Voltage Stability Analysis during Power Network Node Type Conversion

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Abstract: - During the analysis of the power network, a node type conversion will occur under certain extreme conditions. This brings the problem of stability of the power system voltage. This article extracts the characteristic equation of the system voltage critical point by establishing a node voltage-branch current to the power network equation. Therefore, the corresponding solution equation group was in the conversion process of the PQ-PV node. Through this equation group, the difference in the difference between the voltage value of the node and the critical point voltage is formed to determine the system stability after the node type conversion is converted. In the process of calculating the voltage critical point, the improved Newton iteration method is used to avoid Jacques bizarre. This method allows calculations to smoothly reach a state of convergence. The simulation calculation shows that the method proposed in this article is correct and effective.

Key-Words: - static voltage; stability; branch current; characteristic equation; node type conversion.

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1 Introduction

In the process of static voltage stability analysis of the power system, due to the limit of the operating parameters of certain power equipment, the conversion of the node type was generated, which brought more loss of voltage stability [1]. At present, due to the power -free power limit of nodes, the situation of PV nodes into PQ nodes has been more studied. In the case where the system loses stability, the analysis has undergone extreme induced bifurcation or saddle-node bifurcation [2]. The research on the transition of PQ nodes to PV nodes due to the increasing limit of the voltage of the load node is relatively small.

In the process of node type conversion, the study of node voltage loss of voltage is mainly aimed at the solution of the critical point of voltage disability, and then comparative analysis. The calculation methods adopted are mainly indirect and direct methods. The indirect method is performed by continuously changing parameters to form a P-V curve. However, near the critical point, as the elegant matrix tends to be strange, the conventional trend algorithm fails. Therefore, the calculation of the critical point of the voltage is often combined with the pathogenic trend algorithm [3]. The continuous trend method [4,5] Tracks the balanced and dislike of the trendy equation through prediction and correction, and improves the pathological phenomenon and convergence of conventional trend algorithms. It is a more commonly used and effective calculation method. The direct method [6~8] According to the nature of the tide of the critical point, it establishes a trendy equation, and is iterated with Newton-Rafson method to find a more accurate critical point. In addition, the non-linear planning [9~11] method transforms the critical point conditions into an optimized load problem, and uses the optimal conditions of Kun- Tuk to solve it. The above methods also have some shortcomings in the process of practical application. When the power injection is very close to the feasible border, it will greatly increase the number of iterations of the optimal multiplier algorithm leading to the difficulty of solution. At the same time, obtaining sensitivity information requires the left special vector of the Jacques rather than the zero symbol of the matrix, which increases a lot of additional calculations.

This article introduces the branch circuit current representing the trend of the branch, and establishes the equation group of the power network as a variable with the node voltage. By analyzing the trendy equation, the characteristic equation of the node voltage stable boundary is extracted. Using this feature equation can solve the critical point of node voltage discharge. In the process of mutual conversion of the PQ and PV nodes, the comparison of the conversion point voltage and the critical point voltage is used to determine that the node is in the position of the PV curve after the conversion, and then determines the stability of the system after the node type conversion

2 The Power Network Equation is Represented by Node Voltage and Branch Circuit Current as Variables

Under the right-angle coordinate system, when ignoring the ground branch conductivity, the power network can be described as a branch circuit current-a mixed form of node voltage equation [12]:

$$\begin{aligned} i_l^a R_{ij} - i_l^r X_{ij} - e_i + e_j = 0 \\ i_l^a X_{ij} + i_l^r R_{ij} - f_i + f_j = 0 \end{aligned} \tag{1}$$

For node i:

$$\begin{aligned} e_i \sum_{l\in i} i_l^a + f_i \sum_{l\in i} i_l^r = p_i \\ e_i \sum_{l\in i} i_l^r - f_i \sum_{l\in i} i_l^a + (e_i^2 + f_i^2) \sum_{l\in i} B_l = -q_i \end{aligned} \tag{2}$$

