Lotka-Volterra Model with Periodic Harvesting

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Abstract. A closed interaction of predator prey is considered. The interaction is expressed in the Lotka-Volterra model. Two types of Lotka-Volterra models are considered, with and without carrying capacity of the prey. The paper includes a periodic harvesting of predator and/or prey, a function of time which acts to the model. Hence, the model is in the form of a system of non-homogeneous equations. Dynamical properties of the models are investigated. The solutions are computed numerically. Such interaction is in the need of integrated farming on harvesting of predator and/or prey. In this model the number of population in the system is sensitive to the initial value, which can be applied to the integrated farming systems such that the system remains sustainable.

Key-Words: Dynamical properties, integrated farming, Lotka-Volterra model, prey-predator.

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1 Introduction

After the model was introduced by Lotka, [1], and Volterra [2], it attracted many researchers which resulted in many papers. Although Lotka used the model for studying a hypothetical chemical reaction where the chemical concentrations oscillate, and Volterra proposed the model to explain the increase of predator fish in the Adriatic Sea, currently it becomes a standard model, and the simplest model of predator-prey interaction. Often, it is discussed in standard textbooks of ordinary differential equations and mathematical modeling, e.g., [3], [5], [5]. For three population models, the effect on the population dynamics because of these parametric changes in the systems was studied within invariant surfaces and, in terms of stability, about equilibrium points, [6].

In general, the model is in the interest of ecology. The Nicholson formula which is derived from Lotka-Volterra used to determine the area of discovery if the percentage parasitism and adult parasite density are known, to construct as competition curve with relates the percentage parasitism to the fraction of the area covered by the parasites and as a component in a population model in which it represents parasite action, [7]. The Lotka-Volterra model also expresses the tropic interaction which considers the feeding rate as directly proportional to the product of the magnitudes of consumer supply and food supply, [8]. On the other hand, Lotka-Volterra assumed that the response of the populations would be proportional to the product of their biomass densities, [9].

However, it is also in the interest of theoretical physics where Lotka-Volterra method was regarded as the first principle in deriving hydrodynamic equations of motion from the equations of motion of the constituent particles, [10]. Advancement of the model has been conducted by considering several mathematical and physical aspects. Taylor and Crizer's modified Lotka-Volterra model would be better than the classic model if in a biological situation the population had a non-linear effect on each other, [11]. Modified Lotka-Volterra competition model with a non-linear relationship between species, crowding effect has also been considered. Under the Lotka-Volterra competition equation with a nonlinear weak crowding effect, a stable coexistence of many species is plausible, [12], [13].

The effect of diffusion on the model has been discussed in papers, among others, [14], [15], [16]. Diffusion can make the system persistent regardless of the patch dynamics without diffusion.

More recently, Slavik, [17], considered the Lotka-Volterra model on graphs, if both species can move along the edges of the same connected graph G. In a more general model, we might consider two different connected graphs G1, G2, one for each species.

This paper considers a Lotka-Volterra model with a source term. The source term is applied to one variable of the model or both variables. Considering the Lotka-Volterra model as the interaction model of predator-prey, the source term may act as a harvesting scenario that applied merely to the prey, to the predator, or both. The right source term is important for optimal harvesting of the predator and/or the prey.

Understanding of the Lotka-Volterra model is very important for applications in integrated farming systems, especially to determine maximum production and optimality of the system, for example in an integrated farming system consisting of vegetables as prey and fish as predators, [18]. The sustainability of integrated farming systems is interesting to discuss. It is mainly because of the diversity of species and the potential for synergy from integrating crops with livestock. However, the ability of this system to maximize food production has not been widely discussed in the literature and needs to be explored further, [19].

2 Lotka-Volterra Model

In this section a Lotka-Volterra model without carrying capacity will be discussed. Let x, y be functions of time t that represent the populations of prey and predator, respectively. Lotka-Volterra model, then, is given in the form [4]

$$\begin{aligned} \frac{dx}{dt} &= \alpha x - \beta xy \\ \frac{dy}{dt} &= \delta xy - \gamma y \end{aligned} \tag{1}$$

Parameter α represents the intrinsic growth rate of prey, β represents the rate at which predators destroy prey, δ is parameter for the rate at which predator population increases by consuming prey, and γ is the death rate of predators.

System (1) has two critical points, i.e., the trivial equilibrium $E_0 = (0,0)$, and nontrivial equilibrium $E_1 = \left(\frac{\gamma}{\delta}, \frac{\alpha}{\beta}\right)$. On the other hand, the Jacobian matrix of system (1) is

$$J(x,y) = \begin{bmatrix} -\beta y + \alpha & -\beta x \\ \delta y & \delta x - \gamma \end{bmatrix} \tag{2}$$

The Jacobian matrix evaluated at E_0 has characteristic equation

$$-(\alpha - \lambda)(\gamma + \lambda) = 0, \tag{3}$$

which gives eigenvalues $\lambda_1 = \alpha$, and $\lambda_2 = -\gamma$. Since α and γ are greater than 0, this implies that E_0 is not stable. In fact, E_0 it is a saddle point.

For equilibrium point E_1, the Jacobian matrix is in the form

$$J(E_1) = \begin{bmatrix} 0 & \frac{-\beta\gamma)}{\delta} \\ \frac{\delta\alpha}{\beta} & 0 \end{bmatrix}. \tag{4}$$

The Jacobian $J(E_1)$ has characteristic equation

$$\lambda^2 + \alpha\gamma = 0. \tag{5}$$

That gives the eigen value $\lambda_3 = -\sqrt{-\alpha\gamma}$ and $\lambda_4 = \sqrt{-\alpha\gamma}$. Since α and γ are greater than 0, so the eigen values are imaginary numbers, and E_1 is a stable point.

It now will evaluate the trajectory of points governed by system (1). Applying basic calculus by eliminating variable t from (1) one gets

$$\frac{dy}{dx} = \frac{\delta xy - \gamma y}{\alpha x - \beta xy}. \tag{6}$$

Solving (6), one has

$$V = \delta x - \gamma \ln x + \beta y - \alpha \ln y \tag{7}$$

where V is any constant.

