Global Stability of the Positive Equilibrium of a Leslie-Gower Predator-Prey Model Incorporating Predator Cannibalism

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Abstract: - A Leslie-Gower predator prey model with Holling II type cannibalism term on predator species is proposed and studied in this paper. By constructing a suitable Lyapunov function, we show that if the positive equilibrium exist, it is globally asymptotically stable. Our study indicates that suitable cannibalism has no influence on the persistent property of the system, however, cannibalism could reduce the final density of the predator species and increase the final density of the prey species. Excessive cannibalism may enhance the possibility of extinction to the predator species.

Key-Words: Leslie-Gower predator prey model; Cannibalism; Stability

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1 Introduction

The aim of this paper is to investigate the dynamic behaviors of the following Leslie-Gower predator prey model with predator cannibalism

$$\begin{aligned}\frac{dH}{dt} &= (r_1 - a_1 P - b_1 H)H,\\ \frac{dP}{dt} &= \Big(r_2 + c_1 - a_2\frac{P}{H}\Big)P - \frac{fP^2}{d+P},\end{aligned} \tag{1.1}$$

where H and P are the density of prey species and the predator species at time t, respectively. $r_i, i = 1, 2$ are the intrinsic growth rate of the prey and predator species, respectively. r_1/b_1 is the environment carrying capacity of the prey species, f is the cannibalism rate of predator species. $c_1 P$ is the new offsprings due to the cannibalism. Obviously, $c_1 < f$, since it takes depredation of a number of predator by the cannibal to produce one new offspring. Also, it is well known that the flow of energy decreases step by step along the food chain, hence, generally speaking, Only 10 to 20 percent of energy can flow into the next nutritional level, hence c_1 could be restrict to $c_1 \leq \frac{1}{5}f$.

During the last decades, numerous biological modelling were proposed and studied, see [1]-[38] and the references cited therein. Such topics as the influence of Allee effect([1],[2], [14], [17]-[25], the influence of refuge ([7], [13], [37]), the influence of stage structure ([9], [13], [32],[33],[36],[38]), the influence of harvesting ([3], [5],[16]), the influence of commensalism or ammensalism ([10]-[12]), the influence of cannibalism ([27]-[34]) have been extensively studied.

Recently, several scholars began to investigated the dynamic behaviors of the cannibalism, a behavior that consumes the same species and helps to provide food sources, see [27]-[34] and the references cited therein. Indeed, cannibalism often occurs in plankton, fishes, spideres[29] and social insect populations[30].

In 2016, Basheer et al.[30] proposed the prey-predator model with prey cannibalism as follows:

$$\begin{aligned}\frac{du}{dt} &= u(1 + c_1 - u)\\ &\quad -\frac{uv}{u+\alpha v} - c\frac{u^2}{u+d},\\ \frac{dv}{dt} &= \delta v\Big(\beta - \frac{v}{u}\Big),\end{aligned} \tag{1.2}$$

where $c_1 < c$, u and v represent the densities of prey and predator at time t, respectively. The parameters $c_1, \alpha, c, d, \delta$ and β are all nonnegative constants. Here the generic cannibalism term $C(u)$, is added in the prey equation, and is given by

$$C(u) = c \times u \times \frac{u}{u+d},$$

where c is the cannibalism rate. This term has a clear gain of energy to the cannibalistic prey, and this leads to the increase in reproduction in the prey, modeled via adding a $c_1 u$ term to the prey equation. It seems that this is the first time nonlinear cannibalism term were introduced. Previously, cannibalism term were presented in bilinear type ([29], [32]-[33]).

Recently, stimulated by the works of Basheer et al[30], based on the traditional Lotka-Volterra predator prey system, Deng et al[31] investigated the dynamic behaviors of the following predator-prey model with cannibalism for predator:

$$\begin{aligned} \frac{dx}{dt} &= x\Big(b-\alpha x-my\Big), \\ \frac{dy}{dt} &= y\Big(-\beta+c_1+nx\Big)-\frac{cy^2}{y+d}, \end{aligned} \tag{1.3}$$

where $c_1 < c$, x and y are the density of the prey and predator at time t, respectively. The authors showed that cannibalism has both positive and negative effect on the stability of the system, it depends on the dynamic behaviors of the original system.

