Hesitant Fuzzy TOPSIS based Investment Projects Selection Problem

GIA SIRBILADZE[1], IRINA KHUTSISHVILI[2]
Department of Computer Sciences
Ivane Javakhishvili Tbilisi State University
University St. 13, Tbilisi 0186,
GEORGIA
[1]gia.sirbiladze@tsu.ge, [2] irina.khutsishvili@tsu.ge

Abstract: - The present study develops a decision support methodology for investment projects selection problem. The proposed methodology applies the TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) approach under hesitant fuzzy environment.

Selection of investment projects is made considering a set of weighted attributes. To evaluate attributes our approach implies using of experts' assessments. In the proposed methodology the values of the attributes are given by group of experts in the form of lingual assessments - linguistic terms. Then, these lingual assessments are expressed in trapezoidal fuzzy numbers. Consequently, proposed approach is based on hesitant trapezoidal fuzzy TOPSIS decision-making model.

The case when the information on the attributes weights is completely unknown is considered. The attributes weights identification based on De Luca-Termini information entropy is offered in context of hesitant fuzzy sets.

Following the TOPSIS algorithm, first the fuzzy positive-ideal solution (FPIS) and the fuzzy negative-ideal solution (FNIS) are defined. Then the ranking of alternatives is performed in accordance with the proximity of their distances to the both FPIS and FNIS. An example is shown to explain the procedure of the proposed methodology.

Key-Words: - Investment projects selection problem, multiple attribute group decision making, lingual assessments, trapezoidal hesitant fuzzy set, information entropy, hesitant fuzzy TOPSIS approach

1 Introduction

The main objective of the investment projects selection problem is to choose the best project among the feasible projects or to rank all projects, when they are evaluated by a group of experts based on multiple, often conflicting attributes. From this perspective, the selection of investment projects represents a multiple attributes group decision making (MAGDM) problem.

Investment decision making is based on the various special methods. The further development in the field has received the probabilistic approach to the assessment of investment decisions [1],[2]. Along with that, many other methods were developed based on possibility analysis [3] and fuzzy-set approach [4]-[8].

When objective data to make the investment decision aren't present, or they are not enough, experienced experts (decision makers - DMs) are involved to solve the problem. Knowledge and intellectual activities of the experts produce expert evaluations in the decision making process. Thus, the analysis of investment projects involves experts' evaluations that may become dominant in decision making process. Due to the inherent uncertainty of decision makers' preferences, as well as the vagueness and complexity of evaluated objects, expert assessments most often are of fuzzy type.

Processing fuzzy data in decision making models is based on the concept of fuzzy sets introduced by Zadeh and researched by Bellman and Zadeh [4]. As a generalization of a fuzzy set, Torra and Narukawa in [9] and Torra in [10] proposed notion of a hesitant fuzzy set (HFS) and its application in decision-making. In this connection, many well-known MAGDM methods have been extended to take into account fuzzy types of values of attributes and their weights. The latter led to a great number of researches, in which evaluations of attributes involved in the decision making problems frequently are expressed in fuzzy numbers, triangular and trapezoidal fuzzy numbers, intuitionistic fuzzy values, hesitant fuzzy elements and so on [11-14 and others].

However, a more natural representation of decision makers' assessments may be lingual expressions (linguistic terms).

In the proposed methodology the values of the attributes first are given by all decision makers in the form of lingual expressions. Then, these lingual expressions are converted into the trapezoidal fuzzy numbers. Decisions are made using hesitant trapezoidal valued fuzzy TOPSIS method.

The case when the information on the attributes weights is completely unknown is considered. The attributes weights are obtained by applying De Luca-Termini non-probabilistic entropy concept [15], which is offered in context of hesitant fuzzy sets.

Hence, different from other studies, in this paper the novel approach based on hesitant fuzzy TOPSIS decision making model with entropy weights is developed.

