Robust Controller Design for Descriptor-type Time-delay Systems

ALTUĞ İFTAR
Department of Electrical and Electronics Engineering
Eskişehir Technical University
26555 Eskişehir, TURKEY

Abstract - Linear time-invariant descriptor-type time-delay systems are considered. A robust stabilizing controller design approach for such systems is introduced. Uncertainties both in the time-delays and in other system parameters are considered. A frequency-dependent scalar bound on such uncertainties is first derived. Once this bound is found, the controller design is completely based on the nominal model. However, satisfying a scalar frequency-dependent condition, which uses the derived bound, guarantees robust stability. An example is also presented to illustrate the proposed approach.

KeyWords-Time-delaySystems,Descriptor-typeSystems,RobustControl,ControllerDesign,FrequencyShaping

Received: April 3, 2021. Revised: September 5, 2021. Accepted: September 13, 2021. Published: September 17, 2021.

1. Introduction

Systems which involve time-delays are very common in practice [1]. Such systems, which are typically named as *time-delay systems* [2], can be described by delay-differential equations [3]. For some time-delay systems, such as telerobotic systems [4], however, delay-differential equations must be coupled with delay-algebraic equations to describe the dynamics of the system. Such systems are known as *descriptor-type time-delay* systems [5]. Descriptor-type time-delay systems impose a challenge since their response may be discontinuous and even impulsive [6].

A number of controller design methods (e.g., [7], [8], [9], [10], [11], [12]) have already been proposed to design controllers for descriptor-type time-delay systems. Since any model of any physical system would be subject to uncertainties, however, any controller designed for a physical system must be *robust* to uncertainties in the model [13]. This is especially true in the case of time-delay systems, since, in this case, the time-delays are also uncertain in general [14].

In the present work, robust stabilizing controller design is considered for linear time-invariant (LTI) descriptor-type time-delay systems. The design is based on the nominal model of the system. A frequency-dependent bound on the uncertainties, however, is also derived and used in the design in order to guarantee robust stability of the actual closed-loop system. Uncertainties in both time-delays and other system parameters are taken into account. A similar frequency-dependent bound was first derived in [15], [16] for delay-free systems. Such a bound was then derived for retarded time-delay systems in [17] and for neutral time-delay systems in [18]. Systems with distributed time-delay were also considered in [19] and [20]. In [17]–[20], however, only non-descriptor-type systems were considered. Although [21] considers descriptor-type systems with distributed time-delays, uncertainties in time-delays were not considered in [21]. In here, we extend the results of [17] and [18] to descriptor-type time-delay systems. Specifically, we propose an approach to design a stabilizing controller for LTI descriptor-type time-delay systems, which is robust against uncertainties both in the time-delays and in other system parameters.

We state the problem formally in Section 2 and present the proposed approach in Section 3. An example is presented in Section 4 in order to illustrate the proposed approach. Some concluding remarks are included in Section 5.

Throughout the paper, $\mathbf{R}$ denotes the set of real numbers. For positive integers k and l, $\mathbf{R}^k$ and $\mathbf{R}^{k\times l}$ respectively denote the spaces of k dimensional real vectors and $k \times l$ dimensional real matrices. I denotes the identity matrix of appropriate dimensions. $\mathrm{Re}(\cdot)$ denotes the real part of $\cdot$. $\bar{\sigma}(\cdot)$, $\underline{\sigma}(\cdot)$, $\det(\cdot)$, and $\mathrm{rank}(\cdot)$ respectively denote the maximum singular value, the minimum singular value, the determinant, and the rank of $\cdot$. Finally, $i := \sqrt{-1}$ is the imaginary unit.

2. Problem Statement

Consider a LTI descriptor-type time-delay system, described as

$$E\dot{x}(t) = \sum_{j=0}^{\mu} \left(A_j x(t-h_j) + B_j u(t-h_j)\right) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

where $x(t) \in \mathbf{R}^n$, $u(t) \in \mathbf{R}^p$, and $y(t) \in \mathbf{R}^q$ are, respectively, the state, the input, and the output vectors at time t. Furthermore, $h_1, \ldots, h_\mu > 0$ are the time-delays, where μ is the number of independent time-delays. We let $h_0 := 0$

for notational convenience (i.e., $j = 0$ in (1) corresponds to the delay-free part of the system). A_j, B_j, $j = 0, \ldots, \mu$, C, and E are appropriately dimensioned constant real matrices, where C and E are known matrices. We assume that, for $j = 0, \ldots, \mu$, $A_j = A_j^0 + A_j^1$ and $B_j = B_j^0 + B_j^1$, where A_j^0 and B_j^0 are known matrices, representing the nominal part, and A_j^1 and B_j^1 are unknown matrices, representing the uncertainties. It is, however, assumed that

