Two possible approaches for the dynamic Vehicle Routing Problem

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Abstract: The management of goods delivery is becoming very important. The on time delivery is a critical criterion taking into account customers point of view. But the delivery company must also pay attention to the economic considerations. There are many variations on this issue, but all of them are of great computational complexity. It means that the exact solutions are unavailable for large size problem. The paper proposes the Surrogate Method for the Dynamic Vehicle Routing Problem (DVRP). The aim of DVRP is to find a set of routes to serve multiple customers while the travelling time between point to point may vary during the process. The aim is to schedule the vehicle routes minimizing the number of the required vehicles and the completion time. The presented approach uses some common assumptions but different optimization method. Finally, the proposed heuristic is compared with the genetic algorithm.prepare their manuscripts for WSEAS proceedings or journals by means of LaTeX. You will find the format you have to choose, fonts, how to type the title of your paper, the titles of sections, examples of definitions, lemmas, theorems, equations etc.

Key–Words: Delivery problem, vehicle routing problem, heuristics.

1 Introduction

Vehicle routing is obviously significant in the areas of logistics, transportation and related industries. Thus it has attracted a large number of researchers and practitioners from various sectors, which is often referred as VRP (Vehicle Routing Problem). Classical VRP is a well-known combinatorial optimisation problem and has been intensively investigated. VRP involves a set of customers geographically distributed at different locations and a fleet of vehicles. The goal is serving all customers at minimal cost (e.g. travelling distance, time, fuel etc.) while respecting all constraints. Due to the nature of real-world vehicle routing problems, several variants of VRP have been formulated. Each of these VRP variant accommodates certain constraints and factors to reflect one type of real-life scenarios. Well-known VRP variants include: (i) CVRP, the Capacitated Vehicle Routing Problem which has vehicle capacity as a hard constraint; (ii) VRPTW, the Vehicle Routing Problem with Time Windows where each customer can only accept services/deliveries during a predefined time window; and (iii) VRPMT, the Vehicle Routing Problem with Multiple Trips, where a vehicle can take more than one trip/task. In these vehicle routing scenarios, the travel time from one point to another is a constant. In VRP, a fleet of vehicles must service the demands of customers. A vehicle begins and ends its route at the same depot and the sum of the demands of the customers on a route cannot exceed a vehicles capacity. A customer must have all of its demand delivered at one time by a single vehicle. The objective is to minimize the total distance traveled by the fleet. It is still encountered in our days, mainly in the domain of logistics and transport. In the VRP, m vehicles(v_i), with identical capacities (Q), initially located at a central depot (v_0), are to deliver discrete quantities of goods (q_i) to n customers, which are geographically diffused around the central depot. Concurrently, the aim of the VRP, beyond serving customers, is to minimize the travelled distance.

Due to the difficulty the VRP presents and because of its practical applications, many models have been created for solving the problem and many variants of the basic VRP have been compiled, with different parameters, leading to a different structure of the basic VRP. Firstly, the classical VRP is equivalent to the Capacitated VRP (CVRP) in which, the capacity of the vehicle must not be exceeded (see (0; 0; **?**; 0)). However, there is another possibility that the vehicles do not have the same capacities which leads to the Heterogeneous Fleet VRP (HVRP).

Another important variant which was created a decade after the classical VRP was the Multi-Depot VRP (MDVRP) in which, the company has several depots from which it can serve its customers, while the objective is still to service all customers and minimize the number of vehicles and distance travelled (see (0)). At the same period, the Stochastic VRP (SVRP) was created, where, the customers, the demand of each customer or service and travel times are random.

In addition, in the VRP with Pick-up and Delivering (VRPPD), goods are transported, not only from the central depot to customers, but in the opposite direction too. Hence, in VRPPD it is necessary to take into account that the total delivery load and the total pickup load should both fit into the vehicle (see (0)). Furthermore, due to the need for specific arrival time information and for better customer services, an extra restriction to the VRP was added, time windows. The VRP with Time Windows (VRPTW) is the same with the classical VRP, but constrained by a time interval within which the customers have to be supplied (see (0)).

Another variant worth mentioning is the Dynamic VRP (DVRP) in which customer requests, that are trips from an origin to a destination, appear dynamically (see (0)) or when data and information regarding to the VRP such as travel time, change dynamically. Another significant variant which takes into account time is the Time Dependent VRP (TDVRP) in which travel times change as time passes. The reason why this happens is due to traffic congestion. The factors which affect travel times are: (1) the location, and (2) the time of the day.

