Stability analysis of pressure and penetration rate in rotary drilling system

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Abstract: -The purpose of this paper is to stabilize the annular pressure profile throughout the wellbore continuously while drilling. A new nonlinear dynamical system is developed and a controller is designed to stabilize the annular pressure and achieve asymptotic tracking by applying feedback control of the main pumps. Hence, the paper studies the control design for the well known Managed Pressure Drilling system (MPD). MPD provides a closed-loop drilling process in which pore pressure, formation fracture pressure, and bottom-hole pressure are balanced and managed at the surface. Although, responses must provide a solution for critical downhole pressures to preserve drilling efficiency and safety. Our MPD scheme is elaborated in reference to a nontrivial back-stepping control procedure and the effectiveness of the proposed control laws are shown by simulations.

Key-Words: - Drilling - Stability - Back-Stepping - Ordinary Differential Equation

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1 Introduction

In recent century, the gas and oil industry has made great strides in developing drilling techniques and technologies that makes well construction a cost effective and safe enterprize. Also, a new techniques and approaches are developed in several research [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13] . For the modeling, analysis and control of drilling systems, various works studies have investigated the stability properties of the drill string system. There exists three main type of vibrations: axial (bit-bouncing phenomenon), lateral (whirling phenomenon), and torsional (stick-slip oscillations). However, in the literature, many authors were interested in stabilizing the system with different techniques: Backstepping, Flatness, etc. Several authors have investigated the dynamics of oil well drilling, for example [6]. The energy function is proposed by Saldivar *et al.* [6] for the torsional model allows to find a control law that proves the energy dissipation during the drilling. During drilling operations, downhole cuttings need to be transported out of the bore hole. This is done by using a Managed Pressure Drilling (MPD) system. There exists several works were interested in stabilizing the MPD with different ways. An MPD is used to control the annular pressure profile throughout the well bore. The main objectives are to ascertain the downhole pressure environment limits and to manage the annular hydraulic pressure profile accordingly. The MPD is intended to suppress continuous influx of formation fluids to the surface. It is a new technology which has capability of mitigating drilling hazards, increasing production rates, and improving drilling performance. Consequently, the MPD will increase reserves by enabling drilling of areas that were previously economically unbribable. In the literature, different aspects of modeling for MPD have been proposed. Estimation and control design in MPD has been studied by several researchers so far [14]. Various challenges of modeling drilling systems for control and automation are discussed in [15].

In this work, we use the backstepping technique and Lyapunov theory to study our MPD. The backstepping technique is developed in 1990 by Petar V. Kokotovic *et al.* [16, 17] for analyzing the stability of the feedback controls of nonlinear dynamical systems. Then, it has becomes a useful tool in the feedback linearization of nonlinear systems of ODEs. In the Literature, various research are used backstepping method and Lyapunov theory to show the stability of nonlinear systems [16, 18]. Also, several results are invented in the stability studying of ODEs. In [19], the authors are used backstepping method, and Lyapunov theory to find the controller law. In [16], Kristick *et al.* are introduced a new method for nonlinear system with feedback control law. Roger *et al.* are provided the analysis of different techniques: two methods for find feedback control law are proposed and analyzed [20].
The main available variables in this paper are the injected pressure and the pump flow rate of water. Hence, our MPD study is different from the exist-

ing approach in gas and oil industry. This can be explained by the fact that no choke pressure feedback in our drilling system. The most important challenge in these drilling operations is to control bottomhole pressure and penetration rate of the bit.
The paper is organized as follows: in Section 2, we present a hydraulic model (MPD) based on mass balanced and momentum balances that provides the governing equations for flow and pressure in the well in an MPD. We describe and we will formulate our system by a variable change. The main purpose for this section is to control the pump pressure at the bit, the penetration rate, and also the rotational drill string velocity. In section 2, we illustrate the relevance and merits in numerical simulations. Some concluding and remarks are given in this last section.