On the other hand, also stimulated by the works of Basheer et al[30], Lin, Liu, Xie et al[34] proposed the following Leslie-Gower predator prey model with prey cannibalism

$$\begin{aligned} \frac{dH}{dt} &= (r_1+c_1-a_1P-b_1H)H \\ &\quad -\frac{fH^2}{d+H}, \\ \frac{dP}{dt} &= \Big(r_2-a_2\frac{P}{H}\Big)P, \end{aligned} \tag{1.4}$$

where H and P are the density of prey species and the predator species at time t, respectively. By applying the iterative method, the authors obtained a set of sufficient conditions which ensure the global attractive of the positive equilibrium. One may argued that system (1.4) seems much simple than system (1.2), and it is no need to investigated. Indeed, Basheer et al.[30] only investigated the local stability property of the system (1.2), and gave no information about the global stability property of the system. The study of Lin, Liu, Xie et al[34] can be seen as the effort on this direction. They tried to make some insight finding on the cannibalism.

Now, stimulated by the works of Basheer et al.[30], Deng et al[31] and Lin, Liu, Xie et al[34], it is natural to investigate the influence of predator cannibalism to Leslie-Gower predator prey system, this motivated us to propose the system (1.1).

The rest of the paper is arranged as follows. In next section, we will investigate the existence and local stability of the equilibrium of the system (1.1). In Section 3, we will discuss the global stability of the equilibrium by constructing some suitable Lyapunov function. Numeric simulations are presented in Section 4 to show the feasibility of the main results. We end this paper by a briefly discussion.

2 The existence and local stability of the equilibria of system (1.1)

Concerned with the existence of the equilibria of system (1.1), we have the following result.

Theorem 2.1. *System (1.1) admits the boundary equilibrium $A(\frac{r_1}{b_1},0)$; Assume that $c_1-f+r_2>0$, then system (1.1) admits a unique positive equilibrium $B(H^*,P^*)$, where*

$$\begin{aligned} H^* &= \frac{r_1-a_1P^*}{b_1}, \\ P^* &= \frac{-A_2+\sqrt{A_2^2-4A_1A_3}}{2A_1}, \\ A_1 &= (c_1-f+r_2)\,a_1+a_2b_1>0, \\ A_2 &= d\,(r_2+c_1)\,a_1 \\ &\quad +(f-c_1-r_2)\,r_1+da_2b_1, \\ A_3 &= -dr_1(r_2+c_1)<0. \end{aligned} \tag{2.1}$$

Proof. One could easily see that $A(\frac{r_1}{b_1},0)$ is the nonnegative solution of system (2.2). Next we show that there exists a unique positive equilibrium if $c_1-f+r_2>0$. The existence of positive equilibria of the system (1.1) is determined by the following equations:

$$\begin{aligned} r_1-a_1P-b_1H &= 0, \\ r_2+c_1-a_2\tfrac{P}{H}-\frac{fP}{d+P} &= 0. \end{aligned} \tag{2.2}$$

It follows from (2.2) that the prey isocline is l_1 : $r_1-a_1P-b_1H=0$, obviously, l_1 is monotonically decreases, which starts from the point $(0,\frac{r_1}{a_1})$ in the P-axis to the point $(\frac{r_1}{b_1},0)$ in the positive H-axis.

Now let us consider the predator isocline $l_2 : r_2+c_1-a_2\frac{P}{H}-\dfrac{fP}{d+P}=0$. l_2 has no definition on $H=0$, however, we could show that as $H\to 0$, $P\to 0$, indeed, equation $r_2+c_1-a_2\frac{P}{H}-\dfrac{fP}{d+P}=0$ is equivalent to

$$\begin{aligned} &a_2P^2+(Hf-c_1H-r_2H+da_2)P \\ &-Hd(d_1+r_2)=0. \end{aligned} \tag{2.3}$$

From (2.3) and the expression of the solution of quadratic equation, one could easily see that as $H\to 0$, the equation has a solution $P(H)\to 0$. This means that as $H\to 0$, l_2 is lies below the line l_1. On the other hand, now subs $H=\frac{r_1}{b_1}$ to $r_2+c_1-a_2\frac{P}{H}-\dfrac{fP}{d+P}=0$, we could obtain

$$\begin{aligned} &a_2b_1P^2+(a_2b_1d-c_1r_1+fr_1-r_2r_1)P \\ &-(c_1+r_2)r_1d=0. \end{aligned} \tag{2.4}$$

