Research on Bullwhip Effect in Supply Chains with Two Retailers Considering Probability based on the Impact of Price

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Abstract: The bullwhip effect is one of the main problems in supply chain management. The downstream retail price fluctuation is one of the major factors to cause the bullwhip effect. This paper investigates the impact of retail prices variability with a view to probability on the bullwhip effect in a two-echelon supply chain which is composed of one supplier and two retailers. With various probabilities to choose the price, we may simply quantify the relationship between the two prices. The order-up-to inventory policy and the moving average forecasting method are employed respectively by the two retailers. The effects of the price, the probability, the lead time, and the autoregressive coefficient on a bullwhip effect measure are shown finally. And we indentify on which condition the bullwhip effect is lessened based on the price sensitive demands.

Key–Words: Complex Supply chain, Bullwhip effect, Covariance, Price Game, MA Forecasting Method

1 Introduction

The bullwhip effect has received more and more attention in the professional field of modern logistics and supply chain management in recent years. The bullwhip effect refers to that without effectively achieving the sharing of information, when the information flow in the supply chain from the final clients to the original supplier delivery time, the information distortion and enlarge gradually, which leads to a growing volatility in the demand information.

In the condition of industrial dynamics Forrester(1958, 1961) discussed the causes and possible remediation, thus he discovered the existence of this phenomenon firstly. Then, a number of studies also proved the existence of the difference amplification phenomenon in supply chains. Lee et al.(1997a, b) firstly called the amplification phenomenon as the bullwhip effect. He pointed out that demand signal processing, non-zero lead-time, order batching, supply shortages and price fluctuation were five important factors to cause the bullwhip effect. Graves(1999) quantified the bullwhip effect for the supply chain in which demand pattern follows an integrated moving average process. The bullwhip effect for supply chain was quantified by Chen et al.(2000a,b) using the demand forecasts of moving average and exponential smoothing techniques respectively. Chen et al. proposed hypothesis that members of the chain possessed the base stock policy as their inventory system. And found that the order variance would increase with the increasing lead time, number of members in the chain and lower level of information sharing. Zhang(2004) depicted the impact of each parameter on the bullwhip effect with a first-order autoregressive demand process using the MMSE, MA and ES forecasting methods. Ertunga(2008) investigated the reverse bullwhip effect in pricing(RBP) with conditions that create an ampli?cation of price variation moving from the upstream suppliers to the downstream customers in a supply chain. According to the nonserial supply chains hypothesis that Ha, Tong, and Zhang(2011) discussed supply chain coordination and information sharing in two competing supply chains. Nepal et al.(2012) studied an analysis of the bullwhip effect and net-stock amplification in a three-echelon supply chain based on step changes in the production rates during a products lifecycle demand. Sanjita Jaipuria(2013) highlighted an integrated approach of DWT and ANN to improve the forecasting accuracy by comparing with ARIMA model and validated with real-life data. Ma et al.(2013) described a comparison of bullwhip effect under various forecasting techniques with ARMA demand progress in a two stage supply chain with two retailers. Wang, N.M.(2014) discussed the impact of consumer price forecasting behavior on the bullwhip effect and found that consumer forecasting behavior can reduce the bullwhip effect. Ouyang and Yan feng(2014) analysed experiment results to show that advance demand informations could reduce supply chain costs via an experimental study. Akhtar Tanweer*(2014) proposed an optimization model to mitigate the bullwhip effect in a two-echelon supply chain. Ding fei Fu(2014) derived analytic expression of bullwhip effect based on control theoretic concept. He also used new bullwhip metric on conventional and MPC ordering policies for comparison. Finally the conclusion was given that MPC ordering policy outperforms the traditional ordering policies on reducing bullwhip effect. Ma and Bao(2014) had a deeply researchment on the comparision of bullwhip under three different foresting methods considering the market share. Yungao Ma(2015) offered insights into how the bullwhip effect in two parallel supply chains with interacting price-sensitive demands is affected in contrast to the situation of a single product in a serial supply chain.Yongrui Duan(2015) examined the effect of own and substitute products on a focal product's bullwhip effect and estimated the existence and magnitude of the bullwhip effect at the product level. He came to the conclusion that the bullwhip effect is not only affected by a product's own factors but also by those of its substitute products.

This paper mainly discusses the impact of probability of the price fluctuation on bullwhip effect. We finally get the expression of the bullwhip effect by using algebraic analysis and numerical simulation. The impacts of every parameter on the bullwhip effect are also analyzed. Then we come to the conclusion that different probability result in the variation of the bullwhip effect in relation to price, lead-time and demand autocorrelation.

The structure of this article is as follows. Section 2presents a two- echelon supply chain model with two retailers which both follow the price AR(1) process and apply the order-up-to stock policy. In Section 3, we derive the bullwhip effect measure under MA forecasting method. The impacts of every parameter on the bullwhip effect are analyzed in Section4. Finally, Section 5 shows a conclusion of the article and the vision of the future about the bullwhip effect.

2 A supply chain model

2.1 Price autoregressive process

This research will depict a two-echelon supply chain with one supplier and two retailers, both of the two retailers employ the order-up-to inventory policy and an AR (1) price autoregressive model. And we will quantify the bullwhip effect in the simple supply chain. The two retailers order and replenish the stock from the same supplier in each period t.

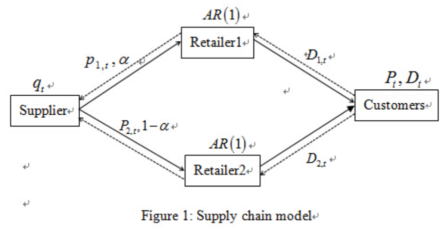

Figure 1: Supply chain model

The supply chain model is shown in Figure 1.

We assume there are only two retailers in the market. Each of the customers possesses their own retail price. Hence, the probability of the customers to select the two retail prices will be an important role to affect the demand process. The differences of the retail price of the two retailers can produce certain influence on the customers' purchase behavior. The probabilities to choose the two retailers for the customers are considered as α and $1-\alpha$ separately.

$$P_t = \delta + \phi P_{t-1} + \varepsilon_t \tag{1}$$

Due to the probability of retail price 1, we consider that the price of retailer 1 employs an AR(1) model as follows:

$$P_{1,t} = \alpha\delta_1 + \phi_1 P_{1,t-1} + \alpha\varepsilon_{1,t} \tag{2}$$

In the above expression, $P_{1,t}$ is the price of period t. δ_1 is the constant of the price autoregressive equation. ϕ_1 is the first-order autocorrelation coefficient and $-1 < \phi_1 < 1$. $\varepsilon_{1,t}$ is the forecast error for period t, $\varepsilon_{1,t}$ is independent and identically distributed from a symmetric distribution with mean 0 and variance σ_1^2. The price autoregressive equation is similar to Ma and Bao (2014).

