}\mathbf{s} + \tilde{\mathbf{f}}(\mathbf{x}) + \boldsymbol{\tau}_Z + \boldsymbol{\delta}, \qquad (24)$$

where velocity tracking error **s** in a significant way will depend on the correct approximation of robot nonlinearities. Approximation of the control compensating for nonlinearities $\mathbf{f}(\mathbf{x})$ is often applied in practice. For the approximation, a fuzzy system may be used. It is convenient to use a fuzzy system linear with respect to the parameters, described in section 4. Then, the nonlinear function approximated by the fuzzy system one can write in the form:

$$\mathbf{f}(\mathbf{x}) = \boldsymbol{\Gamma}^T \mathrm{P}(\mathbf{x}) + \boldsymbol{\varepsilon}, \qquad (25)$$

where $\boldsymbol{\varepsilon}$ is approximation error satisfying condition $\|\boldsymbol{\varepsilon}\| \le \varepsilon_m$, $\varepsilon_m = \text{const} > 0$. The estimate of the $\mathbf{f}(\mathbf{x})$ function can be written as:

$$\hat{\mathbf{f}}(\mathbf{x}) = \hat{\boldsymbol{\Gamma}}^T \mathrm{P}(\mathbf{x}), \qquad (26)$$

where $\hat{\boldsymbol{\Gamma}}$ is the matrix of estimated parameters of an ideal fuzzy system. After using (26) in the control law with the robot's nonlinearities compensation, the control law in the following form is obtained:

$$\boldsymbol{\tau} = \hat{\boldsymbol{\Gamma}}^T \mathbf{P}(\mathbf{x}) + \mathbf{k}_p\mathbf{s} - \boldsymbol{\delta}. \qquad (27)$$

Substitution of (25) and (26) into (24) yields:

$$\mathbf{M}\dot{\mathbf{s}} + \mathbf{k}_{\mathbf{p}}\mathbf{s} = \tilde{\mathbf{f}}(\mathbf{x}) + \boldsymbol{\tau}_Z + \boldsymbol{\delta}, \qquad (28)$$

where $\tilde{\mathbf{f}}(\mathbf{x})$ is an error of approximation of $\mathbf{f}(\mathbf{x})$ function, equal to:

$$\tilde{\mathbf{f}}(\mathbf{x}) = \mathbf{f}(\mathbf{x}) - \hat{\mathbf{f}}(\mathbf{x}) = \boldsymbol{\Gamma}^T \mathrm{P}(\mathbf{x}) - \hat{\boldsymbol{\Gamma}}^T \mathrm{P}(\mathbf{x}) + \boldsymbol{\varepsilon} = \tilde{\boldsymbol{\Gamma}}^T \mathrm{P}(\mathbf{x}) + \boldsymbol{\varepsilon}, \qquad (29)$$

where $\tilde{\boldsymbol{\Gamma}} = \boldsymbol{\Gamma} - \hat{\boldsymbol{\Gamma}}$ is an error in the estimation of weights of the neural network. After using relationship (29), equation (28) is written as:

$$\mathbf{M}\dot{\mathbf{s}} + \mathbf{k}_{\mathbf{p}}\mathbf{s} = \tilde{\boldsymbol{\Gamma}}^T \mathbf{P}(\mathbf{x}) + \boldsymbol{\varepsilon} + \boldsymbol{\tau}_Z + \boldsymbol{\delta}. \qquad (30)$$

The structure of the system for adaptive fuzzy control of robot generalized velocities is shown in Fig. 5. For a derivation of an algorithm of $\hat{\Gamma}$ weights learning, the theory of Lyapunov stability is used.

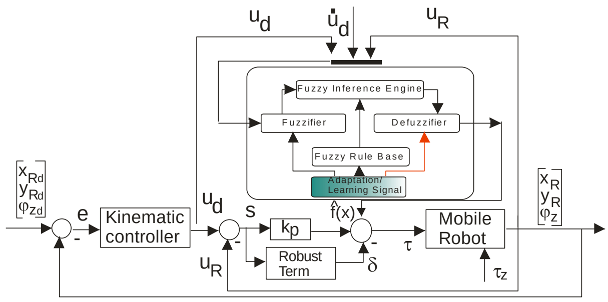

Fig. 5: Adaptive fuzzy feedback control scheme.