$$\bar{\sigma}(A_j^1) \leq \delta_j^a \quad \text{and} \quad \bar{\sigma}(B_j^1) \leq \delta_j^b \ , \tag{3}$$

for some known bounds $\delta_j^a > 0$ and $\delta_j^b > 0$, $j = 0, \ldots, \mu$. Here, it is assumed that all the input-output uncertainties and time-delays are represented at the input. Because of this assumtion, no uncertainties or time-delays appear in the output equation (2). Here, the time-delays, $h_1, \ldots, h_\mu > 0$, are also assumed to be uncertain. Thus, it is assumed that, for $j = 1, \ldots, \mu$, $h_j = h_j^0 + h_j^1$, where h_j^0 is known, representing the nominal part, and h_j^1 is unknown, representing the uncertain part, satisfying

$$|h_j^1| \leq \delta_j^h \tag{4}$$

for some known bounds $\delta_j^h > 0$, $j = 1, \ldots, \mu$.

It is also assumed that $\text{rank}(E) = n_1 < n$. Because of this assumption, the system (1)–(2) is a descriptor-type system [6]. It is, however, asuumed that:

Assumption 1: $\text{rank}(\mathcal{L}A_0\mathcal{R}) = n_2 := n - n_1$, for all A_0^1 satisfying $\bar{\sigma}(A_0^1) \leq \delta_0^a$,

where $\mathcal{L} \in \mathbf{R}^{n_2 \times n}$ and $\mathcal{R} \in \mathbf{R}^{n \times n_2}$ are such that the rows of $\mathcal{L}$ span the left null space of E and the columns of $\mathcal{R}$ span the right null space of E. Because of this assumption, a unique solution to (1), for any suitable initial condition, $x(t + \theta)$, $\theta \in [-h_{max}, 0]$, where $h_{max} := \max_j(h_j)$, is guaranteed [3].

The *characteristic function* of the system (1) is given by

$$\psi(s) := \det(sE - \mathcal{A}(s)) \tag{5}$$

where

$$\mathcal{A}(s) := \sum_{j=0}^{\mu} A_j e^{-sh_j} \tag{6}$$

and the *modes* of the system (1) are the roots of $\psi(s) = 0$. It is known that the system (1) has infinitely many modes, in general [22]. However, under Assumption 1, it has only finitely many modes with real part greater than

$$\nu_f := \sup\{\text{Re}(s) \mid \det(\bar{\mathcal{A}}(s)) = 0\} \tag{7}$$

where $\bar{\mathcal{A}}(s) := \mathcal{L}\mathcal{A}(s)\mathcal{R}$, where $\mathcal{L}$ and $\mathcal{R}$ are as in Assumption 1 and $\mathcal{A}(s)$ is as given in (6) [23].

It is known that (1)–(2) can not be stabilized by a finite-dimensional proper LTI controller unless $\nu_f < 0$ [24]. Thus, since our aim is to stabilize (1)–(2) for all uncertainties satisfying (3) and (4), we make the following assumption:

Assumption 2: $\nu_f < 0$ for any uncertainties satisfying (3) and (4).

This assumption implies that the system (1) has only finitely many unstable modes. A mode is called as an *unstable mode* if it has a non-negative real part. For technical reasons we also make the following assumption:

Assumption 3: For any uncertainties satisfying (3) and (4), the system (1) has the same number of unstable modes.

The problem then is to design a controller, based on the *nominal model*:

$$E\dot{x}(t) = \sum_{j=0}^{\mu} \left(A_j^0 x(t - h_j^0) + B_j^0 u(t - h_j^0) \right) \tag{8}$$

$$y(t) = Cx(t) \tag{9}$$

such that the *actual closed-loop system* obtained by applying this controller to the system (1)–(2) is robustly stable for all uncertainties satisfying the bounds (3) and (4).

3. Controller Design

To solve the above stated problem, we note that the transfer function matrices (TFMs) of the actual system (1)–(2) and of the nominal model (8)–(9) are respectively given by

$$\Gamma(s) = C \left(sE - \mathcal{A}(s) \right)^{-1} \mathcal{B}(s) \tag{10}$$

and

$$\Gamma^0(s) = C \left(sE - \mathcal{A}^0(s) \right)^{-1} \mathcal{B}^0(s) \tag{11}$$

where $\mathcal{A}(s)$ is defined in (6),

$$\mathcal{B}(s) := \sum_{j=0}^{\mu} B_j e^{-sh_j} \tag{12}$$

$$\mathcal{A}^0(s) := \sum_{j=0}^{\mu} A_j^0 e^{-sh_j^0} \tag{13}$$

and

$$\mathcal{B}^0(s) := \sum_{j=0}^{\mu} B_j^0 e^{-sh_j^0} \tag{14}$$

The TFMs $\Gamma(s)$ and $\Gamma^0(s)$ can be related as:

$$\Gamma(s) = \Gamma^0(s) \left(I + \Delta(s) \right) \tag{15}$$

where $\Delta(s)$ is the so-called *multiplicative uncertainty matrix*. Next, we derive a frequency-dependent upper bound on the norm of $\Delta(i\omega)$:

Lemma 1: Let

$$e(\omega) := \frac{n(\omega)}{d(\omega)} \ , \tag{16}$$

where

$$n(\omega) := \delta^b + \sum_{j=1}^{\mu} \bar{\sigma} \left(B_j^0 \right) \theta_j(\omega) + \delta^0(\omega) \tag{17}$$

and

$$d(\omega) := \underline{\sigma} \left(\mathcal{B}^0(i\omega) \right) - \delta^0(\omega) \tag{18}$$

where $\delta^b := \sum_{j=0}^{\mu} \delta_j^b$,

$$\theta_j(\omega) := \begin{cases} 2\sin\left(\frac{|\omega|\delta_j^h}{2}\right) , & |\omega| \leq \frac{\pi}{\delta_j^h} \\ 2 , & |\omega| > \frac{\pi}{\delta_j^h} \end{cases} , \quad j = 1, \ldots, \mu,$$

and

$$\delta^0(\omega) := \left(\delta^a + \sum_{j=1}^{\mu} \bar{\sigma}\left(A_j^0\right) \theta_j(\omega) \right) \bar{\sigma}\left(\Gamma_0(i\omega)\right)$$

where $\delta^a := \sum_{j=0}^{\mu} \delta_j^a$ and

$$\Gamma_0(s) := \left(sE - \mathcal{A}^0(s)\right)^{-1} \mathcal{B}^0(s)$$

Suppose that $d(\omega) > 0$, $\forall \omega \in \mathbf{R}$. Then, for all uncertainties satisfying (3) and (4),

$$\bar{\sigma}(\Delta(i\omega)) \leq e(\omega) , \quad \forall \omega \in \mathbf{R} . \tag{19}$$

Proof: Suppose $\Delta(s)$ satisfies

$$(sE - \mathcal{A}(s))^{-1} \mathcal{B}(s) = \left(sE - \mathcal{A}^0(s)\right)^{-1} \mathcal{B}^0(s) \left(I + \Delta(s)\right) \tag{20}$$

Note that by premultiplying both sides of (20) by C, we obtain (15). This implies that such $\Delta(s)$ can be chosen as the multiplicative uncertainty matrix. Next, premultiply both sides of (20) by $(sE - \mathcal{A}(s))$ and rearrange terms to obtain

$$\mathcal{N}(s) = \mathcal{D}(s)\Delta(s) \tag{21}$$

where

$$\mathcal{N}(s) := \sum_{j=0}^{\mu} B_j^1 e^{-sh_j} + \sum_{j=1}^{\mu} B_j^0 \rho_j(s) + \Delta^0(s)$$

and

$$\mathcal{D}(s) := \mathcal{B}^0(s) - \Delta^0(s)$$

where $\rho_j(s) := e^{-sh_j} - e^{-sh_j^0}$ and

$$\Delta^0(s) := \left[\sum_{j=0}^{\mu} A_j^1 e^{-sh_j} + \sum_{j=1}^{\mu} A_j^0 \rho_j(s) \right] \Gamma_0(s)$$

Note that $|e^{-i\omega h}| = 1$, for any real ω and h, $|\rho_j(i\omega)| = \left| e^{-i\omega(h_j^1 - h_j^0)} - 1 \right| = \left| 2\sin\left(\frac{\omega(h_j^1 - h_j^0)}{2}\right) \right| \leq \theta_j(\omega)$, $j = 1, \ldots, \mu$, and $\bar{\sigma}(\Delta^0(i\omega)) \leq \delta^0(\omega)$, $\forall \omega \in \mathbf{R}$. Thus,

$$\bar{\sigma}\left(\mathcal{N}(i\omega)\right) \leq \delta^b + \sum_{j=1}^{\mu} \bar{\sigma}\left(B_j^0\right) \theta_j(\omega) + \delta^0(\omega) =: n(\omega)$$

and

$$\underline{\sigma}\left(\mathcal{D}(i\omega)\right) \geq \underline{\sigma}\left(\mathcal{B}^0(i\omega)\right) - \delta^0(\omega) =: d(\omega)$$

where we used inequalities $\bar{\sigma}(M \pm N) \leq \bar{\sigma}(M) + \bar{\sigma}(N)$, $\bar{\sigma}(MN) \leq \bar{\sigma}(M)\bar{\sigma}(N)$, and $\underline{\sigma}(M \pm N) \geq \underline{\sigma}(M) - \bar{\sigma}(N)$, for arbitrary matrices M and N [25]. Furthermore, from (21),

