On the Robust Multiple Objective Control with Simultaneous Pole Placement in LMI Regions

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Abstract: - This article studies the problem of designing robust control laws to achieve multiple performance objectives for linear uncertain systems. Specifically, in this study we have selected one of the control objectives to be a closed-loop pole placement in specific regions of the left-half complex plane. As such, a guaranteed cost-based multi-objective control approach is proposed and compared with the H_2/H_∞control by means of an application example.

Key-Words: - robust control, H_2/H_∞, multi-objective control, uncertain systems, regional pole placement, linear matrix inequalities, LMI, LMI framework, control systems, Lyapunov function.

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1 Introduction

In our contemporary society, the upcoming new industrial revolution, the so-called "Industry 4.0" [1] and in particular the increasing number of Internet of Things (IoT) devices that automatically gather information and send them to remote repositories for analysis have increased the need for reliable and efficient systems [2]. Specifically, there is a high demand to design accurate and robust control systems responsible for computing and monitoring the linear and non-linear systems alike to achieve optimal behaviour [4]. Specifically, although artificial neural networks [5] or other dynamical techniques could be considered for system optimization [6] many interesting applications exist regarding control systems such as in robotics [7,8], vehicles [9], trucks [10] solar systems [11,12], wind tunnels [13,14], routing problems [15] and others [16,17].

In many practical design situations, especially in industrial applications implementing Big Data solutions consisting of many automated systems [18] one has to cope not only with uncertainty but also with multiple design specifications. Robust multi-objective control problems are usually formulated in the H_2/H_∞ framework [20,21,22,23,24,25,26].

Additionally, in the LQG context, multiple objectives are expressed in terms of a finite number of quadratic cost functions [27,28,29]. Since the pole locations may be crucial for system's transient response characteristics and stability margins, one of the design objectives is often the closed-loop pole placement into pre-specified regions of the left-half complex plane, for all system uncertainties of a given class (see e.g. [30,31,32,3334], and related references). In most of these approaches, specification of the closed-loop objectives in terms of a common Lyapunov function [35,36] permits reducing the multi-objective controller design to a convex optimization problem; hence, solutions in the LMI framework are sought (see also [37,38,39]). In the present paper, H_2/H_∞ and guaranteed cost multi-objective control approaches with simultaneous pole placement are considered. In particular, a guaranteed cost multi-objective technique that ensures a relative stability degree for all system admissible uncertainties is proposed; furthermore, it is extended to ensure the closed-loop pole clustering into LMI regions.

The paper is organized as follows: In Section 2, the H_2/H_∞ design with regional pole placement is recalled. In Section 3, a guaranteed cost multi-objective technique with simultaneous pole placement is proposed. An application example allows comparing these approaches in Section 4. Finally, conclusions are given in Section 5.

2 H_2/H_∞ Multiple Objective Control

Multi-objective synthesis may consider a mix of time- and frequency-domain specifications, i.e., H_2 and H_∞ performance, regional pole placement, passivity, asymptotic tracking or regulation, and saturation constraints. In particular, stability in presence of unstructured model uncertainties and asymptotic disturbance rejection are met by H_∞ performance while H_2 performance ensures desired closed-loop responses in presence of random noise. If, in addition, the closed-loop pole placement in prespecified regions of the left-half plane is of interest, an H_2/H_∞control problem with simultaneous pole placement is formulated. In this section, a brief overview of existing results is presented.

2.1 Problem Formulation

Consider the linear time-invariant, state-feedback control system of Fig. 1:

