

Global stability of Leslie-Gower Predator-prey Model with Density Dependent Birth Rate on Prey Species and Prey Refuge

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Abstract: - A Leslie-Gower predator prey model with density dependent birth rate on prey species and prey refuge is proposed and studied in this paper. Sufficient condition which ensure the global stable of the positive equilibrium is obtained. Our study indicates density dependent birth rate of prey species has negative effect on the final density of both prey and predator species. Density dependent birth rate may lead to the Allee effect of prey species and enhance the extinction chance of the species. Numeric simulations are carried out to show the feasibility of the main results.

Key-Words: Leslie-Gower predator prey model; Refuge; Stability

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1 Introduction

The aim of this paper is to investigate the dynamic behaviors of the following Leslie-Gower predator prey model with density dependent birth rate on prey species and prey refuge

$$\frac{dH}{dt} = \left(\frac{r_{11}}{c_1 + c_2 H} - r_{12} - b_1 H \right) H - a_1(1 - m)HP, \quad (1.1)$$

$$\frac{dP}{dt} = \left(r_2 - a_2 \frac{P}{(1 - m)H} \right) P,$$

where $m \in [0, 1)$ and $a_i, c_i, i = 1, 2, b_1, r_{11}, r_{12}, r_2$ are all positive constants. where H and P are the density of prey species and the predator species at time t , respectively. $\frac{r_{11}}{c_1 + c_2 H}$ is the birth rate of the prey species, r_{12} is the death rate of the prey species, r_2 is the intrinsic growth rate of the predator species, respectively.

During the past two decades, many scholars investigated the dynamic behaviors of the population modelling ([1]-[40]), specially, due to its dominant importance on the nature, many scholars investigated the dynamic behaviors of the predator prey system, see [1]-[13], [29]-[40] and the references cited therein. Numerous studies has been done on the Leslie-Gower predator prey model, see [5, 8, 9, 12, 32, 33, 34, 35, 36, 37, 38, 39, 40]. There are also many scholars investigated the influence of prey refuge, see [4, 5, 7, 10, 13, 16, 20, 28].

Chen, Chen and Xie[5] proposed a Leslie-Gower predator prey model incorporating prey refuge, which

takes the form:

$$\begin{aligned} \frac{dH}{dt} &= (r_1 - b_1 H)H - a_1(1 - m)HP, \\ \frac{dP}{dt} &= \left(r_2 - a_2 \frac{P}{(1 - m)H} \right) P, \end{aligned} \quad (1.2)$$

where $m \in [0, 1)$ and $r_i, a_i, i = 1, 2, b_1$ are all positive constants. They showed that prey refuge has no influence on the persistent property of the system. They also showed that increasing the prey refuge could increase the final density of the prey species, however, prey refuge has complex influence on the final density of the predator species.

In system (1.2), one could easily see that without the influence of the predator species, the prey species takes the Logistic model

$$\frac{dH}{dt} = (r_1 - b_1 H)H. \quad (1.3)$$

Here, r_1 is the intrinsic growth rate and b_1 is the density dependent coefficient. Obviously, $r_1 = r_{11} - r_{12}$, where r_{11} is the growth rate of the prey species, while r_{12} is the death rate of the prey species. Recently, Chen et al [6] and Zhao et al [22] argued that in some case, the density dependent birth rate of the species is more suitable. Now, stimulated by the work of [6, 22], we also take the famous Beverton-Holt function as the birth rate, then r_{11} in system (1.2) should be replaced by the form $\frac{r_{11}}{c_1 + c_2 x}$ and this leads to the model (1.1). To the best of our knowledge, model (1.1) is first time proposed and studied.

The aim of this paper is to investigate the stability property of the system (1.1), more precisely, we would like to investigate the global stability of the positive equilibrium of the system, since it indicates the long term coexistence of the both species. We also try to find out the influence of the density dependent

birth rate of prey species.

In addition to this section, the rest of the paper is arranged as follows. In next section, we will investigate the existence and local stability of the positive equilibrium of the system (1.1). In Section 3, we will discuss the global stability of the equilibrium by constructing some suitable Lyapunov function. In Section 4, we will discuss the influence of the density dependent birth rate. Numeric simulations are carried out in Section 5 to show the feasibility of the main results. We end this paper by a briefly discussion.