Wherein: $l = 1,2,\cdots,L$ is branch collection, $i, j = 1,2,\cdots,N$ is node collection, i_l^a, i_l^r are the real part and imaginary part of the current of branch l respectively, e_i, f_i are the real part and imaginary part of the voltage of node i respectively, R_{ij}, X_{ij} are the real part and imaginary part of the impedance voltage of branch l respectively, B_l is the 1/2 susceptance to ground of branch l, p_i, q_i are the active and reactive power injected into the node. Assuming $x_i = \sum_{l\in i} i_{li}^a$ and $y_i = \sum_{l\in i} i_{li}^r$ respectively represent the sum of the real and imaginary parts of the node i injected current (excluding the branch current to the ground), $B_{i0} = \sum_{l\in i} B_l$ is the sum of the ground susceptance of the branch l connected to the node i.

The equation (1) and (2) form the power network hybrid equation group with node voltage and branch circuit current. This article analyzes the stability of the power network based on this equation group.

3 Stable Critical Condition of Node Voltage

In the process of converting the PQ-PV node, the comparison of the voltage amplitude of the conversion point needs to be relying on the analysis method of judgment basis. Therefore, the obtaining PQ and PV nodes under the given conditions has become the key. Let's first study the critical condition of stable node voltage.

3.1 PQ node

From the formula (2):

$$\begin{aligned} e_i = \frac{\left\{ \begin{aligned} &\left[2b_{il} p_i x_i - y_i (x_i^2 + y_i^2)\right] \mp \\ &y_i \sqrt{(x_i^2 + y_i^2)^2 - 4b_{il} q_i (x_i^2 + y_i^2) - 4b_{il}^2 p_i^2} \end{aligned} \right\}}{2b_{il}(x_i^2 + y_i^2)} \\ f_i = \frac{\left\{ \begin{aligned} &\left[2b_{il} p_i y_i + x_i (x_i^2 + y_i^2)\right] \pm \\ &x_i \sqrt{(x_i^2 + y_i^2)^2 - 4b_{il} q_i (x_i^2 + y_i^2) - 4b_{il}^2 p_i^2} \end{aligned} \right\}}{2b_{il}(x_i^2 + y_i^2)} \end{aligned} \tag{3}$$

That is to obtain a node voltage explicit expression with a branch current as the parameter. From the formula (3), it can be seen that only:

$$(x_i^2 + y_i^2)^2 - 4b_{il} q_i (x_i^2 + y_i^2) - 4b_{il}^2 p_i^2 \geq 0 \tag{4}$$

which is:

$$(x_i^2 + y_i^2) - 2b_{il} q_i \geq 2b_{il} \sqrt{p_i^2 + q_i^2} \tag{5}$$

When the above conditions are met, the trendy equation exists. And there are the following physical meanings. 1) The solution is about the circle with 0 as the center of the circle and $\sqrt{2b_{il}(q_i + \sqrt{p_i^2 + q_i^2})}$ as the radius. 2) When the amplitude of the node injection current is outside the circle, that is, only ">" is established in the equation (5), and there are multiple solutions. The system can run stably. 3) When the "<" is established, that is, when the square of the node's injection current is in the interior of this circle, there is no solution, and the system cannot run stably. 4) When "=" was established, the only solution existed, and the solution was on the circle, that is, the system's voltage stable boundary. To solve the operating status of the system at this time, the system's voltage stable critical point was found.