Fig. 1 shows the graphics of (7) in $x - y$ Cartesian coordinate for various values of V. The parameters $\alpha = 2$, $\beta = 3$, $\delta = 2$, $\gamma = 1.5$ have been applied. There are three curves of system (7) representing $V = 5, 6, 8$. Each curve represents a trajectory of point $(x(t), y(t))$ for $t \geq 0$. Each trajectory is called an orbit. The orbits are closed curves. It is observed that the larger the value of V, the larger curve.

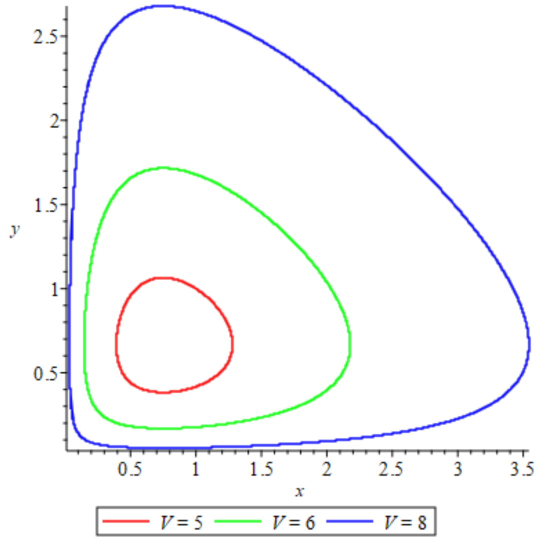

Fig. 1: Graphics of (7), $V = 5, 6, 8$.

For further issue, the evolution of x and y corresponds to the value of V = 5, 6, 8 as shown in Fig. 1 will be presented. Since the analytical solutions of x and y are not straightforward, it will be shown the numerical solutions. To do so, an initial condition i.e., pairs of $(x(0), y(0))$ should be chosen. These pairs must satisfy (7) for corresponding value of V. Table 1 shows the pairs with corresponding value V.

Table 1.Value of $x(0)$ and $y(0)$for corresponding V

V	x (0)	y (0)
5	0.6	0.3994110349
6	1	0.17630435981
8	1.5	0.06696007606

Applying initial values as presented in Table 1, the solutions of x with respect to variable t for various value of V are presented in Fig. 2.

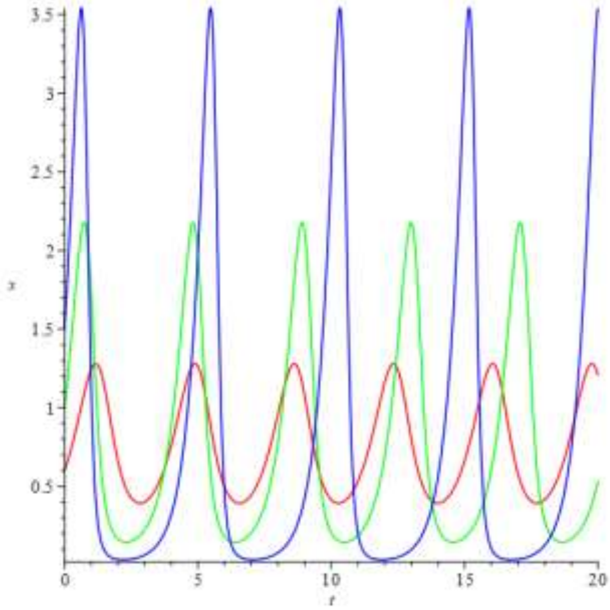

Fig. 2: Solution curves for x of system (1), where $V = 5$ (red), $V = 6$ (green), $V = 8$ (blue).

Similarly, the solutions of y with respect to variable t for various values of V are presented in Fig. 3.

Based on Fig. 2 and Fig. 3, some observations can be made. The larger value of V implies the larger the amplitude of x and y. Similarly, the larger the value of V, the larger the period of x and y. Moreover, the period of x and y are the same, for the same value of V.

For $V = 5$ it is observed that the amplitude of x is approximately 3.5, while the one of y is around 2.7. For $V = 6$, the amplitude of x is 2.2, on the other hand the one of y is around 1.7. Finally, for $E = 8$, the amplitude of x is 1.7 and the amplitude of y is about 1.1.

For $V = 5$ it is observed that the periods of x and y are around 3.72. This means that point $(x(t), y(t))$ requires 3.72 units of time to complete one cycle of the red curve of Fig. 1. For $V = 6$, the period of x and y is approximately 4.08 units of time. Finally, for $E = 8$, the period of x and y is about 4.8 units of time.

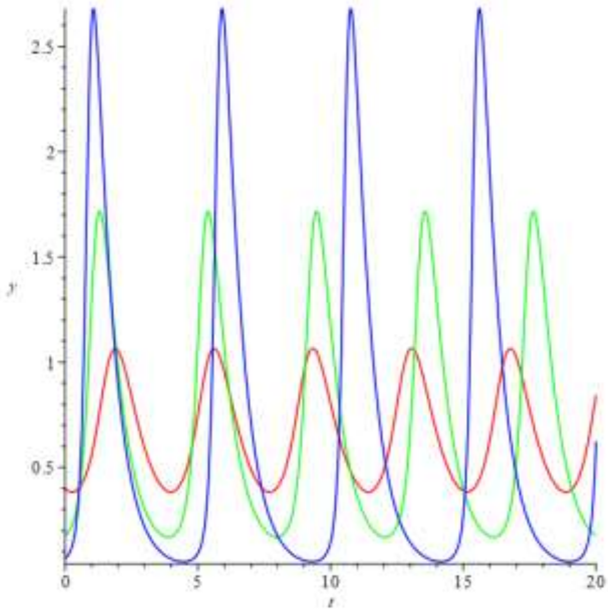

Fig. 3: Solution curves for y of system (1), where $V = 5$ (red), $V = 6$ (green), $V = 8$ (blue).

Period of x as prey and y as predator in this model may provide information when the population prey and predator are at its maximum and minimum. Hence, this may be applied to predict the optimal harvesting.