A hesitant fuzzy TOPSIS method is employed to ranking the alternatives. In the TOPSIS method we identify as optimal with respect to all attributes the alternative with the nearest distance from the so-called fuzzy positive ideal solution (FPIS) and the farthest distance from the fuzzy negative ideal solution (FNIS). Following the TOPSIS method's algorithm, a relative closeness coefficient is defined to determine the ranking order of all alternatives by calculating the distances to the both FPIS and FNIS.

The developed approach is applied to evaluation of investment projects with the aim of their ranking and identification of high-quality projects for investment. The article provides an investment decision making example clearly illustrating the work of the proposed methodology.

2 Preliminaries

2.1 On the trapezoidal fuzzy numbers

A trapezoidal fuzzy number $\widetilde{A}$ can be determined by a quadruple $\widetilde{A}=(a,b,c,d)$. Its membership function is defined as

$$\mu_{\widetilde{A}}(x)=\begin{cases} 0, & if\ \ x<a, \\ \dfrac{x-a}{b-a} & if\ \ a\le x\le b, \\ 1, & if\ \ b\le x\le c, \\ \dfrac{d-x}{d-c} & if\ \ c\le x\le d, \\ 0 & if\ \ x>d, \end{cases}$$

where $a\le b\le c\le d$ [16].

Let's $\widetilde{A}=(a,b,c,d)$ is trapezoidal fuzzy number. Using Graded Mean Integration Representation Method we can get following representation of $\widetilde{A}$ by formula

$$p(\widetilde{A})=(a+2b+2c+d)/6. \tag{1}$$

2.2 On the hesitant fuzzy sets

In HFS the degree of membership of an element to a reference set is presented by several possible fuzzy values. This allows describing situations when decision makers (DMs) have hesitancy in providing their preferences over alternatives. The HFS is defined as follows:

Definition 1. [9,10]. Let $X=\{x_1,x_2,...,x_n\}$ be a reference set, a hesitant fuzzy set H on X is defined in terms of a function $h_H(x)$ when applied to X returns a subset of [0,1]:

$$H=\{<x,h_H(x)>|\,x\in X\},$$

where $h_H(x)$ is a set of some different values in [0,1], representing the possible membership degrees of the element $x\in X$ to H; $h_H(x)$ is called a hesitant fuzzy element (HFE).

Definition 2: [17]. Let M and N be two HFSs on $X=\{x_1,x_2,...,x_n\}$, then the distance measure between M and N is defined as $d(M,N)$, which satisfies the following properties:

1). $0\le d(M,N)\le 1$;
2). $d(M,N)=0$ if and only if $M=N$;
3). $d(M,N)=d(N,M)$.

It is clear that the number of values (length) for different HFEs may be different. Let $l(h_H(x))$ be the length of $h_H(x)$. After arranging the elements of $h_H(x)$ in a decreasing order, let $h_H^{\sigma(j)}(x)$ be the jth largest value in $h_H(x)$. To calculate the distance between M and N when $l(h_M(x_i))\ne l(h_N(x_i))$, it is necessary extend the shorter one by adding any value in it, until both will have the same length. The choice of this value depends on the expert's risk preferences. Optimists experts may add the maximum value from HFE, while pessimists may add the minimal value.

In this work the hesitant weighted Hamming distance is used that is defined by following formula

$$d_{hwh}(M,N)=\sum_{i=1}^{n} w_i\left[\frac{1}{l_{x_i}}\sum_{j=1}^{l_{x_i}}\left|h_M^{\sigma(j)}(x_i)-h_N^{\sigma(j)}(x_i)\right|\right],(2)$$

where $h_M^{\sigma(j)}(x_i)$ and $h_N^{\sigma(j)}(x_i)$ are the jth largest values in $h_M(x_i)$ and $h_N(x_i)$ respectively; $l_{x_i}=\max\{l(h_M(x_i)),l(h_N(x_i))\}$ for each $x_i\in X$; $w_i\ (i=1,2,...,n)$ is the weight of the element $x_i\in X$ such that $w_i\in[0,1]$ and $\sum_{i=1}^{n} w_i=1$.