Since $a_2b_1>0, -(c_1+r_2)r_1d<0$, (2.4) has a unique positive solution $\overline{P}$. This means that as $H\to\frac{r_1}{b_1}$, line l_2 lies above the line l_1. Above analysis shows that l_1 intersect l_2 at least one point. On the other hand, l_2 define a function $P(H)$, by computation, we have

$$\frac{dP}{dH}=\frac{\left(Hfd+P^2a_2+2Pa_2d+a_2\,d^2\right)H}{\left(d+P\right)^2a_2P}>0. \tag{2.5}$$

Hence, l_2 is monotonically increases. Therefore, l_2 intersect l_1 at most one point. Above analysis shows that l_1 and l_2 intersect only one time, consequently, system (2.2) has unique positive solution.

From the first equation of (2.2), we have

$$H=\frac{r_1-a_1P}{b_1}. \tag{2.6}$$

Substituting it into the second equation of (2.2), and simplify, we could obtain the equation

$$A_1P^2+A_2P+A_3=0, \tag{2.7}$$

where A_1, A_2, A_3 are defined by (2.1). (2.7) has a unique positive solution P^*, hence, system (1.1) has the unique positive equilibrium $B(H^*,P^*)$.
This ends the proof of Theorem 2.1.

Theorem 2.2. *$A(\frac{r_1}{b_1},0)$ is unstable equilibrium, Assume that $c_1-f+r_2>0$, then $B(H^*,P^*)$ is locally asymptotically stable.*

Proof. The Jacobian matrix of the system (1.1) is calculated as

$$J(H,P)=\begin{pmatrix} A_{11} & -a_1H \\ \frac{P^2a_2}{H^2} & A_{22}\end{pmatrix}, \tag{2.8}$$

where

$$\begin{aligned} A_{11} &= -2\,Hb_1-Pa_1+r_1,\\ A_{22} &= -\frac{2a_2P}{H}+r_2+c_1\\ &\quad -\frac{2fP}{d+P}+\frac{f\,P^2}{(d+P)^2}.\end{aligned}$$

Then the Jacobian matrix of the system (1.1) about the equilibrium $A(\frac{r_1}{b_1},0)$ is

$$J(A(\frac{r_1}{b_1},0))=\begin{pmatrix} -r_1 & -\frac{a_1r_1}{b_1} \\ 0 & c_1+r_2\end{pmatrix}. \tag{2.9}$$

The eigenvalues of $J(A)$ are $\lambda_1=-r_1<0$, $\lambda_2=c_1+r_2>0$. Thus, $A(\frac{r_1}{b_1},0)$ is a saddle.

Noting that $B(H^*,P^*)$ satisfies the equation

$$\begin{aligned} r_1-a_1P^*-b_1H^* &= 0,\\ r_2+c_1-a_2\tfrac{P^*}{H^*}-\frac{fP^*}{d+P^*} &= 0.\end{aligned} \tag{2.10}$$

The Jacobian matrix of the system (1.1) about the equilibrium $B(H^*,P^*)$ is

$$\begin{aligned} & J(B(H^*,P^*))\\ =& \begin{pmatrix} -H^*b_1 & -a_1H^* \\ a_2\frac{(P^*)^2}{(H^*)^2} & B\end{pmatrix},\end{aligned} \tag{2.11}$$

where $B=-\frac{a_2P^*}{H^*}-\frac{fP^*}{d+P^*}+\frac{f\,(P^*)^2}{(d+(P^*))^2}$.

Then we have

$$\begin{aligned} & DetJ(B(H^*,P^*)\\ =& H^*b_1\Big(\frac{a_2P^*}{H^*}+\frac{fP^*}{d+P^*}-\frac{f\,(P^*)^2}{(d+(P^*))^2}\Big)\\ & +a_1H^*a_2\frac{(P^*)^2}{(H^*)^2}>0,\end{aligned}$$

and

$$\begin{aligned} & TrJ(B(H^*,P^*)|\\ =& -H^*b_1-\frac{dfH^*}{(d+H^*)^2}-\frac{a_2P^*}{H^*}\\ & -\frac{fP^*}{d+P^*}+\frac{f\,(P^*)^2}{(d+(P^*))^2}<0.\end{aligned}$$

So that both eigenvalues of $J(B(H^*,P^*))$ have negative real parts, and $B(H^*,P^*)$ is locally asymptotically stable.