Let us take a scalar positive-definite function:

$$V = \frac{1}{2}\mathbf{s}^T\mathbf{M}\mathbf{s} + \frac{1}{2}\mathrm{tr}\left(\tilde{\boldsymbol{\Gamma}}^T\mathbf{F}^{-1}\tilde{\boldsymbol{\Gamma}}\right) + \mathrm{k_F}\left(\mathrm{e_F^2} + \mathrm{e_L^2}\right) + 2\mathrm{k_o v_{Rd}}\left(1 - \cos(\mathrm{e_o})\right), \qquad (31)$$

where $\mathbf{F} = \mathbf{F}^T > 0$ is a design matrix. A derivative of the *V* function with respect to time, one can write as:

$$\dot{V} = \mathbf{s}^T\mathbf{M}\dot{\mathbf{s}} + \mathrm{tr}\left(\tilde{\boldsymbol{\Gamma}}^T\mathbf{F}^{-1}\dot{\tilde{\boldsymbol{\Gamma}}}\right) + 2\mathrm{k_F}\left(\mathrm{e_F\dot{e}_F} + \mathrm{e_L\dot{e}_L}\right) + 2\mathrm{k_o v_{Rd}\dot{e}_o}\sin(\mathrm{e_o}). \qquad (32)$$

After inserting the expression $\mathbf{M}\dot{\mathbf{s}}$ from equation (30), one obtains:

$$\dot{V} = -\mathbf{s}^T\mathrm{k_p}\mathbf{s} + \mathrm{tr}\left(\tilde{\boldsymbol{\Gamma}}^T\mathbf{F}^{-1}\dot{\tilde{\boldsymbol{\Gamma}}} + \mathbf{P}(\mathbf{x})\mathbf{s}^T\right) + \mathbf{s}^T(\boldsymbol{\varepsilon} + \boldsymbol{\tau}_Z + \boldsymbol{\delta}) + +2\mathrm{k_F}\left(\mathrm{e_F\dot{e}_F} + \mathrm{e_L\dot{e}_L}\right) + 2\mathrm{k_o v_{Rd}\dot{e}_o}\sin(\mathrm{e_o}). \qquad (33)$$

After choosing the law of adaptation of weights as:

$$\dot{\tilde{\boldsymbol{\Gamma}}} = -\mathbf{F}\mathbf{P}(\mathbf{x})\mathbf{s}^T, \qquad (34)$$

and after introducing the robust control signal:

$$\boldsymbol{\delta} = -(\boldsymbol{\varepsilon}_{\mathbf{M}} + \mathbf{Z})\frac{\mathbf{s}}{\|\mathbf{s}\|}, \qquad (35)$$

Relationship (33) is transformed into the form:

$$\dot{V} = -\mathbf{s}^T k_p \mathbf{s} + \mathrm{tr}\left[\tilde{\Gamma}^T (\mathbf{F}^{-1}\dot{\tilde{\Gamma}} + P(x)s^T\right] + s^T(\boldsymbol{\varepsilon} + \boldsymbol{\tau}_Z) + 2k_F(e_F\dot{e}_F + e_L\dot{e}_L) + 2k_o v_{Rd}\dot{e}_o \sin(e_o) - s^T(\boldsymbol{\varepsilon}_m + Z)\frac{s}{\|s\|} \quad (36)$$

After writing in expanded form the error of desired velocities (15) as:

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} u_{d1} - u_{R1} \\ u_{d2} - u_{R2} \end{bmatrix}, \quad (37)$$

and after determining a derivative of error (15), and putting $k_F = k_L k_o \omega_d$, one gets:

$$\dot{V} \le -k_F^2 e_F^2 - k_o^2 v_{Rd}^2 \sin^2(e_o) - k_L^2 v_{Rd}^2 e_L^2 - (k_F e_F - s_1)^2 - (k_o v_{Rd} \sin(e_o) - s_2)^2 - k_{pmin}\|s\|^2 \quad (38)$$

Since V is positive definite for $\|\mathbf{s}\| \neq 0$ and $\dot{V}$ is negative semidefinite, both **s**, $\tilde{\Gamma}$ are bounded according to Lyapunov's theorem. Such a synthesis of the adaptive fuzzy control permits proper operation of the control system with a proportional controller until the fuzzy system starts adapting.