Definition 3: [18] For a HFE $h_H(x)$, the score function $s(h_H(x))$ is defined as follows:

$$s(h_H(x))=\sum_{j=1}^{l(h_H(x))} h_H^{\sigma(j)}(x)\Big/ l(h_H(x)), \qquad (3)$$

where $s(h_H(x))\in[0,1]$.

Let h_1 and h_2 are two HFEs. Based on score function it is possible to make ranking of HFEs according to the following rules: $h_1>h_2$, if $s(h_1)>s(h_2)$; $h_1<h_2$, if $s(h_1)<s(h_2)$ and $h_1=h_2$, if $s(h_1)=s(h_2)$.

3 Formulation of Investment Projects Selection Problem in Hesitant Fuzzy Environment

Consider a MAGDM problem for investment decision making.

Assume that there are m projects – decision making alternatives $A=\{A_1,A_2,\ldots,A_m\}$, and the group $E=\{e_1,e_2,\ldots,e_k\}$ of k DMs evaluates them with respect to an n attributes $X=\{x_1,x_2,\ldots,x_n\}$.

DMs provide evaluations over attributes in form of lingual assessments – linguistic terms. Then, these assessments are expressed in trapezoidal fuzzy numbers (TrFNs) using 5-point linguistic scale (see Table 1):

Table 1. Linguistic scale for rating of alternatives

Linguistic term	Corresponding TrFNs
Very low (VL)	(0, 0.1, 0.2, 0.3)
Low (L)	(0.1, 0.2, 0.3, 0.4)
Medium (M)	(0.3, 0.4, 0.5, 0.6)
High (H)	(0.5, 0.6, 0.7, 0.8)
Very high (VH)	(0.7, 0.8, 0.9, 1.0)

After those transformations of lingual expressions, experts' joint assessments concerning each alternative represent HTrFS:

A HTrFS A_i of the ith alternative - project - on X is given by

$$A_i=\left\{<x_j,\ f_{A_i}(x_j)>|\ x_j\in X\right\},$$

where $f_{A_i}(x_j)$, $i=1,2,\ldots,m$; $j=1,2,\ldots,n$ indicates the possible membership degrees of the ith alternative A_i under the jth attribute x_j, and it can be expressed as a HTrFE $\tilde{t}_{ij}$. All HTrFEs create the aggregate fuzzy hesitant trapezoidal decision matrix $\tilde{T}=(\tilde{t}_{ij})_{m\times n}$.

Considering that the attributes have different importance degrees, the vector of attributes weights we denote by $w=(w_1,w_2,\ldots,w_n)$, where $0\le w_j\le 1$, $\sum_{j=1}^{n} w_j=1$, and w_j is the importance degree of jth attribute.

Then a hesitant MADM problem can be expressed in matrix format as follows

$$\tilde{T}=\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array}\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{bmatrix} \tilde{t}_{11} & \tilde{t}_{12} & \cdots & \tilde{t}_{1n} \\ \tilde{t}_{21} & \tilde{t}_{22} & \cdots & \tilde{t}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{t}_{m1} & \tilde{t}_{m2} & \cdots & \tilde{t}_{mn} \end{bmatrix} \end{array},$$

$$w=(w_1,w_2,\ldots,w_n),$$

$$E=\{e_1,e_2,\ldots,e_k\},$$

where $\tilde{T}$ is the hesitant trapezoidal fuzzy decision matrix, each element of which represents an HTrFE $\tilde{t}_{ij}$.

3.1 Determination of the attributes weights using De Luca-Termini entropy

The complexity and uncertainty of the investment decision problems leads to the information on attributes weights usually being incomplete or completely unknown. Here we consider a case when the attributes weights are unknown.

Let us, we have hesitant decision matrix $H = (h_{ij})_{m\times n}$, each element of which represents a HFE.