3.2 PV Node

Except for PQ nodes, generator nodes are usually defined as PV nodes. For PV nodes, the reactive power formula in the equation (2) are usually replaced by the next formula:

$$e_i^2 + f_i^2 = U_i^2 \tag{6}$$

Wherein: U_i is the amplitude of the voltage of node i. The analysis expression of the PV node voltage from the formula (2) is:

$$\begin{cases} e_i = \dfrac{p_i x_i \mp y_i \sqrt{(x_i^2 + y_i^2)V_i^2 - p_i^2}}{x_i^2 + y_i^2} \\ f_i = \dfrac{p_i y_i \pm x_i \sqrt{(x_i^2 + y_i^2)V_i^2 - p_i^2}}{x_i^2 + y_i^2} \end{cases} \tag{7}$$

For PV nodes, the condition of solution existence is:

$$(x_i^2 + y_i^2)U_i^2 - p_i^2 \geq 0 \tag{8}$$

Its physical significance is similar to PQ nodes. The solution is a circle with 0 as the center and p_i / U_i as the radius. When the amplitude of the node injection current is distributed on the circle or out of the circle, the power network equation has a solution.

4 Analysis of the Loss of Voltage Stability during the Mutual Conversion of the PQ-PV Node

4.1 Judging the Loss of Stability during the Mutual Conversion of the PQ-PV Node

Due to the reactive power limit or voltage limit value, the type of PV and PQ nodes will be converted to each other. During the conversion process, the system may lose stability. Take PV converting PQ nodes as an example here to indicate the loss of stable discrimination during the conversion process. In Figure 1, point A represents the stable critical point of the voltage of a node, and B and C represent the conversion point of the PV node to the PQ node. The horizontal axis indicates the power of the node, and the vertical axis indicates the voltage value of the node.

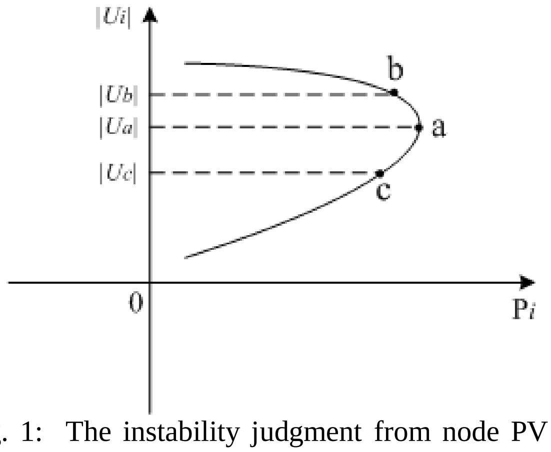

Fig. 1: The instability judgment from node PV to PQ

So the voltage disability judgment can be expressed as:

$$T_i = |U_{tr}| - |U_a| \tag{9}$$

Wherein: $|U_{tr}|$ is the voltage amplitude of node i at the conversion point b or c. $|U_a|$ is the voltage amplitude of the critical point a. If T_i is greater than 0, it means that the conversion point is on the upper half of the PV curve, and the system can still run stably after the conversion. If T_i is less than 0, the system will lose stability in the lower half of the PV curve. Because whether the PV node is converted to a PQ node or the PQ node converted to PV nodes, the voltage amplitude on the conversion point b and c is known as $|U_b|$ and $|U_c|$. Therefore, the voltage amplitude of the voltage stable critical point a is the most critical. It can be used to determine which node that is on the conversion time is on which solution curve. Then judge whether the system is stable.

In the same way, when the node is converted from PQ to PV, there are similar conclusions.

4.2 Analysis of the Loss of Stability during the PV Node Converted to the PQ Node

When the equation is established, in the equation (5), the system's stable boundary is reached, and the characteristic equation of the extraction system is:

$$(x_i^2 + y_i^2) = 2b_{il}(q_i + \sqrt{p_i^2 + q_i^2}) \tag{10}$$

So the node voltage becomes:

$$\begin{aligned} e_i &= \frac{2b_{il} p_i x_i - y_i (x_i^2 + y_i^2)}{2b_{il}(x_i^2 + y_i^2)} \\ f_i &= \frac{2b_{il} p_i y_i + x_i (x_i^2 + y_i^2)}{2b_{il}(x_i^2 + y_i^2)} \end{aligned} \tag{11}$$