2 The existence and local stability of the positive equilibrium of system (1.1)

Concerned with the existence of the positive equilibrium of system (1.1), we have the following result.

Theorem 2.1. Assume that

$$r_{11} > c_1 r_{12} \quad (2.1)$$

holds, then system (1.1) admits a unique positive equilibrium $B(H^*, P^*)$, where

$$\begin{aligned} H^* &= \frac{-B_2 + \sqrt{B_2^2 - 4B_1B_3}}{2B_1}, \\ P^* &= \frac{r_2(1-m)H^*}{a_2}, \\ B_1 &= c_2(r_2(m-1)^2 a_1 + b_1 a_2) > 0, \\ B_2 &= a_1 c_1 r_2 (m-1)^2 + a_2 b_1 c_1 \\ &\quad + a_2 c_2 r_{12}, \\ B_3 &= a_2 (c_1 r_{12} - r_{11}) < 0. \end{aligned} \quad (2.2)$$

Proof. The positive equilibrium of system (1.1) satisfies the equation

$$\begin{aligned} \frac{r_{11}}{c_1 + c_2 H} - r_{12} - b_1 H - a_1(1-m)P &= 0, \\ r_2 - a_2 \frac{P}{(1-m)H} &= 0. \end{aligned} \quad (2.3)$$

From the second equation of (2.2), one has $P = \frac{r_2(1-m)H}{a_2}$. Substituting $P = \frac{r_2(1-m)H}{a_2}$ to the first equation of (2.3) leads to

$$\frac{r_{11}}{c_1 + c_2 H} - r_{12} - b_1 H - a_1(1-m) \frac{r_2(1-m)H}{a_2} = 0. \quad (2.4)$$

Equation (2.4) is equivalent to

$$B_1 H^2 + B_2 H + B_3 = 0, \quad (2.5)$$

where B_1, B_2, B_3 are defined by (2.2). (2.5) has unique positive solution H^* , hence, under the assumption (2.1) holds, system (1.1) admits a unique positive equilibrium $B(H^*, P^*)$.

This ends the proof of Theorem 2.1.

Theorem 2.2. Assume that

$$r_{11} > c_1 r_{12} \quad (2.6)$$

holds, $B(H^*, P^*)$ is locally asymptotically stable.

Proof. Under the assumption (2.6), system (1.1) admits a unique positive equilibrium $B(H^*, P^*)$.

The Jacobian matrix of the system (1.1) is calculated as

$$\begin{aligned} J(H, P) &= \begin{pmatrix} A_{11} & -a_1(1-m)H \\ \frac{P^2 a_2}{(1-m)H^2} & r_2 - 2 \frac{a_2 P}{(1-m)H} \end{pmatrix}, \end{aligned} \quad (2.7)$$

where

$$\begin{aligned} A_{11} &= \frac{r_{11}}{c_2 H + c_1} - r_{12} - b_1 H - a_1(1-m)P \\ &\quad + H \left(-\frac{r_{11} c_2}{(c_2 H + c_1)^2} - b_1 \right). \end{aligned}$$

Noting that at $B(H^*, P^*)$,

$$\begin{aligned} \frac{r_{11}}{c_1 + c_2 H^*} - r_{12} - b_1 H^* - a_1(1-m)P^* &= 0, \\ r_2 - a_2 \frac{P^*}{(1-m)H^*} &= 0. \end{aligned} \quad (2.8)$$

Then the Jacobian matrix of the system (1.1) about the equilibrium $B(H^*, P^*)$ is

$$\begin{aligned} J(B(H^*, P^*)) &= \begin{pmatrix} -B_1 & -a_1(1-m)H^* \\ \frac{P^*}{r_2 H^*} & -r_2 \end{pmatrix}, \end{aligned} \quad (2.9)$$

where

$$B_1 = H^* \left(\frac{r_{11} c_2}{(c_2 H^* + c_1)^2} + b_1 \right).$$

Consequently, we have

$$\text{Det}J(B(H^*, P^*)) = r_2 B_1 + a_1 H^* r_2 \frac{P^*}{H^*} > 0,$$

and

$$\text{Tr}J(B(H^*, P^*)) = -B_1 - r_2 < 0.$$

So that both eigenvalues of $J(B(H^*, P^*))$ have negative real parts, and $B(H^*, P^*)$ is locally asymptotically stable.