De Luca and Termini [15] defined a non-probabilistic entropy formula of a fuzzy set based on Shannon's function on a finite universal set X as:

$$E_{LT} = -k\sum_{i=1}^{n}\left[\mu_A(x_i)\ln\mu_A(x_i) + (1-\mu_A(x_i))\ln(1-\mu_A(x_i))\right],\quad k>0,$$

where $\mu_A : X \to [0,1]$; k is a positive constant.

The attributes weights definition method based on the De Luca-Termini entropy can be described as follows:

Step1: Calculate the score matrix $S = (s_{ij})_{m\times n}$ of hesitant decision matrix H, where $s_{ij} = s(h_{ij})$ is the score value of h_{ij} (see formula (3)).

Step2: Calculate the normalized score matrix $S' = (s'_{ij})_{m\times n}$, where

$$s'_{ij} = s_{ij} \Big/ \sum\nolimits_{i=1}^{m} s_{ij}. \tag{4}$$

Step3: Determine the attributes weights.

By using De Luca-Termini normalized entropy in context of hesitant fuzzy sets

$$E_j = -\frac{1}{m\ln 2}\sum_{i=1}^{m}\left(s'_{ij}\ln s'_{ij} + (1-s'_{ij})\ln(1-s'_{ij})\right),\quad j=1,2,\ldots,n, \tag{5}$$

the definition of the attributes weights is expressed by the formula

$$w_j = \frac{1-E_j}{\sum\nolimits_{j=1}^{n}(1-E_j)},\quad j=1,2,\ldots,n, \tag{6}$$

where the value of w_j represents the relative intensity of x_j attribute importance.

3.2 Evaluation of the Projects using TOPSIS approach

The TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method was developed by Hwang and Yoon [19] and dealt with crisp information in decision making process. Now there are many extensions of TOPSIS taking into account the processing of fuzzy information.

The idea of the TOPSIS method as applied to the MAGDM problem consists in the choice of the best alternative in accordance with distances from both FPIS and FNIS, namely with the nearest distance from FPIS and the farthest from FNIS. Fuzzy TOPSIS has been applied to investment decision making problems by researchers in [20,21 and others].

The algorithm of practical solving an investment MAGDM problem can be formulated as follows:

Step 1: Experts lingual assessments convert into the assessments in a form of trapezoidal fuzzy numbers.

Step 2: Based on the experts' hesitant trapezoidal evaluations construct the aggregate hesitant trapezoidal decision matrix $\tilde{T} = (\tilde{t}_{ij})_{m\times n}$.

Step 3: Transform aggregate hesitant trapezoidal decision matrix $\tilde{T} = (\tilde{t}_{ij})_{m\times n}$ into aggregate hesitant decision matrix $H = (h_{ij})_{m\times n}$ by using Graded Mean Integration Representation Method.

Step 4: Determine the criteria weights $w = (w_1, w_2, \ldots, w_n)$ based on the method given in Section 2 (Subsection 2.3).

Step 5: Determine corresponding hesitant FPIS A^+ and hesitant FNIS A^-.

FPIS is composed of the best performance values for each attribute whereas FNIS consists of the worst performance values.

There are attributes of two types:

a) the benefit type attribute - this means that the bigger attribute's value the better;
b) the cost type attribute - that is, the smaller the attribute's value the better.

Calculate A^+ and A^- by formulas:

$$A^+ = \left\{\max_i\left\langle h_{ij}^{\sigma(\lambda)}\right\rangle \mid j\in J';\ \min_i\left\langle h_{ij}^{\sigma(\lambda)}\right\rangle \mid j\in J''\right\}, \tag{7}$$

$$A^- = \left\{\min_i\left\langle h_{ij}^{\sigma(\lambda)}\right\rangle \mid j\in J';\ \max_i\left\langle h_{ij}^{\sigma(\lambda)}\right\rangle \mid j\in J''\right\}, \tag{8}$$

$$\lambda = 1,2,\ldots,n,$$

where J' is associated with a benefit attribute, and J'' - with a cost attribute.