$$\bar{\sigma}\left(\Delta(i\omega)\right) \leq \frac{\bar{\sigma}\left(\mathcal{N}(i\omega)\right)}{\underline{\sigma}\left(\mathcal{D}(i\omega)\right)} , \quad \forall \omega \in \mathbf{R} ,$$

from which the desired result follows. □

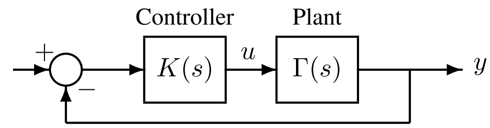

Fig. 1. Controller implementation

Now, suppose that a LTI controller with the TFM $K(s)$, to be implemented in a negative feedback configuration as shown in Fig. 1, is designed to stabilize the nominal model. The *complementary sensitivity matrix* for the nominal closed-loop system is then given as

$$T^0(s) := \Gamma^0(s)K(s)\left[I + \Gamma^0(s)K(s)\right]^{-1} \tag{22}$$

Suppose that this matrix satisfies

$$\bar{\sigma}\left(T^0(i\omega)\right) < \frac{1}{e(\omega)} , \quad \forall \omega \in \mathbf{R} , \tag{23}$$

where $e(\omega)$ is as given in (16). Then, as proved in the following theorem, $K(s)$ also stabilizes the actual system robustly.

Theorem 1: Let Assumptions 1–3 hold. Suppose that the controller $K(s)$ stabilizes the nominal model (8)–(9). Furthermore, suppose that (23) is satisfied. Then, for all uncertainties that satisfy (3) and (4), the controller $K(s)$ robustly stabilizes the actual system (1)–(2).

Proof: Suppose that (23) is satisfied. Then, by (19), for all uncertainties that satisfy (3) and (4),

$$\bar{\sigma}\left(T^0(i\omega)\right) < \frac{1}{\bar{\sigma}\left(\Delta(i\omega)\right)} , \quad \forall \omega \in \mathbf{R} \tag{24}$$

Under Assumptions 1–3, however, (24) implies that any LTI controller that stabilizes the nominal model (8)–(9) also stabilizes the actual system (1)–(2) robustly [13]. Thus, the desired result follows. □

The above theorem implies that, assuming that Assumptions 1–3 hold, we can design a controller, by using any approach, to stabilize the nominal model (8)–(9), while maintaining the bound (23) as well. This controller will then stabilize the actual system (1)–(2) robustly. In the next section, we will illustrate this approach by an example.

4. Example

Consider a system described by (1)–(2) with one input delay, no state delays and no delay-free input channel (thus, $\mu = 1$, $A_1 = 0$, and $B_0 = 0$), where

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} , \quad A_0^0 = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} , \quad B_1^0 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} ,$$

and

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

The bounds on the uncertain parts of A_0 and of B_1 are assumed to be $\delta_0^a = \delta_1^b = 0.1$. The nominal value of the time-delay is $h_1^0 = 0.1$ with 10% uncertainty (i.e., $\delta_1^h = 0.01$).

