Permanence and Global Attractivity of a Non-autonomous Single Species System with Michaelis-Menten-Type Feedback Control

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Abstract: During the past decade, many scholars have investigated the dynamic behaviors of the ecosystem with Michaelis-Menten-type harvesting; however, most of them assume that the harvesting effort does not change with time. Such an assumption has its drawbacks. Generally speaking, the rate of increase in harvesting effort changes with the density of the species. Inspired by this, we put forth a novel form of single-population feedback control model, in which the feedback control variable is of the Michaelis-Menten-type. Sufficient conditions that ensure the permanence and global attractivity of the system are obtained.

Key-Words: species, Michaelis-Menten type feedback control, permanence, global attractivity
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1 Introduction

Throughout this paper, for a continuous and bounded function, we let $f^l = \inf_{t\in R} f(t)$ and $f^u = \sup_{t\in R} f(t)$.

This paper aims to investigate the dynamic behavior of the following model:

$$\begin{aligned} \frac{dx}{dt} &= x\Big(a(t) - b(t)x\Big) \\ &\quad -\frac{q(t)x(t)u(t)}{k_1(t)u(t) + k_2(t)x(t)}, \\ \frac{du}{dt} &= -e(t)u + f(t)x. \end{aligned} \tag{1}$$

In system (1), we always assume:

(H_1) $a(t), b(t), q(t), k_1(t), k_2(t), e(t),$ and $f(t)$ are all continuous and strictly positive functions that satisfy

$$\min\{a^l, b^l, q^l, k_1^l, k_2^l, e^l, f^l\} > 0,$$
$$\max\{a^u, b^u, q^u, k_1^u, k_2^u, e^u, f^u\} < +\infty.$$

During the past two decades, ecosystems with feedback controls have become one of the main topics in the study of population dynamics. One could refer to [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30],[31] and the references cited therein for more information.

Gopalsamy and Weng[23] proposed the following single-species feedback control ecosystem:

$$\begin{aligned} \dot{n} &= rn\Big[1 - \frac{a_1 n(t) + a_2 n(t-\tau)}{K} \\ &\quad -cu(t)], \\ \dot{u} &= -au(t) + bn(t). \end{aligned} \tag{2}$$

They investigated the stability property of the positive equilibrium of the system. For the first time, they introduced the feedback control variable, which can be implemented by means of biological control or some harvesting procedure.

In [26], the authors studied the following single-species feedback control ecosystem:

$$\begin{aligned} \dot{N} &= r(t)N(t)\Big[1 - \frac{N^2(t-\tau_1(t))}{K^2(t)} \\ &\quad -c(t)u(t-\tau_2(t))\Big], \\ \dot{u} &= -a(t)u(t) + b(t)N(t). \end{aligned} \tag{3}$$

Under the assumption that the coefficients of the system are all continuous positive periodic functions, they showed that the system admits at least one positive periodic solution. For the general non-autonomous case, Chen et al. [24] showed that the system is always permanent.

In [31], the authors proposed the following single species model with feedback regulation and distributed time delay:

$$\begin{aligned} \dot{N} &= N\Big(a(t) - b(t)\textstyle\int_0^{+\infty} H(s)N(t-s)ds \\ &\quad -c(t)u(t)\Big), \\ \dot{u} &= -d(t)u(t) + e(t)\textstyle\int_0^{+\infty} H(s)N^2(t-s)ds. \end{aligned} \tag{4}$$

They obtained a set of sufficient conditions to ensure the existence of a positive periodic solution to the above system. By constructing some suitable Lyapunov functionals, Chen [32] obtained a set of sufficient conditions that ensure the permanence and global stability of the positive solution of the system.

In [22], the author argued that it is necessary to consider the stage structure of the species, and he proposed the following non-autonomous single species system with stage structure and feedback control:

$$\begin{aligned} \frac{dx_1(t)}{dt} &= b(t)x_2(t) - d_1(t)x_1(t) \\ &\quad -b(t-\tau)e^{-\int_{t-\tau}^{t} d_1(s)ds}x_2(t-\tau), \\ \frac{dx_2(t)}{dt} &= b(t-\tau)e^{-\int_{t-\tau}^{t} d_1(s)ds}x_2(t-\tau) \\ &\quad -a(t)x_2^2(t) - c(t)x_2(t)u(t), \\ \frac{du(t)}{dt} &= -f(t)u(t) + e(t)x_2(t). \end{aligned} \tag{5}$$

The author showed that system (5) admits at least one positive T-periodic solution if the coefficients are all continuous T-periodic functions. In [23], the authors showed that for the general non-autonomous case, the system (5) is permanent.