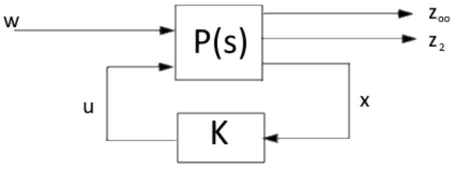

Fig. 1: *State feedback control system*

where $P(s)$ is the transfer function, x is the state vector, u is the control vector, w is a vector of exogenous inputs, and z_∞ , z_2 are the output signals related to the performance of the control system. Denote $T_\infty(s)$ and $T_2(s)$ the transfers from w to z_∞ and z_2 , respectively. Assuming the state vector available for feedback, a linear, constant gain state-feedback control law of the form: $u(t) = Kx(t)$ (1), is sought such that the following closed-loop system objectives are met: (i) The H_∞ - norm of $T_\infty(s)$ remains smaller than a given γ > 0 (ii) The H_2 -norm of $T_2(s)$ remains smaller than a given ν > 0 (iii) The trade-off H_2/H_∞ criterion $\alpha\|T_\infty\|_\infty^2 + \beta\|T_2\|_2^2$ is minimized for some positive weighting scalars α and β (iv) The poles of the closed-loop system are placed in a pre-specified region of the left-half complex plane.

A state-space realization of the closed-loop system of Fig. 1 has the following mathematical form:

$$\begin{aligned} \dot{x} &= (A + B_2K)x(t) + B_1w(t) \\ z_\infty &= (C_1 + D_{12}K)x(t) + D_{11}w(t) \\ z_2 &= (C_2 + D_{22}K)x(t) \end{aligned} \tag{2}$$

where A, B_1 , B_2 , C_1, C_2 , D_{11}, D_{12}, D_{22} are constant matrices of appropriate dimensions. Some of the system matrices may be uncertain but no explicit uncertainty description is used in (2).

2.2 Pole Placement in LMI Regions

By virtue of the Bounded Real Lemma, the control synthesis problem formulated in Section 2.1 can be expressed in terms of a complex optimization problem under LMI constraints [26]. Besides, placing the closed-loop system poles in convex subsets of the left-half complex plane can be expressed as LMI constraints on the Lyapunov matrix, as well [33]. In these references, the following results have been established in terms of necessary and sufficient conditions:

- H_∞ performance: The closed-loop system (2) is stable and the H_∞ -norm of $T_\infty(s)$ is smaller than a given γ > 0, if and only if there exists a symmetric matrix X_∞ such that,

$$\begin{pmatrix} (A+B_2K)X_\infty + X_\infty(A+B_2K)^T & B_1 & X_\infty(C_1+D_{12}K)^T \\ B_1^T & -I & D_{11}^T \\ (C_1+D_{12}K)X_\infty & D_{11} & -\gamma^2 I \end{pmatrix} = H_{loop}$$

Where $H_{loop} < 0$, $X_\infty > 0$ (3)

- $\underline{H_2 \text{ performance}}$: The H_2 -norm of $T_2(s)$ is smaller than a given $\nu > 0$, if and only if there exist two symmetric matrices X_2 and Q, such that,

$$\begin{pmatrix} (A+B_2K)X_2 + X_2(A+B_2K)^T & B_1 \\ B_1^T & -I \end{pmatrix} < 0$$

$$\begin{pmatrix} Q & (C_2+D_{22}K)X_2 \\ X_2(C_2+D_{22}K)^T & X_2 \end{pmatrix} > 0 \quad (4)$$

and tr(Q) < v^2

- <u>Pole placement</u>: The poles of the closed-loop system are placed in the LMI region: $D = \{z \in \mathbb{C}: L + Mz + M^T\bar{z} < 0,$

Where $L = L^T = \{\lambda_{ij}\}_{1\le i,j\le m}$ and $M = \{\mu_{ij}\}_{1\le i,j\le m}$ These equations apply if and only if there exists a symmetric matrix X_{pol}such that:

$$[\lambda_{ij}X_{pol} + \mu_{ij}(A+B_2K)X_{pol} + \mu_{ij}X_{pol} + \mu_{ij}X_{pol}(A+B_2K)^T]_{1\le i,j\le m} = H_{pole\_place}$$

Where $H_{pole\_place} < 0$, $X_{pole} > 0$ (5)

The above design problem is not jointly convex in the variables Q, K, X_∞, X_2, and X_{pol} . However, convexity can be enforced by seeking a common solution $X := X_\infty = X_2 = X_{pol} > 0$, such that conditions (3)-(5) are met. Then, setting Y := KX leads to the multiple objective optimization with LMI constraints,

$$\begin{pmatrix} AX + XA^T + B_2Y + Y^TB_2^T & B_1 & XC_1^T + Y^TD_{12}^T \\ B_1^T & -I & D_{11}^T \\ C_1X + D_{12}Y & D_{11} & -\gamma^2 I \end{pmatrix} = H_{design}$$