On the basis of the first-order autocorrelation property of time series model, for any period t, we can derive the expectation and variance of $P_{1,t}$:

$$\begin{aligned} \mu_{1,p} &= E\left(P_{1,t}\right) = E\left(P_{1,t-1}\right) = \frac{\alpha\delta_1}{1-\phi_1} \\ \sigma_{1,p}^2 &= Var\left(P_{1,t}\right) = Var\left(P_{1,t-1}\right) = \frac{\alpha^2\sigma_1^2}{1-\phi_1^2} \end{aligned} \tag{3}$$

Analogously, in terms of the probability of retail price2, we consider that retailer 2 also employs an AR(1) model as follows:

$$P_{2,t} = (1-\alpha)\,\delta_2 + \phi_2 P_{2,t-1} + (1-\alpha)\,\varepsilon_{2,t} \quad (4)$$

In equation (4), $P_{2,t}$ is the price of period t. δ_2 is the constant of the price autoregressive equation which determines the mean of $P_{2,t}$. ϕ_2 is the first-order autocorrelation coefficient, and we also have $-1 < \phi_2 < 1$. $\varepsilon_{2,t}$ is the forecast error of period t, $\varepsilon_{2,t}$ is independent and identically distributed from a symmetric distribution with mean 0 and variance σ_1^2.

Accordingly, we can also have the expectation and variance of $P_{2,t}$:

$$\mu_{2,p} = E(P_{2,t}) = E(P_{2,t-1}) = \frac{(1-\alpha)\,\delta_2}{1-\phi_2}$$
$$\sigma_{2,p}^2 = Var(P_{2,t}) = Var(P_{2,t-1}) = \frac{(1-\alpha)^2\,\sigma_2^2}{1-\phi_2^2} \quad (5)$$

It is noteworthy that the correlativity of the two retail price is:

$$Cov(P_{1,t}, P_{2,t}) = \frac{\alpha}{1-\alpha} Var(P_{2,t}) \quad (6)$$

2.2 Price Demand process

We all know that price decide to the market demand. So we employ the general price demand process. The two retailers both face the same demand process, and they issue orders to supplier in each period. We depict the demand function model of retailer 1 as follows:

$$D_{1,t} = \mu_1 - \rho_{11} P_{1,t} + \rho_{12} P_{2,t} + \eta_{1,t} \quad (7)$$

In equation (7), $D_{1,t}$ is the demand of period t. μ_1 is the demand constant of the demand model, $P_{1,t}$ and $P_{2,t}$ are the retail price of retailer 1 and retailer 2, ρ_{11} is the self-acting price sensitivity coefficient, ρ_{12} is the inter-acting price sensitivity coefficient. We pronounce that ρ_{11} is non-negative. $\eta_{1,t}$ is the random fluctuation item which is independent and identically distributed from a normally distribution with mean 0 and variance γ^2.

Analogously, the expectation and variance of $D_{1,t}$ can be derived:

$$\begin{aligned}\mu_{1,d} &= E(D_{1,t}) = \mu_1 - \rho_{11}\mu_{1,p} + \rho_{12}\mu_{2,p}\\ \sigma_{1,d}^2 &= Var(D_{1,t})\\ &= \gamma_1^2 + \rho_{11}^2\sigma_{1,p}^2 + \rho_{12}^2\sigma_{2,p}^2 - 2\rho_{11}\rho_{12}\frac{\alpha}{1-\alpha}\sigma_{2,p}^2\end{aligned} \quad (8)$$

Similarly, retailer 2 also employs an price demand model as follows:

$$D_{2,t} = \mu_2 - \rho_{21} P_{2,t} + \rho_{22} P_{1,t} + \eta_{2,t} \quad (9)$$

The parameters in equation (9) have the same meaning with the corresponding parameters in equation (8). Hence, we also have:

$$\begin{aligned}\mu_{2,d} &= E(D_{2,t}) = \mu_2 - \rho_{21}\mu_{2,p} + \rho_{22}\mu_{1,p}\\ \sigma_{2,d}^2 &= Var(D_{2,t})\\ &= \gamma_2^2 + \rho_{21}^2\sigma_{2,p}^2 + \rho_{22}^2\sigma_{1,p}^2 - 2\rho_{21}\rho_{22}\frac{\alpha}{1-\alpha}\sigma_{2,p}^2\end{aligned} \quad (10)$$

The correlativity of the two random fluctuation items (i.e. Covariance) is:

$$Cov(\eta_{1,t}, \eta_{2,t}) = \begin{cases} \gamma_{12}^2 & \text{if } t = t, \\ 0 & \text{else.} \end{cases}$$

2.3 Inventory policy

In this paper, we utilize one of the most common inventory policies called order-up-to inventory policy, so as to meet the dynamic needs of the supply chain model shown in Fig.1. We hold the idea that the two retailers both employ a confirmed order lead time for each order. At the beginning of period t, the order of quantity $q_{1,t}$ send by retailer 1 can be given as follows:

$$q_{1,t} = S_{1,t} - S_{1,t-1} + D_{1,t-1} \quad (11)$$

In equation (11), $S_{1,t}$ is the order-up-to level which is to meet the goal of the inventory policy that maintain inventory levels at the target stock levels $q_{1,t}$, and it can be derived through lead-time demand by:

$$S_{1,t} = \hat{D}_{1,t}^{L_1} + z\hat{\sigma}_{1,t}^{L_1} \quad (12)$$

In the expression above, L_1 is known as the fixed lead time. Lead time is always a multiple of the inventory check interval. $D_{1,t}^{L_1}$ mean value of lead-time demand in supply chain and it can be forecasted on the basis of previous sales data. z is the normal z-score(i.e. a constant)which is often supplied to meet the safe stock can be set based on the desired service level of the inventory policy. $\sigma_{1,t}^{L_1}$ is the standard deviation of lead-time demand forecast error of the lead time.

The same as retailer 1, we consider that retailer 2 also employs the common order-up-to inventory policy:

$$q_{2,t} = S_{2,t} - S_{2,t-1} + D_{2,t-1} \quad (13)$$

Analogously, the order-up-to level $S_{2,t}$ can be applied as follows:

$$S_{2,t} = \hat{D}_{2,t}^{L_2} + z\hat{\sigma}_{2,t}^{L_2} \quad (14)$$

In equation (14), L_2 is known as the fixed lead time of retailer 2. $D_{2,t}^{L_2}$ is the mean value of lead-time demand in supply chain and it can be forecasted on the basis of previous sales data. z is the normal z-score(i.e. a constant) which is often supplied to meet the safe stock can be set based on the desired service level of the inventory policy. $\sigma_{2,t}^{L_2}$ is the standard deviation of lead-time demand forecast error of the lead time.

