Least Square Identification using Noise Reduction Disturbance Observer

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Abstract: - In this article, the application of the identification algorithm of Recursive Least Squares with Forgetting Factor in conjunction with the Noise Reduction Disturbance Observer shows that the effects of noise, which affects input and output signals of the process, can be reduced so that the identification process can be more effective and precise. In order to evaluate the effectiveness of this strategy the results of a case of study in which estimation of a first order process using the Noise Reduction Disturbance Observer is compared to an estimation without the Disturbance Observer.

Key-Words: - Identification, Least Square, Noise Reduction Disturbance Observer.

1 Introduction

It is well known that one of the most important setbacks in the identification of processes is the noise problem. In fact, due to existence of noise, which are normally assume random variables, led to the need to design strategies that minimize their effects resulting, in particular, in schemes or algorithms based on the least square minimization such as the Generalized Least Square or Recursive Least Square with Forgetting Factor (LSFF). However, these strategies assume that the noise ϖ is "white", that is, uncorrelated with $E\{\varpi\} = 0$ and $E\{\varpi\varpi^T\} = \sigma^2 I$. Furthermore, as it is shown in [1], the estimated parameters will have a dispersion of at least σ. Therefore, the combined effect of noise in the measurements of the input and output signals of the process produces unacceptable dispersion or variations in the estimated parameters. Moreover, in a realistic case the noise is normally colored and correlated which may induce also a polarization in the estimation. Under this condition, two strategies are usually applied: to assume a white noise filtered by a coloring filter resulting in a correlated white noise, and the use of the Instrumental Variable Z which is interpreted as a two stage Least Square, [2].

Based on the setbacks induced by the noise an alternative to reduce its effects is to filter input/output signals before been used by any estimation technique. This should be done without introducing additional modes or dynamics that can also alter the parameter estimation. In this context, the Noise Reduction Disturbance Observer (NR-DOB), [3-7], is an excellent alternative to filter the input/output signal of a process such that the effects of the noise are reduced. Although, NR-DOB was stablished for control purposes, its characteristics in estimating and reducing sensor noise and input signal perturbations makes it a good alternative to signal filtering and hence to be applied in conjunction with any on line identification technique.

The paper is divided as follows: In Section 2, a brief description of the NR-DOB is presented together with the well-known LSFF algorithm. In section 3, based on digital simulations, a case of study is presented, including NR-DOB design satisfying stability conditions. Finally, conclusions are presented in Section 4.

2 NR-DOB and LSFF Summary

The block diagram of the noise reduction disturbance observer (NR-DOB) control system is depicted in Fig. 1. Where the shadowed section represents the actual NR-DOB. $R(s)$, $\delta(s)$ and $\eta(s)$ are the reference signal, input perturbation and sensor noise, respectively. $C(s)$, $G(s)$, $\tilde{G}(s)$, and $F(s)$ are the feedback controller, process, process model and a unity steady state gain low-pass filter,

respectively. As it is common in practice, it is also assumed that at low frequency $|\eta(j\omega)| \approx 0$. Whereas at high frequency $|\eta(j\omega)| \neq 0$. This is a normal condition when $\eta(s)$ represents sensor noise. Also, it is assumed that $\delta(j\omega) \approx 0$ when $\omega > \omega_{B_F}$, where ω_{B_F} is filter $F(s)$ bandwidth.
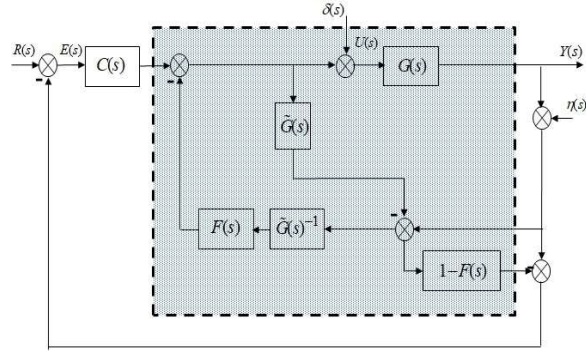