This ends the proof of Theorem 2.2.

3 Global stability

Concerned with the global stability of the positive equilibrium of system (1.1), we have the following result.

Theorem 3.1. *Assume that*

$$r_{11} > c_1 r_{12} \tag{3.1}$$

holds, $B(H^, P^*)$ is globally stable.*

Proof. Under the assumption (3.1) holds, system (1.1) admits a unique positive equilibrium $B(H^*, P^*)$, which satisfies the equalities

$$\begin{aligned} \frac{r_{11}}{c_1 + c_2 H^*} - r_{12} - b_1 H^* - a_1(1 - m)P^* &= 0, \\ r_2 - a_2 \frac{P^*}{(1 - m)H^*} &= 0. \end{aligned} \tag{3.2}$$

Now let us consider the following Lyapunov function:

$$\begin{aligned} V(H, P) &= \ln \frac{H}{H^*} + \frac{H^*}{H} \\ &+ \frac{a_1(1 - m)^2 H^*}{a_2} \left(\ln \frac{P}{P^*} + \frac{P^*}{P} \right). \end{aligned} \tag{3.3}$$

Obviously, $V(H, P)$ is well defined and continuous for all $H, P > 0$. By simple computation, we have

$$\begin{aligned} \frac{\partial V}{\partial H} &= \frac{1}{H} \left(1 - \frac{H^*}{H} \right), \\ \frac{\partial V}{\partial P} &= \frac{a_1(1 - m)^2 H^*}{a_2 P} \left(1 - \frac{P^*}{P} \right). \end{aligned} \tag{3.4}$$

(3.4) shows that the positive equilibrium (H^*, P^*) is the only extremum of the function $V(H, P)$ in the positive quadrant. One could easily verify that

$$\begin{aligned} &\lim_{H \rightarrow 0} V(H, P) \\ &= \lim_{P \rightarrow 0} V(H, P) \\ &= \lim_{H \rightarrow +\infty} V(H, P) \\ &= \lim_{P \rightarrow +\infty} V(H, P) = +\infty. \end{aligned} \tag{3.5}$$

(3.4) and (3.5) show that the positive equilibrium (H^*, P^*) is the global minimum, that is,

$$V(H, P) > V(H^*, P^*) = 1 + \frac{a_1(1 - m)^2 H^*}{a_2} > 0$$

for all $H, P > 0$.

Calculating the derivative of V along the solution

of the system (1.1), by using equalities (3.2), we have

$$\begin{aligned} &\frac{dV}{dt} \\ &= \frac{1}{H} \left(1 - \frac{H^*}{H} \right) \left(\frac{r_{11}}{c_1 + c_2 H} - r_{12} - b_1 H \right. \\ &\quad \left. - a_1(1 - m)P \right) H \\ &\quad + \frac{a_1(1 - m)^2 H^*}{a_2 P} \left(1 - \frac{P^*}{P} \right) \times \\ &\quad \left(r_2 - a_2 \frac{P}{(1 - m)H} \right) P \\ &= \frac{H - H^*}{H} \left(- \frac{r_{11}}{c_1 + c_2 H^*} + b_1 H^* \right. \\ &\quad \left. + a_1(1 - m)P^* + \frac{r_{11}}{c_1 + c_2 H} \right. \\ &\quad \left. - b_1 H - a_1(1 - m)P \right) \\ &\quad + \frac{a_1(1 - m)^2 H^*}{a_2 P} \left(1 - \frac{P^*}{P} \right) \times \\ &\quad \left(a_2 \frac{P^*}{(1 - m)H^*} - a_2 \frac{P}{(1 - m)H} \right) P \\ &= - \frac{b_1}{H} (H - H^*)^2 \\ &\quad + \frac{a_1(1 - m)}{H} (H - H^*)(P^* - P) \\ &\quad + \frac{H - H^*}{H} \frac{r_{11}(c_1 + c_2 H^* - c_1 - c_2 H)}{(c_1 + c_2 H^*)(c_1 + c_2 H)} \\ &\quad + a_1(1 - m)H^* \times \frac{P - P^*}{P} \times \\ &\quad \frac{P^* H - PH + PH - PH^*}{H^* H} \\ &= - \frac{b_1}{H} (H - H^*)^2 \\ &\quad + \frac{a_1(1 - m)}{H} (H - H^*)(P^* - P) \\ &\quad + \frac{H - H^*}{H} \frac{r_{11}c_2(H^* - H)}{(c_1 + c_2 H^*)(c_1 + c_2 H)} \\ &\quad - \frac{a_1(1 - m)}{P} (P - P^*)^2 \\ &\quad + \frac{a_1(1 - m)}{H} (H - H^*)(P - P^*) \\ &= - \frac{b_1}{H} (H - H^*)^2 - \frac{a_1}{P} (P - P^*)^2 \\ &\quad - \frac{r_{11}c_2}{H(c_1 + c_2 H^*)(c_1 + c_2 H)} (H^* - H)^2. \end{aligned} \tag{3.6}$$