This ends the proof of Theorem 2.2.

3 Global stability of the positive equilibrium

Concerned with the global stability property of the positive equilibrium, we have the following result.

Theorem 3.1. *The positive equilibrium $B(H^*,P^*)$ is globally attractive provide that*

$$c_1-f+r_2>0 \tag{3.1}$$

holds.

Proof. $B(H^*,P^*)$ satisfies the equalities

$$\begin{aligned} r_1-a_1P^*-b_1H^* &= 0,\\ r_2+c_1-a_2\frac{P^*}{H^*}-\frac{fP^*}{d+P^*} &= 0.\end{aligned} \tag{3.2}$$

We will adapt the idea of Chen, Chen and Xie[35] to prove Theorem 3.1. More precisely, we construct the following Lyapunov function:

$$V(H,P) = \ln\frac{H}{H^*}+\frac{H^*}{H} + \frac{a_1H^*}{a_2}\Big(\ln\frac{P}{P^*}+\frac{P^*}{P}\Big). \tag{3.3}$$

Obviously, $V(H,P)$ is well defined and continuous for all $H,P>0$. By simple computation, we have

$$\begin{aligned}\frac{\partial V}{\partial H} &= \frac{1}{H}\Big(1-\frac{H^*}{H}\Big),\\ \frac{\partial V}{\partial P} &= \frac{a_1H^*}{a_2P}\Big(1-\frac{P^*}{P}\Big).\end{aligned} \tag{3.4}$$

(3.4) shows that the positive equilibrium (H^*,P^*) is the only extremum of the function $V(H,P)$ in the positive quadrant. One could easily verifies that

$$\begin{aligned}&\lim_{H\to 0}V(H,P)=\lim_{P\to 0}V(H,P)\\ =\ &\lim_{H\to+\infty}V(H,P)=\lim_{P\to+\infty}V(H,P)=+\infty.\end{aligned} \tag{3.5}$$

(3.4) and (3.5) show that the positive equilibrium (H^*,P^*) is the global minimum, that is,

$$V(H,P)>V(H^*,P^*)=1+\frac{a_1H^*}{a_2}>0$$

for all $H,P>0$.

Calculating the derivative of V along the solution of the system (1.1), by using equalities (3.2), we have

$$\begin{aligned}
&\frac{dV}{dt}\\
=\ &\tfrac{1}{H}\Big(1-\tfrac{H^*}{H}\Big)\Big(r_1-a_1P-b_1H\Big)H\\
&+\frac{a_1H^*}{a_2P}\Big(1-\frac{P^*}{P}\Big)\cdot\Big(r_2+c_1\\
&-a_2\tfrac{P}{H}-\frac{fP}{d+P}\Big)P\\
=\ &\frac{H-H^*}{H}\Big(a_1P^*+b_1H^*-a_1P-b_1H\Big)\\
&+\frac{a_1H^*}{a_2}\cdot\Big(1-\frac{P^*}{P}\Big)\cdot\Big(a_2\frac{P^*}{H^*}+\frac{fP^*}{d+P^*}\\
&-a_2\tfrac{P}{H}-\frac{fP}{d+P}\Big)\\
=\ &-\frac{b_1}{H}(H-H^*)^2+\frac{a_1}{H}(H-H^*)(P^*-P)\\
&+a_1H^*\cdot\frac{P-P^*}{P}\cdot\frac{P^*H-PH+PH-PH^*}{H^*H}\\
&+\frac{a_1H^*}{a_2}\cdot\Big(1-\frac{P^*}{P}\Big)\cdot\Big(\frac{fP^*}{d+P^*}-\frac{fP}{d+P}\Big)\\
=\ &-\tfrac{b_1}{H}(H-H^*)^2\\
&+\frac{a_1}{H}(H-H^*)(P^*-P)\\
&-\frac{a_1}{P}(P-P^*)^2\\
&+\tfrac{a_1}{H}(H-H^*)(P-P^*)\\
&+\frac{a_1H^*}{a_2}\frac{P-P^*}{P}\frac{fd(P^*-P)}{(d+P)(d+P^*)}\\
=\ &-\frac{b_1}{H}(H-H^*)^2-\frac{a_1}{P}(P-P^*)^2\\
&-\frac{fd}{H}\frac{(P^*-P)^2}{(d+P)(d+P^*)}.
\end{aligned} \tag{3.6}$$

Obviously, $\frac{dV}{dt}<0$ strictly for all $H,P>0$ except the positive equilibrium (H^*,P^*), where $\frac{dV}{dt}=0$. Thus, $V(H,P)$ satisfies Lyapunov's asymptotic stability theorem, and the positive equilibrium (H^*,P^*) of system (1.1) is globally stable. This ends the proof of Theorem 3.1.