Step 6: Using (2) calculate the separation measures d_i^+ and d_i^- of each alternative A_i from the hesitant FPIS A^+ and the hesitant FNIS A^-, respectively:

$$d_i^+ = \sum\nolimits_{j=1}^{n} d(h_{ij}, h_j^+)\, w_j = \sum_{j=1}^{n} w_j\left[\frac{1}{l}\sum_{j=1}^{l}\left|h_{ij}^{\sigma(j)} - (h_j^{\sigma(j)})^+\right|\right], \tag{9}$$

$$d_i^- = \sum_{j=1}^{n} d(h_{ij}, h_j^-) w_j = \sum_{j=1}^{n} w_j \left[\frac{1}{l} \sum_{j=1}^{l} \left| h_{ij}^{\sigma(j)} - (h_j^{\sigma(j)})^- \right| \right], \quad (10)$$
$$i = 1, 2, \ldots, m .$$

Step 7: Calculate the relative closeness coefficient RC_i of each alternative A_i to the hesitant FPIS A^+:

$$RC_i = d_i^- \big/ (d_i^+ + d_i^-) , \quad i = 1,2,..,m . \quad (11)$$

Step 8: Perform the ranking of the alternatives A_i, $i = 1, 2, \ldots, m$ according to the relative closeness coefficients RC_i, $i = 1, 2, \ldots, m$ by the rule: for two alternatives A_α and A_β we say that A_α is more preferred than A_β, i.e. $A_\alpha \succcurlyeq A_\beta$, if $RC_\alpha \geq RC_\beta$, where $\succcurlyeq$ is a preference relation on A.

4 An example of the Application of Fuzzy Decision Making Approach

Suppose that in the tender for granting investment four construction companies are involved. The group of DMs evaluates the investment projects taking into account the following attributes, by which they will score each candidate project seeking an investment:

x_1 - business profitability;

x_2 - pledge guaranteeing repayment of the credit;

x_3 - investment amount (monetary value);

x_4 - workmanship.

From them only x_3 attribute is of a cost type, the other attributes are of a benefit type.

Assume that there are four decision making alternatives - candidate projects, and the group of the DMs consists of four members.

To evaluate the rating of alternatives with respect to each attribute DMs use the linguistic terms from the Table 1. Aggregated results are presented in Table 2 as the linguistic fuzzy decision matrix.

Table 2. DMs initial assessments - rating of alternatives

	Attributes			
	x_1	x_2	x_3	x_4
A_1	H,VH,VL,VH	H,H,M,VH	L,H,M,VL	VH,VH,M,VM
A_2	L,L,VL,VL	L,M,VL,M	L,M,VL,M	VL,L,VL,VL
A_3	M,VH,H,H	L,H,M,H	L,VL,M,H	M,VH,VH,M
A_4	H,VL,M,M	L,VL,H,L	L,M,VL,L	H,H,H,H

This matrix we convert into HTrF matrix by assigning for each lingual assessment the appropriate TrFN as given in Table 1. Thus, we obtained the following hesitant trapezoidal fuzzy decision matrix (see Table 3).