Fig. 1. NR-DOB control system

An important aspect of NR-DOB is the structure of filter $F(s)$ which is given by:

$$F(s) = \frac{1}{(\lambda s + 1)^n} \qquad (1)$$

That is, a stable unity state gain filter, where n is chosen such that $F(s)\tilde{G}(s)^{-1}$ is causal and $\lambda > 0$. The time constant λ of $F(s)$ is tuned mainly to determine filtering conditions for sensor the noise, $\eta(s)$, and the exogenous perturbation $\delta(s)$. Also, filter $F(s)$ satisfies the following conditions:

$$\begin{aligned} |F(j\omega)| &\approx 1, \quad \omega \in \left[0, \omega_{B_F}\right] \\ |F(j\omega)| &\approx 0, \quad \omega \in \left[\omega_{B_F} + \Delta_\omega, \infty\right) \end{aligned} \qquad (2)$$

The output of the closed-loop configuration of Fig. 1 results in:

$$Y(s) = T_R(s)R(s) + S_\delta(s)\delta(s) + S_\eta(s)\eta(s) \qquad (3)$$

Where

$$T_R(s) = \frac{C(s)\,G(s)\,\tilde{G}(s)}{\left(1 + \tilde{G}(s)C(s)\right)\left[\tilde{G}(s) + F(s)\left(G(s) - \tilde{G}(s)\right)\right]} \quad ;$$

$$S_\delta(s) = \frac{G(s)\tilde{G}(s)\left(1 - F(s)\right)}{\tilde{G}(s) + F(s)\left(G(s) - \tilde{G}(s)\right)} \qquad \text{and} \qquad (4)$$

$$S_\eta(s) = \frac{-G(s)F(s)}{\tilde{G}(s) + F(s)\left(G(s) - \tilde{G}(s)\right)}$$

Assuming denominator $\Lambda(s) = \left(1 + \tilde{G}(s)C(s)\right)\left[\tilde{G}(s) + F(s)\left(G(s) - \tilde{G}(s)\right)\right]$ of equations (4) is Hurwitz and filter $F(s)$ comply with the characteristics defined by eq. (2). Then, at low frequencies, that is, if $\omega \in \left[0, \omega_{B_F}\right]$ the NR-DOB control system defined by eqns. (4) satisfies:

$$|T_R(j\omega)| \approx \left|\frac{C(j\omega)\tilde{G}(j\omega)}{1 + C(j\omega)\tilde{G}(j\omega)}\right|, \; |S_\delta(j\omega)| \approx 0 \text{ and}$$

$$|S_\eta(j\omega)| \approx -1 \qquad (5)$$

Whereas, at high frequencies, i.e. $\omega \in \left[\omega_{B_F} + \Delta_\omega, \infty\right)$, filters magnitude reduces to $|F(j\omega)| \approx 0$, therefore, eqns. (4) reduces to:

$$|T_R(j\omega)| \approx \left|\frac{C(j\omega)G(j\omega)}{1 + C(j\omega)\tilde{G}(j\omega)}\right|, \; |S_\delta(j\omega)| \approx |G(j\omega)|$$