2 Modeling and stability analysis of an MPD

2.1 Model Description

The Managed Pressure Drilling (MPD) system is used to control the pressure throughout the borehole in an oil well drilling. During drilling operation, a carefully designed fluid is pumped down from the mud pit through the drill string system, through the drill bit, up the annulus around the drill string, and back to the mud pit. The goal is not only to transport cuttings in the annulus, but also to manage the pressure in the well so that the unwanted inflow from the surrounding formation or well fracturing can be avoided. The

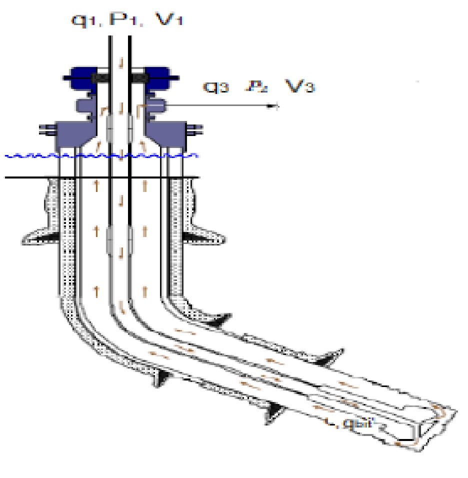

Figure 1: MPD in Rotary Drilling System.

hydraulic model of an MPD system (Fig. 1) derived from mass and momentum balances is described by the following system

$$
\begin{aligned}
M\dot{q}_{bit}(t) &= P_1(t) - P_2(t) - T(q_{bit}) + g(\rho_1 L - \rho_3 y(t)) && (1)\\
\frac{V_1}{\beta_1}\dot{P}_1(t) &= q_1(t) - q_{bit}(t) && (2)\\
(V_0 + Sy(t))\dot{P}_2(t) &= \beta_3\Big[q_{bit}(t) + q_2(t,\Omega) - q_3(t) - \underbrace{\frac{dV_2}{dt}}_{Sv(t)}\Big] && (3)\\
I\dot{\Omega} &= u_4 && (4)\\
\dot{y}(t) &= v(t) && (5)\\
\dot{v}(t) &= u_2(t) && (6)
\end{aligned}
$$

where L is the length of the well from mud pump, $y(t) \in [0, L]$ the spatial coordinate along the flow trajectory (g gravity), $V_2 = V_0 + Sy(t)$ is the crown volume, v the penetration rate of the bit, q_1 the pump flow, q_2 the flow which describes the amount of flow through the surface, S is the annular surface, q_3 is the flow out of the crown, V_1 is the volume of drill string, β_1 and β_3 are the effective bulk modulus, q_{bit} is the flow rate from the tool, M is the integrated density per cross section, P_1 is the pump pressure, P_2 is the pressure when $y = L$, I is the inertia of the drill string per unit length, ρ_1 is the density of the drilling mud in the drill string, ρ_3 is the annulus density, Ω is the rotational velocity of the drill string, u_4 is the control input from the torque applied to the drill string and u_2 is the control input from the penetration rate of the bit.
The total pressure drop due to the friction on the drill string and the annulus is represented by $T(q_{bit})$.
Assumption. The flow q_2 does not depend on the penetration rate, therefore the state v. Thus, it is initially considered that the torsion vibration phenomenon is not affected by the fluid injection.
For flow determination q_2 two cases occur.

2.2 Case where the flow $q_2 \triangleq q_2(t)$

This case presents itself under a constant rotational speed Ω of the train-bit set. Thus, the flow q_2 (sol/ bit interaction) do not depend on the state of the system. Consequently, the system (1) - (6) is transformed to the following

$$
\begin{aligned}
M\dot{q}_{bit}(t) &= P_1(t) - P_2(t) - T(q_{bit}) + g(\rho_1 L - \rho_3 y(t))\\
\frac{V_1}{\beta_1}\dot{P}_1(t) &= q_1(t) - q_{bit}(t)\\
(V_0 + Sy(t))\dot{P}_2(t) &= \beta_3\Big[q_{bit}(t) + q_2(t) - q_3(t) - \underbrace{\frac{dV_2}{dt}}_{Sv(t)}\Big]\\
\dot{y}(t) &= v(t)\\
\dot{v}(t) &= u_2(t)
\end{aligned}
$$