Because when the node voltage approaches the stable critical point, the corresponding Jache compare matrix is strange. It makes the trend calculation based on Newton's method unable to converge. In order to calculate the critical point of the voltage stability, this node can be removed in the node voltage equation, that is, the method of reducing the dimension of the node voltage equation. In the branch circuit current equation, the corresponding node voltage equation is replaced by the equation (10) and (11). As shown in the equation (12), it is assumed that the m node will be converted.

$$\begin{cases} e_i \sum_{l\in i} i_l^a + f_i \sum_{l\in i} i_l^r = p_i \\ e_i \sum_{l\in i} i_l^r - f_i \sum_{l\in i} i_l^a + (e_i^2 + f_i^2) b_{il} = -q_i \end{cases}$$
$$\begin{cases} i_l^a R_{ij} - i_l^r X_{ij} - e_i + e_j = 0 \\ i_l^a X_{ij} + i_l^r R_{ij} - f_i + f_j = 0 \end{cases}$$
$$\begin{cases} e_m = \dfrac{2b_{ml} p_m x_m - y_m (x_m^2 + y_m^2)}{2b_{ml}(x_m^2 + y_m^2)} \\ f_m = \dfrac{2b_{ml} p_m y_m + x_m (x_m^2 + y_m^2)}{2b_{ml}(x_m^2 + y_m^2)} \\ (x_m^2 + y_m^2) = 2b_{ml}(q_m + \sqrt{p_m^2 + q_m^2}) \end{cases}$$

(12)

Wherein: $i \neq m$ is established in node power equation (2). In the branch circuit current equation (1), the voltage of node i is solved by e_m and f_m instead. After solving node m voltage, the voltage difference between the voltage of the PV node and the critical point after the conversion is calculated by the equation (9). Then you can make system stability judgments.

4.3 Analysis of the Loss of Stability during the PQ Node Converted to the PV Node

When the "=" in the equation (8) is established, the feature equation is:

$$(x_i^2 + y_i^2) = \left(\frac{p_i}{U_i}\right)^2 \tag{13}$$

Then the node voltage becomes:

$$\begin{cases} e_i = \dfrac{p_i x_i}{x_i^2 + y_i^2} \\ f_i = \dfrac{p_i y_i}{x_i^2 + y_i^2} \end{cases} \tag{14}$$

When PQ nodes is converted to PV nodes, the voltage amplitude V of the PV node is known. The equation group to be solved is:

$$\begin{cases} e_i \sum_{l\in i} i_l^a + f_i \sum_{l\in i} i_l^r = p_i \\ e_i^2 + f_i^2 = V_i^2 \end{cases}$$
$$\begin{cases} i_l^a R_{ij} - i_l^r X_{ij} - e_i + e_j = 0 \\ i_l^a X_{ij} + i_l^r R_{ij} - f_i + f_j = 0 \end{cases} \tag{15}$$
$$\begin{cases} e_m = \dfrac{p_m x_m}{x_m^2 + y_m^2} \\ f_m = \dfrac{p_m y_m}{x_m^2 + y_m^2} \\ (x_m^2 + y_m^2) = \left(\dfrac{p_m}{U_m}\right)^2 \end{cases}$$

The solution process of this equation group is similar to (12).

5 Calculation Steps

During the conversion of PQ and PV nodes, the calculation steps adopted by static voltage calculation are as follows:

1) The initial value of node voltage V and branch circuit current I is given.

2) PV nodes or PQ nodes m to be solved, use (12) or (15) to form the Jacques matrix with node voltage and branch current as variables, and use Newtonian method to iterate to solve.

3) Calculate the critical point of the PQ or PV node voltage amplitude $|U_a|$.

4) Use formula (9) to calculate the voltage amplitude difference T_i.

5) Determine the loss stability of the system according to the difference T_i.