3 Lotka-Volterra Model with Carrying Capacity

Carrying capacity is the ability of an ecosystem to support the life of organisms in it in a sustainable manner. Or in other words, the carrying capacity is the upper limit of population growth, because the population can no longer be supported by existing facilities, resources, and environment. The Lotka-Volterra model in the previous section assumes that the environment provides unlimited resources for prey growth. In fact, they must compete among themselves for the resources. This underlies the addition of the environmental carrying capacity factor to the Lotka-Volterra model in this section.

Lotka-Volterra model with carrying capacity is in the form

$$\begin{aligned} \frac{dx}{dt} &= \alpha x - \beta xy - \mu x^2 \\ \frac{dy}{dt} &= \delta xy - \gamma y \end{aligned} \tag{8}$$

with μ is a parameter related to the environment carrying capacity of the prey population.

System (8) has three equilibrium points, i.e., the trivial equilibrium $E_2 = (0,0)$, and two nontrivial equilibriums $E_3 = \left(\frac{\alpha}{\mu}, 0\right)$ and $E_4 = \left(\frac{\gamma}{\delta}, \frac{\alpha\delta - \gamma\mu}{\beta\delta}\right)$.

The Jacobian of system (8) is

$$J = \begin{bmatrix} -\beta y - 2\mu x + \alpha & -\beta x \\ \delta y & \delta x - \gamma \end{bmatrix} \tag{9}$$

The Jacobian matrix (9) evaluated at E_2 has characteristic equation

$$-(\gamma + \lambda)(\alpha - \lambda) = 0 \tag{10}$$

which gives eigenvalues $\lambda_5 = -\gamma$ and $\lambda_6 = \alpha$. Since α and γ are greater than 0, this implies that E_2 is not stable. It is a saddle point.

Evaluating the Jacobian matrix (9) at equilibrium point E_3,it has characteristic equation

$$-\frac{(\alpha + \lambda)(\alpha\delta - \gamma\mu - \lambda\mu)}{\mu} = 0 \tag{11}$$

Equation (11) gives eigenvalues $\lambda_7 = -\alpha$ and $\lambda_8 = \frac{\alpha\delta - \gamma\mu}{\mu}$. Since $\alpha > 0$, it implies $\lambda_7 < 0$. But there are three cases for λ_8:
Case 1: $\mu < \frac{\alpha\delta}{\gamma}$.
Case 2: $\mu = \frac{\alpha\delta}{\gamma}$.
Case 3: $\mu > \frac{\alpha\delta}{\gamma}$.

Case 1, $\mu < \frac{\alpha\delta}{\gamma}$ implies $\lambda_8 > 0$. Hence, the equilibrium E_3 is not stable. On the other hand, Case 2, $\mu = \frac{\alpha\delta}{\gamma}$ implies $\frac{\alpha}{\mu} = \frac{\gamma}{\delta}$ and $\frac{\alpha\delta - \gamma\mu}{\beta\delta} = 0$. Therefore, equilibrium E_4 is nothing else, but the equilibrium E_3, meaning that E_3 and E_4 are the same points. Moreover, $\lambda_8 = 0$. Case 3 will be discussed later.

Evaluating Jacobian matrix (9) at E_4, it has characteristics equation

$$\frac{\alpha\delta\gamma + \lambda^2\delta - \gamma^2\mu + \mu\gamma\lambda}{\delta} = 0 \tag{12}$$

where the roots are the eigen values

$$\lambda_9 = \frac{-\mu\gamma + \sqrt{-4\alpha\delta^2\gamma + 4\delta\gamma^2\mu + \mu^2\gamma^2}}{2\delta}, \quad \text{and}$$

$$\lambda_{10} = \frac{-\mu\gamma - \sqrt{-4\alpha\delta^2\gamma + 4\delta\gamma^2\mu + \mu^2\gamma^2}}{2\delta}.$$

Since $\alpha, \delta, \mu, \gamma > 0$, base on the value of $\mathrm{Re}(\lambda_9)$ and $\mathrm{Re}(\lambda_{10})$, there are 3 cases to consider:
Case 1: $\mu < \frac{\alpha\delta}{\gamma}$.
Case 2: $\mu = \frac{\alpha\delta}{\gamma}$.
Case 3: $\mu > \frac{\alpha\delta}{\gamma}$.

Case 1, $\mu < \frac{\alpha\delta}{\gamma}$ implies that $\mathrm{Re}(\lambda_9) < 0$ and $\mathrm{Re}(\lambda_{10}) < 0$. Hence, E_4 it is a stable point. Case 2 results in the collapse of E_4 into E_3. Moreover, $\lambda_9 = 0$ and $\lambda_{10} = -\frac{\mu\gamma}{\delta} = -\alpha < 0$. Observe that the eigen values are similar to the ones of Case 2 of E_3. Investigate is stable point. Finally, Case 3 $\mu > \frac{\alpha\delta}{\gamma}$ implies $\frac{\alpha\delta - \gamma\mu}{\beta\delta} < 0$ meaning that E_4 lying in the fourth quadrant. Hence, the Case 3 is out of discussion of this paper, since *x* and *y* must be non-negative.

For case 1 $\mu < \frac{\alpha\delta}{\gamma}$, the orbit of model (8) cannot be obtained analytically as straight forward as for the previous model. Hence, it will be presented numerically. Fig. 4 shows the phase portrait of model (8) for the case $\mu < \frac{\alpha\delta}{\gamma}$. Initial values are presented in Table 2, and the parameters are $\mu = 0.3$, $\alpha = 2$, $\beta = 3$, $\delta = 2$, $\gamma = 1.5$. Orbit of initial value (1.5,0.066) is red, of (1.7,0.1) is green, and of (2,0.4) is blue. Observe that all orbits tend to E_4.