6 Simulation Results

This section shows some simulation results of the fuzzy logic system using a four-wheeled robot subject to wheel slip whose objective is to follow the given reference trajectory. An adaptive fuzzy system with adaptive learning rules (34) was used in the simulation. The three Gaussian membership functions were selected along each input dimension, therefore 9 fuzzy IF-THEN rules can be generated. The initial and final membership function shapes are shown in Fig. 6.

Remark: We omitted the signal $\dot{u}_d$ in learning the conclusion of the rules because nothing brings on in the process of learning as numerous simulations showed, but the dimensionality of the problem grows considerably.

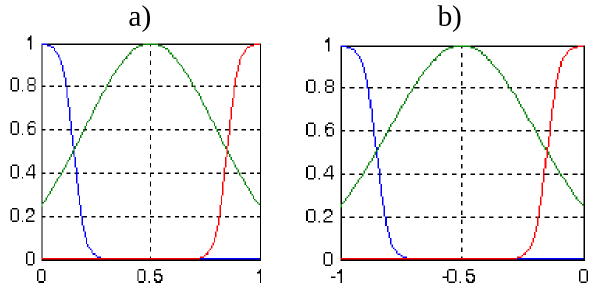

Fig. 6: Membership functions along a) v_R and b) ω dimension

For use in simulation investigations, one assumes the following robot parameters:

- geometric dimensions ($A_1A_3 = A_2A_4 = L$, $A_1A_2 = A_3A_4 = W$ – see Fig. 1), L = 0.35 m, W = 0.386 m, r_i = 0.0965 m, i = {1, …, 4},
- masses of particular bodies: m_0 = 15.02 kg, m_i = 0.66 kg, $m_5 = m_6$ = 0.17 kg,
- rolling resistance coefficient f_r = 0.03,

whereas the constants a_i occurring in equation (7) were determined using the methodology described in works, [3], [4]. The following values of gains for the controller were assumed: k_L = 15, k_F = 10, k_o = 5, k_p = diag(20, 20). Desired motion parameters of the robot's wheels, kinematic parameters of point R, and motion path of point R are shown in Fig. 7. In simulation three phases of motion are assumed: acceleration, motion with constant velocity of the point R ($v_R = 0.3$ m/s), and braking. For an approximation of nonlinearities and variable robot operating conditions, the fuzzy system described in section 4 is used with Gaussian functions describing fuzzy sets, assuming each element of the **f** vector is approximated with 6 rules. In the simulation, parametric disturbance occurring $t \ge 12$ s is assumed in the form of an increase in the rolling resistance coefficient $\nabla f_r = 0.03$, when the characteristic point R of the robot moves along a curvilinear path.

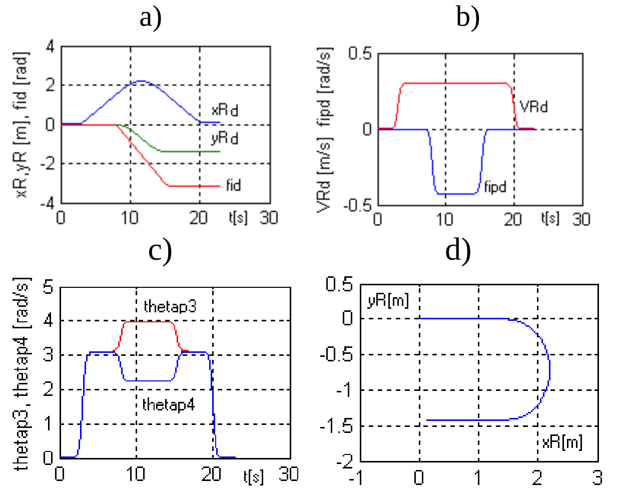

Fig. 7: Desired kinematic quantities used in simulation: a) kinematic parameters of the point *R*, b) desired velocities: linear of the point *R*, and angular of the robot's body, c) angular velocities of driven wheels, d) desired motion path of the point *R*.

Trajectory tracking performances for two cases were considered, firstly with action generation fuzzy compensation of the nonlinearity of the robot according to the scheme shown in Fig. 5 (case 1), and secondly without the compensation of the nonlinearity of the robot and robust term (case 2).