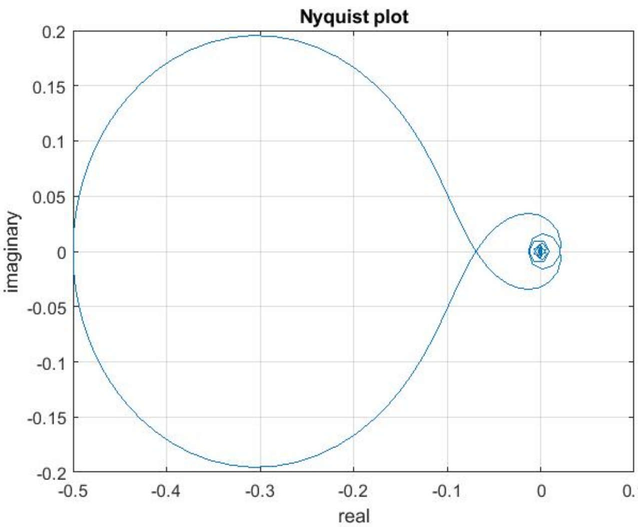

Fig. 2. Nyquist plot of $\Gamma^0(i\omega)$

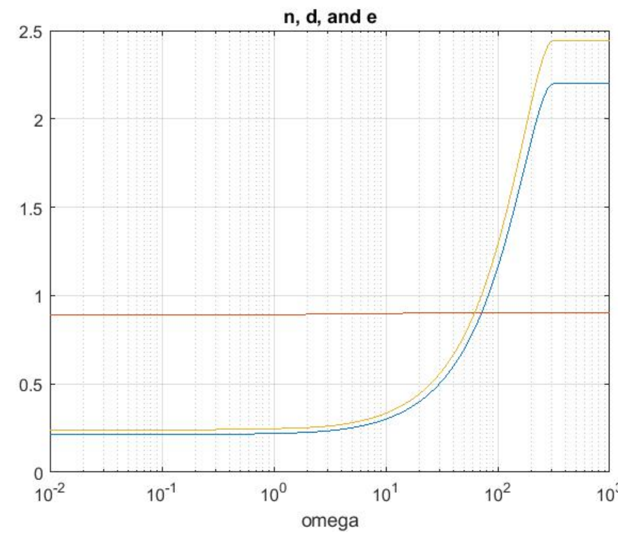

Fig. 3. Plots of $n(\omega)$ (in blue - plot with lowest low-frequency gain), $d(\omega)$ (in red - plot with highest low-frequency gain), and $e(\omega)$ (in cyan - plot with highest high-frequency gain) vs. ω

Note that $\mathcal{L}A_0\mathcal{R} = 1 + a \neq 0$, where a varies between -0.1 and 0.1 as A_0 is varied over all $A_0^0 + A_0^1$ satisfying $\bar{\sigma}\left(A_0^1\right) \leq 0.1$. Thus, Assumption 1 is satisfied. Furthermore, the open-loop system has exactly one unstable mode which varies between 1.8586 and 2.1414 (with nominal value 2) as A_0 is varied over all $A_0^0 + A_0^1$ satisfying $\bar{\sigma}\left(A_0^1\right) \leq 0.1$. Thus, Assumptions 2 and 3 are also satisfied.

The TFM of the nominal system is obtained from (11) as:

$$\Gamma^0(s) = \frac{e^{-0.1s}}{s-2} \tag{25}$$

The Nyquist plot of $\Gamma^0(i\omega)$, for $\omega \in \mathbf{R}$, is shown in Fig. 2. The first two left-most real axis crossings of the Nyquist plot are $\xi_1 = -0.5$ and $\xi_2 = -0.0692$. Thus, the Nyquist plot of $k\Gamma^0(i\omega)$ would encircle the -1 point once in the counter-clockwise direction if k is chosen in the range:

$$\frac{1}{-\xi_1} = 2 < k < 14.45 = \frac{1}{-\xi_2} \tag{26}$$

Therefore, since the nominal system has exactly one unstable mode, by the Nyquist theorem [26], a constant gain controller $K(s) = k$, where k is chosen in the range (26), would stabilize the nominal system.

Next, we calculate $n(\omega)$, $d(\omega)$, and $e(\omega)$, using (17), (18), and (16), respectively. These are plotted in Fig. 3. In particular, we note that $d(\omega) > 0$ for all $\omega \in \mathbf{R}$.

Next, in Figures 4–6, we plot $1/e(\omega)$ and $\bar{\sigma}\left(T^0(i\omega)\right)$ for various k in the range (26). Fig. 4 indicates that the bound (23) is violated in the low-frequency range for

$$2 < k < 2.627 \tag{27}$$

Fig. 6, on the other hand, indicates that the bound (23) is violated in the mid-frequency range for

$$9.566 < k < 14.45 \tag{28}$$

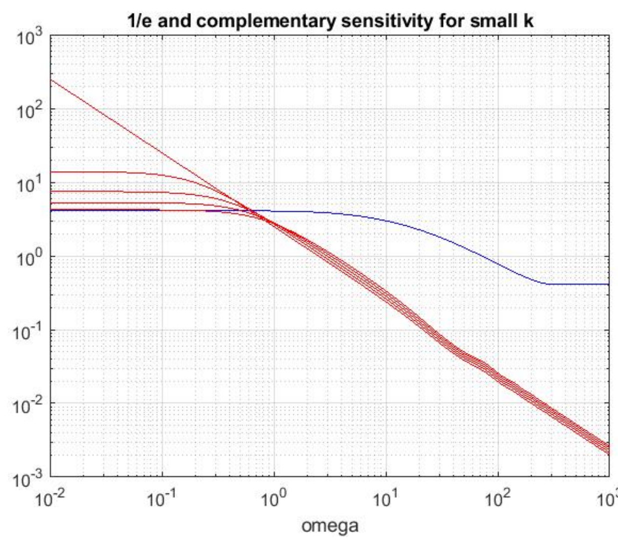

Fig. 4. Plots of $1/e(\omega)$ (in blue - plot with highest high-frequency gain) and of $\bar{\sigma}(T^0(i\omega))$ (in red - plots with lower high-frequency gain) vs. ω for various k in the range (27)

However, Fig. 5 indicates that the bound (23) is satisfied for all $\omega \in \mathbf{R}$, for

$$2.627 < k < 9.566 \tag{29}$$

Therefore, by Theorem 1, a constant gain controller $K(s) = k$, where k is chosen in the range (29), would robustly stabilize the given system as long as the bounds (3) and (4) are satisfied.