Obviously, $\frac{dV}{dt} < 0$ strictly for all $H, P > 0$ except the positive equilibrium (H^*, P^*) , where $\frac{dV}{dt} = 0$. Thus, $V(H, P)$ satisfies Lyapunov's asymptotic stability theorem, and the positive equilibrium (H^*, P^*) of system (1.1) is globally stable. This ends the proof of Theorem 3.1.

4 The influence of density dependent birth rate

From Theorem 2.1 and 3.1, it seems that c_2 has no influence on the existence and stability property of the positive equilibrium. Now let us take a in-depth insight on this matter.

Noting that $B(H^*, P^*)$ satisfies the equation Under the assumption (3.1) holds, system (1.1) admits a unique positive equilibrium $B(H^*, P^*)$, which satisfies the equalities

$$\begin{aligned} \frac{r_{11}}{c_1 + c_2 H^*} - r_{12} - b_1 H^* - a_1(1 - m)P^* &= 0, \\ r_2 - a_2 \frac{P^*}{(1 - m)H^*} &= 0. \end{aligned} \tag{4.1}$$

From the second equation of (4.1), we could obtain

$$P^* = \frac{r_2(1 - m)H^*}{a_2}. \tag{4.2}$$

Substituting above equality into the first equation of (4.1), leads to

$$\begin{aligned} \frac{r_{11}}{c_1 + c_2 H^*} - r_{12} - b_1 H^* - a_1(1 - m) \frac{r_2(1 - m)H^*}{a_2} &= 0. \end{aligned} \tag{4.3}$$

Now let us denote

$$\begin{aligned} F(H^*, c_2) &= \frac{r_{11}}{c_1 + c_2 H^*} - r_{12} - b_1 H^* - a_1(1 - m) \frac{r_2(1 - m)H^*}{a_2}, \end{aligned}$$

then equation (4.3) can be rewrite in the form

$$F(H^*, c_2) = 0. \tag{4.4}$$

Since

$$\begin{aligned} \frac{\partial F}{\partial H^*} &= -\frac{r_{11}c_2}{(c_2 H^* + c_1)^2} - b_1 - \frac{a_1(1 - m)^2 r_2}{a_2} < 0, \end{aligned} \tag{4.5}$$

$$\frac{\partial F}{\partial c_2} = -\frac{r_{11}H^*}{(c_2 H^* + c_1)^2} < 0, \tag{4.6}$$

from (4.4)-(4.6) and the implicit function theorem, it immediately follows that

$$\frac{dH^*}{dc_2} = -\frac{F_{c_2}}{F_{H^*}} < 0. \tag{4.7}$$

(4.7) shows that H^* is the decreasing function of c_2 .

From (4.2) one could easily see that P^* is also the decreasing function of c_2 .

From (4.3) we could also draw an interesting finding, $H^* \rightarrow 0$ as $c_2 \rightarrow +\infty$. Otherwise, assume that there exists a $\delta > 0$ such that $H^* > \delta$ as $c_2 \rightarrow +\infty$. Then one could easily see that

$$\frac{r_{11}}{c_1 + c_2 H^*} \rightarrow 0 \text{ as } c_2 \rightarrow 0.$$

Consequently, $F(H^*, c_2) < 0$, which is contradict to equation (4.2).