Table 3. The hesitant trapezoidal fuzzy decision matrix $\tilde{T}$

	Attributes			
	x_1	x_2	x_3	x_4
A_1	(0.5,0.6,0.7,0.8) (0.7,0.8,0.9,1.0) (0.0,0.1,0.2,0.3) (0.7,0.8,0.9,1.0)	(0.5,0.6,0.7,0.8) (0.5,0.6,0.7,0.8) (0.3,0.4,0.5,0.6) (0.7,0.8,0.9,1.0)	(0.1,0.2,0.3,0.4) (0.5,0.6,0.7,0.8) (0.3,0.4,0.5,0.6) (0,0.1,0.2,0.3)	(0.7,0.8,0.9,1.0) (0.7,0.8,0.9,1.0) (0.3,0.4,0.5,0.6) (0.7,0.8,0.9,1.0)
A_2	(0.1,0.2,0.3,0.4) (0.1,0.2,0.3,0.4) (0, 0.1,0.2,0.3) (0, 0.1,0.2,0.3)	0.1,0.2,0.3,0.4) (0.3,0.4,0.5,0.6) (0,0.1,0.2,0.3) (0.3,0.4,0.5,0.6)	(0.1,0.2,0.3,0.4) (0.3,0.4,0.5,0.6) (0,0.1,0.2,0.3) (0.3,0.4,0.5,0.6)	(0, 0.1, 0.2, 0.3) (0.1,0.2,0.3,0.4) (0, 0.1, 0.2, 0.3) (0, 0.1, 0.2, 0.3)
A_3	(0.3,0.4,0.5,0.6) (0.7,0.8,0.9,1.0) (0.5,0.6,0.7,0.8) (0.5,0.6,0.7,0.8)	(0.1,0.2,0.3,0.4) (0.5,0.6,0.7,0.8) (0.3,0.4,0.5,0.6) (0.5,0.6,0.7,0.8)	(0.1,0.2,0.3,0.4) (0,0.1,0.2,0.3) (0.3,0.4,0.5,0.6) (0.5,0.6,0.7,0.8)	(0.3,0.4,0.5,0.6) (0.7,0.8,0.9,1.0) (0.7,0.8,0.9,1.0) (0.3,0.4,0.5,0.6)
A_4	(0.5,0.6,0.7,0.8) (0, 0.1, 0.2, 0.3) (0.3,0.4,0.5,0.6) (0.3,0.4,0.5,0.6)	(0.1,0.2,0.3,0.4) (0, 0.1, 0.2, 0.3) (0.5,0.6,0.7,0.8) (0.1,0.2,0.3,0.4)	(0.1,0.2,0.3,0.4) (0.3,0.4,0.5,0.6) (0,0.1,0.2,0.3) (0.1,0.2,0.3,0.4)	(0.5,0.6,0.7,0.8) (0.5,0.6,0.7,0.8) (0.5,0.6,0.7,0.8) (0.5,0.6,0.7,0.8)

Then the constructed matrix we transform into hesitant fuzzy decision matrix using equation (1). If the evaluation values of any attribute given by experts are coincident, then such values are included in HFE only once. We assume that the experts are pessimistic, and the hesitant fuzzy data in HFEs are changed by adding the minimal values. Hence, the hesitant fuzzy decision matrix H looks like Table 4:

Table 4. The hesitant fuzzy decision matrix H

	Attributes			
	x_1	x_2	x_3	x_4
A_1	(0.65,0.85, 0.15,0.15)	(0.65,0.45, 0.85,0.45)	(0.25,0.65, 0.45,0.15)	(0.45,0.85, 0.45,0.45)
A_2	(0.25,0.15, 0.15,0.15)	(0.25,0.15, 0.45,0.15)	(0.15,0.25, 0.45, 0.15)	(0.15,0.25 0.15,0.15)
A_3	(0.45,0.85 0.65,0.45)	(0.25,0.65 0.45,0.25)	(0.25,0.15, 0.45,0.65)	(0.45,0.85, 0.45,0.45)
A_4	(0.65,0.15, 0.45,0.15)	(0.15,0.25, 0.65,0.15)	(0.25,0.45, 0.15,0.15)	(0.65,0.65, 0.65,0.65)

According to the method of determining the attributes weights given in Subsection 3.1, we first calculate the score matrix S of hesitant decision matrix H based on equation (3):

$$S = \begin{bmatrix} 0.45 & 0.6 & 0.375 & 0.55 \\ 0.175 & 0.25 & 0.25 & 0.175 \\ 0.6 & 0.4 & 0.375 & 0.55 \\ 0.35 & 0.3 & 0.25 & 0.65 \end{bmatrix}$$

Secondly, we obtain the normalized score matrix S' using equation (4):

$$S' = \begin{bmatrix} 0.2857 & 0.3871 & 0.3 & 0.2857 \\ 0.1111 & 0.1613 & 0.2 & 0.0909 \\ 0.3809 & 0.2581 & 0.3 & 0.2857 \\ 0.2222 & 0.1935 & 0.2 & 0.3376 \end{bmatrix}$$