Let us introduce $q_1 = u_1$ as a control input in forces. We use the variable change $z(t) = \frac{1}{V_0 + Sy(t)}$, then we get the following system

$$
\begin{aligned}
\dot{q}_{bit}(t) &= c_1 P_1(t) - c_1 P_2(t) - c_1 T(q_{bit}) + h(z) && (7)\\
\dot{P}_1(t) &= c u_1(t) - c q_{bit}(t) && (8)\\
\dot{P}_2(t) &= R(t)z(t) - \beta_3 z(t) S v(t) && (9)\\
\dot{z}(t) &= -Sz^2(t)v(t) && (10)\\
\dot{v}(t) &= u_2(t) && (11)
\end{aligned}
$$

where $c = \frac{\beta_1}{V_1}$, $h(z) = c_1 g(\rho_1 L - \frac{\rho_3}{S}(\frac{1}{z(t)} - V_0))$, $c_1 = \frac{1}{M}$, $R(t) = \beta_3\Big[q_{bit}(t) + q_2(t) - q_3(t)\Big]$.
The equilibrium point of the system (7)-(11) is $(0, 0, 0, \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L}, 0)$.
The key contribution in this section, is the stability study of the ODE which describes the MPD under (u_1, u_2) as control inputs from the injected fluid (the pump pressure and the penetration rate of the bit). Hence our goal is to control the pump pressure and the penetration rate.
Now, we study the result that leads to the control laws of the hydraulic system.

Theorem 1 *Let consider the system (7)-(11). Choosing $A < 0$ such that $\beta_3 P_2(t)Sz(t) + Sz^2(t)(z(t) - \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L}) + A$ is never to zero. Then the both feedback control laws*

$$\begin{aligned} u_1(t) &= -\frac{P_1 - \Psi_1(q_{bit}, P_2, z)}{c} + \frac{c - c_1}{c} q_{bit}(t) \\ &+ \frac{1}{c}\dot{\Psi}_1(q_{bit}, P_2, z) \end{aligned}$$

and

$$\begin{aligned} u_2(t) &= Sz^2(t)(z(t) - \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L}) - v(t) + \Psi_2(P_2, z) \\ &+ \dot{\Psi}_2(P_2, z) + \beta_3 Sz(t)P_2(t) \end{aligned}$$

asymptotically stabilize the system (7)-(11) where

$$\Psi_1(q_{bit}, P_2, z) = P_2 - (\frac{1}{c_1} q_{bit} - T(q_{bit})) - \frac{h(z)}{c_1}$$

and

$$\Psi_2(P_2, z) = \frac{P_2^2(t) + (z(t) - \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L})^2 + (\frac{P_2(t)}{2} + R(t)z(t))^2}{\beta_3 P_2(t)Sz(t) + Sz^2(t)(z(t) - \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L}) + A}$$

in which

$$\Psi_2(0, \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L}) = 0, \quad \Psi_1(0, 0, \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L}) = 0$$

In order to prove this theorem, we use the backstepping technique and the Lyapunov theory.

Proof 1 *First, we consider the following sub-system*

$$\dot{q}_{bit}(t) = c_1 P_1(t) - c_1 P_2(t) - c_1 T(q_{bit}) + h(z)$$

and introduce a virtual feedback control law which satisfies

$$\Psi_1(q_{bit}, P_2, z) = P_2 - (\frac{1}{c_1} q_{bit} - T(q_{bit})) - \frac{h(z)}{c_1}$$

in which $\Psi_1(0, 0, \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L}) = 0$. Then we obtain

$$\dot{q}_{bit}(t) = -q_{bit}$$

We introduce the following Lyapunov function $L_1(t) = \frac{1}{2} q_{bit}^2$, then $\dot{L}_1(t) = -2L_1(t)$. This proves that (7) is asymptotically stable at the equilibrium $q_{bit} = 0$.
Second, let consider the virtual state variable

$$\zeta_1 = P_1 - \Psi_1(q_{bit}, P_2, z)$$

Then we obtain

$$\begin{aligned} \dot{q}_{bit}(t) &= c_1 \zeta_1(t) - q_{bit}(t) \\ \dot{\zeta}_1(t) &= -\dot{\Psi}_1(q_{bit}, P_2, z) + c u_1(t) - c q_{bit}(t) \end{aligned}$$

We choose the following Lyapunov function

$$L_2(t) = L_1(t) + \frac{1}{2}\zeta_1^2 = \frac{1}{2}(q_{bit}^2 + \zeta_1^2)$$

The time derivative of L_2 is given by

$$\begin{aligned} \dot{L}_2 &= \dot{q}_{bit} q_{bit} + \dot{\zeta}_1 \zeta_1 \\ &= q_{bit}(c_1 \zeta_1 - q_{bit}) + \zeta_1(-\dot{\Psi}_1(q_{bit}, P_2, z) + c u_1(t) \\ &- c q_{bit}(t)) \\ &= -q_{bit}^2 - \zeta_1^2 + \zeta_1(\zeta_1 + (c_1 - c) q_{bit} - \dot{\Psi}_1(q_{bit}, P_2, z) \\ &+ c u_1(t)) \end{aligned}$$