Case 1.

In Fig. 8 are shown obtained control signals, and in Fig. 9, errors of neural control of position and heading of the robot. The obtained control signals τ_3, τ_4 (i.e., desired torques for driven wheels) that realize desired trajectory of motion of the point *R* of the mobile robot are shown in Fig .8a. Values of torques are the largest during motion of the mobile robot along a circular trajectory, their value is constant until the occurrence of a parametric disturbance. This corresponds to robot motion with constant velocity. At the moment of occurrence of the parametric disturbance, the value of the τ_3 torque increases whereas the value of the $|\tau_4|$ torque decreases, which results from an increase in the adopted motion resistance. For time $t \geq 12$ s values of torques decrease, which corresponds to the phase of braking and finishing motion along the rectilinear path.

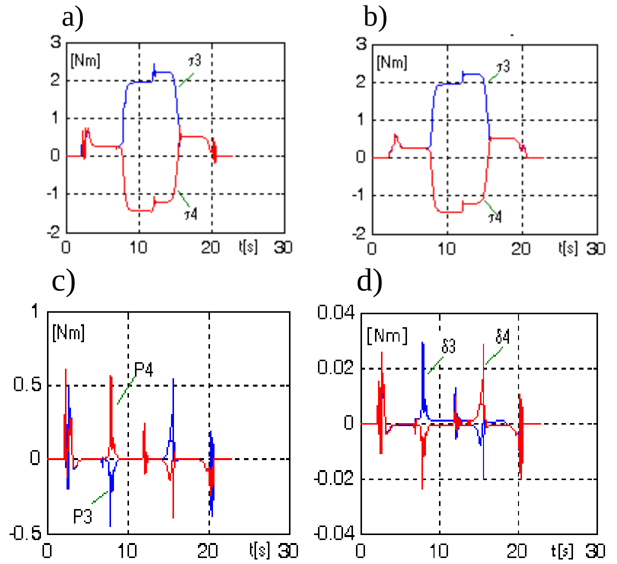

Fig. 8: Control signals according to a relationship (23)

The discussed total control signals are generated based on control signals compensating for robot nonlinearities shown in Fig. 8b, signals generated by a P-type regulator (Fig. 8c), and robust control signals (Fig. 8d). The fuzzy compensation control has the largest influence on the total control signal, as far as level and character are concerned. In turn, the stabilizing P control and robust δ control have the largest values during periods of occurrence of disturbances associated with wheels' slips or resulting from the character of desired velocity, desired motion path, or the occurring parametric disturbances. It follows from the fact that in those motion states, the fuzzy compensation adapts to changing operating conditions of the robot, and only after the adaptation period the fuzzy system generates dominant control signals. This fact of the significance of the influence of fuzzy compensating control on the overall quality of control is confirmed by results shown in Fig. 9a-c, in which errors of neural control of position and heading of the robot are presented. Error-values are the largest during the period of motion along a circular trajectory, and then as the process of fuzzy adaptation progresses, they decrease. The occurring parametric disturbance as well as changing robot operating conditions, excite the proposed control structure, which as a result generates control signals that make the control errors $e_{xR}, e_{yR}, e_{\varphi}$ bounded, which confirms the theoretical considerations. In Fig. 9d are shown the desired and actual paths realized with small errors, marked as 'trajd' and 'traj', respectively.

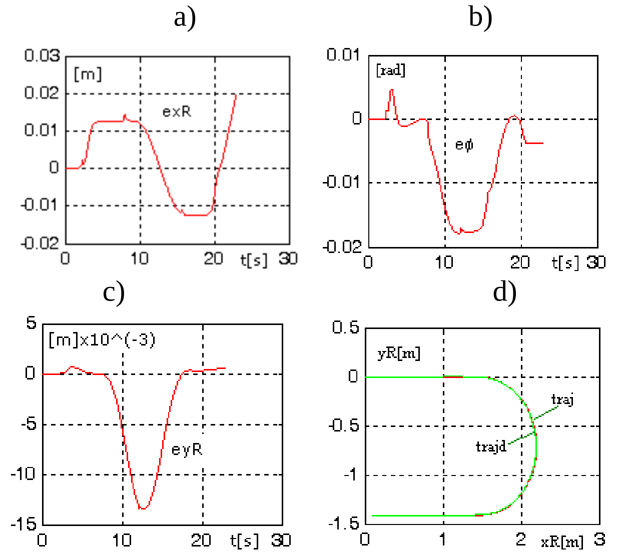

Fig. 9: Errors of fuzzy control of robot's position and heading

For quantitative evaluation of the generated control signals and realized tracking motion, the following quality indices are introduced:

- maximum values of the errors e_{xRmax}, e_{yRmax} in (m) and $e_{\varphi max}$ in (rad), $e_{(.)max} = \text{abs}(e_{(.)}(k))$, $k = 1,2,\ldots n$,

  the square root of the mean squared error (RMSE) of motion realization

$$e_{xR} = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left(x_{Rd}(k)\text{-}x_R(k)\right)^2} \text{ (m)},$$

$$e_{yR} = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left(y_{Rd}(k)\text{-}y_R(k)\right)^2} \text{ (m)},$$

$$e_{\varphi} = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left(\varphi_d(k)\text{-}\varphi(k)\right)^2} \text{ (rad)},$$

where k is the ordinal number of a discrete value and $n = 23\,000$ is the total number of discrete values. Values of all quality indices of realization of tracking motion are given in Table 1.

Table 1. Values of the introduced quality indices

e_{xR}	e_{yR}	e_{φ}	e_{xRmax}	e_{yRmax}	$e_{\varphi max}$
0.0101	0.005403	0.01001	0.02004	0.01345	0.01787

Case 2.

To gauge the effectiveness of the fuzzy compensation of the nonlinearity of the robot it is useful to compare the performance of the closed-loop system without the output of action generating compensation of the nonlinearity of the robot and without robust term. The output tracking performance, in this case, is shown in Fig. 10. The output tracks corresponds to the desired trajectory with the bigger values of the introduced quality indices,

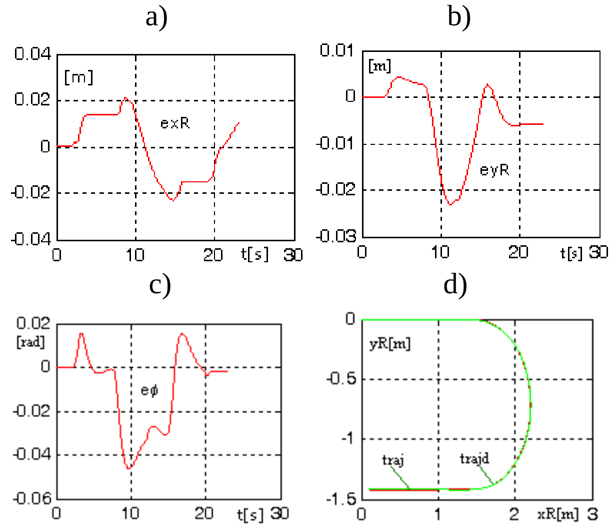

Fig. 10: Errors of fuzzy control of the robot's position and heading

which are given in Table 2, because of the non-linearity of the robot dynamics. There exist state errors resulting from the nonlinear appearance of the system.

Table 2. Values of the introduced quality indices

e_{xR}	e_{yR}	e_{φ}	e_{xRmax}	e_{yRmax}	$e_{\varphi max}$
0.01324	0.009465	0.01001	0.02278	0.02314	0.04633

7 Conclusions

In the article, a stable algorithm of control of position and heading in tracking the motion of a four-wheeled mobile robot is designed. In the algorithm, the fuzzy system linear with respect to estimated parameters is used. The algorithm does not require prior knowledge of the dynamic properties of the controlled object and is robust to occurring longitudinal and lateral slips of wheels as well as to parametric disturbances. After the fuzzy logic system has compensated partially for the non-linearity of the controlled system through adaptive learning, the output tracking of the plant follows the reference trajectory quite satisfactorily. The same controller works even if the behavior or structure of the system has changed. Results of conducted simulation investigations lead to the conclusion that intelligent control with a correctly designed kinematic controller significantly increases the accuracy of the realization of tracking motion. Additionally, the proposed fuzzy control algorithm operates online and does not require initial learning of fuzzy parameters.

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Conflict of Interest

The authors have no conflict of interest to declare.