Since we are interesting in the influence of density dependent birth rate, above analysis shows that with the increasing of c_2 . the density of both prey and predator are decreasing, and if c_2 is enough large, the final density of prey species will approach to zero, which increasing the extinction property of the prey species.

5 Numeric simulations

Now let's consider the following two examples.

Example 5.1

$$\begin{aligned} \frac{dH}{dt} &= \left(\frac{2}{1 + H} - 1 - H \right) H - 1 \cdot (1 - 0.5)HP, \end{aligned} \tag{5.1}$$

$$\frac{dP}{dt} = \left(1 - 1 \cdot \frac{P}{(1 - 0.5)H} \right) P,$$

where corresponding to system (1.1), we take $r_{11} = 2, c_1 = c_2 = r_{12} = b_1 = a_1 = r_2 = a_2 = 1, m = 0.5$, then,

$$r_{11} = 2 > 1 = c_1 r_{12},$$

hence, it follows from Theorem 3.1 that the unique positive equilibrium $B(0.3689, 0.1844)$ of system (5.1) is globally stable. Fig. 1 and 2 support this assertion.

Example 5.2

$$\begin{aligned} \frac{dH}{dt} &= \left(\frac{2}{1 + c_2 H} - 1 - H \right) H - 1 \cdot (1 - 0.5)HP, \end{aligned} \tag{5.2}$$

$$\frac{dP}{dt} = \left(1 - 1 \cdot \frac{P}{(1 - 0.5)H} \right) P,$$

where all the coefficients are the same as Example 5.1, only take c_2 as the variable coefficients, then,

$$r_{11} = 2 > 1 = c_1 r_{12},$$

it follows from Theorem 3.1 that the system (5.2) always admits a unique positive equilibrium $B(H^*, P^*)$, which is globally stable. Obviously, H^* and P^* are the function of c_2 . In this case, H^* satisfies the equation

$$\frac{2}{c_2 H^* + 1} - 1 - 1.25 H^* = 0.$$

Numeric simulation (Fig.3) shows that with the increasing of c_2 , H^* is decreasing and finally H^* is approach to zero.

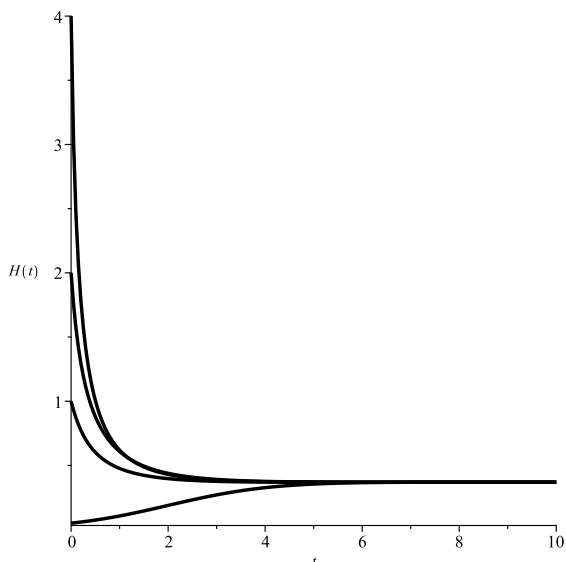


Figure 1: Dynamic behaviors of the first species in system (5.1), the initial condition $(H(0), P(0)) = (1.5, 1.5), (1.5, 0.3), (0.2, 0.1)$ and $(0.4, 1.5)$, respectively.

6 Discussion

Stimulated by the works of Chen et al[5], Chen et al[6] and Zhao et al[22], based on the model (1.2), we further incorporate the density dependent birth rate to the prey species, and this result in the system (1.1).