Where $H_{design} < 0$ and $\begin{pmatrix} Q & C_2X + D_{22}Y \\ XC_2^T + Y^TD_{22}^T & X \end{pmatrix} > 0$

$$[\lambda_{ij} + \mu_{ij}(AX + B_2Y) + \mu_{ij}(XA^T + Y^TB_2^T)]_{1\le i,j\le m} = H_{final}$$

Where $H_{final} < 0$ (6) and $tr(Q) < v_0^2$, $\gamma^2 < \gamma_0^2$

In this case, the multiple objective problem reduces to the minimization of $\gamma^2 + \beta tr(Q)$, where Y, X, Q, and γ^2 satisfy inequalities (6). The optimal solution is denoted $(X^*, Y^*, Q^*, \gamma^*)$and the resulting state-feedback gain: $K^* = Y^*(X^*)^{-1}$ (7), guarantees a satisfactory worst-case performance corresponding to:

$$\|T_\infty\|_\infty \le \gamma^* \ , \ \|T_2\|_2 \le \sqrt{tr(Q^*)} \quad (8)$$

(see [33] for details and proof).

In the output-feedback case, nonlinear terms arise at the synthesis procedure formulas and hence the problem is intractable via LMIs. A change of the controller's variables leads to approaches that are more sophisticated. Besides, since seeking for a common solution X in (6) implies some conservatism [33], recent works attempt to remove it by using BMI optimization techniques with output-feedback dynamic controllers [40, 41].

Consider now the case where structured uncertainties in system (2) are taken into account in terms of either the polytopic description,

$$A = \sum_{j=1}^{k} A_j a_j \ , \sum_{j=1}^{k} a_j = 1 \ , \ a_j \ge 0 \quad (9)$$

or the affine description:

$$A = A_{nom} + \sum_{j=1}^{k} A_j r_j \quad (10)$$

in which the uncertain parameters r_j belong to known bounded intervals and where for simplicity it has been assumed that the state matrix is denoted by A. In ref. [34], the previous synthesis results were extended to the uncertain cases (9), (10); furthermore, existing quadratic D-stability results [31,42] were generalized to arbitrary LMI regions.

Note that when dealing with structured uncertainties, the resulting regional pole placement conditions are only sufficient. In order to reduce the related conservatism, the Lyapunov matrix was chosen as an affine function of the uncertain parameters, in [29]. Then, uncertainty levels were determined for which stability is preserved. The analysis results were used for synthesis purposes in order to achieve regional pole assignment via output-feedback.

3 Guaranteed cost multiple objective control

When dealing with linear state-space descriptions with structured uncertainties, the LQR context allows achieving desirable performances in terms of guaranteed cost control (GCC) (see e.g. [43,44,45,46] and related references). Moreover, as shown in [44], the nominal system's optimality is preserved despite uncertainty, in the sense of the well-known stability margins ensured by the optimal linear quadratic regulator [47]. In order to reduce the usual conservatism, an upper bound of the quadratic cost function has to be minimized. Besides, in the quadratic *D*–stability approaches [31,32] it is straightforward to deduce that an associated guaranteed cost can be minimized, as well. Hence, the multi-objective control with simultaneous pole placement for linear systems in state-space descriptions with structured uncertainties can naturally be considered in the LQR/GCC context. The multiple objectives are expressed in terms of multiple quadratic performance indices that have to be as small as possible by the same controller while the closed-loop poles have to be placed in a pre-specified region of the left-half plane. Recall that the GCC can tolerate unstructured uncertainties affecting the system input [48]. This fact allows extending the GCC application to the class of uncertain system affected by external disturbances, as well.