Recently, [29], took the stocking as a feedback control variable, and she proposed a single species stage-structured model with positive feedback control. The system takes the form:

$$\begin{aligned} \frac{dx_1}{dt} &= \alpha x_2 - \beta x_1 - \delta_1 x_1, \\ \frac{dx_2}{dt} &= \beta x_1 - \delta_2 x_2 - \gamma x_2^2 + d x_2 u, \\ \frac{du}{dt} &= g - eu - f x_2. \end{aligned} \tag{6}$$

The author showed that if the positive feedback control is large enough, the species will finally live in the long run.

In [27], the authors proposed the following single-species discrete model with feedback control:

$$\begin{aligned} N(n+1) &= N(n)\exp\Big\{r(n)\Big(1-\tfrac{N(n-m)}{K} \\ &\quad -c(n)\mu(n)\Big\}, \\ \Delta\mu(n) &= -a(n)\mu(n) + b(n)N(n-m). \end{aligned} \tag{7}$$

They investigated the positive periodic solution of the system. In [28], the author gave sufficient conditions to ensure the permanence of the system (7).

Recently, in [2], the authors argued that in some cases, species may have the Allee effect, since endangered species may have difficulties finding mates when population density is low; hence, they proposed the following single-species feedback control system:

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= x(1-x)(x-m) - axu, \\ \frac{\mathrm{d}u}{\mathrm{d}t} &= -bu + cx, \end{aligned} \tag{8}$$

where a,b,c are all positive constants and $0 < m < 1$, m represents the Allee constant. They showed that the system may have saddle-node bifurcation and Bogdanov-Takens bifurcation of codimension 2.

In [1], the authors proposed the following Logistic model with additive Allee effect and feedback control:

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= x(1-x-\frac{m}{x+a}) - bxu, \\ \frac{\mathrm{d}u}{\mathrm{d}t} &= -u + cx. \end{aligned} \tag{9}$$

The authors showed that the system may have saddle-node bifurcation and transcritical bifurcation. The dynamical behaviors of the system are richer and more complex than those in the traditional logistic model with feedback control. Both the Allee effect and feedback control can increase the species' extinction property.

It brings to our attention that in systems (2)-(9), the feedback control variable represents the harvesting effect of human beings or biological control, such an idea comes from the pioneering work of [23]. The idea of [23], comes from linear harvesting. However, such a kind of harvesting is not a sound one. As we know, nonlinear harvesting is more realistic from biological and economic points of view. In [33], the author first proposed a harvesting term $h = \dfrac{qEx}{cE + lx}$, which is named Michaelis-Menten type functional form of catch rate. Recently, many scholars have investigated the dynamic behaviors of the ecosystem with Michaelis–Menten-type harvesting. For example, in [34], the authors proposed a two-species amensalism model with Michaelis-Menten-type harvesting and a cover for the first species:

$$\begin{aligned} \frac{dx}{dt} &= a_1x(t) - b_1x^2(t) - c_1(1-k)x(t)y(t) \\ &\quad -\frac{qE(1-k)x(t)}{m_1E + m_2(1-k)x(t)}, \\ \frac{dy}{dt} &= a_2y(t) - b_2y^2(t). \end{aligned} \tag{10}$$

Chen[35] studied the following Lotka-Volterra commensal symbiosis model of two populations with Michaelis-Menten-type harvesting for the first species:

$$\begin{aligned} \frac{dx}{dt} &= r_1x\Big(1-\frac{x}{K_1}+\alpha\frac{y}{K_1}\Big) \\ &\quad -\frac{qEx}{m_1E+m_2x}, \\ \frac{dy}{dt} &= r_2y\Big(1-\frac{y}{K_2}\Big). \end{aligned} \tag{11}$$

They showed that there are two saddle-node bifurcations and two transcritical bifurcations under suitable conditions. One could refer to [34], [35], [36], [37], [38], [39], for more works in this direction.