$$\text{and } |S_\eta(j\omega)| \approx 0 \qquad (6)$$

From eqns. (5) and (6) is clear that the effects of sensor noise $\eta(s)$, which as indicated above is assumed negligible at low frequency, can be effectively reduced at output $Y(s)$. Whereas, input perturbation $\delta(s)$ is also reduced. It must be noted that although $\delta(s)$ is not reduced at high frequency, as shown in (6), it was assumed $\delta(j\omega) \approx 0$ when $\omega > \omega_{B_F}$. Hence, NR-DOB can reduce the effects of sensor noise and input perturbation at the input signal $\bar{U}(s)$ and output signal $\bar{Y}(s)$. In this context, LSFF will render a better estimation $\hat{G}(z)$ of the discretization $G(z)$ of $G(s)$ using filtered signals $\bar{Y}(s)$ and $\bar{U}(s)$ rather than $Y(s)$ and $U(s)$, as described in Figure (2).
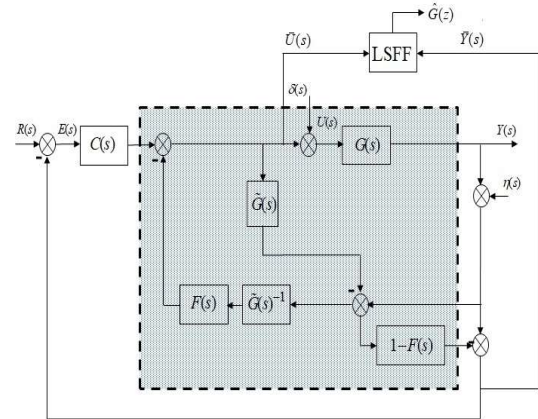

Fig. 2. NR-DOB control system with LSFF

Least square with forgetting factor (LSFF)

Let a discrete process or system whose output $y(k)$ in the instant k is given by:

$$y(k) = \theta^T \phi(k-1) + \varpi(k) \tag{7}$$

Where, θ represents the parameters vector, $\phi(k-1)$ a vector of measurements of process input/output signals $u(k)$, $y(k)$ and, $\varpi(k)$ a perturbation.

The recursive estimation $\hat{\theta}(k)$ of θ that minimizes the quadratic error with forgetting factor criteria $J\{\hat{\theta}\}$, equation (8),

$$J\left(\hat{\theta}(i)\right) = \sum_{i=1}^{k}\left(\prod_{j=i}^{k}\lambda(j)\right)\left(y(i) - \hat{\theta}(i)^T \phi(i-1)\right)^2 ; \tag{8}$$

$$\lambda(k) = \lambda, \forall k$$

is given by:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \lambda F(k+1)\phi(k-1)e(k)$$
$$e(k) = y(k) - \hat{\theta}(k-1)^T \phi(k-1) \tag{9}$$
$$F(k+1) = \frac{1}{\lambda}\left(F(k) - \frac{F(k)\phi(k-1)\phi(k-1)^T F(k)}{1 + \phi(k-1)^T F(k)\phi(k-1)}\right)$$

Where $0 < \lambda \leq 1$ the forgetting factor and $F(1) > 0$ a positive definite matrix.

The LSFF converges that is, $\hat{\theta}(k) \to \theta$ if the following conditions are satisfied, [8]
- The number of poles and zeros of $G(z)$ are known.
- $u(k)$ is a persistent excitation signal.
- $u(k)$ and $y(k)$ bounded
- $\varpi(k)$ a decorrelated white noise

3 Case of Study
Consider the following first order transfer function in the S domain.

$$\frac{Y(s)}{U(s)} = G(s) = \frac{36.9677}{(s + 13.8629)} \tag{10}$$

From equations (4) - (6), the design of the NR-DOB control system for $G(s)$, which satisfies performance and stability specifications, assuming perfect match between process $G(s)$ and model process $\tilde{G}(s)$, requires:

- Control $C(s)$ stabilizes process model $\tilde{G}(s)$ complying with performance specification.
- Filter $F(s)$ with sufficient roll-off to assure $F(s)G(s)^{-1}$ causal.

