

Numerical Investigation and Factor Analysis of Two-Species Spatial-Temporal Competition System after Catastrophic Events

YOUWEN WANG¹, MARIA VASILYEVA¹, SERGEI STEPANOV², ALEXEY SADOVSKI¹

¹Department of Mathematics and Statistics,
Texas A&M University - Corpus Christi,
Corpus Christi, Texas,
UNITED STATES OF AMERICA

²Institute of Mathematics and Information Science,
North-Eastern Federal University,
Yakutsk, Republic of Sakha (Yakutia),
RUSSIA

Abstract: - The interaction of species in an ecological community can be described by coupled system partial differential equations. To analyze the problem numerically, we construct a discrete system using finite volume approximation by space with semi-implicit time approximation to decouple a system. We first simulate the converges of the system to the final equilibrium state for given parameters (reproductive rate, competition rate, and diffusion rate), boundaries, and initial conditions of population density. Then, we apply catastrophic events on a given geographic position with given catastrophic sizes to calculate the restoration time and final population densities for the system. After that, we investigate the impact of the parameters on the equilibrium population density and restoration time after catastrophe by gradually releasing the hold of different parameters. Finally, we generate data sets by solutions of a two-species competition model with random parameters and perform factor analysis to determine the main factors that affect the restoration time and final population density after catastrophic events.

Key-Words: - Multi-species competition model, Numerical investigation, Factor analysis, Catastrophic event, Population dynamics, Ecosystem, Spatial-Temporal model, Lotka–Volterra model, Finite difference approximation

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1 Introduction

Natural and artificial catastrophes disturb human and natural environments, [1], and cast impacts on species. For instance, unrestrained hunting in Sabah (Malaysia) between 1930 and 1950 caused a drastic population decline of the Sumatran rhino, [2], an outbreak of yellow fever in Argentina between 2007 and 2009 threatened endangered brown howler monkey populations, [3]. In the marine environment, human-caused oil spills can have devastating ecological effects, as evidenced after the Ixtoc blowout in the Gulf of Mexico during 1979-1980, zooplankton decreased in biomass levels by almost four orders of magnitude more than observed in 1972, [4]. This work focuses on studying predictive factors for species restoration time after the catastrophe event. Finding factors impacting restoration time can provide better conservation decisions and minimize recovery time, [5]. The

following are some factors that can affect species' populations after catastrophes according to previous studies: species life-history strategies (i.e., the tradeoff between growth, survival, and reproduction. For example, fast-lived species are better than slow-lived species in terms of recovering after climate or land-use change), spatial area of human-assigned natural reserves, communities' political, social, and financial capitals, the age distribution of species, dispersal distance, connectivity, catastrophic mortality, initial population size, environmental stochasticity, demographic stochasticity, density, sex ratio, harvest, genetic variation, etc., [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19].

In this paper, we study the key factors that influence population dynamics. With the improvement of computational power, more factors can be included in the prediction model. However,

the increased number of factors creates complexity and difficulty in the simulation of the species restoration process. Furthermore, we also need to have methods that allow us to identify which factors are the most important in species recovery so that we can allocate our conservation resources and minimize costs. This search for key factors has been intensively studied via the combination of field data, [20], and simulation techniques such as population viability analysis (PVAs), [21]. The PVAs include various key habitat factors to predict the population dynamic and risk of extinction of species using mathematical models, [22]. The PVA approach has been a core methodology in conservation science over the last three decades. It can utilize at least three types of models, [23]: (1) simple occupancy models for metapopulation, which are parameterized using data on the presence or absence of a species in habitat patches but ignoring demographic data (sex, age, stage, etc.); (2) structured population models, which incorporate the spatial structure of habitat patch and species' internal dynamic (age structure, immigration, density, etc.), [24]; (3) most complex individual-based population models, in which individual dispersal, survival, and reproduction vary with respect to their demographic characteristics, [25], [26]. Multiple PVA packages can serve the simulation purpose. For example, the ZooRisk package supports faster analysis of ex-situ populations, while the VORTEX package can be used when the data, expertise, and time is adequate to explore complex individual-based metapopulation models, [27]. After PVA simulation using data of species, sensitivity analysis is applied to determine the key factors that affect species' survival, [28], [29], [30], [31], [32]. However, there are some criticisms of the PVA approach, for instance, significant differences were noticed in terms of prediction by different PVA packages, [33], although catastrophe is verified to have a strong effect on PVA outcome, the proportion of studies that examined this effects did not increase over time, [34], additionally, PVA is effective for evaluating the relative extinction risks of different species, but it shouldn't be used to estimate the likelihood that a certain species would become extinct, [35].

In this work, instead of using the PVA approach considering multiple habitat factors, we simulate multispecies competition based on the Lotka–Volterra model, which is used to describe the population dynamics of species competing for some common resource, [36]. We also combine the multispecies model with the simulation of the effect of catastrophe. In this way, we can study the

dominant factors of species recovery after the catastrophe event. Population viability analysis (PVA) and Lotka–Volterra multispecies competition model are methods for simulating population dynamics, but they differ in their goals, assumptions, and complexity. PVA is designed to predict population persistence or extinction under different scenarios. In contrast, the Lotka–Volterra model is designed to simulate species interactions and the potential for extinction due to competition. The Lotka–Volterra competition model uses an interaction matrix to describe the dynamics of multiple species interacting pairwise. It has been used in many areas: Industry Competition, Genetic Drift, Ecology, Epidemiology, Game Theory, Sociology, etc., [37], [38], [39], [40], [41]. In the Population Dynamic of species, this model has been intensively used to study the impact of the shift of environment, [42], [43], [44], [45], [46], [47], [48], [49], [50]. It's a powerful tool for studying the dynamic of species after the catastrophe in that it can model population recovery, [51], the connection between climate feedback and mass extinction under the competition for limited resources, [52], the connection between spatial heterogeneity and robustness of ecosystem after catastrophe, [53], feedback loops, [54], etc.