4 The influence of cannibalism

Let

$$\begin{aligned}F(H^*,P^*,c_1,f) &= r_1-a_1P^*-b_1H^*,\\ G(H^*,P^*,c_1,f) &= r_2+c_1-a_2\tfrac{P^*}{H^*}\\ &\quad -\frac{fP^*}{d+P^*},\end{aligned}$$

then rewrite equation (3.2) in the form

$$\begin{cases}F(H^*,P^*,c_1,f) = 0,\\ G(H^*,P^*,c_1,f) = 0.\end{cases} \tag{4.1}$$

By simple computation, we have

$$\begin{aligned}J &= \frac{D(F,G)}{D(H^*,P^*)}\\ &= \begin{vmatrix} F_{H^*} & F_{P^*}\\ G_{H^*} & G_{P^*}\end{vmatrix}\\ &= \begin{vmatrix} -b_1 & -a_1\\ \dfrac{a_2P^*}{(H^*)^2} & -E_1\end{vmatrix}\\ &= b_1E_1+\frac{a_1a_2P^*}{(H^*)^2}>0.\end{aligned} \tag{4.2}$$

where

$$C_1=\frac{a_2}{H^*}+\frac{f}{d+P^*}-\frac{fP^*}{(d+P^*)^2}.$$

Using implicit function set theorem, the equation (4.1) has an unique solution in the neighborhood of $B(H^*, P^*)$

$$H^* = H^*(c_1, f), \quad P^* = P^*(c_1, f). \tag{4.3}$$

and

$$\begin{aligned} \frac{\partial H^*}{\partial c_1} &= -\frac{1}{J}\frac{D(F,G)}{D(c_1,P^*)}, \\ \frac{\partial P^*}{\partial c_1} &= -\frac{1}{J}\frac{D(F,G)}{D(H^*,c_1)}, \end{aligned} \tag{4.4}$$

$$\begin{aligned} \frac{\partial H^*}{\partial f} &= -\frac{1}{J}\frac{D(F,G)}{D(f,P^*)}, \\ \frac{\partial P^*}{\partial f} &= -\frac{1}{J}\frac{D(F,G)}{D(H^*,f)}. \end{aligned} \tag{4.5}$$

By computation, we have

$$\begin{aligned} & \frac{D(F,G)}{D(c_1,P^*)} \\ = & \begin{vmatrix} F_{c_1} & F_{P^*} \\ G_{c_1} & G_{P^*} \end{vmatrix} \\ = & \begin{vmatrix} 0 & -a_1 \\ 1 & -\frac{a_2}{H^*} - \frac{f}{d+P^*} + \frac{fP^*}{(d+P^*)^2} \end{vmatrix} \\ = & \ a_1 > 0, \\ & \frac{D(F,G)}{D(H^*,c_1)} \\ = & \begin{vmatrix} F_{H^*} & F_{c_1} \\ G_{H^*} & G_{c_1} \end{vmatrix} \\ = & \begin{vmatrix} -b_1 & 0 \\ \frac{a_2P^*}{(H^*)^2} & 1 \end{vmatrix} \\ = & -b_1 < 0, \\ & \frac{D(F,G)}{D(f,P^*)} \\ = & \begin{vmatrix} F_f & F_{P^*} \\ G_f & G_{P^*} \end{vmatrix} \\ = & \begin{vmatrix} 0 & -a_1 \\ -\frac{P^*}{d+P^*} & -\frac{a_2}{H^*} - \frac{f}{d+P^*} + \frac{fP^*}{(d+P^*)^2} \end{vmatrix} \\ = & -\frac{a_1P^*}{d+P^*} < 0, \end{aligned}$$