Next, we choose $k = 6$, which is about the mid-point of range (29). Plots of $1/e(\omega)$ and of $\bar{\sigma}\left(T^0(i\omega)\right)$ for this k are shown in Fig. 7. The closed-loop modes with real part greater than or equal to -40 under this controller for the nominal system are shown in Fig. 8 (we use QPmR [27] to calculate the closed-loop modes). The right-most closed-loop

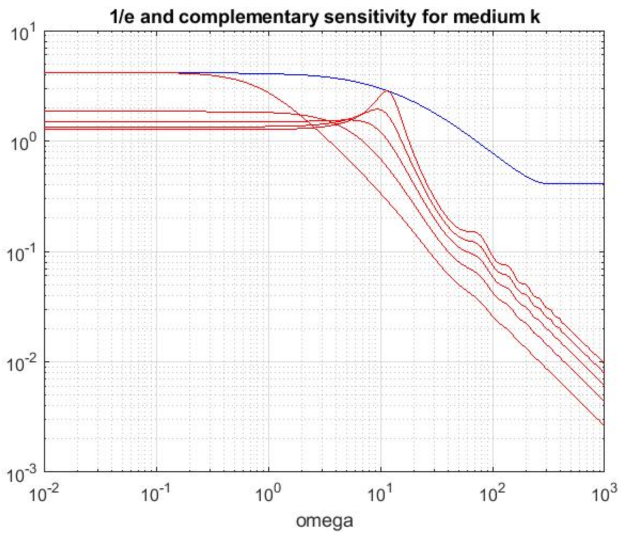

Fig. 5. Plots of $1/e(\omega)$ (in blue - plot with highest high-frequency gain) and of $\bar{\sigma}(T^0(i\omega))$ (in red - plots with lower high-frequency gain) vs. ω for various k in the range (29)

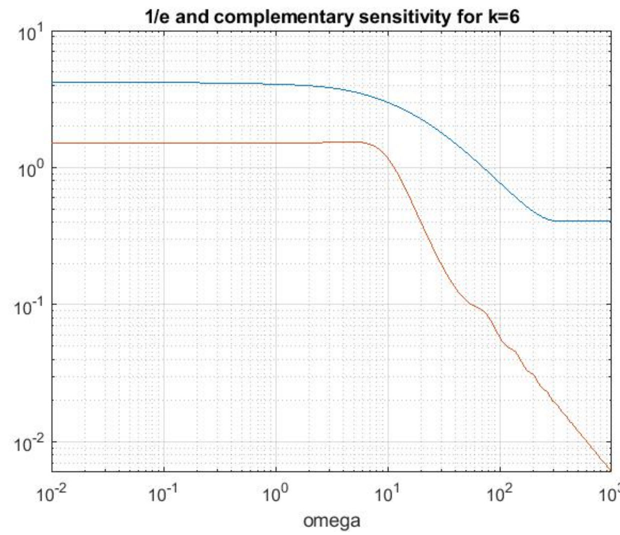

Fig. 7. Plots of $1/e(\omega)$ (in blue - plot with highest high-frequency gain) and of $\bar{\sigma}(T^0(i\omega))$ (in red - plot with lower high-frequency gain) vs. ω for $k = 6$

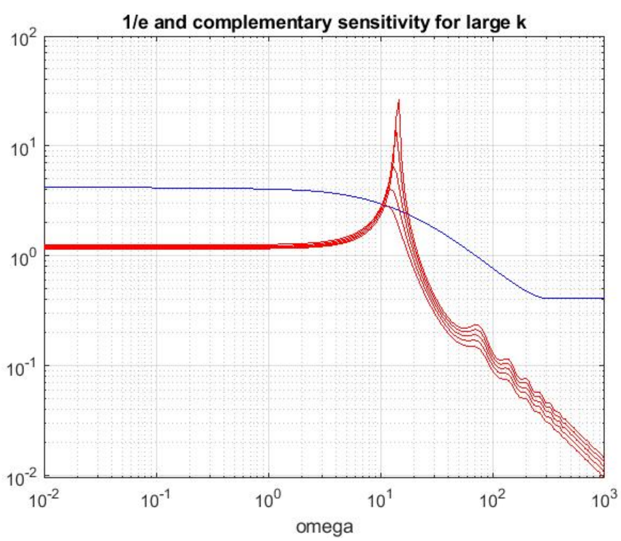

Fig. 6. Plots of $1/e(\omega)$ (in blue - plot with highest high-frequency gain) and of $\bar{\sigma}(T^0(i\omega))$ (in red - plots with lower high-frequency gain) vs. ω for various k in the range (28)