In the simulation result analysis, statistical techniques such as factor analysis and sensitivity analysis are used to identify the main factors that affect the restoration time or equilibrium population after the catastrophe. However, they differ in purpose and approach. While sensitivity analysis is used to identify the most important input variables that affect the output or response of a particular model or system, [55], factor analysis is used to identify underlying factors that explain the variation in a set of measured variables, [56]. Note that sensitivity analysis is widely used in analyzing ecosystem datasets, but applying factor analysis on a nonlinear system is rarely studied. Therefore, in this work, we first numerically investigated the post-catastrophe ecosystem from a perspective of species competition, then we used numerical simulation to generate a dataset with random values of different factors, at last, we Therefore, this work applies factor analysis to simulated nonlinear system datasets and interprets the simulated dataset in a new way. In this paper, we employed a four-stage methodology to investigate the dynamics of a two-species competition model and the impact of catastrophic events on system recovery. First, we simulated the convergence of the system to its final equilibrium state using given parameters, boundaries, and initial population densities. Next,

we introduced catastrophic events at specific locations with specific geographic sizes and calculated restoration time and final population densities. We then analyzed the effect of model parameters on equilibrium population density and restoration time by gradually releasing their hold. Lastly, we generated data sets using random values of factors and performed factor analysis to identify key factors influencing restoration time and final population density after catastrophes.

The paper is organized as follows. Section 2 describes our problem formulation, introduces the mathematical model used for simulation, and presents numerical results with some fixed sets of parameters. In Section 4, we make factor analyses of simulated datasets. Section 6 concludes the work and discusses future works.

2 Problem Formation

We consider a two-species competition model in one-dimensional domains $\Omega = [0, 1]$. The mathematical model is described by the following coupled system of equations, [57], [58]:

$$\begin{aligned} \frac{\partial u_1}{\partial t} - \varepsilon_1 \frac{\partial^2 u_1}{\partial x^2} &= r_1 u_1 (1 - u_1) - \alpha_{12} u_1 u_2, \\ x \in \Omega, \quad t > 0, \\ \frac{\partial u_2}{\partial t} - \varepsilon_2 \frac{\partial^2 u_2}{\partial x^2} &= r_2 u_2 (1 - u_2) - \alpha_{21} u_1 u_2, \\ x \in \Omega, \quad t > 0 \end{aligned} \quad (1)$$

with some given initial condition

$$u_1 = u_{10}, \quad u_2 = u_{20}, \quad x \in \Omega, \quad t = 0$$

and fixed boundary conditions for both species,

$$u_1 = u_2 = 0, \quad x \in \partial\Omega, \quad t > 0.$$

Here $u_1(x, t)$ and $u_2(x, t)$ are the population of the first and second species, ε_1 and ε_2 are the diffusion coefficient, r_1 and r_2 are the first and second species reproductive growth rate, α_{12} and α_{21} are the interaction coefficient due to competition.

We define a uniform mesh:

$$\Omega_h = \{x_i = ih, 1 \leq i \leq N\},$$

where N is a positive integer and $h = 1/N$. Let τ be a time step, and $t_n = \tau n$ for $n \geq 0$. For numerical solution, we use a finite difference approximation by space with a semi-implicit scheme for time approximation, Then, for $u_1(x_i, t_n) = u_{1,i}^n$ and $u_2(x_i, t_n) = u_{2,i}^n$, we obtain the following discrete form

$$\begin{aligned} \frac{u_{1,i}^n - u_{1,i}^{n-1}}{\tau} - \varepsilon_1 \frac{u_{1,i+1}^n - 2u_{1,i}^n + u_{1,i-1}^n}{h^2} &= r_1 u_{1,i}^{n-1} (1 - u_{1,i}^{n-1}) \\ &\quad - \alpha_{12} u_{1,i}^{n-1} u_{2,i}^{n-1}, \quad 2 \leq i \leq N - 1, \\ \frac{u_{2,i}^n - u_{2,i}^{n-1}}{\tau} - \varepsilon_2 \frac{u_{2,i+1}^n - 2u_{2,i}^n + u_{2,i-1}^n}{h^2} &= r_2 u_{2,i}^{n-1} (1 - u_{2,i}^{n-1}) \\ &\quad - \alpha_{21} u_{1,i}^{n-1} u_{2,i}^{n-1}, \quad 2 \leq i \leq N - 1, \end{aligned} \quad (2)$$

with $u_{1,1} = u_{1,N} = u_{2,1} = u_{2,N} = 0$.

To simulate pre and post catastrophic cases, we have the following algorithm:

- *Pre-catastrophic case:* Solve the system (2) with some given constant initial condition:

$$u_1 = s_1, \quad u_2 = s_2, \quad t = 0,$$

to find an equilibrium state ($u_1(x, T_{pre})$ and $u_2(x, T_{pre})$) and the time needed to reach it (T_{pre}).