3.1 Problem Formulation

Consider the linear uncertain system,

$$\dot{x}(t) = (A + \Delta A)\dot{x(t)} + (B + \Delta B)u(t), \in [0, \infty),\quad x(0) = x_0 \tag{11}$$

where A and B are the state and control matrices, respectively, having appropriate dimensions. Let the uncertainty description be of the affine type,

$$\Delta A = \sum_{i=1}^{k} A_i r_i, \Delta B = \sum_{i=1}^{l} B_i p_i \tag{12}$$

where A_i , $i = 1,2, \ldots, k$ and B_i , $i = 1,2, \ldots, l$ are constant matrices that determine the uncertainty structure and r_i , p_i are scalar uncertain parameters. It is assumed that the corresponding uncertain parameter vectors belong to known and bounded compact sets,

$$\mathfrak{R} \coloneqq \{r \in R^k : |r_i| \le \bar{r},\ i = 1,2, \ldots, k\};\ \bar{r} > 0, \tag{13}$$

$$\mathfrak{J} \coloneqq \{r \in R^l : |p_i| \le \bar{p},\ i = 1,2, \ldots, l\};\ \bar{p} > 0$$

An equivalent normalized description of (12) with $\bar{r} = \bar{p} = 1$ can be used without loss of generality. Moreover, linearity of (12) allows the uncertainty matrices A_i, B_i to have unity rank and thus be decomposed in terms of vector products of appropriate dimensions,

$$A_i = d_i e_i^T,\ i = 1,2, \ldots, k\ , B_i = f_i g_i^T,\ i = 1,2, \ldots, l \tag{14}$$

For the design purpose define the following symmetric positive definite matrices (15):

$$D \coloneqq [d_1 \ldots d_k]\ , E \coloneqq [e_1 \ldots e_k]\ ,$$
$$F \coloneqq [f_1 \ldots f_l]\ , G \coloneqq [g_1 \ldots g_l]\ ,$$
$$\tilde{S} \coloneqq diag(\sigma_1 \ldots \sigma_\kappa)\ , \tilde{T} \coloneqq diag(\tau_1 \ldots \tau_l)$$

where *j j* σ τ are positive scalars. Since the rank-1decomposition (14) is not unique, these scalars may be chosen to determine the appropriate one, in order to satisfy design requirements.

The control design specifications are described in terms of multiple quadratic performance indices of the form,

$$J_i(x_0, \Delta A, \Delta B, t) = \int_0^\infty [x^T(t) Q_i x(t) + u^T(t) R_i u(t)]dt \tag{16}$$

where $Q_i > 0$, $R_i > 0$, $i = 1, \ldots, \rho$.

The *guaranteed cost multiple objective control* problem consists of finding a linear state-feedback control law of the form (1) such that the corresponding values of *all* performance indices (16) are made as small as possible and remain upper bounded for all *admissible uncertainties* i.e., consistent with (12)-(13). It is obvious that for ρ =1 this problem reduces to the well-known GCC problem. In the sequel, pole placement constraints will be considered as well.

3.2 An LMI approach for GC multi-objective control

Following to the GCC theory, the *guaranteed cost control* with respect to the i^{th}performance index is: $u(t) = -\delta_i R_i^{-1} B^T P x(t)$ (17)
and results from the solution of the generalized Riccati equation:

$$PA + A^T P - P(\delta_i B R_i^{-1} B^T - \delta_i^2 B R_i^{-1} G T_i^{-1} G^T R_i^{-1} B^T - FT_i F^T - DS_i D^T)P + ES_i E^T + Q_i = 0 \quad (18)$$

Furthermore, it ensures that $J_i(x_0, \Delta A, \Delta B, t)t \leq \widetilde{J_\iota} = x_0^T P x_0$, for all admissible uncertainties, $\widetilde{J_\iota}$ being the *guaranteed cost*. To avoid the dependency of the cost functions (16) on initial conditions, one can assume x_0 uniformly distributed over a unit sphere and thus randomized with zero mean and identity covariance. Then, by considering the expected values of (16), the corresponding guaranteed cost values are $\widetilde{J_\iota} \leq tr(P)$, i.e.,

$$J_i(x_0, \Delta A, \Delta B, t) \leq tr(P) \quad (19)$$

Consider now the case where a degree of relative stability is to be ensured to the closed-loop system. By multiplying (16) by e^{2at} , α > 0 and replacing A by $\bar{A} = A + aI$ in the generalized Riccati equation, the guaranteed cost control law places the closed-loop system's poles at the left of −α on the complex plane, for all admissible uncertainties [49]. In consequence, if a common positive definite solution of the set of generalized Riccati equations,