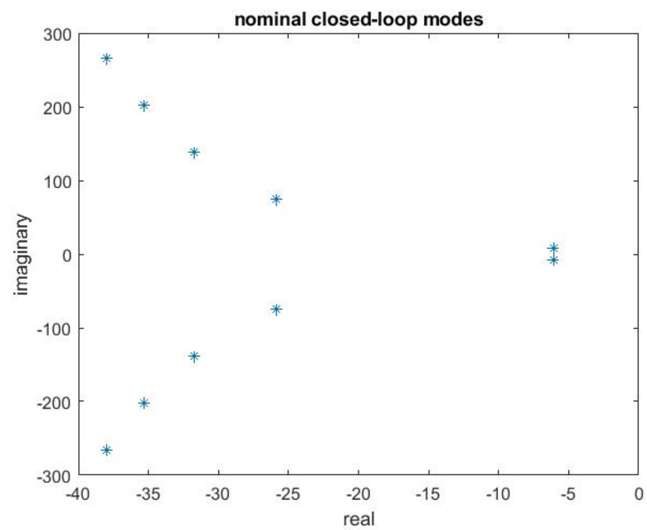

Fig. 8. Closed-loop modes with real part greater than or equal to -40 for the nominal system ($k = 6$)

modes are at $-6.0597 \pm i7.4834$. Under this controller, the worst case perturbations (which satisfy (3) and (4)), in the sense that the largest real part of the right-most closed-loop modes is obtained, are

$$A_0^1 = \begin{bmatrix} 0.0707 & 0.0707 \\ -0.0707 & 0.0707 \end{bmatrix}, \quad B_1^1 = \begin{bmatrix} 0.0707 \\ -0.0707 \end{bmatrix}, \quad h_1^1 = 0.01 \tag{30}$$

The closed-loop modes with real part greater than or equal to -40 for the system with these perturbations under this controller are shown in Fig. 9. The right-most closed-loop modes are at $-4.0015 \pm i8.6821$. This also shows that the designed controller robustly stabilizes the given system.

5. Conclusions

A stabilizing controller design approach for LTI descriptor-type time-delay systems, which is robust against uncertainties both in the time-delays and in other system parameters, has been proposed. A frequency-dependent bound (16) on the uncertainties has first been derived. It has then been shown that, under Assumptions 1–3, any controller which stabilizes the nominal model (8)–(9) and which satisfies the bound (23), robustly stabilizes the actual system (1)–(2). Once the bound (16) is found, the controller design is completely based on the nominal model. Thus, any controller design method, such as [7]–[12], can be used together with the bound (16) to design the controller.

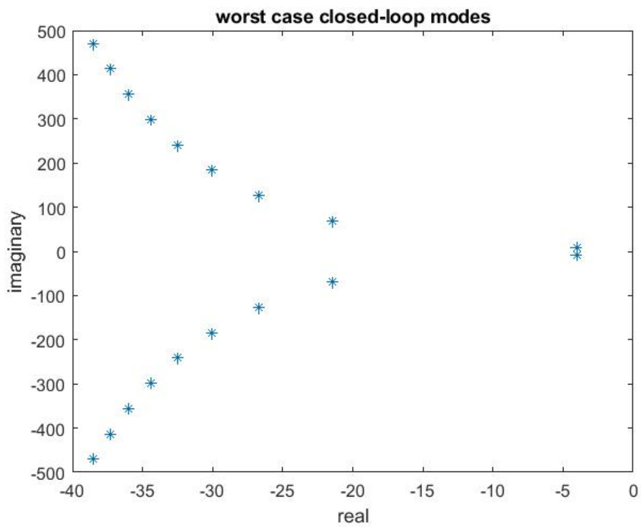

Fig. 9. Closed-loop modes with real part greater than or equal to -40 for the system with perturbations given in (30) ($k = 6$)

Note that, even for complicated systems, checking the condition (23) is easy, since (16) is a scalar function. Furthermore, since (16) is a function of frequency, the proposed approach can also be used for frequency shaping [28].

Acknowledgement

This work has been supported by the Scientific Research Projects Commission of Eskişehir Technical University under grant number 20ADP235.