It brings to our attention that in systems (10)-(11), the authors all assume that the harvesting effect E is a constant, such an assumption has its drawbacks. Generally speaking, the rate of increased harvesting effort changes with the density of the species. Now, if we take the harvesting effort as the feedback control variable, the Logistic equation with nonlinear feedback control could be proposed:

$$\begin{aligned} \frac{dx}{dt} &= x(a-bx) - \frac{qux}{k_1u+k_2x}, \\ \frac{du}{dt} &= -eu + fx. \end{aligned} \tag{12}$$

It is well known that a non-autonomous system is more suitable since circumstances change with time. This stimulated us to propose the system (1).

This paper's goal is to investigate the dynamic behaviors of the system (1). In the next section, we will investigate the persistent property of the system, and then, in Section 3, by constructing some suitable Lyapunov functions, we will investigate the global attractivity of the system. Some numeric simulations are carried out to show the feasibility of the main results in Section 4. We end this paper with a brief discussion.

2 Permanence

To investigate the persistent and extinct properties of the system, we need the following lemmas, which are Lemma 2.2 and 2.3 of [30], respectively.

Lemma 2.1. If $a>0, b>0$ and $\dot{x} \geq b-ax$, when $t \geq 0$ and $x(0)>0$, we have

$$\liminf_{t\to+\infty} x(t) \geq \frac{b}{a}.$$

If $a>0, b>0$ and $\dot{x} \leq b-ax$, when $t \geq 0$ and $x(0)>0$, we have

$$\limsup_{t\to+\infty} x(t) \leq \frac{b}{a}.$$

Lemma 2.2. *If $a>0, b>0$ and $\dot{x} \geq x(b-ax)$, when $t \geq 0$ and $x(0)>0$, we have*

$$\liminf_{t\to+\infty} x(t) \geq \frac{b}{a}.$$

If $a>0, b>0$ and $\dot{x} \leq x(b-ax)$, when $t \geq 0$ and $x(0)>0$, we have

$$\limsup_{t\to+\infty} x(t) \leq \frac{b}{a}.$$

Concerned with the persistent property of the system (4), we have the following result.

Theorem 2.1. *Assumes that*

$$a^l > \frac{q^u}{m_1^l} \tag{13}$$

holds, then system (1) is permanent.

Proof. From the first equation of the system (1), one has

$$\begin{aligned} \frac{dx}{dt} &= x\Big(a(t)-b(t)x\Big) \\ &\quad -\frac{q(t)x(t)u(t)}{k_1(t)u(t)+k_2(t)x(t)} \\ &\leq x\Big(a^u - b^l x\Big). \end{aligned} \tag{14}$$

Applying Lemma 2.2 to inequality (14) leads to

$$\limsup_{t\to+\infty} x(t) \leq \frac{a^u}{b^l} \overset{def}{=} M_1. \tag{15}$$

For $\varepsilon>0$ enough small, from (15) there exits a $T_1>0$ such that

$$x(t) < \frac{a^u}{b^l} + \varepsilon \ \ for\ \ all \ \ t \geq T_1. \tag{16}$$

From the second equation of (1) and (16), for $t>T_1$, one has

$$\begin{aligned} \frac{du}{dt} &= -e(t)u + f(t)x. \\ &\leq -e^l u + f^u\Big(\frac{a^u}{b^l}+\varepsilon\Big). \end{aligned} \tag{17}$$

Applying Lemma 2.1 to inequality (17) leads to

$$\limsup_{t\to+\infty} u(t) \leq \frac{f^u\Big(\frac{a^u}{b^l}+\varepsilon\Big)}{e^l}. \tag{18}$$

Since ε is an arbitrary small positive constant, setting $\varepsilon \to 0$ in (18) leads to

$$\limsup_{t\to+\infty} u(t) \leq \frac{f^u a^u}{b^l e^l} \overset{def}{=} M_2. \tag{19}$$

From the first equation of the system (1), we also have

$$\begin{aligned} \frac{dx}{dt} &= x\Big(a(t)-b(t)x\Big) \\ &\quad -\frac{q(t)x(t)u(t)}{k_1(t)u(t)+k_2(t)x(t)} \\ &\geq x\Big(a(t)-b(t)x\Big) - \frac{q(t)x(t)u(t)}{k_1(t)u(t)} \\ &\geq x\Big(a^l - \tfrac{q^u}{k_1^l} - b^u x\Big). \end{aligned} \tag{20}$$