$$P\bar{A} + \bar{A}^T P - P(\delta_i B R_i^{-1} B^T - \delta_i^2 B R_i^{-1} G T_i^{-1} G^T R_i^{-1} B^T - FT_i F^T - DS_i D^T)P + ES_i E^T + Q_i = 0 \quad (20)$$

for $= 1, \ldots, \rho$, exists, then the guaranteed cost control law: $u(t) = -\delta_i R_i^{-1} B^T P x(t)$ (21)

ensures the upper bound: $J_{ai} \leq tr(P)$ (22)

of the performance indices

$$J_{ai}(\Delta A, \Delta B, t) = E\{\int_0^\infty e^{2at}[x^T(t) Q_i x(t) + u^T(t) R_i u(t)]dt\} \quad (23)$$

Moreover, it ensures the closed-loop system's pole positioning at the left of −α on the complex plane, for all admissible uncertainties.

Generalized Riccati equations of the form (18), (20) lack of analytic solutions (see [46] for an overview of solutions and features). A computational solution can be found by using LMIs, provided such a solution exists. In addition, the positive scalars σ_i, τ_i can be treated as free parameters to the solution search. Since the common solution should be such that the corresponding guaranteed cost is minimized, one can solve an LMI minimization problem.

3.3 Theorems

In the next sections we provide in the theorems and proofs for the above-mentioned mathematical equations.

3.3.1 Theorem 1

Consider the uncertain system (11) with random initial conditions and multiple performance indices (23). If the minimization problem

$$\min_{(M, W, \delta_i, S_i, T_i)} tr(M) \quad (24)$$

with LMI constraints: $\begin{bmatrix} M & I \\ I & W \end{bmatrix} > 0$ and (25)

$$\begin{bmatrix} -\bar{A}W - W\bar{A}^T + \delta_1 B R_{-1}^1 B^T - DS_1 D^T - FT_1 F^T & WE & \delta_1 B R_{-1}^1 G & W \\ E^T W & S_1 & 0 & 0 \\ \delta_1 G^T R_{-1}^1 B^T & 0 & T_1 & 0 \\ W & 0 & 0 & \bar{Q}_1^{-1} \end{bmatrix} > 0 \quad (26)$$

$$\begin{bmatrix} -\bar{A}W - W\bar{A}^T + \delta_\rho B R_\rho^1 B^T - DS_\rho D^T - FT_\rho F^T & WE & \delta_\rho B R_\rho^1 G & W \\ E^T W & S_\rho & 0 & 0 \\ \delta_1 G^T R_\rho^1 B^T & 0 & T_\rho & 0 \\ W & 0 & 0 & \bar{Q}_\rho^{-1} \end{bmatrix} > 0$$

admits a non-empty set of feasible solutions $(M, W, \delta_i, S_i, T_i)$ where M, W are symmetric and positive definite matrices, *S*, *T* are diagonal positive definite matrices and $\bar{Q_\iota} < Q_i$, $i =$

$1, \dots, \rho$, then the positive definite matrix, $P = W^{-1}$ (27), satisfies the set of generalized algebraic Riccati equations (20). The guaranteed cost control law is: $u^*(t) = -\delta_i R^1_{-i} B^T W^{-1} x(t)$ (28) and minimizes the guaranteed cost: $J^* = tr(P)$ (29), for all admissible uncertainties. Furthermore, the closed-loop system's poles are placed at the left of $-\alpha$ on the complex plane, for all admissible uncertainties.