Here, we select the actual feedback control law

$$u_1(t) = -\frac{\zeta_1}{c} + \frac{c - c_1}{c} q_{bit}(t) + \frac{1}{c}\dot{\Psi}_1(q_{bit}, P_2, z)$$

This proves that $\dot{L}_2(t) = -2L_2(t)$. Then the system (7)-(8) is asymptotically stable at the equilibrium $(q_{bit}, P_1) = (0, 0)$.
Now, we consider the following sub-system

$$\begin{aligned} \dot{P}_3(t) &= R(t)z(t) - \beta_3 z(t) S v(t) && (12) \\ \dot{z}(t) &= -Sz^2(t) v(t) && (13) \\ \dot{v}(t) &= u_2(t) && (14) \end{aligned}$$

As before, let consider

$$\begin{aligned} \dot{P}_3(t) &= R(t)z(t) - \beta_3 S z(t) v(t) \\ \dot{z} &= -S v(t) z^2(t) \end{aligned}$$

where $R(t) = \beta_3(q_{bit}(t) + q_2(t) - q_3(t))$, and introduce a virtual feedback control law which might $\Psi_2(P_2, z) = \frac{P_2^2(t) + (z(t) - \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L})^2 + (\frac{1}{2} P_2(t) + R(t)z(t))^2}{\beta_3 P_2(t)Sz(t) + Sz^2(t)(z(t) - \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L}) + A}$ in which $\Psi_2(0, \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L}) = 0$.
Then we obtain the following system

$$\begin{aligned} \dot{P}_3(t) &= R(t)z(t) - \beta_3 z(t) S \Psi_2(P_2, z) && (15) \\ \dot{z} &= -Sz^2(t) \Psi_2(P_2, z) && (16) \end{aligned}$$

We use the following Lyapunov function

$$L_3(t) = \frac{1}{2}(P_2^2 + (z - \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L})^2)$$

The time derivative of L_3 is given by

$$\begin{aligned} \dot{L}_3(t) &= \dot{P}_2 P_2 + \dot{z}(z - \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L}) \\ &= R(t)z(t)P_2(t) - \beta_3 S z(t) P_2(t) \Psi_2(P_2, z) \\ &- Sz^2(t)(z - \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L}) \Psi_2(P_2, z) \\ &\leq R(t)z(t)P_2(t) - \Big[P_2^2(t) + (z - \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L})^2 \\ &+ (\frac{1}{2} P_2(t) + R(t)z(t))^2\Big] \\ &\leq -P_2^2(t) - (z - \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L})^2 \\ &\leq -2L_3(t) \end{aligned}$$

This proves that the system (12)-(13) is asymptotically stable at the equilibrium $(P, z) = (0, \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L})$.
Second, we use the following virtual state variable, $\zeta_2(t) = v(t) - \Psi_2(P, z)$ *to transform the system (12)-(14) in this form*

$$\begin{aligned}\dot{P}_3(t) &= R(t)z(t) - \beta_3 Sz(t)\zeta_2(t) - \beta_3 z(t)S\Psi_2(P_2, z)\\ \dot{z}(t) &= -Sz^2(t)\zeta_2(t) - Sz^2(t)\Psi_2(P_2, z)\\ \dot{\zeta}_2(t) &= u_2(t) - \dot{\Psi}_2(P_2, z)\end{aligned}$$

We know that the first sub-system (15)-(16) is asymptotically stable at the equilibrium $(P, z) = (0, \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L})$. *Here, we use the following Lyapunov function*

$$L_4(t) = L_3(t) + \frac{1}{2}\zeta_2^2.$$

We differentiate L_4 *with respect to time, we get*

$$\begin{aligned}\dot{L}_4(t) &= \dot{P}_2(t)P_2(t) + \dot{z}(t)(z(t) - \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L}) + \dot{\zeta}_2(t)\zeta_2(t)\\ &\leq -2L_3 + \dot{\zeta}_2(t)\zeta_2(t) - \beta_3 Sz(t)\zeta_2(t)P_2(t)\\ &- Sz^2(t)(z(t) - \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L})\zeta_2(t)\\ &\leq -2L_3(t) - \zeta_2^2(t) + (\zeta_2(t) + u_2(t) - \dot{\Psi}_2(P_2, z))\zeta_2(t)\\ &- \beta_3 Sz(t)\zeta_2(t)P_2(t) - Sz^2(t)(z(t) - \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L})\zeta_2(t)\\ &\leq -2L_4(t) + (\zeta_2(t) + u_2(t) - \dot{\Psi}_2(P_2, z)\\ &- \beta_3 Sz(t)P_2(t) - Sz^2(t)(z(t) - \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L}))\zeta_2(t)\end{aligned}$$