- *Post-catastrophic case:* We use the previous (pre-catastrophic) solution and apply catastrophic events in some subdomain $\Omega_{cat} \in \Omega$

$$g_1 = \begin{cases} 0, & x \in \Omega_{cat} \\ u_1(x, T_{pre}), & x \in \Omega/\Omega_{cat} \end{cases}$$

$$g_2 = \begin{cases} 0, & x \in \Omega_{cat} \\ u_2(x, T_{pre}), & x \in \Omega/\Omega_{cat} \end{cases}$$

Then solve the system (2) with the initial condition:

$$u_1 = g_1, \quad u_2 = g_2, \quad t = T_{pre},$$

until the system reaches an equilibrium state ($u_1(x, T_{post})$ and $u_2(x, T_{post})$) and records the restoration time (T_{post}). When the change in population density is less than $tol = 10^{-3}$, the equilibrium is considered reached.

Next, we performed numerical simulations based on the presented algorithm. Before we release control of values of all parameters during simulation, we first controlled the value of all parameters (reproductive rate, competition rate, diffusion rate, boundaries, initial conditions of population density, and catastrophic size) in section 2.1.

We consider two cases:

- Case 1 (one species survive)

$$D_1 = 0.035, \quad D_2 = 0.014,$$

$$r_1 = 0.074, \quad r_2 = 0.084,$$

$$\alpha_{12} = 0.074, \quad \alpha_{21} = 0.013.$$
- Case 2 (both species survive)

$$D_1 = 0.016, \quad D_2 = 0.014,$$

$$r_1 = 0.083, \quad r_2 = 0.081,$$

$$\alpha_{12} = 0.053, \quad \alpha_{21} = 0.049.$$

with regular diffusion $\varepsilon = D$ and small diffusion $\varepsilon = D/10$. In simulations, we used a grid with $N = 100$ nodes and performed simulations with $\tau = 1$ with initial conditions $s_1 = s_2 = 0.5$ for pre-catastrophic cases.

After that, we gradually released the control of catastrophe size (section 2.2) and diffusion rate (section 2.3) to understand the dynamic of the system. Finally, we released control of all parameters and simulated catastrophic events (section 3).

2.1 Solution and Dynamic for Two-Species Competing Model Pre- and Post-Catastrophe

2.1.1 Case 1 (One Species Survives)

Figure 1 presents the case in which only one species survives when we control all parameters. We observed that, in comparing regular diffusion with small diffusion, the latter takes more time to reach equilibrium, both before and after the catastrophe.

Specifically, species with small diffusion take 876-time steps to reach equilibrium before the catastrophe, whereas species with regular diffusion take only 269-time steps. After the catastrophe, species with small diffusion take $2689 - 876 = 1813$ -time steps to reach equilibrium, while species with regular diffusion take only $637 - 269 = 368$ -time steps, which is five times faster.

Furthermore, the boundary constraint has less effect on small diffusion. If diffusion is small, at the final equilibrium state, the central highly populated area is larger than with regular diffusion. At equilibrium, around 40% of the central geographic domain has a population density above 0.8 with regular diffusion, while around 80% has a population density above 0.8 with small diffusion for surviving species.

2.1.2 Case 2 (Two Species Survive)

We also examined the case in which both species survive, as is shown in Figure 2. We observed that small diffusion leads to a shorter time to reach equilibrium compared to regular diffusion, both before and after the catastrophe. This situation is the

opposite of what we observed in case 1, where species with small diffusion take more time to reach equilibrium.

Specifically, before the catastrophe, species with small diffusion reached equilibrium in 226-time steps, slightly faster than species with regular diffusion, which took 276-time steps. After the catastrophe, species with small diffusion reach equilibrium in $488 - 314 = 174$ -time steps, which is twice as fast as species with regular diffusion, reaching equilibrium in $637 - 276 = 361$ -time steps.

Same as in case 1, the boundary constraint has less effect on small diffusion. At equilibrium, around 20% - 50% of the central geographic domain has a population density above 0.5 with regular diffusion, while around 80% - 85% has a population density above 0.5 with small diffusion for surviving species.

2.2 Effect of the Catastrophic Size

As is shown in Figure 3, we controlled all other parameters and only released the control of catastrophe size. We found that as the catastrophe size becomes larger, the restoration time slightly increases, both in case 1, and case 2.

Specifically, we observed that under case 1, when we place a catastrophe to the system after the system has reached its pre-catastrophe equilibrium (at time step 269), the number of time steps to reach equilibrium after a catastrophe is 368, no matter what the size of the catastrophe is (5, 25, 50, 75%). Under case 2, when we place a catastrophe to the system after the system has reached its pre-catastrophe equilibrium (at time step 276), the time steps for the system to reach equilibrium slightly increase from 345 (when the size of the catastrophe is 5) to 387 (when the size of the catastrophe is 75).

2.3 Effect of the Diffusion

As is shown in Figure 4, we controlled all parameters and only released control of diffusion, and we observed that diffusion is the key factor for determining the survival status groups.

Specifically, similar diffusion rate combinations before and after the catastrophe lead to similar survival statuses for both species. Borderline combinations require more time steps to reach equilibrium. After the catastrophe, diffusion rates still determine survival status, with little change except for borderline combinations.