During the last decade, many researchers have tried, while solving the VRP, to minimize carbon (or fuel emissions) as carbon dioxide (CO2) emitted by trucks is the main greenhouse gas. In addition, the Green VRP has been strengthened due to the technical developments and the road traffic information which allows planning vehicle routes and schedules and taking time-varying speeds into account ((0; **?**; **?**)). In the same field belongs the Hybrid VRP where vehicles can work both electrically and with petroleum-based fuel.

The Dynamic Vehicle Routing Problem (DVRP) is a complex variation of classical Vehicle Routing Problem (VRP). The aim of DVRP is to find a set of routes to serve multiple customers at minimal total travelling cost while the travelling time between point to point may vary during the process because of factors like traffic congestion. To effectively handle DVRP, a good algorithm should be able to adjust itself to the changes and continuously search for the best solution under dynamic environments. Because of this dynamic nature of DVRP, evolutionary algorithms (EAs) appear highly appropriate for DVRP as they search in a parallel manner with a population of solutions. Solutions scattered over the search space can better capture the dynamic changes. Solutions for new changes are not built from scratch as they can inherit problem-specific knowledge from parent solutions. However, the performance of EA is highly dependent on the utilised configuration

Travel-time plays an important role in the distribution of the perishable goods, since its fluctuations may extend the time that the goods spend on the vehicles. Different representations of the fluctuations of the travel-times between the customers have been reported and different extensions of VRP have been proposed to address the fluctuations in travel-times. Routing problems with stochastic travel-times are presented by ((0),(0),(0)). In (0), they presented a TSP in which they have considered a zone in the city center with traffic jams in the afternoon and show how simulated annealing and threshold-accepting algorithms are able to handle such time-dependent problems. In (0) Park presented the time-dependent VRP in which the travel speed between two locations depends on the route and the time of the day. They proposed a model for estimating the time varying travel speed. In (0), they proposed a time-dependent model for the VRPTW. The model that they developed is based on time-dependent travel speeds and satisfies the first-in-first-out (FIFO) property. They extended the tabu search heuristic to solve the problem and showed that the time-dependent model provides substantial improvements over a model based on fixed travel-times.

Classical VRP and its variants are known as -hard in terms of the problem complexity. That means finding the optimal solution could be impractical for VRP instance of reasonable size due to the prohibitive computational resource required. Exact methods, which guarantee optimal solutions, are only advisable to be used on small instances. In reality small instances have little practical values as real world problems often are large in size. Thus, meta-heuristic algorithms are better alternatives in these scenarios, as they can often generate solutions of good quality within an acceptable amount of time. This kind of method offers no guarantee of optimality but high application value as the good solutions generated by them are often not far from the optimal solutions. Typical meta-heuristic algorithms include tabu search, simulated annealing, evolutionary algorithms, ant colony and variable neighbourhood algorithms (see (0; 0; 0; 0; 0).

In reality most of logistics and transportation problems are dynamic by nature. Only limited information is available at the beginning of a trip. New information arrives over time. For example, a new order from customer may appear whilst vehicles are already on road serving customers. Another important factor is traffic condition, which may vary dramatically in different time of a same day, or a same time in different days. It is difficult to foretell the level of traffic congestion (see (0)). This leads to a challenging and realistic dynamic variant of the VRP, which is denoted as DVRP, referring the vehicle routing problem with traffic congestion. Dynamic order occurs much less often in comparison. In DVRP the exact travel time between customers is not known in advance and subject to the level of the traffic congestion on the path, meaning that after generating a set routing plans and after the vehicles have left the depot to serve customers, the travel time between customers may change. When a change occurs, the optimisation algorithm should adapt to it and find new solutions at minimal cost. However, DVRP is relatively unexplored despite its theoretical importance and practical values.

The following section of the paper contain short problem description and its mathematical formulation, then the solution method is described, results for a case study are presented and finally some conclusions are discussed.

2 Problem description

In this section, we first formally describe the classical VRP, then the dynamic VRP variant. In classical VRP, there is a set of geographically spreaded customers with known demands and a fleet of vehicles of fixed capacity. VRP can be formulated as a mathematical model as follows. Let G(V,E) be a complete directed graph where $V = \{v_0, v_1, ..., v_n\}$ is a set of nodes. Node v_0 is the depot which has m vehicles and nodes $v_1; ...; v_n$ represent a set of customers. Vehicles have identical capacity, $Q = Q_1, ..., Q_m$. E represents a set of edges connecting customers v_i and v_j, $E_{i,j} = \{(v_i, v_j)\ v_i, v_j \in V, i \leq j\}$. Each edge $E_{i,j}$ has a non-negative value which is the cost e.g. the travel time between v_i and v_j. The cost is defined by a matrix $C = (c_{i,j})$. An entry $c_{i,j}$ of the matrix C represents the shortest path between customers v_i and v_j. Each customer v_i is associated with a value representing q_i goods to be delivered or picked at this customer. Each delivery has a service time t_i. The goal of DVRP optimisation is to find a set of vehicle routes to serve all customers at minimal cost while satisfying the following constraints:

- All vehicles must start and end their routes, $R_1, ..., R_m$, at the depot v_0;
- The total demand assigned for each vehicle should not exceed the vehicle capacity;
- Each customer is visited only once in the delivery plan;
- The total duration of each route should not exceed the given global upper bound;

In real-life situations, the travel time between nodes depends on traffic condition of the current road network. Traffic could vary significantly depending on the time of the day. For example, the travel time during rush hour would be multiple times higher than the time travelling at midnight, see Fig.1. For this reason, we consider time dependent travel times that vary respect to the time slice of the day considering the traffic condition of the network and possible evolution on the basis of historical data.

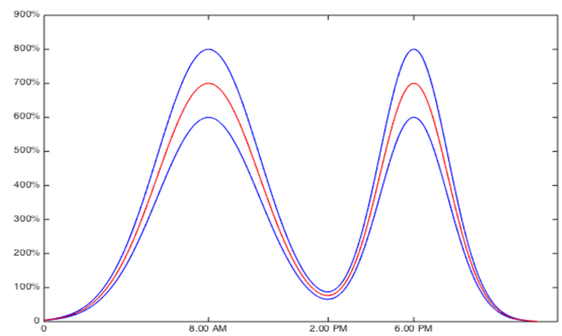

Figure 1: Trend of traffic during the day

For each pair (v_i, v_j) a set $S_{i,j}$ of shortest paths from v_i position to v_j is calculated, one for each considered time slice. Note that, when constructing the shortest path, the release date of v_i and the traffic condition in a specific time slice effect its shortest path to reach v_j.

The objective function takes into account: i) the duration of delivery process ii) the number of used vehicles iii) the satisfaction of costumers .

Let be the set of routes in a solution, each associated to the vehicle v(i) used for route i. The cost of route i is given by three quantities: (i) the fixed cost associated to the usage of vehicle v, (ii) the variable cost c i (i) associated to length of route i , and (iii) the penalty cost Open image in new window for late deliveries. The objective function of the problem is therefore: Open image in new window Duration time of the delivery for a vehicle is defined as the time required for visiting all the planned nodes on its route, it is measured from the departure from the source point to the end of the service of the last node on the route. This time includes also the nodes service time for all planned nodes for each vehicle route. Only the nodes service time is time independent. This completion time for delivery (C_r) is compared with the established time limit (T_{MAX}), and the following rules are assumed:

- if C_r is less than the limit T_{MAX}, then the objective function is linearly dependent on C_r plus the constant cost for the truck ;

- if C_r is longer by more than half of the established limit, then the cost is bigger and it is given by the constant cost for the truck plus ten times the extraordinary cost for C_D (unsatisfactory time) ;

- in the other cases the objective function is given by the constant cost for the truck plus the ordinary cost that is linear dependent on T_{MAX} plus the extraordinary cost that is linear dependent on $C_r - T_{MAX}$.

For this purpose these rules are summarized as:

$$OF_{(C_r)} = \begin{cases} C_v + C_O * C_r, C_r \leq T_{MAX} \\ C_v + C_O * T_{MAX} + C_E * (C_r - T_{MAX}), \\ T_{MAX} \leq C_r \leq 1.5 T_{MAX} \\ C_v + 10 * C_E * C_D, C_D \geq 1.5 T_{MAX} \end{cases} \quad (1)$$

To calculate the penalty function the following rules are considered:

- if t_a is less than the due date plus a threshold of the costumer, then the delivery process ends in advance and the dissatisfaction of the customer is minimal;

- if t_a is longer than the due date plus a threshold of the costumer, then the delivery process ends late and the dissatisfaction of the customer is maximum ;

- in the other cases the costumer is satisfied.