$$\begin{aligned} & \frac{D(F,G)}{D(H^*,f)} \\ = & \begin{vmatrix} F_{H^*} & F_f \\ G_{H^*} & G_f \end{vmatrix} \\ = & \begin{vmatrix} -b_1 & 0 \\ \frac{a_2P^*}{(H^*)^2} & -\frac{P^*}{d+P^*} \end{vmatrix} \\ = & \frac{b_1P^*}{P^*(d+P^*)} > 0, \end{aligned} \tag{4.6}$$

thus,

$$\begin{aligned} \frac{\partial H^*}{\partial c_1} &= -a_1\frac{1}{J} < 0, \\ \frac{\partial P^*}{\partial c_1} &= \frac{1}{J}b_1 > 0, \\ \frac{\partial H^*}{\partial f} &= \frac{1}{J}\frac{a_1P^*}{d+P^*} > 0, \\ \frac{\partial P^*}{\partial f} &= -\frac{1}{J}\frac{b_1P^*}{P^*(d+P^*)} < 0. \end{aligned} \tag{4.7}$$

Above analysis shows that both H^* is the decreasing function of c_1 and increasing function of f, while P^* is the increasing function of c_1 and the decreasing function of f. Noting the fact $c_1 < f$, which can be seen that the cannibalism has negative effect on the final density of the predator species, while it has the positive effect on the density of the prey species. Such a result seems naturally, since cannibalism can be seen as the predator has other food resource, and this reduce the direct predating of prey species. However, cannibalism means that the predator species take itself as food resource, this certainly has negative effect on predator species.

5 Numeric simulations

Now let's consider the following two examples.

Example 5.1

$$\begin{aligned} \frac{dH}{dt} &= \Big(1-H\Big)H - HP, \\ \frac{dP}{dt} &= \Big(1 - 1\cdot\frac{P}{H} + 0.1P\Big)P \\ & \quad -0.5\frac{P^2}{1+P}, \end{aligned} \tag{5.1}$$

where corresponding to system (1.1), we take $r_1 = b_1 = a_1 = r_2 = a_2 = 1, f = 0.5, c_1 = 0.1$, then,

$$r_2 + c_1 - f = 0.6 > 0,$$

hence, it follows from Theorem 4.1 that the unique positive equilibrium $B(0.55, 0.45)$ of system (5.1) is

globally stable. Fig. 1 and 2 support this assertion.
Example 5.2

$$\begin{aligned}\frac{dH}{dt} &= \Big(1-H\Big)H - HP,\\ \frac{dP}{dt} &= \Big(1 - 1\cdot\frac{P}{H} + 0.2fP\Big)P \\ &\quad -f\frac{P^2}{1+P},\end{aligned} \qquad (5.2)$$

where all the coefficients are the same as Example 5.1, only take f as the variable coefficients, also, choose $c_1 = 0.2f$, then, if

$$r_2 + c_1 - f = r_2 - 0.8f = 1 - 0.8f > 0,$$

i.e.,

$$f < 1.25,$$

it follows from Theorem 3.1 that the system (5.2) always admits a unique positive equilibrium $B(H^*, P^*)$, which is globally stable. Obviously, H^* and P^* are the function of f. In this case, P^* satisfies the equation

$$1 - \frac{P^*}{1-P^*} - 0.8fP^* = 0.$$

Numeric simulation (Fig.3) shows that with the increasing of f, P^* is decreasing and finally P^* is approach to zero.

6 Discussion

Recently, Deng et al [31] incorporated the Basheer type cannibalism [30] to the traditional Lotka-Volterra predator prey system, this led to the system (1.3). They showed that if system (1.3) admits the positive equilibrium, then the equilibrium is globally stable. On the other hand, Lin et al[34] also incorporated the Basheer type cannibalism [30] to the prey species in Leslie-Gower predator prey system, by applying the iterative method, they also obtained a set of sufficient conditions which ensure the globally attractive of positive equilibrium of the system. Stimulated by their works, we incorporating predator cannibalism to the Leslie-Gower predator prey system, this leads to system (1.1).

Noting that condition (3.1) is enough to ensure the existence of the positive equilibrium of system (1.1), and the proof of Theorem 3.1 is independent of the condition (3.1), hence, we can draw the conclusion: Once system (1.1) admits a unique positive equilibrium, it is globally stable. Such a property is similar to the traditional Leslie Gower predator prey system[35].