2.4 Forecasting method

As we can see from the above inventory equation which consists of order-up-to level S_t and demand forecasting value D_t^L.In order to quantify the order-up-to level of the two retailers, the lead-time demand forecasting value D_t^L must be estimated. In this research, both of the two retailers apply the MA (i.e. Moving average) forecasting method to predict the lead-time demand. Moving average method a method in order to forecast method that is based on time series, item by item, then calculate contains a certain number of chronological average. The MA forecasting method is short for moving average. Under the MA forecasting method, with is the span K (number of periods) for the MA forecasting method. D_{t-i} is the actual demand in period $t - i$, the lead-time demand can be expressed as follows:

$$\hat{D}_t^L = \frac{L}{k}\sum_{i=1}^{k} D_{t-i} \quad (15)$$

3 The Measure of the Bullwhip Effect

According to the given price autocorrelation model, price demand model and order-up-to inventory policy, this section mainly discusses the measure of the bullwhip effect under the MA forecasting method.

Input equation (12) into equation (11), we have:

$$q_{1,t} = \hat{D}_{1,t}^{L_1} - \hat{D}_{1,t-1}^{L_1} + z\hat{\sigma}_{1,t}^{L_1} - z\hat{\sigma}_{1,t-1}^{L_1} + D_{1,t-1} \quad (16)$$

We have known that $\hat{\sigma}_{1,t}^{L_1}$ has nothing to do with t, so expression (16) can be deprived as:

$$q_{1,t} = \hat{D}_{1,t}^{L_1} - \hat{D}_{1,t-1}^{L_1} + D_{1,t-1} \quad (17)$$

Under the MA forecasting method, the lead-time demand of retailer 1can be depicted as below:

$$\hat{D}_{1,t}^{L_1} = \frac{L_1}{k}\sum_{i=1}^{k} D_{1,t-i} \quad (18)$$

Using equation (18) in equation (17), the order quantity of retailer 1 can be derived:

$$\begin{aligned} q_{1,t} &= \frac{L_1}{k}\sum_{i=1}^{k} D_{1,t-i} - \frac{L_1}{k}\sum_{i=1}^{k} D_{1,t-i-1} + D_{1,t-1} \\ &= \left(1 + \frac{L_1}{k}\right) D_{1,t-1} - \frac{L_1}{k} D_{1,t-k-1} \quad (19) \end{aligned}$$

We can get the order quantity expression of retailer 2 with the same method as retailer 1:

$$q_{2,t} = \left(1 + \frac{L_2}{k}\right) D_{2,t-1} - \frac{L_2}{k} D_{2,t-k-1} \quad (20)$$

Hence, the total order quantity of the two retailers is the summation meter of $q_{1,t}$ and $q_{2,t}$:

$$\begin{aligned} q_t &= q_{1,t} + q_{2,t} \\ &= \left(1 + \frac{L_1}{k}\right) D_{1,t-1} - \frac{L_1}{k} D_{1,t-k-1} \\ &+ \left(1 + \frac{L_2}{k}\right) D_{2,t-1} - \frac{L_2}{k} D_{2,t-k-1} \quad (21) \end{aligned}$$

By calculating the variance of the total order quantity, we derive:

$$\begin{aligned} &\mathrm{Var}(q_t) \\ &= \left(1 + \frac{L_1}{k}\right)^2 \mathrm{Var}(\mathrm{D}_{1,t-1}) + \left(\frac{L_1}{k}\right)^2 \\ &\mathrm{Var}(\mathrm{D}_{1,t-k-1}) + \left(1 + \frac{L_2}{k}\right)^2 \mathrm{Var}(\mathrm{D}_{2,t-1}) \\ &+ \left(\frac{L_2}{k}\right)^2 \mathrm{Var}(\mathrm{D}_{2,t-k-1}) \\ &-2\left(1 + \frac{L_1}{k}\right)\frac{L_1}{k} Cov(D_{1,t-1}, D_{1,t-k-1}) \\ &+2\left(1 + \frac{L_1}{k}\right)\left(1 + \frac{L_2}{k}\right) Cov(D_{1,t-1}, D_{2,t-1}) \end{aligned}$$

$$-2\left(1+\frac{L_1}{k}\right)\frac{L_2}{k}Cov\left(D_{1,t-1},D_{2,t-k-1}\right)$$
$$-2\frac{L_1}{k}\left(1+\frac{L_2}{k}\right)Cov\left(D_{1,t-k-1},D_{2,t-1}\right)$$
$$+2\frac{L_1}{k}\frac{L_2}{k}Cov\left(D_{1,t-k-1},D_{2,t-k-1}\right)$$
$$-2\left(1+\frac{L_2}{k}\right)\frac{L_2}{k}Cov\left(D_{2,t-1},D_{2,t-k-1}\right) \quad (22)$$

Due to the expression before, we can easily proof that:

$$\begin{aligned}
&Cov\left(P_{1,t-1},P_{1,t-k-1}\right)=\phi_1^k\mathrm{Var}\left(\mathrm{P}_{1,t}\right),\\
&Cov\left(P_{1,t-1},P_{2,t-1}\right)=\frac{\alpha}{1-\alpha}Var\left(P_{2,t}\right),\\
&Cov\left(P_{1,t-1},P_{2,t-k-1}\right)=\phi_1^k\frac{\alpha}{1-\alpha}Var\left(P_{2,t}\right),\\
&Cov\left(P_{1,t-k-1},P_{2,t-1}\right)=\phi_2^k\frac{\alpha}{1-\alpha}Var\left(P_{2,t}\right),\\
&Cov\left(P_{1,t-k-1},P_{2,t-k-1}\right)=\frac{\alpha}{1-\alpha}Var\left(P_{2,t}\right),\\
&Cov\left(P_{2,t-1},P_{2,t-k-1}\right)=\phi_2^k\mathrm{Var}\left(\mathrm{P}_{2,t}\right).
\end{aligned} \quad (23)$$