Hence, we select the actual control law given by

$$\begin{aligned}u_2(t) &= Sz^2(t)(z(t) - \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L}) - \zeta_2(t)\\ &+ \dot{\Psi}_2(P_2, z) + \beta_3 Sz(t)P_2(t)\end{aligned}$$

Then $\dot{L}_4(t) \leq -2L_4(t)$.
Consequently, let introduce the following Lyapunov function

$$\begin{aligned}V(t) &= \frac{1}{2}\Big(q_{bit}^2 + \Big(P_1 - \Psi_1(q_{bit}, P_2, z)\Big)^2\Big)\\ &+ \frac{1}{2}\Big(P_3^2(t) + \Big(z(t) - \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L}\Big)^2\Big)\\ &+ \frac{1}{2}\Big(v(t) - \Psi_2(P_2, z)\Big)^2\end{aligned}$$

Then

$$\dot{V} = \dot{L}_2 + \dot{L}_4 \leq -2(L_2 + L_4) \leq -2V$$

This conclude that the system (7)-(11) is asymptotically stable at the equilibrium.

Remark 1 *The torsional vibration phenomenon is considered to be independent of the injection system, this implies that the penetration rate of the bit is not impacted by the pressure of the injected fluid. Indeed, in the contrary case, we will have to deal with a model coupling the dynamics of torsion as well as the fluid behavior injected by the hydraulic system to the surface.*

2.3 Case where the flow $q_2 \triangleq q_2(t, \Omega)$

This case occurs at a non constant rotational velocity Ω of the drill string. Then, the flow q_2 (bit / rock interaction) depends on the state of the system. So, we can express:

$$\begin{aligned}q_2(t, \Omega) &= \rho_3 Sr(\Omega(t) + d(t))\\ I\dot{\Omega} &= u_1(t)\end{aligned}$$

where $d(t)$ is the disturbance, I the inertia of the drill string per unit length, r the annular radius, u_4 the torque applied to the drill string. Our main is to control the pressure at the bit, the penetration rate, and also the rotational drill string velocity.
Consequently, we consider the following model

$$\begin{aligned}\dot{q}_{bit}(t) &= c_1 P_1(t) - c_1 P_2(t) - c_1 T(q_{bit}) + h(z) && (17)\\ \dot{P}_1(t) &= cu_1(t) - cq_{bit}(t) && (18)\\ \dot{P}_3(t) &= R(t)z(t) + \beta_3\rho_3 Sr(w(t) + d(t))z(t)\\ &- \beta_3 z(t)Sv(t) && (19)\\ \dot{\Omega} &= \frac{1}{I}u_4(t) && (20)\\ \dot{z}(t) &= -Sz^2(t)v(t) && (21)\\ \dot{v}(t) &= u_3(t) && (22)\end{aligned}$$

where $R(t) = \beta_3\Big[q_{bit} - q_3\Big]$, u_1 is considered an input control, u_4 is the torque applied to the drill string which control the amount of flow through a surface (tool/ground) at the bottomhole and u_3 is the penetration rate.

Theorem 2 *The three feedback control laws*

$$\begin{aligned}u_1(t) &= -\frac{P_1 - \Psi_1(q_{bit}, P_2, z)}{c} + \frac{c - c_1}{c}q_{bit}(t) + \frac{1}{c}\dot{\Psi}_1(q_{bit}, P_2, z)\\ u_3 &= -v + \Psi_4 + Sz^2(z - \frac{\rho_3}{\rho_3 V_0 + \rho_1 SL}) + \dot{\Psi}_4\\ u_4(t) &= I\Big(-\rho_3 Sr\beta_3 P_2(t)z(t) + \dot{\Psi}_3 - \Omega + \Psi_3\Big)\end{aligned}$$

asymptotically stabilize the system (17)-(22) at the equilibrium $(q_{bit}, P_1, P_2, \Omega, z, v) = (0, 0, 0, 0, \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L}, 0)$ *where*

$$\begin{aligned}\Psi_1(q_{bit}, P_2, z) &= P_2 - (\frac{1}{c_1}q_{bit} - T(q_{bit})) - \frac{h(z)}{c_1}\\ \Psi_4(z) &= \frac{z - \frac{\rho_3}{\rho_3 V_0 + \rho_1 SL}}{Sz^2(t)}\end{aligned}$$