Applying Lemma 2.2 to inequality (20) leads to

$$\liminf_{t\to+\infty} x(t) \geq \frac{a^l - \frac{q^u}{k_1^l}}{b^u} \stackrel{def}{=} m_1. \tag{21}$$

For $\varepsilon_1 > 0$ enough small, without loss of generality, we may assume that $\varepsilon_1 < \frac{1}{2}m_1$, from (21) there exits a $T_3 > T_2$ such that

$$x(t) > m_1 - \varepsilon_1 \ \ for \ \ all \ \ \ t \geq T_3. \tag{22}$$

From the second equation of (1) and (22), for $t > T_3$, one has

$$\begin{aligned} \frac{du}{dt} &= -e(t)u + f(t)x. \\ &\geq -e^u u + f^l\Big(m_1 - \varepsilon_1\Big). \end{aligned} \tag{23}$$

Applying Lemma 2.1 to inequality (23) leads to

$$\liminf_{t\to+\infty} u(t) \geq \frac{f^l\Big(m_1 - \varepsilon_1\Big)}{e^u}. \tag{24}$$

Since ε_1 is an arbitrary small positive constant, setting $\varepsilon_1 \to 0$ in (24) leads to

$$\liminf_{t\to+\infty} u(t) \geq \frac{f^l m_1}{e^u} \stackrel{def}{=} m_2. \tag{25}$$

(15), (19), (21) and (25) show that under the assumption (13) holds, the system is permanent.

This ends the proof of Theorem 2.1.

3 Global attractivity

The following lemma is from [40], and will be employed in establishing the global asymptotic stability of (1).

Lemma 3.1. *Let h be a real number and f be a nonnegative function defined on $[h; +\infty)$ such that f is integrable on $[h; +\infty)$ and is uniformly continuous on $[h; +\infty)$, then $\lim\limits_{t\to+\infty} f(t) = 0$.*

Theorem 3.1 *Let $(x^*(t), u^*(t))$ be a bounded positive solution of system (1). If*

$$b^l > f^u + \frac{q^u k_2^u M_2}{(k_1^l m_2 + k_2^l m_1)^2} \tag{26}$$

and

$$e^l > \frac{q^u k_2^u M_1}{(k_1^l m_2 + k_2^l m_1)^2} \tag{27}$$

hold, where m_1, m_2, M_1, M_2 are defined by (15), (19), (21) and (25) respectively. Then $(x^*(t), u^*(t))$ is globally asymptotically stable.

Proof. Conditions (26) and (27) imply that for a sufficiently small positive constant $\varepsilon > 0$ (without loss of generality, we may assume that $\varepsilon < \frac{1}{2}\min\{m_2, m_1\}$), the following inequality holds:

$$b^l > \frac{q^u k_2^u (M_2 + \varepsilon)}{\Big(k_1^l(m_2 - \varepsilon) + k_2^l(m_1 - \varepsilon)\Big)^2} \tag{28}$$

and

$$e^l > \frac{q^u k_2^u (M_1 + \varepsilon)}{\Big(k_1^l(m_2 - \varepsilon) + k_2^l(m_1 - \varepsilon)\Big)^2}. \tag{29}$$

Let $(x(t), u(t))^T$ be any solution of (1) with a positive initial value. It then follows from condition (26) and Theorem 2.1 that for above $\varepsilon > 0$, there exists a $T > 0$ such that for all $t \geq T$,

$$\begin{aligned} m_1 - \varepsilon < x(t), x^*(t) < M_1 + \varepsilon, \\ m_2 - \varepsilon < x(t), x^*(t) < M_2 + \varepsilon. \end{aligned} \tag{30}$$

Consider a Lyapunov function defined by

$$\begin{aligned} V(t) &= |\ln\{x(t)\} - \ln\{x^*(t)\}| \\ &\quad + |u(t) - u^*(t)|, t \geq 0. \end{aligned} \tag{31}$$