On the basis of equation (23), we derive the expression bellow:

$$\begin{aligned}
&Cov\left(D_{1,t-1},D_{1,t-k-1}\right)\\
&=\rho_{11}^2\phi_1^kVar\left(p_{1,t}\right)+\rho_{12}^2\phi_2^kVar\left(p_{2,t}\right)\\
&-\rho_{11}\rho_{12}\left(\phi_1^k+\phi_2^k\right)\frac{\alpha}{1-\alpha}Var\left(p_{2,t}\right),\\
&Cov\left(D_{1,t-1},D_{2,t-1}\right)\\
&=-\rho_{11}\rho_{22}Var\left(p_{1,t}\right)-\rho_{12}\rho_{21}Var\left(p_{2,t}\right)\\
&+\left(\rho_{11}\rho_{21}+\rho_{12}\rho_{22}\right)\frac{\alpha}{1-\alpha}Var\left(p_{2,t}\right)+\gamma_{12}^2,\\
&Cov\left(D_{1,t-1},D_{2,t-k-1}\right)\\
&=-\rho_{11}\rho_{22}\phi_1^kVar\left(p_{1,t}\right)-\rho_{12}\rho_{21}\phi_2^kVar\left(p_{2,t}\right)\\
&+\left(\rho_{11}\rho_{21}\phi_1^k+\rho_{12}\rho_{22}\phi_2^k\right)\frac{\alpha}{1-\alpha}Var\left(p_{2,t}\right),\\
&Cov\left(D_{1,t-k-1},D_{2,t-1}\right)\\
&=-\rho_{11}\rho_{22}\phi_1^kVar\left(p_{1,t}\right)-\rho_{12}\rho_{21}\phi_2^kVar\left(p_{2,t}\right)\\
&+\left(\rho_{11}\rho_{21}\phi_2^k+\rho_{12}\rho_{22}\phi_1^k\right)\frac{\alpha}{1-\alpha}Var\left(p_{2,t}\right),\\
&Cov\left(D_{1,t-k-1},D_{2,t-k-1}\right)\\
&=-\rho_{11}\rho_{22}Var\left(p_{1,t}\right)-\rho_{12}\rho_{21}Var\left(p_{2,t}\right)\\
&+\left(\rho_{11}\rho_{21}+\rho_{12}\rho_{22}\right)\frac{\alpha}{1-\alpha}Var\left(p_{2,t}\right)+\gamma_{12}^2,\\
&Cov\left(D_{2,t-1},D_{2,t-k-1}\right)\\
&=\rho_{22}^2\phi_1^kVar\left(p_{1,t}\right)+\rho_{21}^2\phi_2^kVar\left(p_{2,t}\right)\\
&-\rho_{21}\rho_{22}\left(\phi_1^k+\phi_2^k\right)\frac{\alpha}{1-\alpha}Var\left(p_{2,t}\right),\\
&Var\left(D_{1,t-1}\right)=Var\left(D_{1,t-k-1}\right)\\
&=\rho_{11}^2Var\left(p_{1,t}\right)+\rho_{12}^2Var\left(p_{2,t}\right)\\
&-2\rho_{11}\rho_{12}\frac{\alpha}{1-\alpha}Var\left(p_{2,t}\right)+\gamma_1^2,\\
&Var\left(D_{2,t-1}\right)=Var\left(D_{2,t-k-1}\right)\\
&=\rho_{22}^2Var\left(p_{1,t}\right)+\rho_{21}^2Var\left(p_{2,t}\right)\\
&-2\rho_{21}\rho_{22}\frac{\alpha}{1-\alpha}Var\left(p_{2,t}\right)+\gamma_2^2.
\end{aligned} \quad (24)$$

The derivation of equation (23) and equation (24)can be seen in the appendix.

Then, replacing equation (24) into equation (22), we have:

$$\begin{aligned}
&Var\left(q_t\right)=Var\left(p_{1,t}\right)\\
&\left\{\begin{array}{l}
\rho_{11}^2\left[2A_1^2+2A_1+1-2\phi_1^k\left(A_1^2+A_1\right)\right]\\
+\rho_{22}^2\left[2A_2^2+2A_2+1-2\phi_1^k\left(A_2^2+A_2\right)\right]\\
+2\rho_{11}\rho_{22}\left(\phi_1^k-1\right)\left(2A_1A_2+A_1+A_2\right)\\
-2\rho_{11}\rho_{22}
\end{array}\right\}\\
&+Var\left(p_{2,t}\right)\\
&\left\{\begin{array}{l}
\rho_{12}^2\left[2A_1^2+2A_1+1-2\phi_2^k\left(A_1^2+A_1\right)\right]\\
+\rho_{21}^2\left[2A_2^2+2A_2+1-2\phi_2^k\left(A_2^2+A_2\right)\right]\\
+2\rho_{12}\rho_{21}\left(\phi_2^k-1\right)\left(2A_1A_2+A_1+A_2\right)\\
-2\rho_{12}\rho_{21}
\end{array}\right\}\\
&\left(2\rho_{11}\rho_{12}B_1+2\rho_{21}\rho_{22}B_2+2\rho_{11}\rho_{21}B_3+2\rho_{12}\rho_{22}B_4\right)\\
&\frac{\alpha}{1-\alpha}Var\left(p_{2,t}\right)\\
&+\left(2A_1^2+2A_1+1\right)\gamma_1^2+\left(2A_2^2+2A_2+1\right)\gamma_2^2\\
&+2\left(2A_1A_2+A_1+A_2+1\right)\gamma_{12}^2
\end{aligned} \quad (25)$$

Where, $\frac{L_i}{k}=A_i,(i=1,2)$

$$\begin{aligned}
&B_1=\left(\phi_1^k+\phi_2^k\right)\left(A_1^2+A_1\right)-\left(2A_1^2+2A_1+1\right)\\
&B_2=\left(\phi_1^k+\phi_2^k\right)\left(A_2^2+A_2\right)-\left(2A_2^2+2A_2+1\right)\\
&B_3=\left(2A_1A_2+A_1+A_2+1\right)\\
&-\phi_1^k\left(A_1A_2+A_2\right)-\phi_2^k\left(A_1A_2+A_1\right)\\
&B_4=\left(2A_1A_2+A_1+A_2+1\right)\\
&-\phi_2^k\left(A_1A_2+A_2\right)-\phi_1^k\left(A_1A_2+A_1\right)
\end{aligned}$$

Equation (25) can be simply written as:

$$\begin{aligned}
Var\left(q_t\right) &= G_1Var\left(p_{1,t}\right)+G_2Var\left(p_{2,t}\right)\\
&+ G_3\frac{\alpha}{1-\alpha}Var\left(p_{2,t}\right)+G_4
\end{aligned} \quad (26)$$

In equation (26),G_1 is the coefficient of $\mathrm{Var}\left(\mathrm{P}_{1,\mathrm{t}}\right)$, G_2 is the coefficient of $\mathrm{Var}\left(\mathrm{P}_{2,\mathrm{t}}\right)$,$G_3$ is the coefficient of $\frac{\alpha}{1-\alpha}Var\left(P_{2,t}\right)$, G_4 represents the disturbance variance.