A simpler I controller $C(s)$ for model process $\tilde{G}(s)$ assuring adequate bandwidth and, gain and phase margins is given by:

$$C(s) = \frac{1.25}{s} \tag{12}$$

In Figure 3, Bode plots of $C(s)\tilde{G}(s)$ show a bandwidth $\omega_B = 3.49 rad/sec$ with appropriate gain and phase margins of $M_p = 75.9°$ and $M_g \to \infty dBs$, respectively. These conditions were set in order to obtain sufficient stability margins and a steady state response of approximately $t_{ss} = 4sec$. Therefore, the first requirement for performance and stability is complied.
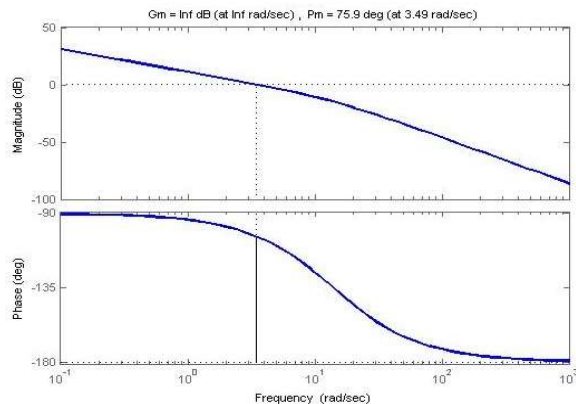

Fig. 3. Bode plots of $C(s)\tilde{G}(s)$

As model process $\tilde{G}(s)$ is first-degree filter $F(s)$ must have a degree equal or greater than two. Also, in order to filter sensor noise at frequencies $\omega > 10 rad/sec$, $F(s)$ bandwidth is set $\omega_{B_F} = 10/rad/sec$ resulting in:

$$F(s) = \frac{1}{(0.1s+1)^2} \qquad (13)$$

Time responses $\bar{y}(t)$ and $\bar{u}(t)$, Figure 4, resulting from substituting $G(s)$, $\tilde{G}(s)$, $C(s)$ and $F(s)$ in the NR-DOB control system of Figure 2, with reference signal $R(s)$ a sequence of square pulses, input perturbation $\delta(t) = 0.5\cos(0.1t)$ and, sensor noise $\delta(t) = 0.5\cos(15t)$.
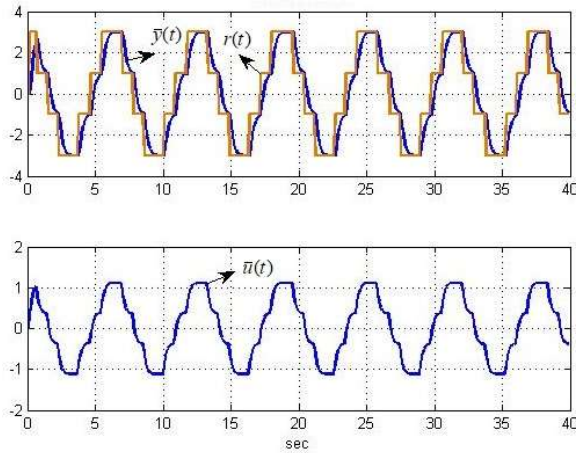

Fig. 4. Time responses of $\bar{y}(t)$ and $\bar{u}(t)$ with NR-DOB

On the other hand, Figure 5 shows the time responses $u(t)$ and $y(t)$ of the control system without NR-DOB; that is, a control system based only on the controller $C(s)$ and process $G(s)$ of equations (12) and (13).
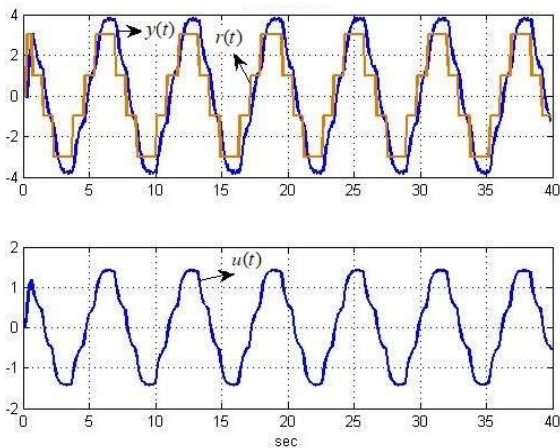

Fig. 5. Time responses of $y(t)$ and $u(t)$ without NR-DOB

Figure 4 shows how the NR-DOB highly reduces the perturbations in the input/output signals,

compared with the responses without NR-DOB of Figure 5. Therefore, it is projected that using the filtered signals $\bar{y}(t)$ and $\bar{u}(t)$, rather than $u(t)$ and $y(t)$, will render a better estimation of process.