Our study shows that under some very nature assumption, more precisely, for the prey species, the birth rate is larger than the death rate, then the system could exits a unique positive equilibrium, which is globally stable. Obviously, if we assume that $c_1 = 1, c_2 = 0$, then system (1.1) is reduced to the system

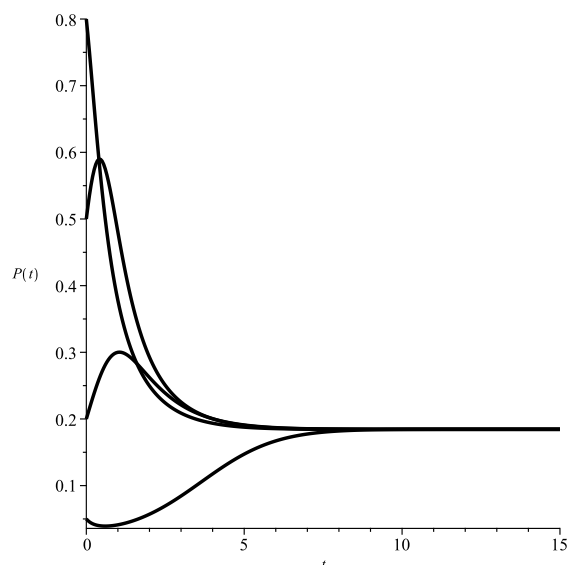


Figure 2: Dynamic behaviors of the second species in system (5.1), the initial condition $(H(0), P(0)) = (1.5, 1.5), (1.5, 0.3), (0.2, 0.1)$ and $(0.4, 1.5)$, respectively.

considered in [5], and Theorem 3.1 is degenerate to Theorem 2.1 in [5], it is in this sense, we generalize the main result of Chen et al[5].

It is curiously that Theorem 2.1 and 3.1 are independent of the coefficient c_2 , however, one could easily see that H^* is the implicit function of c_2 , our study shows that H^* and P^* are both the decreasing function of c_2 . Also, $H^* \rightarrow 0, P^* \rightarrow 0$ as $c_2 \rightarrow +\infty$. It is well known that if the amount of the species is less than a threshold, then, many endangered species will have Allee effect[10, 15, 23], which means that the population size will decrease if it is too sparse, this will enhance the possibility of the extinction of prey species.

To sum up, by introducing the density dependent birth rate of prey species, we show that generally speaking, the system could still be coexist in a stable state. However, with the increasing influence of the density dependent birth rate, the final density of both predator and prey species will reduced, and this may have negative effect on the long time survival of the prey and predator species.

References:

- [1] Chen F. D., Chen W. L., et al, Permanece of a stage-structured predator-prey system, *Appl. Math. Comput.*, Vol 219, No. 17, 2013, pp. 8856-8862.