In section 2, we suppose there are two retailers in the supply chain. Hence, the total demand which the two retailers face is the summation meter of the two lead-time demand:

$$D_t = D_{1,t} + D_{2,t} \tag{27}$$

It is known to all that $\frac{P_{1,t}}{P_{2,t}} = \frac{\alpha}{1-\alpha}$ and $Cov\left(D_{1,t}, D_{2,t}\right) = Cov\left(D_{1,t-1}, D_{2,t-1}\right)$.

Taking the variance of the total demand, we have:

$$\begin{aligned} &Var\left(D_t\right) \\ &= Var\left(D_{1,t}\right) + Var\left(D_{2,t}\right) + 2Cov\left(D_{1,t}, D_{2,t}\right) \\ &= \left(\rho_{11} - \rho_{22}\right)^2 Var\left(p_{1,t}\right) + \left(\rho_{12} - \rho_{21}\right)^2 Var\left(p_{2,t}\right) \\ &+ \left(\rho_{11} - \rho_{22}\right)\left(\rho_{21} - \rho_{12}\right)\frac{\alpha}{1-\alpha}Var\left(p_{2,t}\right) \\ &+ \gamma_1^2 + \gamma_2^2 + \gamma_{12}^2 \end{aligned} \tag{28}$$

In equation (28),M_1 is the coefficient of $\mathrm{Var}\left(\mathrm{P}_{1,\mathrm{t}}\right)$, M_2 is the coefficient of $\mathrm{Var}\left(\mathrm{P}_{2,\mathrm{t}}\right)$,$M_3$ is the coefficient of $\frac{\alpha}{1-\alpha}Var\left(P_{2,t}\right)$, M_4 represents the disturbance variance.

To sum up above all, we finally derive the expression of BWE achieving the quantization of the BWE under MA forecasting method. The expression can be written as follows:

$$\begin{aligned} BWE &= \frac{Var\left(q_t\right)}{Var\left(D_t\right)} \\ &= \frac{G_1\left(\frac{\alpha}{1-\alpha}\right)^2 + G_2 + G_3\frac{\alpha}{1-\alpha} + {G_4}/{\mathrm{Var}\left(P_{2,t}\right)}}{M_1\left(\frac{\alpha}{1-\alpha}\right)^2 + M_2 + M_3\frac{\alpha}{1-\alpha} + {M_4}/{\mathrm{Var}\left(P_{2,t}\right)}} \end{aligned} \tag{29}$$

4 Behavior of the bullwhip effect measure and numerical simulation

As we described before, the bullwhip effect is a phenomenon that the information distortion and enlarge gradually when the information flow in the supply chain from the final clients to the original supplier. According to the expression of the bullwhip effect under MA forecasting methods, we can try our best to mitigate the bullwhip effect by algebraic analysis and numerical simulation. Then the analysis of parameters under the MA forecasting method will be conducted.

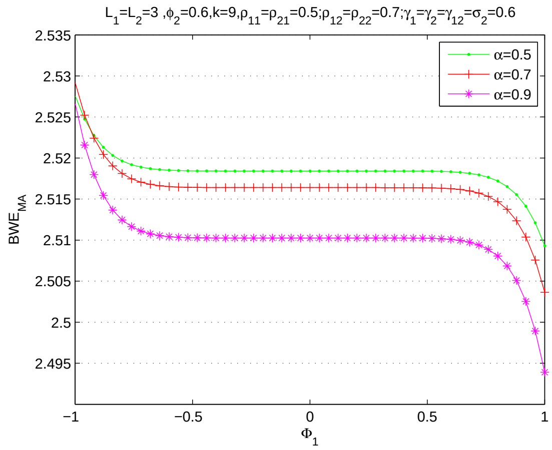

Figure 2: Impact of ϕ_1 on bullwhip effect for different α under the MA

The pictures below simulate the changing process of the bullwhip effect under MA forecasting method which depict the impact of parameters on the whole supply chain bullwhip effect vividly.

4.1 The analysis of autoregressive coefficient

Figures 2-4 emulate equation (29) with the curve iconically. These figures depict the effect of autoregressive coefficient on the bullwhip effect under moving average forecasting method.

Figure 2 shows that the bullwhip effect decreases quickly as ϕ_1 varying from -1 to -0.8. When the value of ϕ_1 comes between -0.8 and 0.8, the bullwhip effect is a stable constant all the time with the increase of ϕ_1.Gradually, the bullwhip effect gets down swiftly with ϕ_1 is more than 0.8. So α hardly affects the bullwhip effect in the circumstance of different ϕ_1. But we may also find that the bigger of the value of α, the larger the bullwhip effect will be. Therefore, we may choose the bigger α appropriately.

According to Figure 3, by transforming the span (number of periods) k of retailer 1, we can come to the conclusion that the impact of ϕ_1 on bullwhip effect for different k under the MA is very similar to the trend of ϕ_1 for different α under the moving average forecasting method. However, it is unlike figure 2.As k comes larger, the bullwhip effect is becoming smaller and smaller. k has reverse effect on bullwhip effect.

Figure 4 which is similar to the two figures above shows the impact of ϕ_1 on the bullwhip effect with the variation of different L_1. The bullwhip effect is a stable constant with ϕ_1 shift between -0.8 and 0.8. In addition to that, the bullwhip effect drops rapidly all the time with the increase of ϕ_1. And we know that L_1 can obviously influences the BWE, L_1 has a positive