Table 2. Initial value

	x(0)	*y*(0)
Initial 1	1.5	0.066
Initial 2	1.7	0.1
Initial 3	2	0.4

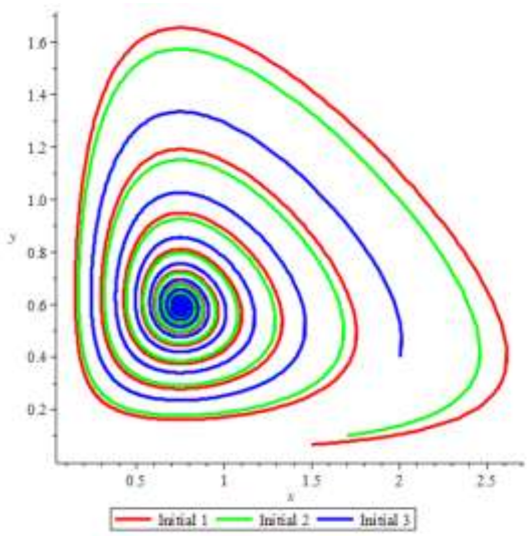

Fig. 4: Phase portrait system (8)

Fig. 5 shows the solution of x corresponds to the orbits of the same color as in Fig. 4. All solutions tend to the same value, i.e., x about 0.8.

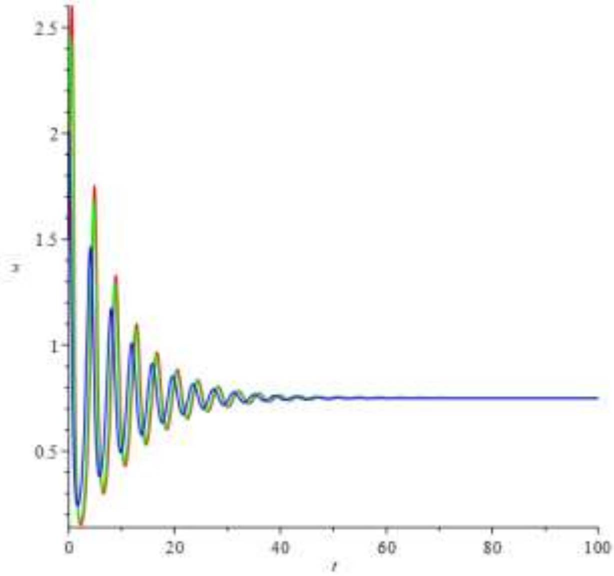

Fig. 5: Solution curves of x of system (8) correspond to the orbits in Fig. 4.

Fig. 6 shows the solution of y corresponds to the orbits of the same color as in Fig. 4. All solutions tend to the same value, i.e., y about 0.6.

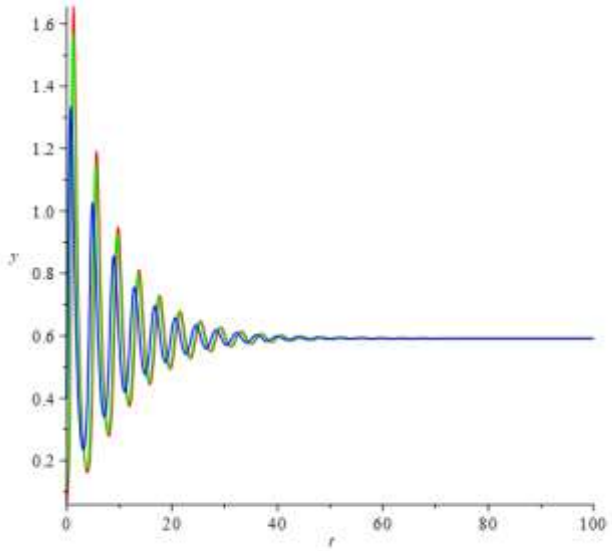

Fig. 6: Solution curves of y of system (8) correspond to the orbits in Fig. 4.

Phase portrait for case 2, $\mu = \frac{8}{3}$, is presented in Figure 7. The initial values of the orbits are presented in Table 2. Orbit of initial value (1.5, 0.066) is red, of (1.7,0.1) is green, and of (2,0.4) is blue. Observe that all orbits tend to E_3 which is the same point as E_4.

Fig. 8 shows solutions of x (green) and y (red) where the initial condition is (2,0.4). In this case, x tends to be a carrying capacity parameter, while y becomes extinct.

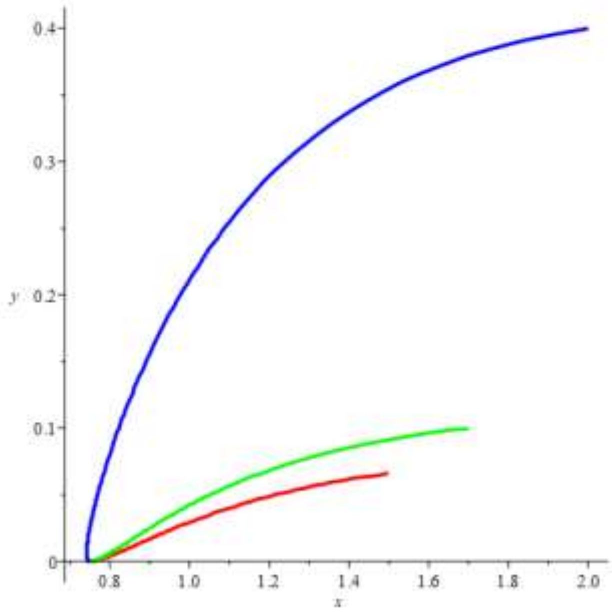

Fig. 7: Phase portrait for $\mu = \frac{\alpha\delta}{\gamma}$

The solution of x (red) and y (green) for case 2 correspond to the initial value in Table 2 are presented in Fig. 8.

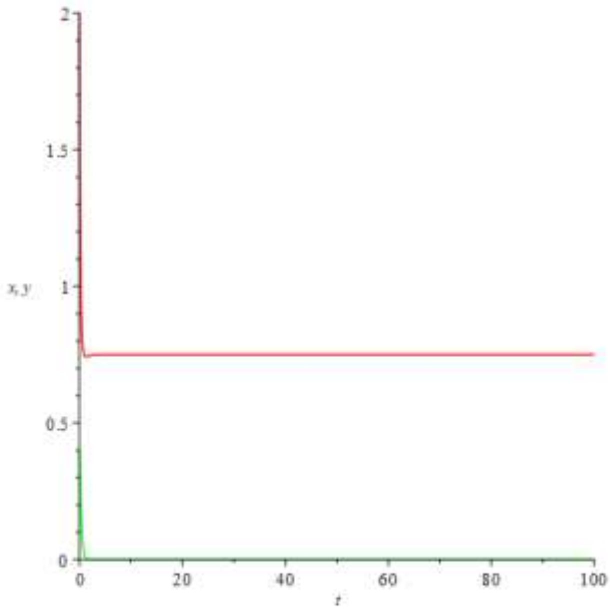

Fig. 8: Solution curve x (red) and y (green) for case 2, $\mu = \frac{\alpha\delta}{\gamma}$.