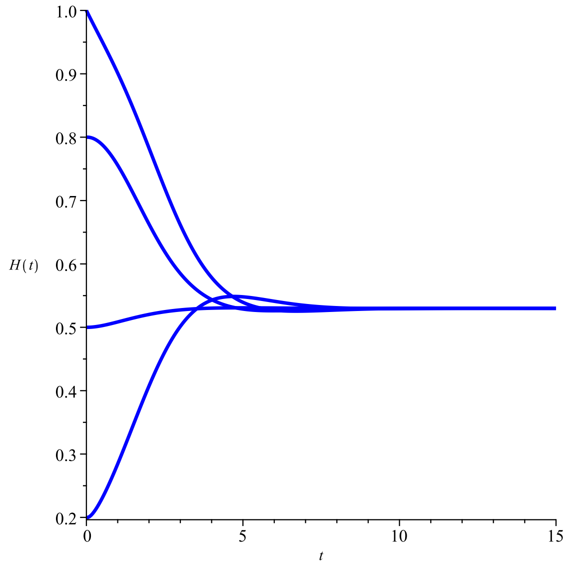

Figure 1: Dynamic behaviors of the first species in system (4.1), the initial condition $(H(0), P(0)) = (1.5, 1.5)$, $(1.5, 0.3)$, $(0.2, 0.1)$ and $(0.4, 1.5)$, respectively.

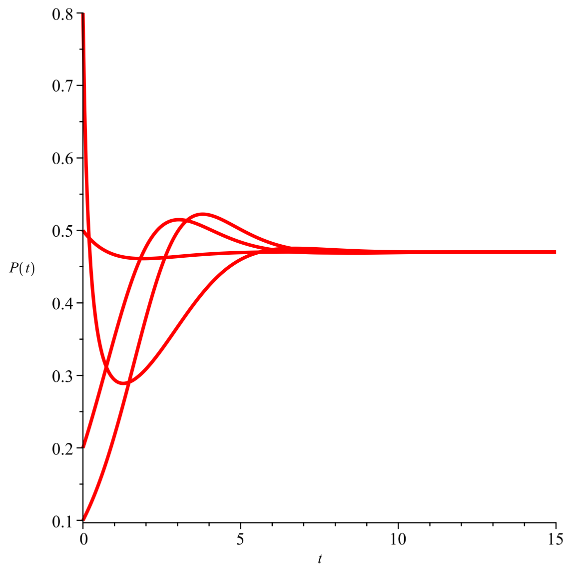

Figure 2: Dynamic behaviors of the second species in system (4.1), the initial condition $(H(0), P(0)) = (1.5, 1.5)$, $(1.5, 0.3)$, $(0.2, 0.1)$ and $(0.4, 1.5)$, respectively.

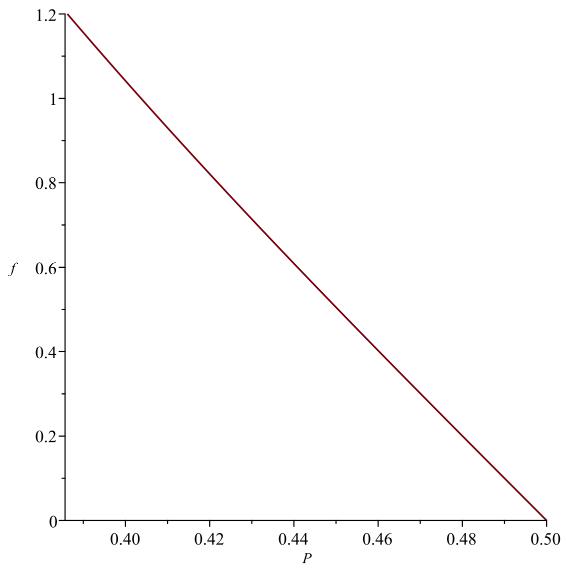

Figure 3: Relationship of P^* and f in system (5.2).

Our study also indicates that the cannibalism of predator species has negative effect on the predator species and positive effect on the prey species, since with the increasing of cannibalism, the final density of predator species is reduced and the final density of prey species is increasing.

We would like to mention here that to this day, still seldom did scholars investigate the dynamic behaviors of the nonautonomous cannibalism predator prey model, we will do some study on this direction in the future.

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