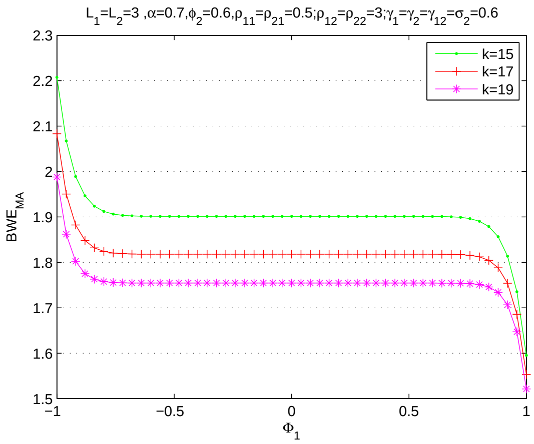

Figure 3: Impact of ϕ_1 on bullwhip effect for different k under the MA

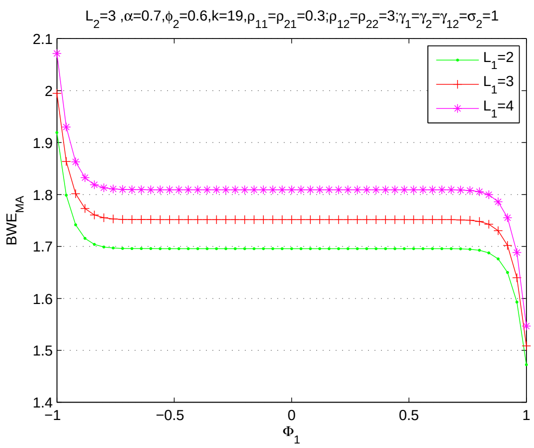

Figure 4: Impact of ϕ_1 on bullwhip effect for different L_1 under the MA

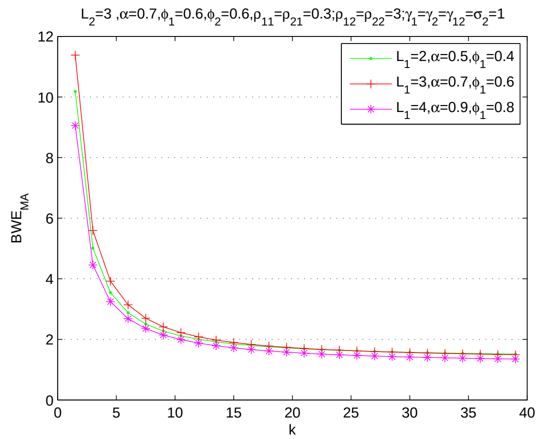

Figure 5: Impact of k on bullwhip effect for different L_1,α,ϕ_1 under the MA

effect on bullwhip effect.

4.2 The analysis of span k

Figure 5 indicates the effect of the price forecasting span on the bullwhip effect. The bullwhip effect decreases rapidly as the span going longer. A longer span mitigates the bullwhip effect dramatically. Then the bullwhip effect trends to 1along with the span more than 20. This phenomenon is just like the saying that it is noted that lead-time demand can be forecasted more accurately by more historical demand date.

4.3 The analysis of probability to choose the price

The analysis of probability to choose the price will be described through Figure 6 and figure 7. In this section, we will ferret out the influence of various α on the whole supply chain by simulate the expression of bullwhip effect.

Figure 6 reveals that the bullwhip effect is increased first to the maximum and declined gradually with the increase of α. As α being zero, the bullwhip effect under three different L_1 is not the same value. However, as α goes to maximum, the bullwhip effect turn into a constant which is approach to1.45. It is obvious that the longer the lead time is, the larger the bullwhip effect will be. This appearance explains one of the reasons to cause BWE commendably.

Impact of on bullwhip effect for different k under the MA is illustrated in Figure 7 accordingly. Firstly, the bullwhip effect grows up little by little and it reaches the culminating point when α gets to mid-value. Then the bullwhip effect sinks lower and lower as α rise higher and higher. We are surprised to find

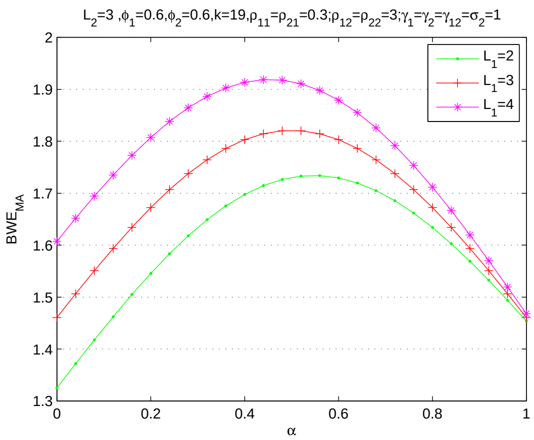

Figure 6: Impact of α on bullwhip effect for different L_1 under the MA

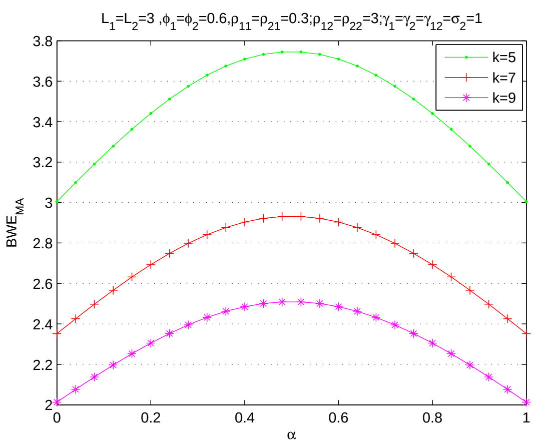

Figure 7: Impact of α on bullwhip effect for different k under the MA

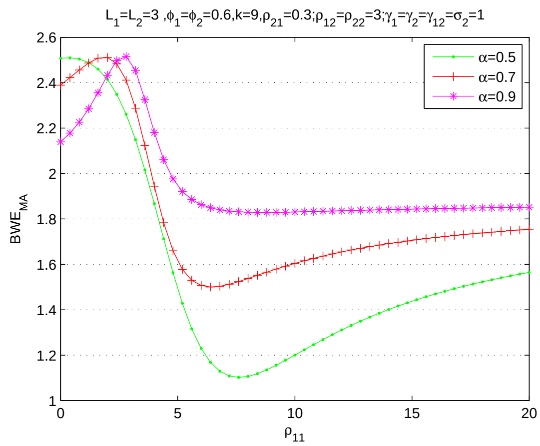

Figure 8: Impact of ρ_{11} on bullwhip effect for different α under the MA

the shape of bullwhip effect is symmetrical with respect to the value of α goes to 0.5. With different k, t is also changing timely. A relatively long k may mitigate the bullwhip effect abstractly. And α is a key factor to the bullwhip effect in the supply chain management.

4.4 The analysis of the self-acting price sensitivity coefficient

Figures 8-11 illustrate the expression of bullwhip effect under moving average forecasting method by analyzing the self-acting price sensitivity coefficient. We can derive the characteristic of the self-acting price sensitivity coefficient through the following discussion.

From Figure 8 we can see that the smaller self-acting price sensitivity coefficient does not always result in the lower bullwhip effect which can be found with ρ_{11} is less than 2, but the much greater self-acting price sensitivity coefficient does lead to the higher bullwhip effect which can be proved with ρ_{11} is approach to7. Compared with the autoregressive coefficient in Figure 2-4, the lead-time is a little harder to be controlled to mitigate the bullwhip effect.

According to Figure 9, the bullwhip effect rise to the highest point firstly with ρ_{11} is 3. Then it falls to the lowest point rapidly with ρ_{11} is 6. After that the three curves rise slowly and steadily. With the same self-acting price sensitivity coefficient, the three curves reach the maximum and minimum respectively no matter what the span of forecasting is.