In its application to integrated farming systems, this can be used by farmers in determining the right time to renew the carrying capacity of the environment, so that the prey does not become extinct. Extinction of prey can result in a decrease in the number of predators or even extinction due to a lack of food sources.

4 Lotka-Volterra Model with periodic Harvesting of Predator

In a predator-prey system, in addition to the interaction between prey and predators, it is possible to harvest only the predators, prey only or both prey and predators at the same time.

In this section, the Lotka-Volterra model with a periodic harvesting of predators is considered. The model is in the form

$$
\begin{aligned}
\frac{dx}{dt} &= \alpha x - \beta xy \\
\frac{dy}{dt} &= \delta xy - \gamma y - \varepsilon y(1 + \sin \omega t)
\end{aligned}
\tag{13}
$$

where $\varepsilon(1 + \sin(\omega t))$ is a harvesting function for the predator, which is always positive, ε is a parameter related to the number of harvesting and ω is the harvesting period. Fig. 9 shows a phase portrait of the system (13) for various initial values based on Table 2 and $\varepsilon = 0.5$, $\omega = 1$. The red, green and blue curves use the initial values $x(0)$ and $y(0)$ in Table 2 for initial 1, initial 2 and initial 3, respectively. The solutions of x and y correspond to the orbits in Fig. 9 are presented in Fig. 10 and Fig. 11, respectively.

The number of peaks of green and red curves in Fig. 9 are 5, but the number of peaks of the blue curve is 6.This means that the 'period' of the blue curve is smaller than the ones of green and red curves. Observe that the peaks (the troughs) are not always at the same height. Similar phenomenon also happens for the y solution curves displayed in Fig. 11.

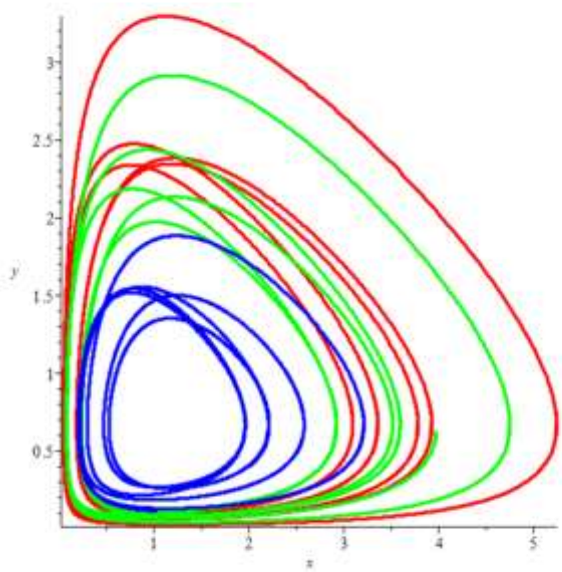

Fig. 9: Phase portrait system (13), where the initial values presented in Table 2

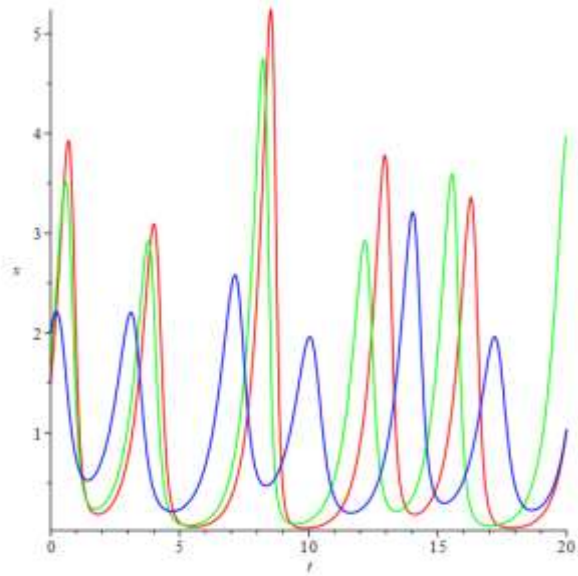

Fig. 10: x solution of system (13) for the initial values presented in Table 2

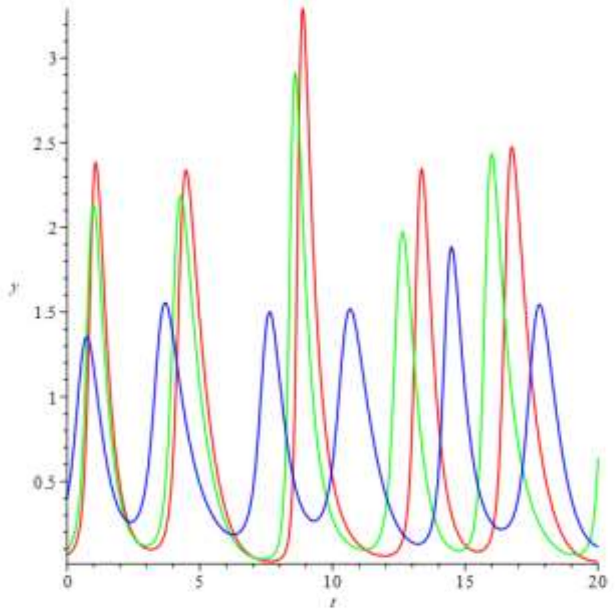

Fig. 11: y solution of system (13) for initial values presented in Table 2

Periodic harvesting of predators does not make prey or predators extinct. In each period the maximum value decreases. The difference in determining the initial value affects the maximum population size for each period. This simulation is important to support the decision making of adding the initial value in the next period so that the maximum population number achieved in the next period remains the same as the initial period.