3 Factor Analysis for Random Parameters

We performed 100k simulations with random input parameters, with the following scale for reproduction rate, competition rate, diffusion rate, and initial condition of population density.

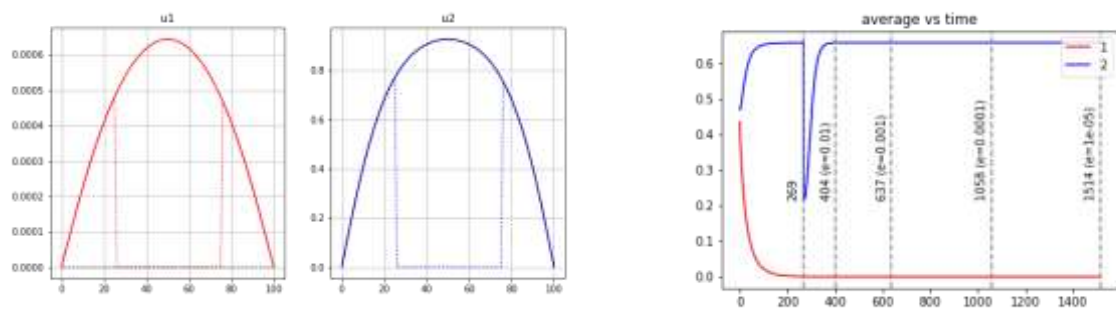
$$0.01 < r_k, \alpha_{kl}, D_k < 0.1, \quad 0.01 < u_0^{(k)} < 0.99,$$

Next, we take a simulation that leads to the cases where at least one species survives (73k). Table 1 presents the proportion of the survival group before the catastrophe stroke. In more than 80 % of the scenario, only one species survives. Finally, we apply catastrophic events with random length: $0.1 < L_{cat} < 0.6$.

Table 1. Proportion of the survival group before the catastrophe stroke. In more than 80 % of the scenario, only one species survives.

	01	10	11
N	31088	30619	11293
%	42.5	41.9	15.4699

Regular diffusion, $L_{cat} = 50\%$



Small diffusion, $L_{cat} = 50\%$

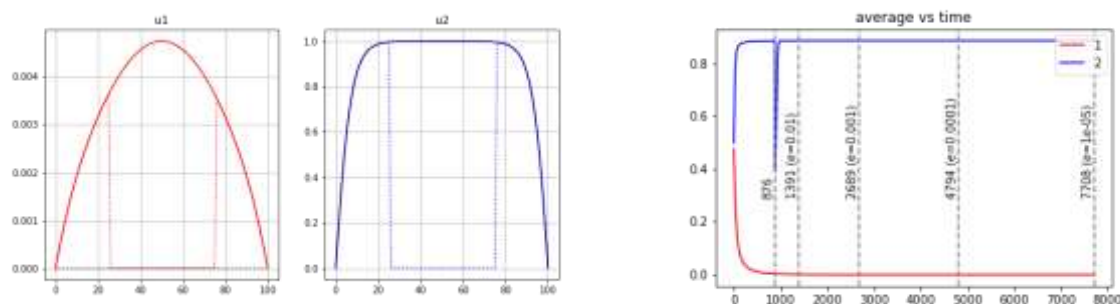


Fig. 1: Case 1: One species survives. Solutions at the final time in the first and second columns, the solution average over the domain versus time in the third column, under regular diffusion with $\varepsilon = D$ (first row) and small diffusion with $\varepsilon = D/10$ (second row). Solid lines represent before the catastrophe, while dashed lines represent after the catastrophe with a size of the catastrophe $L_{cat} = 50\%$. The red line represents species 1, and the blue line represents species 2.