References

[1] S.-I. Niculescu, *Delay Effects on Stability: A Robust Control Approach,* Lecture Notes in Control and Information Sciences, No. 269. London: Springer-Verlag, 2001.
[2] J.-P. Richard, "Time-delay systems: an overview of some recent advances and open problems," *Automatica*, vol. 39, pp. 1667–1694, 2003.
[3] J. K. Hale and S. M. Verduyn-Lunel, *Introduction to Functional Differential Equations*. New York: Springer-Verlag, 1993.
[4] N. Mollet, ed., *Remote and Telerobotics*. InTech, 2010.
[5] H. E. Erol and A. İftar, "Stabilization of decentralized descriptor-type neutral time-delay systems by time-delay controllers," *Automatica*, vol. 64, pp. 262–269, 2016.
[6] G.-R. Duan, *Analysis and Design of Descriptor Linear Systems*. New York: Springer, 2010.
[7] S. Gümüşsoy and W. Michiels, "Fixed-order H-infinity control for interconnected systems using delay differential algebraic equations," *SIAM Journal on Control and Optimization*, vol. 49, pp. 2212–2238, 2011.
[8] W. Michiels, "Spectrum-based stability analysis and stabilisation of systems described by delay differential algebraic equations," *IET Control Theory and Applications*, vol. 5, pp. 1829–1842, 2011.
[9] A. İftar, "Controller design for descriptor-type systems with distributed time-delay using extension," *WSEAS Transactions on Circuits and Systems*, vol. 20, pp. 27–32, 2021.
[10] A. İftar, "Robust tracking and disturbance rejection for descriptor-type neutral time-delay systems," *WSEAS Transactions on Systems*, vol. 20, pp. 48–53, 2021.
[11] H. E. Erol and A. İftar, "Decentralized time-delay controller design for systems described by delay differential-algebraic equations," *Transactions of the Institute of Measurement and Control*, 2021. Published online, DOI: 10.1177/01423312211015961.
[12] S. M. Özer and A. İftar, "Eigenvalue optimisation-based centralised and decentralised stabilisation of time-delay systems," *International Journal of Control*, 2021. Published online, DOI: 10.1080/00207179.2021.1906446.
[13] K. Zhou, J. C. Doyle, and K. Glover, *Robust and Optimal Control*. Englewood Cliffs: Prentice Hall, 1996.
[14] Q. C. Zhong, *Robust Control of Time-Delay Systems*. London: Springer-Verlag, 2006.
[15] A. İftar and Ü. Özgüner, "Decentralized LQG/LTR controller design for interconnected systems," in *Proceedings of the American Control Conference*, (Minneapolis, MN, U.S.A.), pp. 1682–1687, June 1987.
[16] A. İftar and Ü. Özgüner, "Local LQG/LTR controller design for decentralized systems," *IEEE Transactions on Automatic Control*, vol. AC–32, pp. 926–930, 1987.
[17] A. İftar, "Robust control of time-delay systems: A Nyquist approach," in *Proceedings of the IASTED International Conference on Intelligent Systems and Control*, (Marina del Rey, CA, U.S.A.), pp. 232–237, Nov. 2013.
[18] A. İftar, "Robust controller design for neutral time-delay systems," in *Preprints of the 12th IFAC Workshop on Time-delay Systems*, (Ann Arbor, MI, USA), pp. 180–184, June 2015.
[19] A. İftar, "A robust controller design approach for systems with distributed time-delay," in *Proceedings of the European Control Conference*, (Strasbourg, France), pp. 85–90, June 2014.
[20] A. İftar, "Robust controller design for neutral systems with distributed time-delay," in *Preprints of the 13th IFAC Workshop on Time-delay Systems*, (Istanbul, Turkey), pp. 88–93, June 2016.
[21] A. İftar, "A robust controller design approach for descriptor-type systems with distributed time-delay," in *Proceedings of the European Control Conference*, (Rotterdam, Netherlands), pp. 429–433, June 2021.
[22] W. Michiels and S.-I. Niculescu, *Stability and Stabilization of Time-Delay Systems*. Philadelphia: SIAM, 2007.
[23] W. Michiels and S.-I. Niculescu, *Stability, Control, and Computation for Time-Delay Systems: An Eigenvalue-Based Approach*. Philadelphia: SIAM, 2014.
[24] J. J. Loiseau, M. Cardelli, and X. Dusser, "Neutral-type time-delay systems that are not formally stable are not BIBO stabilizable," *IMA Journal of Mathematical Control and Information*, vol. 19, pp. 217–227, 2002.
[25] X. Zhan, *Matrix Inequalities*. Berlin: Springer, 2002.
[26] H. Özbay, *Introduction to Feedback Control Theory*. Boca Raton: CRC Press, 1999.
[27] T. Vyhlidal and P. Zitek, "QPmR - Quasi-polynomial root-finder: Algorithm update and examples," in *Delay Systems: From Theory to Numerics and Applications,* T. Vyhlidal, J.-F. Lafay, and R. Sipahi (Eds.), (Springer, New York), pp. 299–312, 2014.
[28] D. Pilbauer, T. Vyhlidal, and W. Michiels, "Spectral design of output feedback controllers for systems pre-compensated by input shapers," in *Preprints of the 12th IFAC Workshop on Time-delay Systems*, (Ann Arbor, MI, USA), June 2015.