5 Lotka-Volterra Model with Periodic Harvesting of Prey and Predator

The Lotka-Volterra model with a periodic harvesting of prey and predator that depends on time is represented by

$$\frac{dx}{dt} = \alpha x - \beta xy - \rho x(1 + \sin(\Omega t))$$
$$\frac{dy}{dt} = \delta xy - \gamma y - \varepsilon y(1 + \sin(\omega t)) \qquad (14)$$

where ρ is a parameter related to the number of harvestings of prey, Ω is periodic time for x and $\rho(1 + \sin(\Omega t))$ is a harvesting function for prey.

Phase portrait of the system (14), when $\alpha = 2$, $\beta = 3$, $\delta = 2, \gamma = 1.5, \varepsilon = 0.5$, $\rho = 0.2, \omega = 1, \Omega = 1$ and the initial values of x and y are given in Table 2. Phase portrait system (14) with various initial values is shown in Fig. 12.The red, green, and blue curves use the initial values $x(0)$ and $y(0)$ in Table 2 for initial 1, initial 2 and initial 3, respectively.

The x and y solution of system (14) correspond to orbits presented in Fig. 12 are shown in Fig. 13 and Fig. 14, respectively. Similar to the previous section, the number of peaks of green and red curves in Fig. 13 and Fig. 14 are 5, but the number of peaks of blue curves are 6.

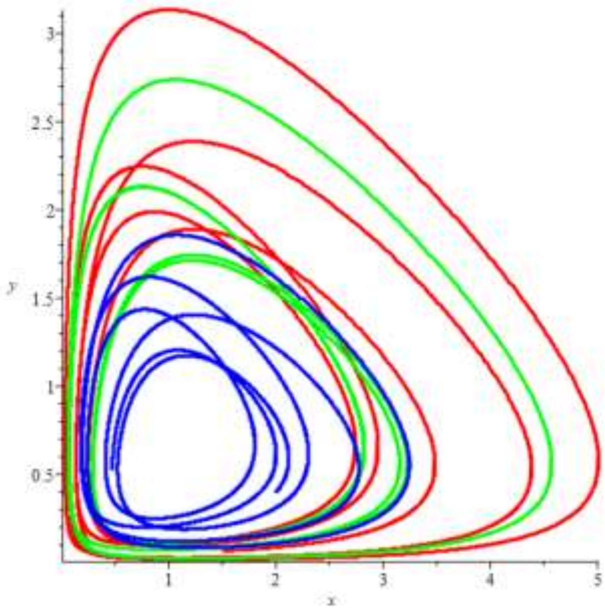

Fig. 12: Phase portrait of system (14), for the initial values given in Table 2

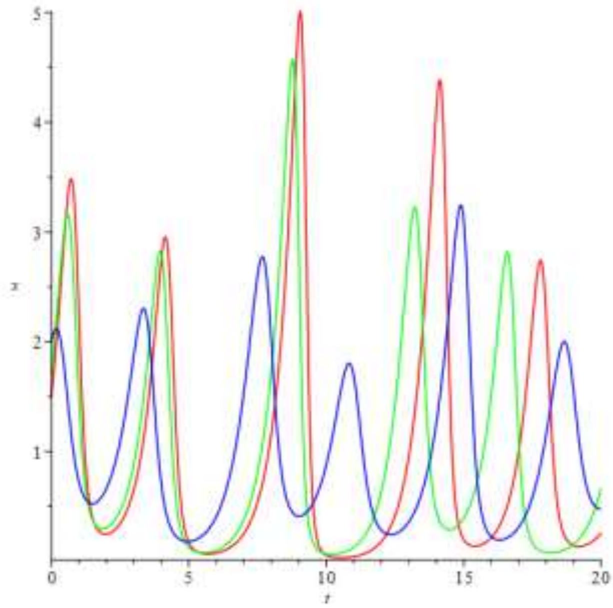

Fig. 13: x solution of system (14) with initial values given in Table 2

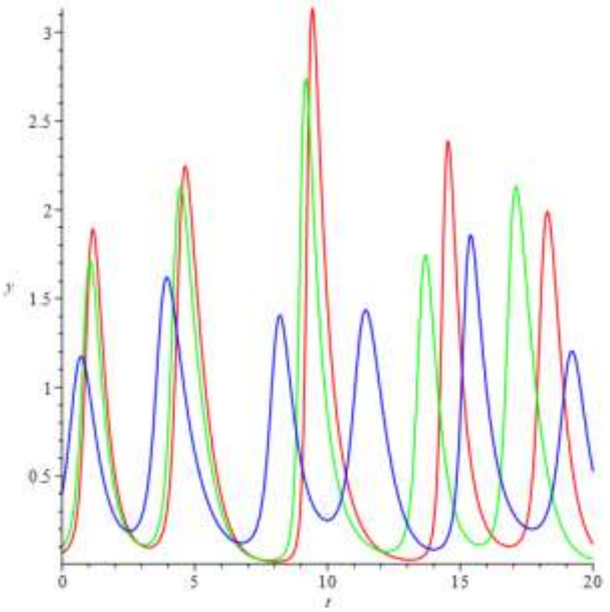

Fig. 14: y solution of system (14) with initial values given in Table 2

Periodic harvesting of prey and predators simultaneously can provide multiple benefits to an integrated farming system, [20]. However, it can be seen in Fig. 12 that the harvesting of prey and predators at the same time can cause a decrease in the number of prey and predator populations in the following period.

A sustainable seed supply is one of the keys to the success of an integrated farming system, [21], so that the farmer can decide when to add seeds as an initial value to get a maximum value of prey and predators in the next period will be the same as the previous period. The right time to add these seeds can be predicted using the simulation of system (14).

Diversification of production between horticultural crops and livestock in an integrated manner, sustainable supply of inputs, and efficient use of natural resources are important factors in an integrated farming system, [21]. The supply of inputs to plants, in this case acting as prey, can be in the form of the carrying capacity of the environment which limits the development of prey. This is added to the model which is discussed in the following section.