Proof: By applying the Schur complement to the multiple LMIs (26) and then pre and post-multiplying by $P = W^{-1}$, one obtains the following set of inequalities:

$$PA + A^T P - P(\delta_i B R_i^{-1} B^T - \delta_i^2 B R_i^{-1} G T_i^{-1} G^T R_i^{-1} B^T - FT_i F^T - DS_i D^T)P + ES_i E^T + \overline{Q}_\iota < 0 \quad (30)$$

These inequalities are satisfied for $\overline{Q}_\iota < Q_i$, if the generalized algebraic Riccati equations (20) are satisfied. Furthermore, application of the Schur complement to (25) yields $M > W^{-1} = P$. In consequence, minimization of tr (M) implies minimization of tr(P). Since P is the common solution of the set of (20), $tr(P)$ is the minimal guaranteed cost bound of the multiple performance objectives. The closed-loop pole positioning follows from (20) and (23) according the results in [45,49]. The optimality of the solution follows from the convexity of the objective function and of the constraints [50].

Based on Theorem 3.1, the guaranteed cost multi-objective control with pole placement in LMI regions can be formulated as follows:

3.3.2 Theorem 2

Consider the uncertain system (11) with random initial conditions and multiple performance indices (16). If the minimization problem

$$\min_{(M,W,\delta_i,S_i,T_i)} tr(M) \quad (31)$$

with LMI constraints: $\begin{bmatrix} M & I \\ I & W \end{bmatrix} > 0$ and (32)

$$\begin{bmatrix} -AW - WA^T + \delta_1 B R^1_{-1} B^T - DS_1 D^T - FT_1 F^T & WE & \delta_1 B R^1_{-1} G & W \\ E^T W & S_1 & 0 & 0 \\ \delta_1 G^T R^1_{-1} B^T & 0 & T_1 & 0 \\ W & 0 & 0 & \bar{Q}_1^{-1} \end{bmatrix} > 0$$

$$\vdots \quad (33)$$

$$\begin{bmatrix} -AW - WA^T + \delta_\rho B R^1_\rho B^T - DS_\rho D^T - FT_\rho F^T & WE & \delta_\rho B R^1_\rho G & W \\ E^T W & S_\rho & 0 & 0 \\ \delta_1 G^T R^1_\rho B^T & 0 & T_\rho & 0 \\ W & 0 & 0 & \bar{Q}_\rho^{-1} \end{bmatrix} > 0$$

$$[\lambda_{\kappa\nu} W + \mu_{\kappa\nu}(AW - \delta_i R^1_{-i} B^T) + \mu_{\kappa\nu}(WA^T - \delta_i R^1_{-i} B^T)]_{1 \le \kappa,\nu \le m} < 0 \quad (34)$$

admits a non-empty set of feasible solutions $(M, W, \delta_i, S_i, T_i)$ where M, W are symmetric and positive definite matrices, S, T are diagonal positive definite matrices and $\overline{Q}_\iota < Q_i$, $i = 1, \dots, \rho$, then the positive definite matrix,

$$P = W^{-1} \quad (35)$$

satisfies the set of generalized algebraic Riccati equations (18). The guaranteed cost control law is the following:

$$u^*(t) = -\delta_i R^1_{-i} B^T W^{-1} x(t) \quad (36)$$

and minimizes the guaranteed cost: $J^* = tr(P)$ (37), for all admissible uncertainties. Furthermore, the closed-loop system's poles are placed in the LMI region $D = \{z \in \mathbb{C}: L + Mz + M^T \bar{z} < 0\}$ where $L = L^T = \{\lambda_{\kappa\nu}\}_{1 \le \kappa,\nu \le m}$ and $M = \{\mu_{\kappa\nu}\}_{1 \le \kappa,\nu \le m}$, for all admissible uncertainties.

Proof: The proof follows from Theorem 3.1 and the results in [33].

Remark In the control laws (28) and (36), the values of R_i , $i = 1, \dots, \rho$ yield different values of feedback gains obtained for the same P; this seems to deviate from the initial problem statement. In effect, for any choice of R_i the control law guarantees the same upper bound overall performance indices. Moreover, by choosing the suitable feedback gain between them, it is possible to satisfy further design specifications.

4 Design Example

The control approaches presented in previous sections are illustrated and compared by means of a design example. All LMI-related computations were performed with the function *hinfmix* from the *LMI Control Toolbox* [51].