For this purpose these rules are summarized as:

$$OF_{(p_i)} = \begin{cases} C_T, t_{ai} - dd_i \geq \delta \\ C_E, dd_i - t_{ai} \geq \delta \\ 0, dd_i - \delta \leq t_a \leq dd_i + \delta \end{cases} \quad (2)$$

The total objective function is calculated by summing these two elements, and the problems becomes:

$$OF(X) = min\{\sum_{j=1}^{V} OF_{(C_{rj})} + \sum_{i=1}^{N} OF_{(p_i)}\} \quad (3)$$

The scheduling of deliveries and the routing of vehicles in urban areas is affected by traffic conditions which have a significant impact on travel time and consequently on delivery efficiency and customer service. A solution is a set of routes one for each vehicle and all costumers are satisfied. Each solution variant is evaluated, where the assessment takes into account both the duration of the delivery process and the satisfaction of costumers. Duration time of the delivery for a vehicle is defined as the time required for visiting all the planned nodes on its route, it is measured from the departure from the source point to the end of the service of the last node on the route.The satisfaction of the customers is considered comparison the arrival time of the vehicle node with due date.

The presented problem is similar to the travelling salesman problem: the shortest path which visits the given set of the graph nodes should be found. If all the nodes to be visited are numbered in a range from 1 to N (where N denotes the total number of customers) and the starting point is marked as 0, each series of such numbers is a valid schedule of visiting the nodes:

$$(0, K_1, K_2; \ldots, K_N) \quad (4)$$

where k_i number of a node, which will be serviced as i-th.

3 Heuristics

In this section the description of the heuristic procedures is provided. A greedy heuristic with a local search algorithm is presented. It is very fast and easily addresses the simultaneous minimization of operational costs and of customers' discomfort costs. A basic genetic algorithm has been implemented to test the procedure embedded in several simulation tools. Its performance have been compared to the results obtained by a standard genetic algorithm. Both the procedures are sketched below.

3.1 Greedy heuristic + local search

Before briefly describing the main steps of the heuristic procedure, the definition of set X_c is provided. Given a customer c the set X_c is formed by all the customers c' that are at a distance that respects their due dates including threshold. The heuristic can be summed up in the steps in Algorithm 1. It is composed by two phases, the first implementing the greedy procedure and the second executing the local search procedure. The greedy phase tries to assign a set of customers in X_c for each vehicle starting from the costumer c with smallest due date. The customers are fixed in a route is the capacity constrain is satisfied. The local search phase tries to improve the solution.

Algorithm 1 Greedy heuristic and Local search

Given the N ▷ Set of customers
Given $|V|$ ▷ Total number of vehicles
Given C_v ▷ C_v is the capacity of vehicle
$K = 1$ ▷ Counter K: it counts the number of assigned vehicle/route ▷ **Greedy heuristic phase**
Order N with respect to the increasing due date
for (each $c \in N$) **do**
  Generate X_c; ▷ X_c is the set of all the costumers $c' \in N$ that can be reached by c satisfied the due date considering also the threshold
end for
for (each $c \in N$) **do**
  if $q_c \leq C_k$ **then**
$R_k = \{c\}$
▷ c is added at the R_k, it is the kth route
N=N-{c}
$C_k = C_k - q_c$
k=k+1;

for (each $c' \in X_c$) **do**
  if $q_{c'} \leq C_k$ **then**
    Assign to c' a route r_v ;
    $R_k = R_k + \{c'\}$
▷ c' is added at the R_k, it is the kth route
    N=N-c'
    $C_k = C_k - q_{c'}$
  else k=k+1;
  end if
end for
while $(K > V)$ **do**
  Remove the vehicle k with bigger C_k
  Assign the costumers in R_k to others vehicles respecting the capacity constrain
end while
▷ **Local search phase**
Fix L ▷ L is the maximum number of iterations
for $(l \leq L)$ **do**
  Select randomly a customers c;
  Assign it to a route of a vehicle in X_c;
  Select the routing with minimum obcetive function
end for
Return the best routing. =0

3.2 Genetic heuristic

This heuristic is a standard genetic algorithm, whose main steps are reported in Algorithm 2. After generating the initial population of cardinality S, by a random assignment of the vehicles to all costumers, a cyclic subroutine selects two parents that will generate the new population, till the stop criteria are met. The probability of each individual to be selected for the reproduction is proportional to their Fitness value. In such way the best individuals have a higher probability to transmit their genetic inheritance. The two operators that generate the new population from the parents are:

- The Crossover operator combines the genetic inheritance of the parents to generate new individuals. Each children is composed by two parts, each one belonging to one of the two parents. The CR is the rate of crossover. Each crossover operation generates two children.
- The Mutation operator changes casually one or more components of a individual. The mutation operator is applied only with a certain probability, called mutation rate MR. The number of children that are generated by the mutation operator depends on the number of individuals generated by the crossover operator. The new population will have the same cardinality of the initial one (S).

This heuristic is a standard genetic algorithm. The initial solution is generated considering first the number of vehicles in service is randomly drawn, then the nodes to be serviced are sequentially assigned to vehicles. After generating the initial population by a random assignment, a cycle, composed by the following steps, runs till the stop criteria are met.