6 Calculation Example

Take the IEEE118 node system as an example. Among them, the 69th balance nodes and node 118 are confrontation, and the power factor of the stable critical point of the voltage is 0.9. Table 1 is the analysis of the stable situation of the system when the PV node reaches the upper limit of the reactive power to the upper limit of the PQ node, where $Q_{\lim it}$ represents the limit of the reactive power. Table 2 is the calculation of the PQ node which the voltage reaches the lower limit to the PV node. Table 1 and Table 2 both selected representative calculation results.

Table 1. The stability analysis of node PV transforming to PQ

No.	$\lvert U_{tr} \rvert$	$\lvert U_a \rvert$	$Q_{\lim it}$	T_i
18	0.9668	0.9520	0.500	0.0148
42	0.9881	0.9670	3.000	0.0211
49	0.9903	0.9611	2.100	0.0292
54	0.9692	0.9551	3.000	0.0141
55	0.9511	0.9624	0.230	-0.0113
56	0.9528	0.9598	0.150	-0.0070
59	0.9789	0.9619	1.800	0.0170
61	0.9812	0.9662	3.000	0.0150
72	0.9771	0.9572	1.000	0.0199
74	0.9562	0.9611	0.390	-0.0041
76	0.9505	0.9529	0.530	-0.0024
77	0.9871	0.9670	0.700	0.0201
105	0.9592	0.9638	0.300	-0.0046
107	0.9501	0.9529	2.000	-0.0019
110	0.9619	0.9555	0.430	0.0064

It can be seen from the calculation results of Table 1 that the T_i value of 55, 56 and other nodes is negative. This shows that when the node type is converted from PV to PQ, the conversion point is on the lower half of the PV curve, so the system is in an unstable state after the conversion. The parameters of the system can be seen that the reactive power limit value of such nodes are generally small and the adjustment capabilities are poor. And the nodes are in the heavy load area, the voltage stable critical point will be reached easily. Therefore, the voltage amplitude ($\lvert U_a \rvert$) of the critical point is large, so the conversion point is easier to fall into the lower half of the PV curve, resulting in the system's loss of stability.

Table 2 The stability analysis of node PQ transforming to PV

No.	$\lvert U_{tr} \rvert$	$\lvert U_a \rvert$	T_i
20	0.9600	0.9533	0.0067
21	0.9600	0.9562	0.0038
22	0.9600	0.9711	-0.0111
43	0.9600	0.9641	-0.0041
44	0.9600	0.9508	0.0092
45	0.9600	0.9633	-0.0033
51	0.9600	0.9691	-0.0091
52	0.9600	0.9579	0.0021
53	0.9600	0.9638	-0.0038
82	0.9600	0.9666	-0.0066
83	0.9600	0.9613	-0.0013

In Table 2, the lower limit of the node's voltage is 0.96, and T_i is the negative value indicates that the system is in a state of losing stability after the conversion. Observing the unstable node will find that the cause of the disability is similar to Table 1. Therefore, in the process of system stability adjustment, first of all, nodes such as 45 and 51 should be adjusted to adjust their reactive power reserves and reduce the load rate.

7 Conclusion

This article establishes the power network equation based on node voltage and branch current as variables. The voltage stability conditions corresponding to the PQ and PV nodes are proposed. Then analyze the system voltage stability during the mutual conversion of the PQ-PV node. The following conclusions are obtained through simulation calculations:

1) The method proposed in this article can be applied to the stability analysis of the node type conversion of the power system, which is used to determine the disability of the static voltage.

2) This article proposes a critical point feature equation and an unstable judgment of voltage stability. It provides a new way for the use of node voltage amplitude values in the use of node conversion as a method.

3) During the mutual conversion of the PV and PQ nodes, nodes with low reactive power reserves are more likely to occur type conversion. The possibility of the lower half of the PV curve is great during the conversion of the heavy load area. This can provide a basis for system stable adjustment.

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