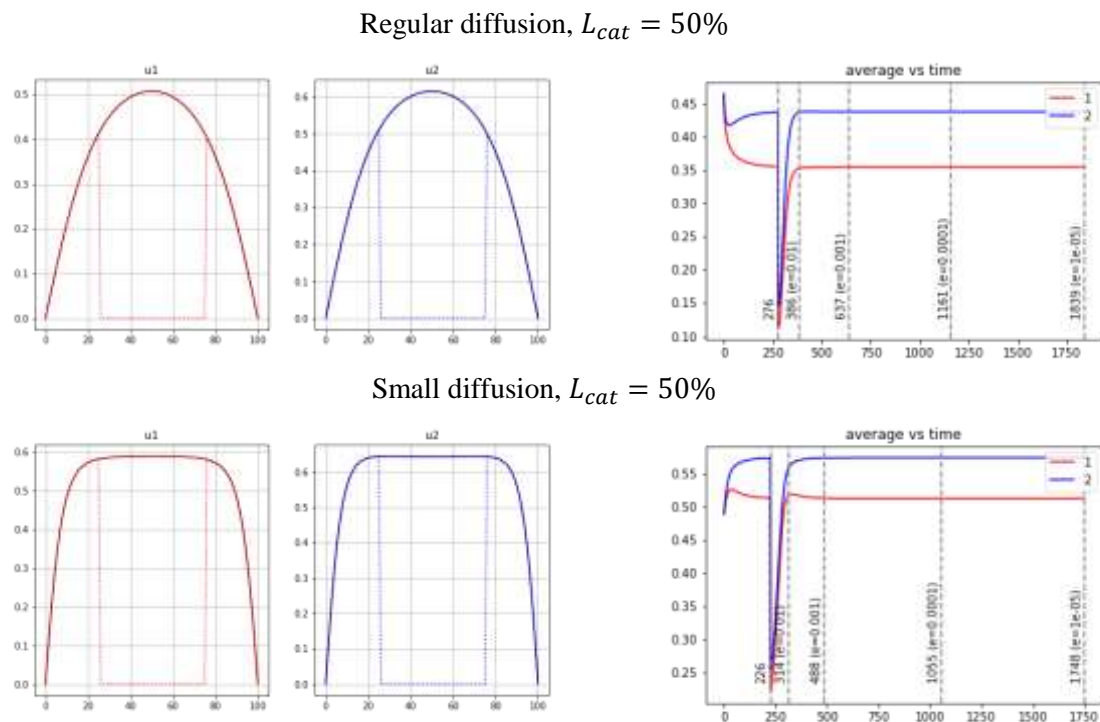


Fig. 2: Case 2: Both species survive. Solutions at the final time in the first and second columns, the solution average over the domain versus time in the third column, under regular diffusion with $\varepsilon = D$ (first row) and small diffusion with $\varepsilon = D/10$ (second row). Solid lines represent before the catastrophe, while dashed lines represent after the catastrophe with the size of the catastrophe $L_{cat} = 50\%$. The red line represents species 1, and the blue line represents species 2.

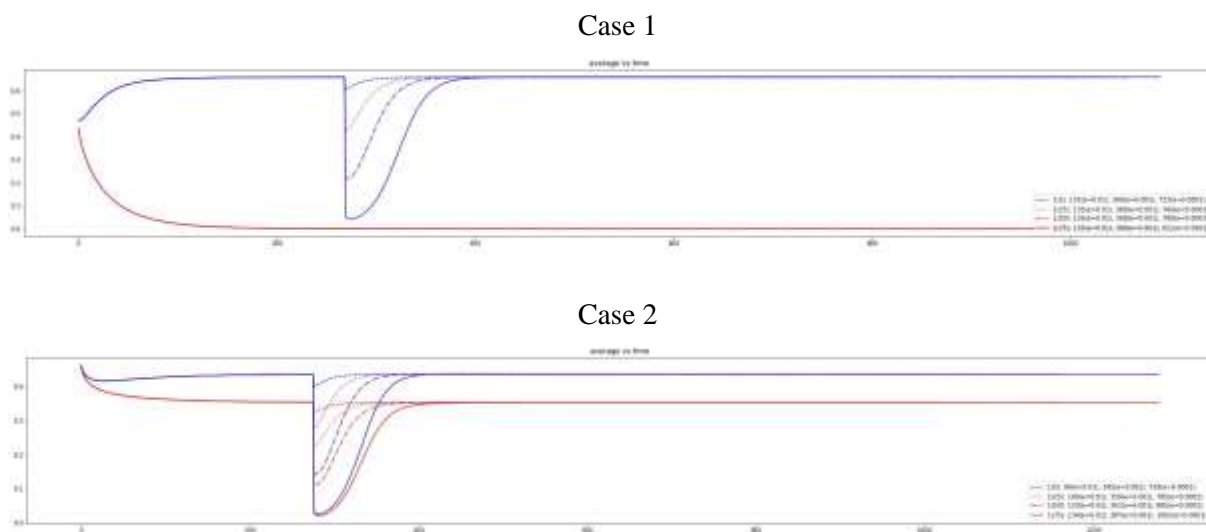
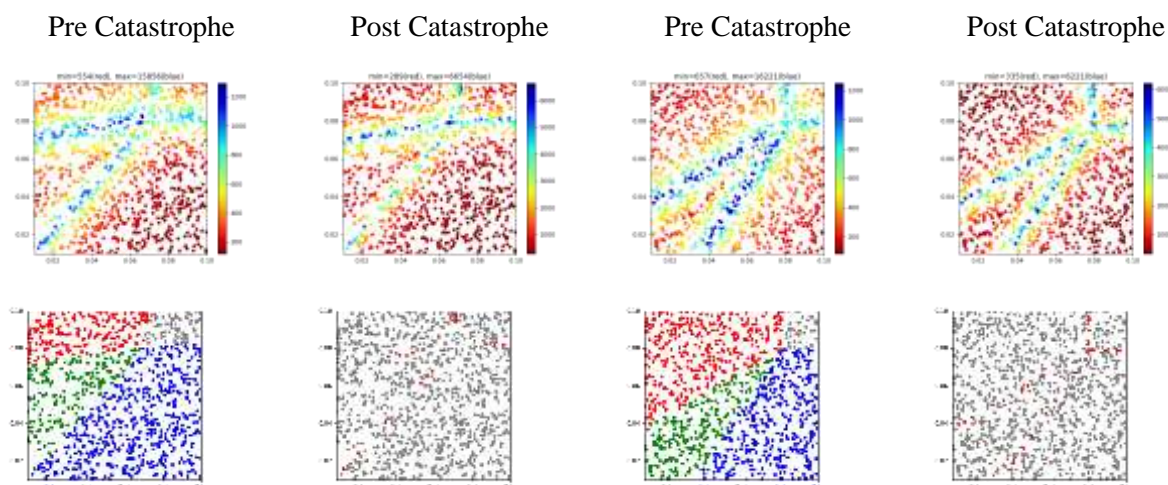


Fig. 3: Restoration time for different catastrophic sizes $L_{cat} = 5,25,50,75\%$ under regular diffusion (which means it has the same scale as other parameters such as birth rate and competition rate). Case 1 (only one species survives) is presented in the first row, and case 2 (both species survive) is presented in the second row.