4.1 Satellite's attitude control

This example is adapted from the satellite's attitude control problem in [52]. The design purpose is to control the pointing direction of a satellite in orbit about the earth. The system is modelled as two masses connected by a spring with torque constant k and viscous-damping constant f that can vary because of temperature fluctuations in the range,

$$0.09 < k < 0.4,\ 0.0038 < f < 0.04$$

Their nominal values are $k = 0.245$, $f = 0.0188$. The satellite's motion is affected by a disturbance w. The system's state-space description is,

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k & k & -f & f \\ k & -k & f & -f \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} (w + T)\ ,$$

$$y = \begin{bmatrix} z_\infty \\ z_2 \end{bmatrix}\ \text{, with}\quad z_\infty = \theta_2\ \text{ and}$$

$$z_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} T$$

where θ_1 , θ_2 are the angles of the main body and of the sensor, respectively and T is the control torque on the main body. The open-loop nominal system poles are $(0, 0, -0.0188 \pm j0.7)$ and thus it is practically unstable. Furthermore, the poles of the open-loop uncertain system for all possible values of the uncertain parameters in the above range lie to a region located close to the imaginary axis, shown in Fig. 2.

For this system, a robust control law that ensures closed-loop performances and regional pole placement in a cycle with equation $(x + 6.9)^2 + y^2 = 46.24$ for all admissible parameter variations is sought. Application of the H_∞ design with pole placement constraints results to extremely large controller gains that may cause undesirable effects, such as actuator saturation, to the closed-loop system. In the sequel, both the H_2/H_∞ and the guaranteed cost multi-objective control are applied to the satellite's model.

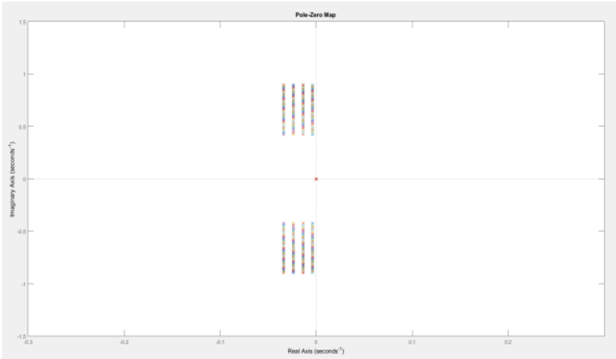

Fig. 2: *Open-loop pole locations of the uncertain system*

The controller gain K = [-5.8861 -7.2805 -3.6630 -23.2995] has been obtained from the trade-off between minimizing H_2 and H_∞ -norms of the uncertain system. The closed-loop nominal system's poles are $(-1.12 \pm j0.584, -0.727 \pm j1.22)$. Furthermore, the closed-loop pole locations of the uncertain system for all admissible parameter variations are shown in Fig. 3.

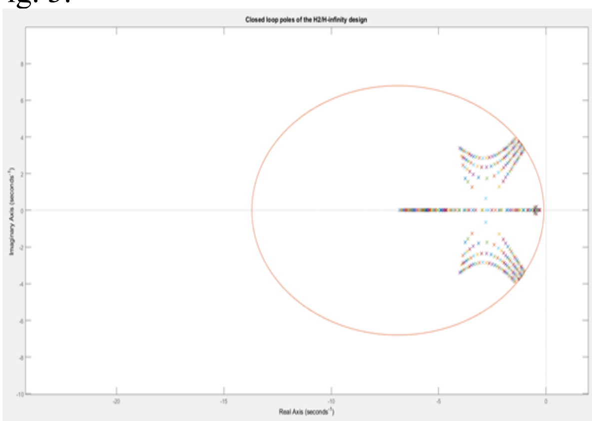

Fig. 3: *Closed-loop pole locations obtained by the H_2/H_∞control*

5 Conclusion

The guaranteed cost multi-objective approach proposed in the present paper considered the general case of uncertain systems in which uncertain parameters may affect both the state and control input matrices. The uncertainty decomposition was treated as an optimization tool. Minimization of the GC allowed reducing conservatism related to Lyapunov based methods. The non-uniqueness of the obtained controller provides several degrees of freedom to the design procedure: Since all of the controllers are stabilizing, the most adequate one can be chosen for the practical system under consideration. In a future work, an inverse problem will be formulated and solved to establish algorithms for the choice of the most adequate solution with respect to the desirable performances.

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