$\Psi_3(P_2, z, v) = \frac{-P_2 - R(t)z(t) - \rho_3 Sr\beta_3 d(t)z(t) + \beta_3 Sz(t)v(t)}{\rho_3 Sr\beta_3 z(t)}$
in which $\Psi_1(0, 0, \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L}) = \Psi_3(0, \frac{\rho_3}{\rho_3 V_0 + S\rho_1 L}, 0) = 0$ *and* $0 = \Psi_4(\frac{\rho_3}{\rho_3 V_0 + S\rho_1 L})$.

Proof 2 *We recall that the system (17)-(18) is asymptotically stable at the equilibrium* $(q_{bit}, P_1) = (0, 0)$ *(see proof of Theorem 1), consequently the system (17)-(18) is asymptotically stable at the same equilibrium.*
Now, let consider the following sub-system

$$\begin{aligned}\dot{P}_3(t) &= R(t)z(t) + \beta_3\rho_3 Sr(\Omega(t) + d(t))z(t) - \beta_3 z(t)Sv(t)\\ \dot{\Omega} &= \frac{1}{I}u_4(t)\end{aligned}$$

We introduce the virtual control law
$\Psi_3(P_2, z, v) = \frac{-P_2 - R(t)z(t) - \rho_3 Sr\beta_3 d(t)z(t) + \beta_3 Sz(t)v(t)}{\rho_3 Sr\beta_3 z(t)}$. *We*

propose $L_5(t) = \frac{1}{2}P_2^2$*, then, we get* $\dot{L_5}(t) \leq -P_2^2$.
Let $\xi_3(t) = \Omega(t) - \Psi_3$ *the virtual state variable, then we obtain the following sub-system*

$$\begin{aligned}\dot{P}_3(t) &= R(t)z(t) + \beta_3\rho_3 Sr(\xi_3(t) + \Psi_3 + d(t))z(t) \\ &- \beta_3 z(t)Sv(t) \\ \dot{\xi}_3 &= -\dot{\Psi}_3 + \frac{1}{I}u_4(t)\end{aligned}$$

Here, we introduce the following Lyapunov function

$$L_6(t) = \frac{1}{2}\Big(P_2^2(t) + \xi_3^2(t)\Big).$$

The time derivative of L_6 *is given by*

$$\begin{aligned}\dot{L}_6(t) &= \dot{P}_3 P_2 + \dot{\xi}_3\xi_3 \\ &= P_2(R(t)z(t) + \beta_3\rho_3 Srz(t)\Psi_3 + \beta_3\rho_3 Srd(t)z(t) \\ &- \beta_3 Sz(t)v(t)) + \xi_3(t)(P_2(t)\rho_3 Sr\beta_3 z(t) \\ &- \dot{\Psi}_3 + \frac{1}{I}u_4(t))\end{aligned}$$

We select the actual control law

$$u_4(t) = I\Big(- \rho_3 Sr\beta_3 P_2(t)z(t) + \dot{\Psi}_3 - \xi_3(t)\Big)$$

Then $\dot{L}_6(t) \leq -2L_6(t)$.
Finally, let consider the last sub-system

$$\begin{aligned}\dot{z}(t) &= -Sz^2(t)v(t) \\ \dot{v}(t) &= u_3(t)\end{aligned}$$

We consider $\Psi_4 = \frac{z - \frac{\rho_3}{\rho_3 V_0 + \rho_1 SL}}{sz^2(t)}$ *as the virtual control law and the lyapunov function* $L_7(t) = \frac{1}{2}(z - \frac{\rho_3}{\rho_3 V_0 + \rho_1 SL})^2$*, we obtain* $\dot{L}_7(t) \leq -2L_7$*. Let* $\xi_4 = v - \Psi_4$ *the virtual state variable. Then we get the following sub-system*

$$\begin{aligned}\dot{z}(t) &= -Sz^2(t)\xi_4(t) - Sz^2(t)\Psi_4 \\ \dot{\xi}_4(t) &= -\dot{\Psi}_4 + u_3(t)\end{aligned}$$