1. For each solution evaluate the objective function.
2. Select a set of best solutions on the basis of step 2 and save them.
3. Combine them to create a new population.

When the cycle stops the best solution is given in output.

4 Results analysis

At this point, a serious problem arises for the service provider, how to deliver the desired goods to a large number of customers waiting for the delivery at various points of the city, at a specified time, with at low

Algorithm 2 Genetic heuristic

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Given I   ▷ I is the maximum number of iteration of the algorithm
Given RC   ▷ CR is the rate of crossover
Generate P = {p_1, p_2, ... p_S}   ▷ The initial population of feasible solutions of cardinality S
for (i ≤ I) do
    for (s ≤ S) do
        Compute Fitness(p_s)   ▷ The Fitness function is the objective function of the problem
        Select the two parents p_m and p_f in P with the best Fitness function values
        Update p*   ▷ p* is the individual with the best Fitness function value
        CC = Crossover (p_m, p_f, CR); ▷ CC set of individuals derived from the Crossover
        M = Mutation (MR)   ▷ M set of individuals derived from the Mutation
        P = CC ∪ M
    end for
    i = i + 1
end for
Return p* =0
```

cost as possible. The presented case study makes use of the data published by (0).

In Fig.2, the graph presents the simplified structure of the transportation network and the layout of customers and the location of the supplier company. There is a driving time assigned to each edge and a service time assigned to each node. It is represents the network and the time for a specific time slice. Given the stochastic nature of the problem, it was assumed that a solution is the average on 20 runs of optimization procedure. T_{MAX} is equal to 40 minutes and V is equal to 30.

As was mentioned in the introduction section the considered problem is computationally very complex, therefore the genetic algorithm (GA) is considered and compared with the *GHLS*.

The *GA* and the *SM* try to minimize number of required vehicles and the time needed for service. Some preliminary results are reported The trends of the two algorithms for the different runs is reported in Fig. 3. The variation respected to the run is very limited for both algorithms.

In table 1, a comparison between heuristics is reported, other indices are considered. V represents the number of vehicles used, D_v represents the weight of the vehicle routing, UC represents how many costumers are served late.

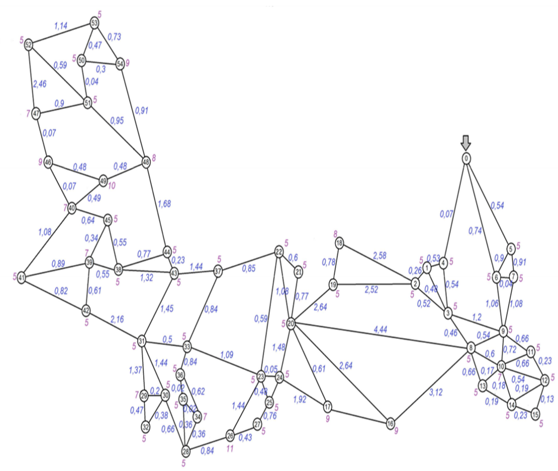

Figure 2: Test Network for a time slice

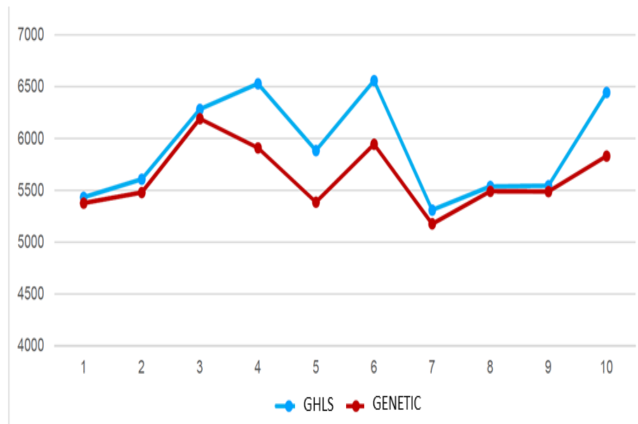

Figure 3: Trend of the objective function

The *GH* finds a solution with minor completion time and better satisfied the time constrains. This preliminary results seams to demonstrate also for this type of problem the ability of Genetic algorithm to find optimal solution for complex problem.

5 Conclusion

Scheduling deliveries in a large catering company may require the use of quick methods for determining the routes of vehicles. This study is at begging phase and a more complete analysis is under study.

HEURISTIC	OF	V	D_v	UC
GHLS	5.924	6	145	8,3
GA	5.627	6	125	6,5

Table 1: comparison between GHLS and GA

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