Now we are calculating and estimating the upper right derivative of $V(t)$ along the solutions of system (9), for $t > T$, it follows that:

$$\begin{aligned} & D^+V(t) \\ = \ & sgn(x(t) - x^*(t))\Big[a(t) - b(t)x(t) \\ & -\frac{q(t)u(t)}{k_1(t)u(t) + k_2(t)x(t)} \\ & -a(t) + b(t)x^*(t) \\ & +\frac{q(t)u^*(t)}{k_1(t)u^*(t) + k_2(t)x^*(t)}\Big] \\ & +sgn(u(t) - u^*(t))\Big[e(t)u^*(t) - f(t)x^*(t) \\ & -e(t)u(t) + f(t)x(t)\Big]. \end{aligned} \tag{32}$$

Noting that for $t > T$, by applying (30), one has

$$\frac{q(t)u^*(t)}{k_1(t)u^*(t) + k_2(t)x^*(t)} - \frac{q(t)u(t)}{k_1(t)u(t) + k_2(t)x(t)}$$

$$\begin{aligned} &= \frac{q(t)u^*(t)\Delta_2 - q(t)u(t)\Delta_1}{\Delta_1\Delta_2} \\ &= \frac{q(t)k_2(t)(u^*(t)x(t) - u(t)x^*(t))}{\Delta_1\Delta_2} \\ &= \frac{q(t)k_2(t)(u^*(t)x(t) - u(t)x(t))}{\Delta_1\Delta_2} \\ &\quad + \frac{q(t)k_2(t)(u(t)x(t) - u(t)x^*(t))}{\Delta_1\Delta_2} \\ &\leq \frac{q^u k_2^u (M_1+\varepsilon)|u^*(t) - u(t)|}{\Delta_3^2} \\ &\quad + \frac{q^u k_2^u (M_2+\varepsilon)|x(t) - x^*(t)|}{\Delta_3^2}, \end{aligned} \tag{33}$$

where

$$\begin{aligned} \Delta_1 &= k_1(t)u^*(t) + k_2(t)x^*(t), \\ \Delta_2 &= k_1(t)u(t) + k_2(t)x(t), \\ \Delta_3 &= \Big(k_1^l(m_2-\varepsilon) + k_2^l(m_1-\varepsilon)\Big)^2. \end{aligned} \tag{34}$$

Hence,

$$\begin{aligned} &D^+V(t) \\ \leq\ & -\Big(b^l - f^u - \frac{q^u k_2^u(M_2+\varepsilon)}{\Delta_3^2}\Big)|x(t) - x^*(t)| \\ & -\Big(e^l - \frac{q^u k_2^u(M_1+\varepsilon)}{\Delta_3^2}\Big)|u^*(t) - u(t)| \\ \leq\ & -\mu(|x(t) - x^*(t)| + |u(t) - u^*(t)|), \end{aligned} \tag{35}$$

where

$$\begin{aligned} \mu \;=\; \min\Big\{& b^l - f^u - \frac{q^u k_2^u(M_2+\varepsilon)}{\Delta_3}, \\ & e^l - \frac{q^u k_2^u(M_1+\varepsilon)}{\Delta_3}\Big\}. \end{aligned} \tag{36}$$

Hence, for $t \geq T$, one has

$$D^+V(t) \leq -\mu\Big(|x(t) - x^*(t)| + |u(t) - u^*(t)|\Big). \tag{37}$$

Integrating on both sides of (37) from T to t produces

$$\begin{aligned} V(t) \quad &+\mu\int_T^t \Big(|x(s) - x^*(s)| + |u(s) - u^*(s)|\Big)ds \\ &\leq V(T) < +\infty, \ \ t \geq T. \end{aligned}$$

Then

$$\begin{aligned} &\int_T^t \Big(|x(s) - x^*(s)| + |u(s) - u^*(s)|\Big)ds \\ \leq\ & \mu^{-1}V(T) < +\infty, \ \ t \geq T, \end{aligned}$$

and hence,

$$|x(t) - x^*(t)| + |u(t) - u^*(t)| \in L^1([T, +\infty)).$$

The boundedness of $x^*(t)$ and $u^*(t)$ and the ultimate boundedness of $x(t)$ and $u(t)$ imply that $x(t), x^*(t)$, $u(t)$, and $u^*(t)$ all have bounded derivatives for $t \geq T$(from the equations satisfied by them). Then it follows that $|x(t) - x^*(t)| + |u(t) - u^*(t)|$ is uniformly continuous on $[T, +\infty)$. By Lemma 3.1, we have

$$\lim_{t\to+\infty}\Big(|x(t) - x^*(t)| + |u(t) - u^*(t)|\Big) = 0.$$

The proof is completed.