(a) Case 1

(b) Case 2

Fig. 4: Each scatter plot compares D_1 and D_2 diffusion rates for two species. The size of the catastrophe is set at $L_{cat} = 50\%$. Column 1 and 3 is pre-catastrophe, while columns 2 and 4 show post-catastrophe with different stopping thresholds (for example. Row 1 shows the time to reach equilibrium, while rows 2 show the final survival groups and changed survival groups after the catastrophe: 00 (grey), 01 (blue), 10 (red), and 11 (green). Here 00 means no species survive, and 01 means only the second species survive. In Row 2, the red dots on the boundaries of groups show the change in survival groups after the catastrophe.

3.1 Mean and Standard Deviation of Time Until Equilibrium by Categories

Table 2 and Figure 5 show the mean and standard deviation of time steps until equilibrium after categories.

From Table 2, we observed that the restoration time varies with catastrophe size, with larger catastrophes leading to longer restoration times and higher variability.

From Figure 5, we observed that when the catastrophe size is larger than 0.3, the variation of restoration time experiences a surge.

In summary, restoration time varies with catastrophe size, with larger catastrophes leading to longer restoration times and higher variability, especially when catastrophe size is larger than 0.3. This indicates that it is more difficult to predict the restoration time as the catastrophe size increases.

3.2 Mean and Standard Deviation of Equilibrium Population Density Solution Differences pre- and post-catastrophe

Table 3 and Figure 6 show the mean and standard deviation of solution differences before and after categories.

In Table 3, we observed that mean solution differences are consistent across catastrophic sizes, but standard deviation and maximum values vary greatly. Catastrophe sizes between 0.3 and 0.4 lead to the highest standard deviation, making it difficult to predict equilibrium population density in specific scenarios.

In Figure 6, we observed that there is an extreme outlier in the interval [0.3,0.4]. This might account for the high standard deviation in this catastrophic group. Yet we can still see that when catastrophe size is larger, the variation of solution difference is larger, hence harder to predict.

In summary, the mean solution differences are consistent across catastrophic sizes, but standard deviation and maximum values vary greatly. Catastrophe sizes between 0.3 and 0.4 lead to the highest standard deviation, making it difficult to predict equilibrium population density in specific scenarios.

3.3 Regrouping

Table 4 shows the percentage of regrouping of survival status after the catastrophe. Survival group changes occur at a consistently low rate of 2.5-3%. The probability of regrouping is highest in catastrophe sizes between 0.3-0.4 and 0.5-0.6, making predicting species' survival status challenging.

Table 2. Mean and standard deviation of time steps until equilibrium after categories. N = number of simulations

Category	1 [0.1, 0.2]	2 [0.2, 0.3]	3 [0.3, 0.4]	4 [0.4, 0.5]	5 [0.5, 0.6]
N	14717	14517	14575	14521	14670
mean	700.48	751.22	801.19	869.46	969.42
std	709.26	758.35	803.44	892.47	1001.84

Table 3. Box plots of equilibrium population density solution differences of pre vs. post catastrophe

$$(\text{difference} = \sqrt{(u_{pre1} - u_{post1})^2 + (u_{pre2} - u_{post2})^2}),$$

under different catastrophe sizes

Category	1 [0.1, 0.2]	2 [0.2, 0.3]	3 [0.3, 0.4]	4 [0.4, 0.5]	5 [0.5, 0.6]
mean	0.14	0.14	0.14	0.14	0.14
std	0.18	0.22	0.37	0.20	0.23
max	2.51	13.35	34.87	6.37	11.82

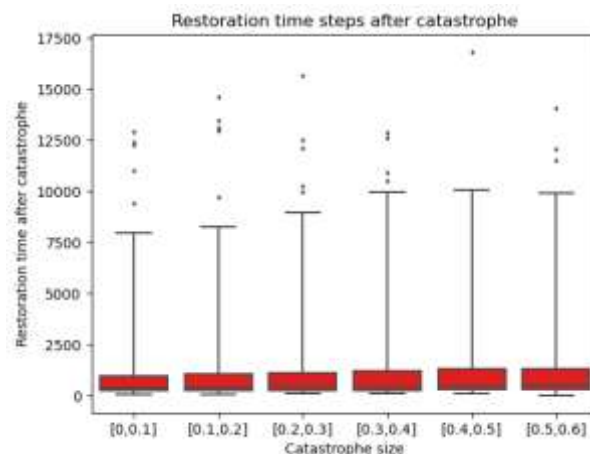


Fig. 5: Post Catastrophe Restoration time steps, under different catastrophe sizes