We introduce the following Lyapunov function
$L_8(t) = L_7(t) + \frac{1}{2}\xi_4^2(t)$*. Differentiating* L_8 *with respect to time, we find*

$$\begin{aligned}\dot{L}_8(t) &= \dot{L}_7(t) + \xi(t)\dot{\xi}(t) \\ &\leq -2\dot{L}_7(t) + \xi_4(-Sz^2(z - \frac{\rho_3}{\rho_3 V_0 + \rho_1 SL}) \\ &- \dot{\Psi}_4 + u_3(t)) \\ &\leq -2\dot{L}_8(t) + \xi_4(\xi_4 - Sz^2(z - \frac{\rho_3}{\rho_3 V_0 + \rho_1 SL}) \\ &- \dot{\Psi}_4 + u_3(t))\end{aligned}$$

Then selecting the actual control law u_3

$$u_3 = -\xi_4 + Sz^2(z - \frac{\rho_3}{\rho_3 V_0 + \rho_1 SL}) + \dot{\Psi}_4$$

Hence, let introduce the following Lyapunov function

$$\begin{aligned}\mathfrak{L} &= \frac{1}{2}[q_{bit}^2 + (P_1 - \Psi_1)^2 + P_2^2 + (\Omega - \Psi_3)^2 \\ &+ (z - \frac{\rho_3}{\rho_3 V_0 + \rho_1 SL})^2 + (v - \Psi_4)^2]\end{aligned}$$

Then

$$\begin{aligned}\dot{\mathfrak{L}} &= \dot{L}_2 + \dot{L}_6 + \dot{L}_8 \\ &\leq -2(L_2 + L_6 + L_8) \leq -2\mathfrak{L}\end{aligned}$$

Finally, we conclude that the overall system (17)-(22) is asymptotically stable.

3 Simulation

Drilling is an essential part of the oil industry and penetration rate must be enhanced to ensure speedy completion of drilling processs. Torque on bit, pressure, rotary speed, weight on bit, drill bit type, formation characteristics and mud properties are the basic factors that affect the penetration rate of a bit. The focus of this work is the stability of the MPD system under the control of fluid and penetration rate of the bit. Hence, in this section, we test the effectiveness of the control laws found for the stability of the MPD. As expected by Theorems 1 and 2, the proposed controller stabilizes all the drilling variables including the downhole pressure in the well. The following physical parameters are used in simulation [21, 4, 14]:

Variable	Value
L	$2000\ m$
I	$0.095\ kg.m$
$\rho_1 = \rho_3$	$1250\ kg.m^{-3}$
M	$8300\ kg.m^{-4}$
$\beta_1 = \beta_3$	$24750\ bar$
V_0	$110\ m^3$
g	$9.81\ ms^{-2}$
S	$\pi \times (0.25)^2\ m^2$
c_d	0.61
T_a	$0.003.10^6\ \frac{bar.s^2}{m^6}$

Table 1: Different physical parameters

The flow rate due to the mud leaving through the open annulus is given by [22],

$$q_3 = c_d S\sqrt{\frac{2}{\rho_3}(P_{dh} - P_2 + \rho_3\frac{g}{S}(\frac{1}{z} - V_0))}$$

The displacement of the bit, and the bit characteristics permit to construct the bottomhole pressure

$$P_{dh} = P_2 - \frac{\rho_3 g}{S}(\frac{1}{z} - V_0) + T_a q_{bit}^2$$

with T_a denotes the friction factor in the annulus. The stabilizing controller results to MPD are presented in figures (Fig. 2-7). Clearly, all the simulations imply an adequate convergence of the system variables to their expected values. For example, one notes that y converges to $\frac{\rho_1 L}{\rho_3} = 2000.m$ (Fig. 2). By applying the control laws (Fig. 7-8), the equilibrium is reached asymptotically stable (Fig. 3-6). Consequently, under the proposed control input, the hydraulic vibration system avoids an excessive increase (saturation) in pressures.