4 Numeric simulations

Now let's consider the following example:

Example 4.1

$$\begin{aligned} \frac{dx}{dt} &= x\Big(5 - x\Big) \\ &\quad - \frac{(1+\frac{1}{2}\sin(t))x(t)u(t)}{(2+\sin(t))u(t) + x(t)}, \\ \frac{du}{dt} &= -e(t)u + f(t)x, \end{aligned} \tag{38}$$

where corresponding to system (1), we take $a(t) = 5, b(t) = 1, q(t) = 1 + \frac{1}{2}\sin(t), k_1(t) = 2 + \sin(t), k_2(t) = 1, e(t) = 6+\cos(t), f(t) = 2-\sin(t)$. Then, by simple computation, one has

$$M_1 = 1, M_2 = \tfrac{3}{5}, m_1 = \tfrac{7}{2}, m_2 = \tfrac{1}{2}. \tag{39}$$

Thus

$$b^l = 5 > 3 + \frac{9}{160} = f^u + \frac{q^u k_2^u M_2}{(k_1^l m_2 + k_2^l m_1)^2} \tag{40}$$

and

$$e^l = 5 > \frac{3}{32} = \frac{q^u k_2^u M_1}{(k_1^l m_2 + k_2^l m_1)^2} \tag{41}$$

hold. Hence, it follows from Theorems 2.1 and 3.1 that the system is permanent and the positive solutions of the system are globally attractive. Fig. 1 shows the globally asymptotically stable of the species x and Fig. 2 shows the the globally asymptotically stable of the feedback control variable u.

5 Discussion

Over the last two decades, two topics in population dynamics have received a lot of attention: the feedback control ecosystem and the system with

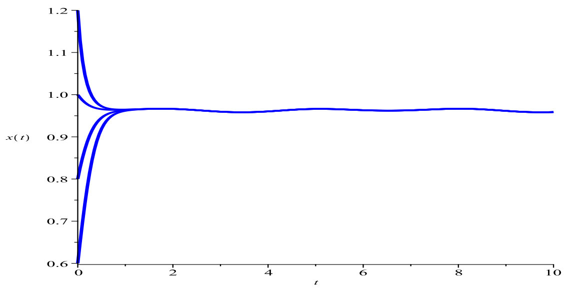

Figure 1: Dynamic behaviors of the first component x in system (38) with the initial condition $(x(0), y(0)) = (1.2, 0.4)$, $(1, 0.2)$, $(0.8, 0.8)$, and $(0.6, 0.6)$, respectively.

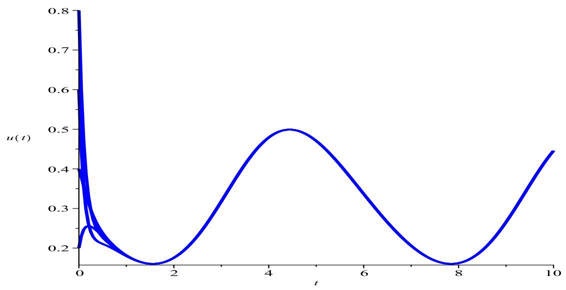

Figure 2: Dynamic behaviors of the second component u in system (38) with the initial condition $(x(0), y(0)) = (1.2, 0.4)$, $(1, 0.2)$, $(0.8, 0.8)$, and $(0.6, 0.6)$, respectively.

Michealis-Menten-type harvesting. However, there are some drawbacks in these two areas. Firstly, the study of the feedback control ecosystem comes from the pioneering work of [23], whose study was based on linear harvesting. Since then, most of the studies on the feedback control ecosystems are based on the linear harvesting of the system. Seldom did scholars consider the other types of harvesting. Secondly, recently, many scholars argued that Michaelis-Menten type harvesting is more suitable, and they investigated the dynamic behaviors of the ecosystem with nonlinear harvesting. However, all of their works assume that the harvesting coefficient is constant, which is unrealistic since, generally speaking, the harvesting effect will change according to the density of the species. To overcome those two drawbacks, we proposed the system (1). Sufficient conditions that ensure the permanence and global attractivity of the system are obtained.

We mention here that for species with generation overlap, it is appropriate to model them with discrete systems. We will try to propose a discrete single-species model with nonlinear feedback control, and investigate the dynamic behavior of the system.

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