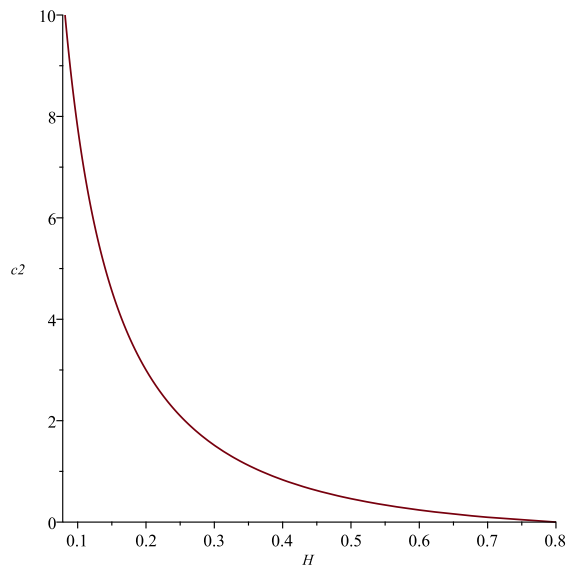


Figure 3: Relationship of H^* and c_2

- [2] Chen F. D., Xie X. D., et al, Partial survival and extinction of a delayed predator-prey model with stage structure, *Appl. Math. Comput.*, Vol. 219, No.8, 2012, pp. 4157-4162.
- [3] Chen F. D., Wang H. N., Lin Y. H. , Chen W. L., Global stability of a stage-structured predator-prey system, *Appl. Math. Comput.*, Vol. 223, No.1, 2013, pp. 45-53.
- [4] Chen F., Ma Z., Zhang H., Global asymptotical stability of the positive equilibrium of the Lotka-Volterra prey-predator model incorporating a constant number of prey refuges, *Nonlinear Analysis: Real World Applications*, Vol.13, No. 6, 2012, pp. 2790-2793.
- [5] Chen F., Chen L., Xie X., On a Leslie-Gower predator-prey model incorporating a prey refuge, *Nonlinear Analysis: Real World Applications*, Vol.10, No.5, 2009, pp. 2905-2908.
- [6] Chen F., Xue Y. , Lin Q., et al, Dynamic behaviors of a Lotka-Volterra commensal symbiosis model with density dependent birth rate, *Advances in Difference Equations*, Vol. 2018, 2018, Article ID 296.
- [7] Ma Z., Chen F., Wu C., et al, Dynamic behaviors of a Lotka-Volterra predator-prey model incorporating a prey refuge and predator mutual interference, *Applied Mathematics and Computation*, Vol.219, No.15, 2013, pp.7945-7953.
- [8] Yu S. B., Effect of predator mutual interference on an autonomous Leslie-Gower predator-prey model, *IAENG International Journal of Applied Mathematics*, Vol.49, No.2, 2019, pp.229-233.
- [9] Yu S., Almost periodic solution for a modified Leslie-Gower system with single feedback control, *IAENG International Journal of Applied Mathematics*, Vol.52, No.1, 2022, pp. 1-6.
- [10] Huang Y. , Zhu Z., Li Z., Modeling the Allee effect and fear effect in predator-prey system incorporating a prey refuge, *Advances in Difference Equations*, Vol. 2020, 2020, pp. 1-13.
- [11] Li Z., Han M., et al, Global stability of a predator-prey system with stage structure and mutual interference, *Discrete and Continuous Dynamical Systems-Series B (DCDS-B)*, Vol.19, No.1, 2014, pp. 173-187.
- [12] Lin X., Xie X. , et al, Convergences of a stage-structured predator-prey model with modified Leslie-Gower and Holling-type II schemes, *Advances in Difference Equations*, Vol. 2016, 2016, Article ID 181.
- [13] Xiao Z., Li Z., Zhu Z., et al. Hopf bifurcation and stability in a Beddington-DeAngelis predator-prey model with stage structure for predator and time delay incorporating prey refuge, *Open Mathematics*, Vol.17, No.1, 2019, pp.141-159.
- [14] Yue Q., Permanence of a delayed biological system with stage structure and density-dependent juvenile birth rate, *Engineering Letters*, Vol.27, No.2, 2019, pp.1-5.
- [15] Lv Y., Chen L., Chen F., Stability and bifurcation in a single species logistic model with additive Allee effect and feedback control, *Advances in Difference Equations*, Vol.2020, 2020, Article ID 129.
- [16] Lei C. Q., Dynamic behaviors of a stage structure amensalism system with a cover for the first species, *Advances in Difference Equations*, Vol. 2018, 2018, Article ID 272.
- [17] Wu R., Li L., Permanence and global attractivity of the discrete predator-prey system with Hassell Varley Holling III type functional response, *Discrete Dynamics in Nature and Society*, Volume 2013, 2013, Article ID 393729, 9 pages.
- [18] Xue Y., Xie X. , et al. Global attractivity and extinction of a discrete competitive system with infinite delays and single feedback control, *Discrete Dynamics in Nature and Society*, Volume 2018, 2018, Article ID 1893181, 14 pages.