Table 4. Post Catastrophe Percentage of regrouping of survival status

Category	1 [0.1, 0.2]	2 [0.2, 0.3]	3 [0.3, 0.4]	4 [0.4, 0.5]	5 [0.5, 0.6]
Pre Catastrophe					
01	42.54	42.20	42.59	42.87	42.74
10	42.07	42.24	41.81	41.82	41.77
11	15.39	15.56	15.60	15.31	15.49
Post Catastrophe					
Diff	2.71	2.73	2.96	2.73	2.98
00	1.54	1.69	1.64	1.58	1.71
01	42.18	41.85	42.43	42.59	42.51
10	42.01	41.92	41.58	41.58	41.51
11	14.26	14.54	14.35	14.25	14.27

Variance explained by each factor out of the total variance:

	Factor1	Factor2	Factor3	Factor4	Factor5
SS Loading	1.615	1.236	1.196	1.154	0.998
Prop Var	0.161	0.124	0.120	0.115	0.100
Cum Var	0.161	0.285	0.405	0.520	0.620

The correlation coefficient for the variable and factor:

	Factor1	Factor2	Factor3	Factor4	Factor5
r1	0.271	-0.178	0.207	0.554	-0.058
r2	-0.121	0.981	0.112	-0.084	-0.001
a12	-0.036	0.022	-0.004	-0.001	-0.002
a21	-0.013	0.009	-0.021	0.058	0.006
d1	-0.130	0.111	0.960	-0.213	0.014
d2	0.971	0.171	-0.133	0.058	-0.043
lcat	-0.005	0.009	0.001	0.015	0.118
u1	0.295	-0.201	-0.406	0.830	-0.135
u2	-0.681	0.385	0.192	-0.250	-0.167
npost3	0.118	-0.110	0.008	-0.193	0.966

Variance explained by each factor out of the total variance:

	Factor1	Factor2	Factor3	Factor4	Factor5	Factor6
SS Loading	1.519	1.459	1.458	1.418	1.033	0.634
Prop Var	0.152	0.146	0.146	0.142	0.103	0.063
Cum Var	0.152	0.298	0.444	0.585	0.689	0.752

The correlation coefficient for the variable and factor:

	Factor1	Factor2	Factor3	Factor4	Factor5	Factor6
r1	-0.084	-0.165	0.972	0.037	0.082	-0.097
r2	-0.031	-0.058	0.007	0.997	-0.010	-0.007
a12	0.054	0.016	0.028	-0.013	0.995	-0.017
a21	-0.046	0.007	0.059	0.105	-0.026	0.710
d1	0.149	0.639	0.675	-0.063	-0.126	0.068
d2	0.808	0.163	-0.048	0.542	0.036	-0.140
lcat	0.008	0.007	-0.008	-0.011	0.000	0.033
u1	0.092	-0.953	0.112	0.010	-0.039	-0.092
u2	-0.875	0.144	-0.010	0.244	-0.040	-0.177
npost3	0.238	0.252	-0.197	-0.234	0.117	0.235

Fig. 7: Factor Analysis of restoration time steps in two species systems. Dominant (threshold: $\text{corr} > 0.7$) parameters in each factor are highlighted in yellow. The number of factors is determined by the number of eigenvalues greater than 1. Left: case 1. Right: case 2.

Variance explained by each factor out of the total variances:

	Factor1	Factor2	Factor3	Factor4	Factor5
SS Loading	2.387	2.105	1.509	1.088	0.370
Prop Var	0.217	0.191	0.137	0.099	0.034
Cum Var	0.217	0.408	0.546	0.644	0.678

The correlation coefficient for the variable and factor:

	Factor1	Factor2	Factor3	Factor4	Factor5
r1	0.308	-0.197	0.138	0.893	-0.209
r2	-0.208	0.631	0.165	-0.067	0.039
a12	-0.022	0.028	-0.007	0.007	-0.076
a21	0.046	-0.007	-0.006	0.006	-0.001
d1	-0.645	0.271	-0.254	0.462	0.477
d2	0.111	-0.167	0.962	0.085	0.148
lcat	-0.002	0.001	0.002	-0.001	0.003
u1	0.926	-0.274	0.195	0.159	0.037
u2	-0.226	0.841	-0.446	-0.084	-0.182
e3y1	0.926	-0.273	0.194	0.159	0.039
e3y2	-0.226	0.841	-0.444	-0.083	-0.184

Variance explained by each factor out of the total variances:

	Factor1	Factor2	Factor3	Factor4	Factor5	Factor6
SS Loading	2.417	2.411	1.430	1.430	0.741	0.688
Prop Var	0.220	0.219	0.130	0.130	0.067	0.063
Cum Var	0.220	0.439	0.569	0.699	0.766	0.829

The correlation coefficient for the variable and factor:

	Factor1	Factor2	Factor3	Factor4	Factor5	Factor6
r1	0.092	0.244	0.946	0.003	0.077	0.002
r2	0.235	0.094	0.005	0.957	-0.038	0.114
a12	-0.037	-0.005	0.047	0.023	0.831	-0.039
a21	-0.036	-0.053	0.013	0.025	-0.016	0.800
d1	-0.146	-0.584	0.726	-0.017	-0.158	0.039
d2	-0.590	-0.144	-0.013	0.712	0.035	-0.163
lcat	0.012	0.015	0.009	0.006	-0.017	-0.003
u1	-0.109	0.989	0.044	0.002	-0.058	-0.033
u2	0.989	-0.100	0.008	0.049	-0.067	-0.044
e3y1	-0.114	0.989	0.046	-0.001	-0.055	-0.035
e3y2	0.989	-0.105	0.007	0.050	-0.068	-0.040

Fig. 8: Factor Analysis of final population density in two species systems after the catastrophe. Dominant (threshold: $\text{corr} > 0.7$) parameters in each factor is highlighted in yellow. The number of factors are determined by the number of eigenvalues greater than 1. Left: case 1. Right: case 2.