By observing Figure 10, we realize that the bullwhip effect has the same trend of that in Figure 9. Which is different from Figure 9 is that the maximums of the three curves are very close to the others. But

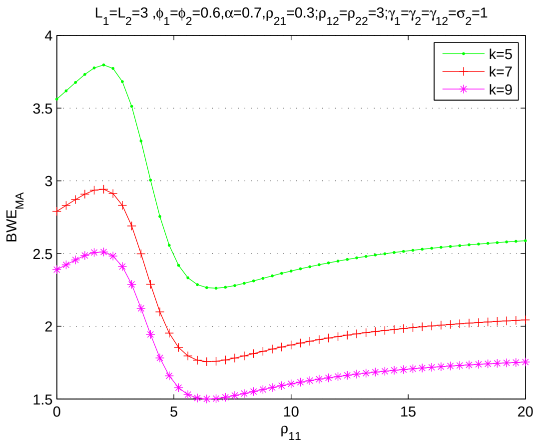

Figure 9: Impact of ρ_{11} on bullwhip effect for different k under the MA

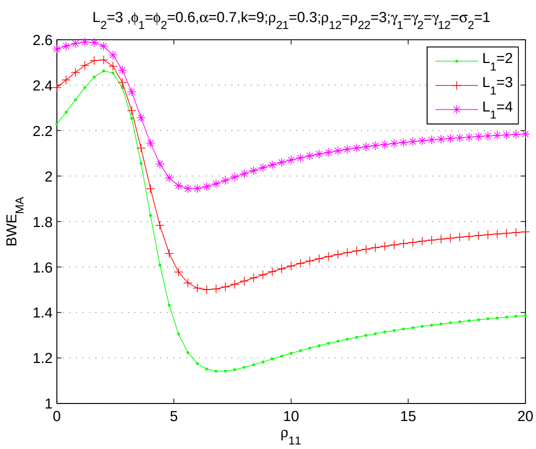

Figure 10: Impact of ρ_{11} on bullwhip effect for different L_1 under the MA

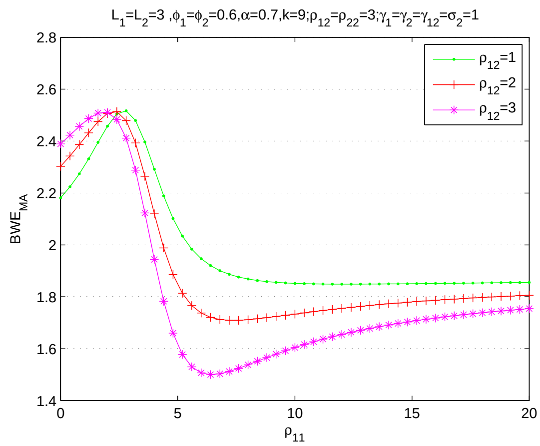

Figure 11: Impact of ρ_{11} on bullwhip effect for different ρ_{12} under the MA

the minimums of the three curves turn into completely different. When the self-acting price sensitivity coefficient is less than 6, there is so little difference on the bullwhip effect for different L_1 under the MA. However, the bullwhip effect is not approach any more with ρ_{11} more than 6. The shorter the lead time is, the lower the bullwhip effect is.

Figure 11 declares the impact of ρ_{11} on bullwhip effect for different ρ_{12} under the MA. The result reveals that before the bullwhip effect reaches the maximum the larger ρ_{12} is, the lower the bullwhip effect is. However, the smaller inter-acting price sensitivity coefficient does not always result in the lower bullwhip effect. After the maximum of the bullwhip effect, the larger ρ_{12} is, the higher the bullwhip effect is.

4.5 The analysis of the inter-acting price sensitivity coefficient

Figures 12-14simulate the expression of the bullwhip effect under the MA which depicts the impact of ρ_{12} on bullwhip effect for different parameters.

Figure 12 shows the impact of ρ_{12} on bullwhip effect under various probabilities. As the probability goes on, the trend of the bullwhip effect turns into different. We set the probability as 0.5 and 0.7 separately. In these two situations the curves increase to the maximum rapidly. Then they decrease slowly and placidly. While the probability increases to 0.9, the bullwhip effect increases smoothly all the time.

As can be seen from Figure 13, we consider the span of forecasting as 5,7and 9 respectively. We may result in that the bullwhip effect with $k = 9$ is the lowest of all and the bullwhip effect with $k = 5$ is the highest. Therefore, a relatively long span must be employed to forest the price under moving average

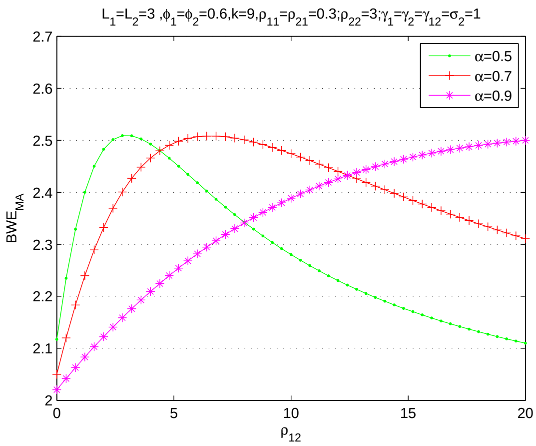

Figure 12: Impact of ρ_{12} on bullwhip effect for different α under the MA

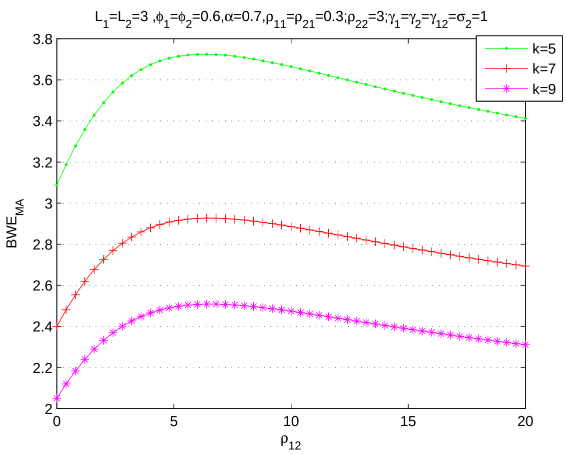

Figure 13: Impact of ρ_{12} on bullwhip effect for different k under the MA

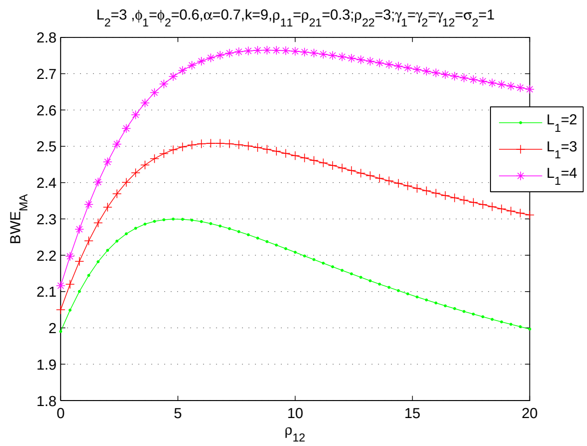

Figure 14: Impact of ρ_{12} on bullwhip effect for different L_1 under the MA

forecasting method.