- [19] Xue Y., Xie X., et al. Almost periodic solution of a discrete commensalism system, *Discrete Dynamics in Nature and Society*, Volume 2015, 2015, Article ID 295483, 11 pages.
- [20] Xie X., Xue Y., et al. Permanence and global attractivity of a nonautonomous modified Leslie-Gower predator-prey model with Holling-type II schemes and a prey refuge, *Advances in Difference Equations*, Vol. 2016, 2016, Article ID 184.
- [21] Xie X. D., Chen F. D. , et al, Note on the stability property of a cooperative system incorporating harvesting, *Discrete Dynamics in Nature and Society*, Volume 2014, 2014, Article ID 327823, 5 pages.
- [22] Zhao L., Qin B., Sun X. , Dynamic behavior of a commensalism model with nonmonotonic functional response and density-dependent birth rates, *Complexity*, Volume 2018, 2018, Article ID 9862584.
- [23] Lin Q., Stability analysis of a single species logistic model with Allee effect and feedback control, *Advances in Difference Equations*, Vol. 2018, 2018, Article ID 190.
- [24] Chen L. , Wang Y., et al, Influence of predator mutual interference and prey refuge on Lotka-Volterra predator-prey dynamics, *Communications in Nonlinear Science & Numerical Simulations*, Vol.18, No.11, 2013, pp.3174-3180.
- [25] He M. , Chen F., Extinction and stability of an impulsive system with pure delays, *Applied Mathematics Letters*, Vol. 91, No.2019, pp.128-136.
- [26] Wu R., Li L., Zhou X., A commensal symbiosis model with Holling type functional response, *Journal of Mathematics and Computer Science-JMCS*, Vol.16, No.3, 2016, pp.364-371.
- [27] Chen B., The influence of commensalism on a Lotka-Volterra commensal symbiosis model with Michaelis-Menten type harvesting, *Advances in Difference Equations*, Vol. 2019, 2019. pp. 1-14.
- [28] Chen B., Dynamic behaviors of a non-selective harvesting Lotka-Volterra amensalism model incorporating partial closure for the populations, *Advances in Difference Equations*, Vol. 2018, 2018, Article ID 111.
- [29] Walters C. , Christensen V. , Fulton B., et al., Predictions from simple predator-prey theory about impacts of harvesting forage fishes, *Ecological modelling*, Vol.337, No.2, 2016, pp.272-280.
- [30] Kang Y., Rodriguez-Rodriguez M. , Evlisor S., Ecological and evolutionary dynamics of two-stage models of social insects with egg cannibalism, *Journal of Mathematical Analysis and Applications*, Vol.430, No.1, 2015, pp. 324-353.
- [31] Zhang F. , Chen Y., Li J., Dynamical analysis of a stage-structured predator-prey model with cannibalism, *Mathematical Biosciences*, Vol. 307. No.1, 2019, pp. 33-41.
- [32] Basheer A., Quansah E., Bhowmick S. , et al., Prey cannibalism alters the dynamics of Holling-Tanner-type predator-prey models, *Nonlinear Dynamics*, Vol.85, No.4, 2016, pp. 2549-2567.
- [33] Basheer A., Parshad R. D., Quansah E., et al., Exploring the dynamics of a Holling-Tanner model with cannibalism in both predator and prey population, *International Journal of Biomathematics*, Vol. 11, No.01, 2018, Article ID 1850010.
- [34] Deng H., Chen F., Zhu Z., et al, Dynamic behaviors of Lotka-Volterra predator-prey model incorporating predator cannibalism, *Advances in Difference Equations*, Vol. 2019, 2019, Article ID 359.
- [35] Zou R., Guo S., Dynamics of a diffusive Leslie-Gower predator-prey model in spatially heterogeneous environment, *Discrete & Continuous Dynamical Systems-B*, Vol.25, No.11, Article ID 4189.
- [36] Leslie P. H., A stochastic model for studying the properties of certain biological systems by numerical methods, *Biometrika*, Vol. 45, No.1, 1958, pp.16-31.
- [37] Korobeinikov A., A Lyapunov function for Leslie-Gower predator-prey models, *Appl. Math. Lett.*, Vol. 14, No.6, 2001, pp. 697-699.
- [38] Mishra P. , Raw S. N., Tiwari R., Study of a Leslie-Gower predator-prey model with prey defense and mutual interference of predators, *Chaos, Solitons & Fractals*, Vol.120, No.1, 2019, pp. 1-16.
- [39] X. Wang, X. Tan, Y. Cai, et al, Impact of the fear effect on the stability and bifurcation of a Leslie-Gower predator-prey Model, *International Journal of Bifurcation and Chaos*, 2020, 30(14): 2050210.
- [40] Arancibia-Ibarra C., Flores J., Dynamics of a Leslie-Gower predator-prey model with Holling type II functional response, Allee effect and a generalist predator, *Mathematics and Computers in Simulation*, Vol.188, No.2021, pp. 1-22.

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Sijia Lin, Yanbo Chong wrote the draft.
Shangming Chen carried out the simulation.
Fengde Chen proposed the problem.

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

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