3.4 Factor Analysis of Time Steps in Two Species System

Factor analysis is used to reveal any latent variables that cause the manifest variables to covary and can help us to see the trend driving the system, [59]. A survey of over 1700 PsycINFO studies, including Factor Analysis, suggested that over 50% of surveyed researchers used Varimax rotation and decided the number of factors to be retained for rotation by Kaiser criterion (all factors with eigenvalues greater than one), [60]. In this case, the observed variables are the various factors that contribute to the species' restoring time steps. Factor analysis can help identify the most important factors driving the variation in the observed variables.

Figure 7 shows the Factor Analysis of restoration time steps in two species systems. In both survival cases, the reproduction rate, diffusion rate, and the equilibrium population density before the catastrophe are the most dominant factors. It can be indicated that to impact the restoration time of species in the aftermath of a catastrophe, it is important to examine the diffusion rate and the level of species population density before the catastrophe. However, other dominant factors differ between the two cases. In case 1, the reproductive rate of species 1 is more important than the diffusion and equilibrium population density of species 1. Additionally, competition efficiency is not among the dominant factors in case 1. This suggests that the underlying mechanisms driving the species' restoration time steps may differ in each survival case. When both species are to survive together, competition efficiency matters.

Case 1 top factors:

- Diffusion of species 2
- Reproduction of species 2
- Diffusion of species 1
- Pre-catastrophe population of species 1
- Restoration time

Case 2 top factors:

- Diffusion of species 2 and Pre-catastrophe population of species 2
- Pre catastrophe population of species 1
- Reproduction of species 1
- Reproduction of species 2
- Competition Efficiency of species 1
- Competition Efficiency of species 2

3.5 Factor Analysis of Final Population Density in Two Species System

Figure 8 shows the Factor Analysis of the final population density in two species systems. We observed that there is a difference in the dominant factors between the two survival cases. While in both cases, the most important driving factor is the pre- and post-population density, followed by diffusion and reproductive rates, competition efficiency is not among the dominant factors in case 1, whereas it is a dominant factor for both species in case 2. It can be indicated that to ensure the survival of both species in the aftermath of a catastrophe, it is important to examine the competition efficiency.

Case 1 top factors:

- Pre and Post catastrophe population of species 1
- Pre and Post catastrophe population of species 2
- Diffusion of species 2
- Reproduction of species 1

Case 2 top factors:

- Pre and Post catastrophe population of species 2
- Pre and Post catastrophe population of species 1
- Reproduction and Diffusion of species 1
- Reproduction and Diffusion of species 2
- Competition Efficiency of species 1
- Competition Efficiency of species 2

4 Conclusions

In our simulation study, we numerically investigated the impact of various parameters (reproductive rate, competition rate, and diffusion rate) on the restoration time and final population densities of a two-species competition model after catastrophic events. This research holds positive repercussions for the scientific and academic communities, as it not only enhances understanding of the post-catastrophe driving factors of species survival and recovery dynamics but also presents a methodology of applying factor analysis to ecosystem restoration process analysis, instead of applying the sensitivity analysis.

We first compared the time dynamic and final population density solutions between two survival cases (case 1: only one species survives; case 2: both species survive) under regular and small diffusion rates. We found that the restoration time is different for the two survival statuses. For case 1, it

takes more time to reach equilibrium when both species have small diffusion. For case 2, it takes more time to reach equilibrium when both species have regular diffusion. We also observed that boundary constraints have less effect on small diffusion for both survival statuses.

We then investigated the impact of catastrophic event size and diffusion rate on the restoration time and final population densities. We found that as the catastrophic event size increases (especially when greater than 0.3), the restoration time and final population density do not change much, but the variation increases, making it potentially harder to predict. The diffusion rate is the key factor for determining the survival status group. Similar diffusion rate combinations before and after the catastrophe lead to similar survival statuses for both species. After the catastrophe, diffusion rates still determine survival status, with little change except for borderline combinations.

Finally, we performed factor analysis on the restoration time and final population density data sets generated by solutions of a two-species competition model with random values of parameters. We observed that for different survival statuses, the dominant factors and the order of the factors are different. The dominant factor is usually reproduction rate, diffusion, and population density before the catastrophe. However, competition efficiency is an important factor to consider if both species are to survive together (case 2), while it is not the main factor under case 1. This observation suggested that if our goal is to have both species survive together, we need to pay attention to the competitive rates.

In future works, we will study more about the modeling of catastrophic events. In the real world, events such as hurricanes, oil spills, disease outbreaks, hypoxic events, harmful algal blooms, and coral bleaching all can cause massive species mortality, [7], [61], however, their simulation may differ due to variations in spatial patterns. We plan to randomize catastrophe locations within the spatial domain, rather than keeping them centralized. Additionally, we will model various catastrophe scenarios, accounting for differing species mortality rates. Lastly, as our current study indicates that predicting equilibrium population density and restoration time is more challenging for catastrophes of larger size, we will employ deep neural networks to forecast the final state and recovery time of competing species systems, [62], [63].

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