From equation (9), is clear that LSFF identifies the parameters of discrete models based on difference equations. In this context, discretization $G(z)$ of $G(s)$ including the zero order hold with a sampling period $T = 0.1sec$ results in:

$$\frac{Y(z)}{U(z)} = G(z) = \frac{2}{(z-0.25)} \qquad (11)$$

The difference equations of $G(z)$ is given by:

$$y(k) = 0.25y(k-1) + 2u(k-1)$$
$$y(k) = \begin{bmatrix} 0.25, & 2 \end{bmatrix} \begin{bmatrix} y(k-1) \\ u(k-1) \end{bmatrix} \qquad (12)$$

That is, parameter vector θ and regression vector $\phi(k-1)$ are given by:

$$\theta^T = \begin{bmatrix} \theta_1, & \theta_2 \end{bmatrix} = \begin{bmatrix} 0.25, & 2 \end{bmatrix}$$
$$\phi(k-1) = \begin{bmatrix} y(k-1), & u(k-1) \end{bmatrix}^T \qquad (13)$$

On line samples of $\bar{y}(t)$ and $\bar{u}(t)$, Figure 2, with sampling period $T = 0.1sec$, were used in the LSFF assuming the following initial conditions:

$$F_1 = \begin{bmatrix} 40 & 0 \\ 0 & 50 \end{bmatrix}; \qquad \theta(0) = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^T \text{ and}$$

$\lambda = 0.95$

Figure 6, shows the evolution of the estimated parameter vector $\hat{\theta}(k) = \begin{bmatrix} \hat{\theta}_1(k) & \hat{\theta}_2(k) \end{bmatrix}^T$; meanwhile, Figure 7 shows the estimation of $\hat{\theta}(k)$ without NR-DOB under the same conditions; that is, with the same initial conditions, forgetting factor sampling period and, using samples of $u(t)$ and $y(t)$
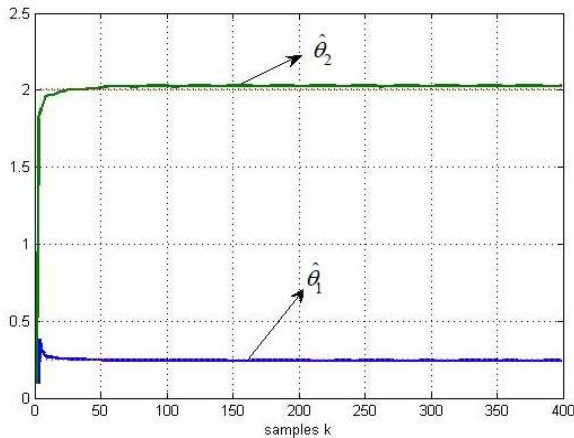
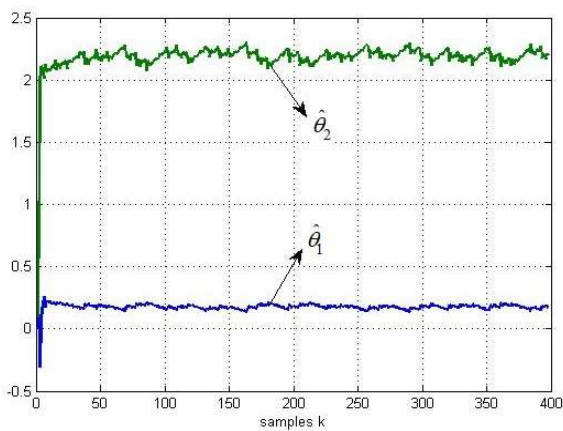Fig. 6. Estimations of $\hat{\theta}_1$ and $\hat{\theta}_2$ with NR-DOB

Fig. 7. Estimations of $\hat{\theta}_1$ and $\hat{\theta}_2$ without NR-DOB

Comparison between Figures 6 and 7 shows that thanks to the reduction of noise and perturbations by NR-DOB the estimation of a process via LSFF results more effective and precise.