Then the vector of attributes weights is determined using equations (5) and (6):

$$w = \left(0.261469,\ 0.24894,\ 0.227837,\ 0.261754\right).$$

Following the hesitant fuzzy TOPSIS method, we determine the hesitant FPIS A^+ and the hesitant FNIS A^- by equations (7) and (8), respectively:

$$A^+ = \{ (0.65,0.85,0.65,0.45),(0.65,0.65,0.85,0.45),$$
$$(0.15,0.15,0.15,0.15),(0.65,0.85,0.65,0.65)\};$$

$$A^- = \{ (0.25,0.15,0.15,0.15),(0.15,0.15,0.45,0.15),$$
$$(0.25,0.65,0.45,0.65),(0.15,0.25,0.15,0.15)\}.$$

Then we calculate the distances d_i^+ and d_i^- of each alternative A_i from the hesitant FPIS A^+ and the hesitant FNIS A^- by equations (9) and (10), respectively:

$$d_1^+ = 0.155267,\ d_2^+ = 0.383978,\ d_3^+ = 0.165835,$$
$$d_4^+ = 0.201441;$$

$$d_1^- = 0.291894,\ d_2^- = 0.063183,\ d_3^- = 0.281326,$$
$$d_4^- = 0.24572\ .$$

Using equation (11) to calculate the relative closeness coefficient RC_i of each alternative A_i to the hesitant FPIS A^+ we obtain:

$$RC_1 = 0.652771,\ RC_2 = 0.141297,$$
$$RC_3 = 0.629139,\ RC_4 = 0.549511.$$

Finally, we perform the ranking of the alternatives A_i, $i = 1, 2, \dots, 4$ according to the relative closeness coefficients RC_i and obtain:

$$A_1 \succ A_3 \succ A_4 \succ A_2\,.$$

From the obtained ranking of projects, it is possible to make a conclusion that the project A_1 will be the most preferable, while the project A_2 will be the least preferable choice of the decision. That means that when investing the capital only in one project, DMs prefer the investment project A_1, i.e. the project A_1 receives investment.

5 Conclusions

In the present work a novel approach for solving MAGDM problem based on hesitant trapezoidal valued fuzzy TOPSIS method with entropy weights is developed.

Our methodology provides experts with the opportunity to manifest intellectual activity of a high level. Securing the freedom of experts' subjective evaluations, the methodology, however, allows for developing experts' joint decision, for instance, on selection of the best project among a set of candidate projects seeking investment.

The new aspects in the TOPSIS approach have been used. We proposed a new attributes weighting method based on De Luca-Termini information entropy to express the relative intensities of attribute importance and determine the attributes weights. The latter distinguishes our methodology from the others.

It should also be noted that in the real problem of selection of investment projects, practically have been processed the attributes of both the benefit and the cost types.

Based on proposed methodology we have developed software package and used it to rank investment projects in the real investment decision making problem. The application and testing of the software was carried out based on the data provided by the "Bank of Georgia". The results are illustrated in the example.

References:

[1] J. M. Merigó, Probabilistic Decision Making with the OWA Operator and its Application in Investment Management, *In Proceedings of the Conference European Society for Fuzzy Logic and Technology – EUSFLAT*, Lisbon, Portugal, 2009, pp. 1364-1369.

[2] J. M, Merigó, Fuzzy Multi-Person Decision Making with Fuzzy Probabilistic Aggregation

Operators, *Int. Journal of Fuzzy Systems*, 13, 2011, pp. 163-174.

[3] D. Dubois and H. Prade, *Possibility Theory: An Approach to Computerized Processing of Uncertainty*, New-York: Plenum Press, 1988.

[4] R. E. Bellman and L. A. Zadeh, Decision-Making in a Fuzzy Environment, *Management Science*, Vol.17, No.4, 1970, B141-B164.

[5] D. Ruan, J. Kacprzyk and M. Fedrizzi, *Soft Computing for Risk Evaluation and Management: Applications in Technology, Environmenf and Finance. Studies in Fuzziness and Soft Computing*, Heidelberg, New York: Physica-Verlag, 2001.