6 Lotka-Volterra Model with Periodic Harvesting of Predator and Carrying Capacity

In systems (13) and (14) there is no self-competition for x, while in the real world the growth of a population is limited by the carrying capacity of the environment. Therefore, this section discusses the Lotka-Volterra model with harvesting on predators with the presence of parameters related to the carrying capacity of x population.

Modification of system (13) by adding the carrying capacity of the environment that limits x is obtained Lotka-Volterra model with periodic harvesting of predator that depends on time may be represented by

$$\begin{aligned} \frac{dx}{dt} &= \alpha x - \mu x^2 - \beta xy \\ \frac{dy}{dt} &= \delta xy - \gamma y - \varepsilon y(1 + \sin \omega t) \end{aligned} \tag{15}$$

The initial value data from Table 2 are still considered. Corresponding orbits of system (15) with these initial values are displayed in Fig. 15. In this figure $\mu = 0.3$ and t = 0, 1...,100. The red, green and blue curves use the initial values $x(0)$ and $y(0)$ in Table 2 for initial 1, initial 2 and initial 3, respectively.

The x solutions of the system (15) that correspond to the orbits in Fig. 15 are presented in Figure16. It can be seen in the portrait phase in Fig. 15 that the portrait phase leads to a fixed phase trajectory. It can also be seen that in the solution curve in Fig. 16 and Fig. 17, the solution curve at about t between 20 and 40, the population size decreases and starts to form the same curves.

To increase the population, carrying capacity improvements are needed and can be predicted using a simulation of the system (15) with parameter values adjusted to the types of livestock and plants that are integrated.

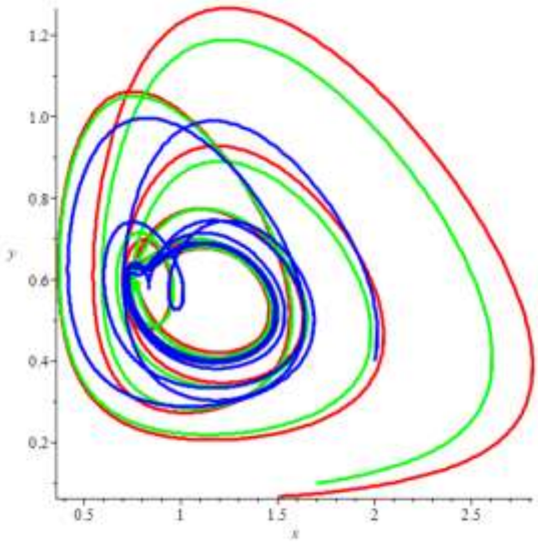

Fig. 15: Phase portrait system (15)

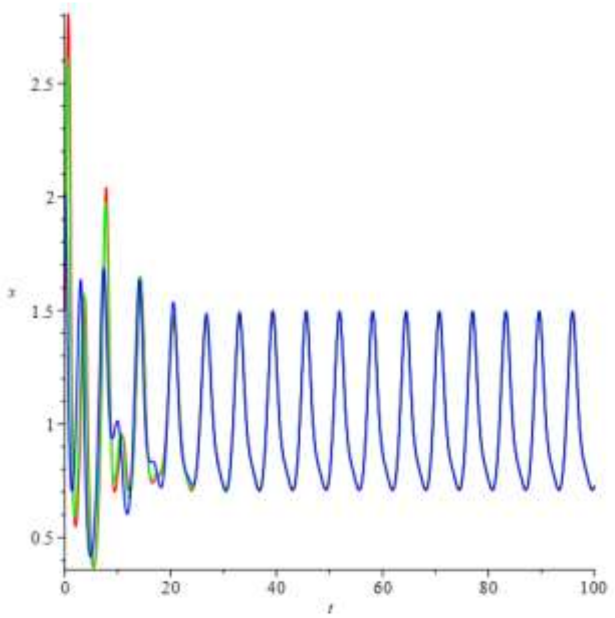

Fig. 16: Solution curves for *x* of system (15), with initial value in Table 2

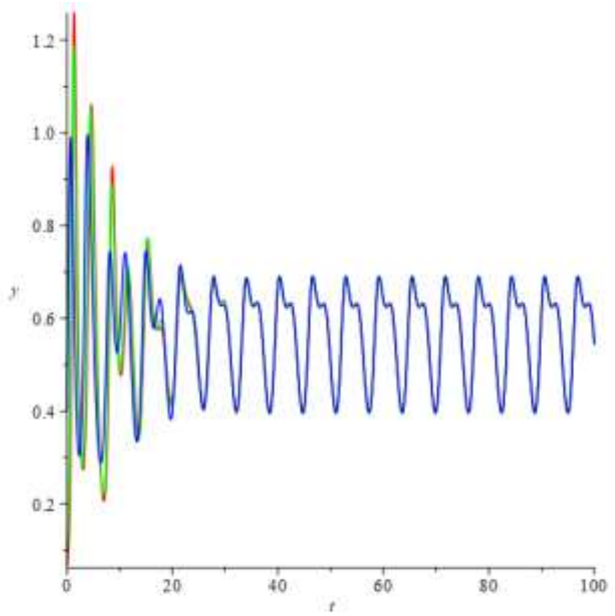

Fig. 17: Solution curves for *y* of system (15), with initial value in Table 2

7 Lotka-Volterra Model with Periodic Harvesting of Prey and Predator and Carrying Capacity

Lotka-Volterra model with periodic harvesting of prey and predator that depends on time and carrying capacity of the environment that limits *x* can be find by modification of system (14) may be represented by

$$\frac{dx}{dt} = \alpha x - \mu x^2 - \beta xy - \rho x(1 + \sin(\Omega t))$$
$$\frac{dy}{dt} = \delta xy - \gamma y - \varepsilon y(1 + \sin(\omega t)) \quad (16)$$

Using the initial value data from Table 2, $\mu = 0.3$ and t = 0, 1,..., 100 and the other parameters follow parameters of the previous section then obtained phase portrait system (16) is shown in Fig. 18. It can be seen in the portrait phase in Fig. 18, the portrait phase leads to a fixed phase trajectory. It can also be seen that in the solution curve in Fig. 19 and Fig. 20, the solution curve at about *t* between 30 and 40 begins to form the same curve.