However, input/output signals $\bar{y}(t)$ and $\bar{u}(t)$ were obtained based on the unrealistic case of perfect match between process $G(s)$ and model process $\hat{G}(s)$ in the design of the NR-DOB control system.

In order to assess the effects of mismatch between $G(s)$ and model process $\hat{G}(s)$, process model was modified as:

$$\tilde{G}(s) = \frac{40}{\left(s+10\right)} \qquad (14)$$

That is, with an increment in gain and the pole at lower frequency rendering a reduction in the phase margin.

The resulting Bode plots of modified process model $\tilde{G}(s)$ and controller $C(s)$, equation (12), depicted in Figure 8, shows that stability is preserved. Therefore, and assuming filter $F(s)$ as described in equation (13), the NR-DOB control system is stable.

Figure 9 shows NR-DOB control system responses $\bar{y}(t)$, $\bar{u}(t)$ and $r(t)$, using the modified model process of equation (14). These responses show a small performance degradation due to the reduction of phase margin $M_p = 67.1°$. These, responses are due also because NR-DOB is in fact an Internal Model Control (IMC) control system were output $\bar{y}(t)$ is driven, at frequencies $\omega \in \left[0, \omega_{B_F}\right]$, by model process $\tilde{G}(s)$ as indicated by transfer function $T_R(s)$ in equation (5).
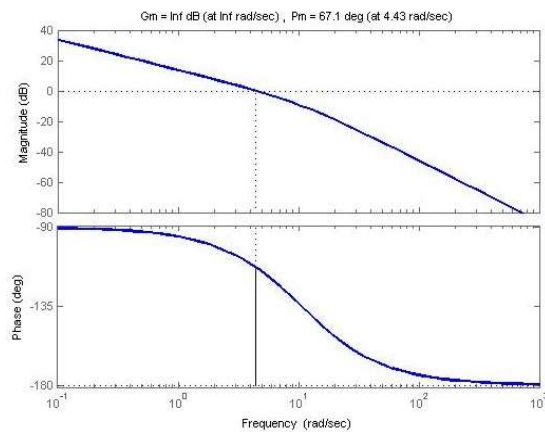

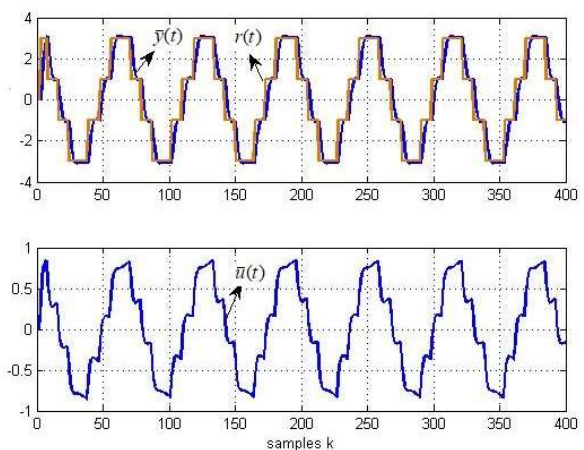Fig. 8. Bode plots of $C(s)\tilde{G}(s)$ with modified $\tilde{G}(s)$

Fig. 9. NR-DOB $\bar{y}(t)$ and $\bar{u}(t)$ time responses with modified $\tilde{G}(s)$

Setting conditions for the LSFF identical to previous experiments, estimated parameters $\hat{\theta}(k) = \begin{bmatrix} \hat{\theta}_1(k) & \hat{\theta}_2(k) \end{bmatrix}^T$, using the modified model process $\tilde{G}(s)$, are shown in Figure 10.