[6] R. R. Yager, J. Kacprzyk and G. Beliakov, *Recent Development in the ordered Weighted Averaging Operations: Theory and Practice. Studies in Fuzziness and Soft Computing 265*, Berlin: Springer-Verlag, 2011.

[7] I. Khutsishvili and G. Sirbiladze, Decision Support's Precising Technology in the Investment Project Risk Management, *In Proceedings of the 11th WSEAS International Conference on Automatic Control, Modelling and Simulation (ACMOS '09)*, Istanbul, Turkey, 2009, pp. 303-311.

[8] G. Sirbiladze, I. Khutsisvili and P. Dvalishvili, Decision Precising Fuzzy Technology to Evaluate the Credit Risks of Investment Projects, *In Proceedings of IEEE 10-th International Conference on Intelligent Systems Desigh and Applications (ISDA 2010)*, Cairo, Egypt, 2010, pp. 103-108.

[9] V. Torra and Y. Narukawa, On hesitant fuzzy sets and decision, *In: The 18th IEEE International Conference on Fuzzy Systems*, Jeju Island, Korea, 2009, pp. 1378–1382.

[10] V. Torra, Hesitant fuzzy sets, *International Journal of Intelligent Systems*, Vol, 25, 2010, pp. 529–539.

[11] L. Li and G. Fan, Fuzzy MADM with triangular numbers for project investment model based on left and right scores, *Research Journal of Applied Sciences, Engineering and Technology*, Vol.7, 2014, pp. 2793-2797.

[12] G. Sirbiladze, I. Khutsishvili and B. Ghvaberidze, Multistage decision-making fuzzy methodology for optimal investments based on experts' evaluations, *European Journal of Operational Research*, Vol.232, No.1, 2014, pp. 169–177.

[13] G. Sirbiladze, New Fuzzy Aggregation Operators Based on the Finite Choquet Integral – Application in the MADM Problem, *International Journal of Information Technology & Decision Making*, Vol.15, No. 03, 2016, pp. 517-551.

[14] G. Sirbiladze and O. Badagadze, Intuitionistic Fuzzy Probabilistic Aggregation Operators Based on the Choquet Integral: Application in Multicriteria Decision-Making, *International Journal of Information Technology & Decision Making*, Vol.16, No.01, 2017, pp. 245-279.

[15] A. De Luca and S.Termini, A Definition of a Nonprobabilistic Entropy in the Setting of Fuzzy Sets Theory, *Information and Control,* Vol.20, 1972, pp. 301-312.

[16] T. Rashid and S. M. Husnine, Multicriteria Group Decision Making by Using Trapezoidal Valued Hesitant Fuzzy Sets, *The Scientific World Journal*, 2014, pp. 1-8.

[17] Z. Xu, M. Xia, Distance and similarity measures for hesitant fuzzy sets, *Information Sciences*, Vol.181, 2011, pp. 2128–2138.

[18] M. Xia, Z. Xu, Hesitant fuzzy information aggregation in decision making, *International Journal of Approximate Reasoning*, Vol.52, No.3, 2011, pp. 395–407.

[19] C. L. Hwang and K.Yoon, *Multiple attribute decision making: methods and applications*, Berlin, Heidelberg: Verlag, 1981.

[20] I. Khutsishvili and G. Sirbiladze, A Multiple-Attributes Decision Making in Hesitant Fuzzy Environment: Application to Evaluation of Investment Projects, *Georgian Electronic Scientific Journal: Computer Science and Telecommunications*, No.3 (43), 2014, pp. 47-53.

[21] I. Khutsishvili, G. Sirbiladze, B. Ghvaberidze, TOPSIS based Hesitant Fuzzy MADM for Optimal Investment Decisions, *In: Recent Advances in Systems - Proceedings of the 19th International Conference on Systems (part of CSCC '15)*, Zakynthos Island, Greece, 2015, pp. 119-124.