Periodic harvesting of prey and predators with limited carrying capacity of prey may double farmers' income, from prey harvesting and predator harvesting. However, the decline in the number of populations caused by the decrease of carrying capacity of the prey population can be corrected so that the prey and predator populations can increase again. In this system, the difference in the initial value affects the maximum population of the system in each period, but at certain times it becomes the same because of the influence of carrying capacity.

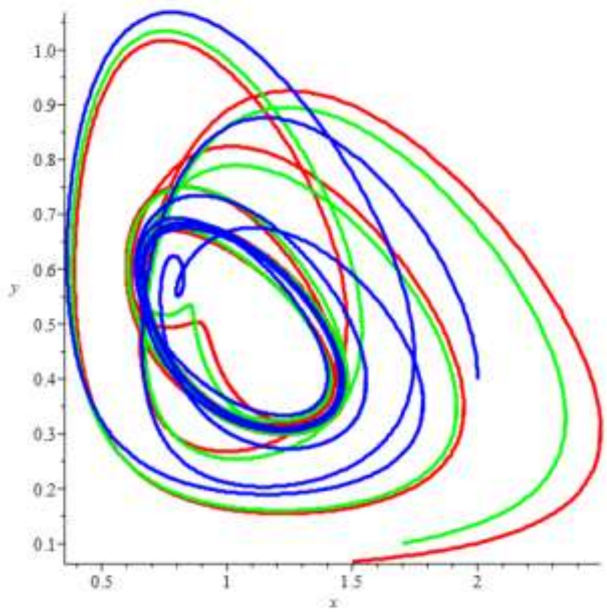

Fig. 18: Phase portrait system (16)

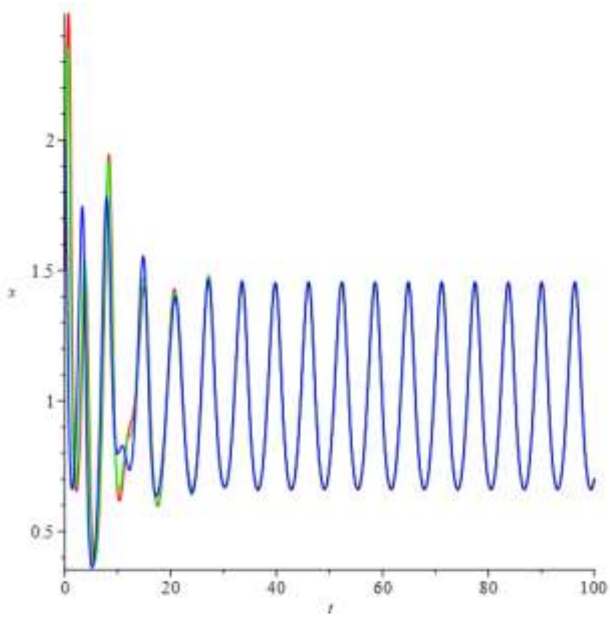

Fig. 19: Solution curves for *x* of system (16), with initial value in Table 2.

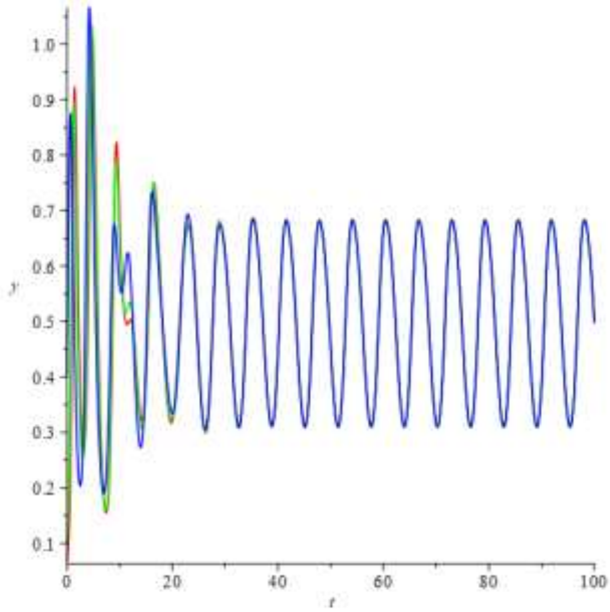

Fig. 20: Solution curves for *y* of system (16) with the initial value in Table 2

Integration of several types of commodities may increase agricultural productivity, profitability and sustainability compared to the cultivation of one commodity. The use of residues and by-products adds value to integrated farming systems. Besides that, an integrated farming system may increase farm income and implement sustainable agriculture, [22], [23].

System (16) can be used to determine when to add carrying capacity and seeds as initial values so that a sustainable integrated farming system can be implemented.

8 Concluding Remarks and Open Problem

This research has modified several variations of the Lotka-Volterra models, namely modeling with carrying capacity, periodic harvesting of predator, periodic harvesting of prey-predator, periodic harvesting of predator with carrying capacity, and periodic harvesting of prey-predators with carrying capacity. Numerical simulation is to describe the consistency and behavior of these models using the same initial value. Modeling with carrying capacity illustrates that the higher the initial value for the predator, the stability is achieved in a short time, and vice versa. Modeling with periodic harvesting of predators provides an overview of monitoring harvest limits. This provides important information because it maintains the sustainability of the ecosystem. Harvesting criteria are needed, the predators are harvested and remain consistent in the ecosystem. Modeling for harvesting prey and predator provides an indicator the optimum harvesting is carried out, for example, harvesting in trajectory points. Other information is to provide scenarios for intervention of prey and predator. The harvesting of predator with carrying capacity in a steady state condition will still have interactions between prey and predator as well as dynamically based on the available capabilities. For periodic harvesting of prey-predator and carrying capacity modeling is sensitive to the initial value. These models provide an overview of optimal harvesting scenarios for further work. For example, we are determining the precision of the initial value to determine optimal yields.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

-Norma Muhtar (PhD student) carried out mathematical modeling, analysis, and finishing the paper.
-Edi Cahyono(promotor) is responsible for the main idea of the research, and supervising the process.
-R. Marsuki Iswandi and Muhidin (co-promotors) are responsible for the motivation of the research, interpretation and future applications in agriculture.