As expected, Figure 10 shows a deterioration in the estimation of parameter vector $\hat{\theta}(k)$ due to the reduction of NR-DOB filtering capabilities by mismatch between process and model process. Nevertheless, $\hat{\theta}(k)$ estimation is still more accurate than in the case without NR-DOB.
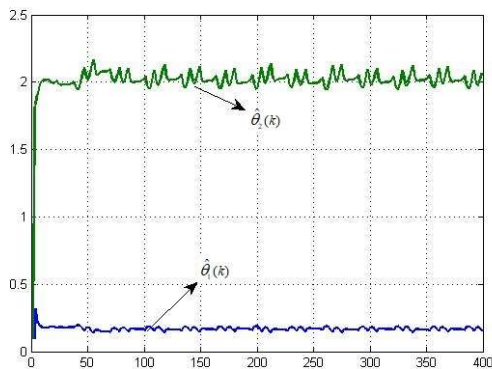

Fig. 10. Estimations of $\hat{\theta}_1$ and $\hat{\theta}_2$ with modified $\tilde{G}(s)$

4 Conclusions

Noise or perturbations at processes input/output signals introduce problems in the identification of processes, especially when the perturbations do not comply with been "white noise" and if its variance is high. Although several approaches can be used to reduce these problems (Maximum Likelihood or Instrumental Variable) implementing a filtering scheme that reduces noise or disturbances without introducing additional modes could improve parameter estimation, regardless of the identification strategy used. In this article, the proposed filtering strategy is the Noise Reduction Disturbance Observer (NR-DOB) which has proven to be an excellent control strategy capable of reducing the effects of noise and disturbances. This characteristic is used to filter process input/output signals so that the identification is less affected. Through a case study, based on the identification of a first order system, the used of the NR-DOB and the Least Square with Forgetting Factor identification algorithm, it is shown that the use of NR-DOB helps to generate a more accurate and effective process identification.

References:
[1] Ioan D. Landau and Gianluca Zito, (2006). *"Digital Control Systems. Design, Identification and Implementation",* Springer, ISBN 978-1-84628-056-6
[2] G. Imbens and J. Angrist, (1994). *"Identification and estimation of local average treatment effects".* Econometrica 62 (2): 467-476. JSTOR 2951620I.
[3] Wei Xie, (2010). *"High frequency measurement noise rejection based on disturbance observer".* Journal of the Franklin Institute 347 pp. 1825–1836
[4] Nam H. Jo and Hyungbo Shim, (2013), *"Robust Stabilization via Disturbance Observer with Noise Reduction".* European Control Conference (ECC) Zürich, Switzerland.
[5] Emre Sariyildiz and Kouhei Ohnishi, (2015). *"Stability and Robustness of Disturbance-Observer-Based Motion Control Systems".* IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS, VOL. 62, NO. 1, JANUARY 2015
[6] Hyungbo SHIM, Gyunghoon PARK, Youngjun JOO, Juhoon BACK, Nam Hoon JO, (2016). *"Yet Another Tutorial of Disturbance Observer: Robust Stabilization and Recovery of Nominal Performance Control".* Theory_Tech, Vol.14, No.3, pp.237–249, August 2016 Control Theory and Technology, http://link.springer.com/journal/11768
[7] Noriaki Hirose, Ryosuke Tajima, Nagisa Koyama, Kazutoshi Sukigara, Minoru Tanaka. (2016). *"Following control approach based on model predictive control for wheeled inverted pendulum robot".* Advanced Robotics 30:6, pages 374-385.
[8] Graham C. Goodwin and Kwai Sang Sin, (2009). *"Adaptive Filtering Prediction and Control"* Dover Publications, Inc. New York, NY, USA ©2009 ISBN:0486469328 9780486469324