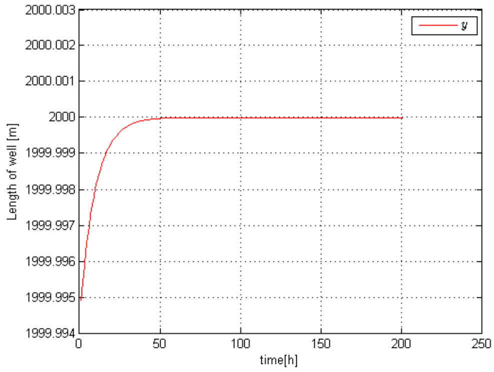

Figure 2: Stabilization of the state y

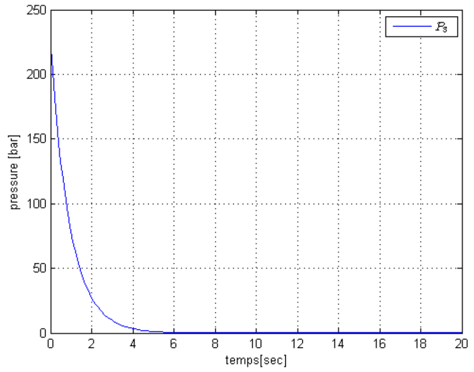

Figure 3: Stabilization of the pressure P_2

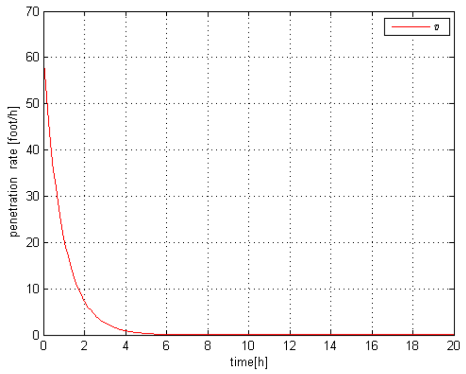

Figure 4: Stabilization of the penetration rate of the bit v

Conclusion

In this work, a dynamic model based on mass and momentum balances for MPD which describes by five and six ODE is proposed. The most important task in this paper is to control the pressure at the bit, the penetration rate, and bottom-hole pressure during drilling operations. The proposed controller laws asymptotically stabilizes all the drilling variables including the downhole pressure in the well. Future work should focus on the interaction between MPD system and torsional vibrations.

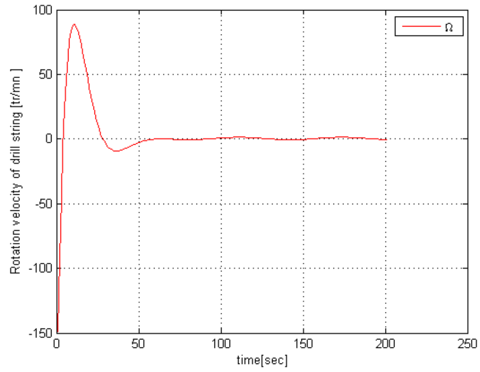

Figure 5: Stabilization of the rotation velocity of the drill string Ω

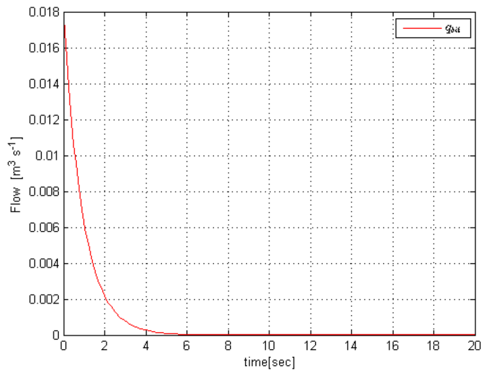

Figure 6: Stabilization of the the flow rate from the tool q_{bit}

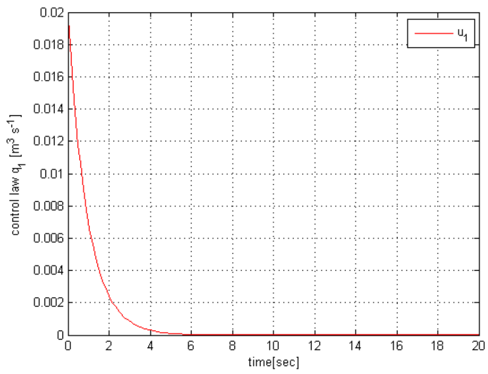

Figure 7: Stabilization of the control law u_1

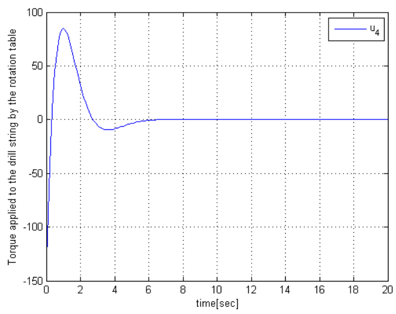

Figure 8: Stabilization of the control law u_4

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