Figure 14 indicates that if only the self-acting price sensitivity coefficient is in the price expression, the bullwhip effect with various lead time turn into closely to each other. While the inter-acting price sensitivity coefficient increases to 6, the curves reach their maximum value. As ρ_{12} increasing persistently, the bullwhip effect begins to decrease gradually. The result reveals the inter-acting price sensitivity coefficient influents the bullwhip a lot.

5 Conclusions

In this article, we depict the bullwhip effect in a two-echelon supply chain which is composed of one supplier and two retailers. The impact of retail prices variability with a view to probability on the bullwhip effect is illustrated in this paper. Even though most of the present researches have shown the bullwhip effect considering the demand forecasting, this paper carries a study on the impact of price on the bullwhip effect. Whats more the probability to choose the price is also illustrated in this research. The effect of autoregressive coefficient, span (number of periods), probability, self-acting price sensitivity coefficient and inter-acting price sensitivity coefficient on the bullwhip effect in a two-stage supply chain have been conducted in the above sections.

The result shows that no matter the value of probability, span and lead time, the larger the autoregressive coefficient changes, the lower the bullwhip effect in the supply chain will be. Hence, a bigger is extremely needed. Which means the price of the two adjacent periods must be closed to each other. A large fluctuation on price shouldnt be conducted in supply chain management. The bullwhip effect decreases quickly with the span going longer. A longer span mitigates the bullwhip effect obviously. From the point of view of management more historical demand date may actually reduce the bullwhip effect. And probability to choose the price is also a key factor to influent the bullwhip effect. When the probability reaches the mid-value, the expression acquires the maximum value. Considering the impact of the other parameters on bullwhip effect, it is better for the probability to choose a properly big value. Then we talk about the self-acting price sensitivity coefficient. The bullwhip effect fluctuates with the self-acting price sensitivity coefficient obviously. We are surprised to find that while the self-acting price sensitivity coefficient comes at about 7, the bullwhip effect will be the smallest. Finally, in the same way we discuss the inter-acting price sensitivity coefficient. And we come to the conclusion that with a smaller inter-acting price sensitivity coefficient the bullwhip effect is lower. It means that the more prices are influenced by competitors, the bigger the bullwhip effect we get. So the retailer should take measures to reduce the influence of competitors prices for their products.

At last, we all know that quantifying the bullwhip effect through inventory policy and forecasting method even with the price are helpful to mitigate the bullwhip effect in supply chain management. However, we must point out that: Firstly, a multi-stage supply chain must be done additional research while this paper pays attention to a two-stage supply chain. Secondly, the impact on the two retailers chaos game behavior on the bullwhip effect can be conducted in the later research. Hence, more and more research must be done in this research field. We should do more to mitigate the bullwhip effect in the supply chain management urgently.

Appendix

1.The proof of equation (23) is as follows:

After iteration computation for equation (2), we have:

$$\begin{aligned} P_{1,t-1} &= \left(1+\phi_1 + +\phi_1^{k-1}\right)\alpha\delta_1 \\ &+ \phi_1^k P_{1,t-k-1} + \alpha\phi_1^{k-1}\varepsilon_{1,t-k} \\ &+ \alpha\phi_1^{k-2}\varepsilon_{1,t-k+1} + \cdots + \alpha\varepsilon_{1,t-1} \\ &= \frac{1-\phi_1^{i+1}}{1-\phi_1}\alpha\delta_1 + \phi_1^k P_{1,t-k-1} \\ &+ \sum_{j=0}^{k-1}\alpha\phi_1^{k-1-j}\varepsilon_{1,t-k+j} \end{aligned} \tag{A.1}$$

So, we can get:

$$\begin{aligned} & Cov\left(P_{1,t-1}, P_{1,t-k-1}\right) \\ & = Cov\left(\begin{array}{l} \frac{1-\phi_1^{i+1}}{1-\phi_1}\alpha\delta_1 + \phi_1^k P_{1,t-k-1} \\ + \sum\limits_{j=0}^{k-1}\alpha\phi_1^{k-1-j}\varepsilon_{1,t-k+j}, P_{1,t-k-1} \end{array}\right) \\ & = \phi_1^k \mathrm{Var}\left(\mathrm{P_{1,t}}\right), \\ & Cov\left(P_{1,t-1}, P_{2,t-k-1}\right) \\ & = Cov\left(\begin{array}{l} \frac{1-\phi_1^{i+1}}{1-\phi_1}\alpha\delta_1 + \phi_1^k P_{1,t-k-1} \\ + \sum\limits_{j=0}^{k-1}\alpha\phi_1^{k-1-j}\varepsilon_{1,t-k+j}, P_{2,t-k-1} \end{array}\right) \\ & = \phi_1^k Cov\left(P_{1,t-k-1}, P_{2,t-k-1}\right) \\ & = \phi_1^k \tfrac{\alpha}{1-\alpha} Var\left(P_{2,t}\right). \end{aligned} \tag{A.2}$$

Analogously, we may get:

$$\begin{aligned} & Cov\left(P_{1,t-k-1}, P_{2,t-1}\right) = \phi_2^k \tfrac{\alpha}{1-\alpha} Var\left(P_{2,t}\right), \\ & Cov\left(P_{2,t-1}, P_{2,t-k-1}\right) = \phi_2^k \mathrm{Var}\left(\mathrm{P_{2,t}}\right). \end{aligned} \tag{A.3}$$

2. Since we know the equation(23), the proof of equation (24) can be done bellow:

$$\begin{aligned} & Cov\left(D_{1,t-1}, D_{1,t-k-1}\right) \\ & = Cov\left(\begin{array}{l} \mu_1 - \rho_{11}P_{1,t-1} + \rho_{12}P_{2,t-1} \\ +\eta_{1,t-1}, \mu_1 - \rho_{11}P_{1,t-k-1} \\ +\rho_{12}P_{2,t-k-1} + \eta_{1,t-k-1} \end{array}\right) \\ & = \rho_{11}^2\phi_1^k Var\left(p_{1,t}\right) + \rho_{12}^2\phi_2^k Var\left(p_{2,t}\right) \\ & - \rho_{11}\rho_{12}\left(\phi_1^k + \phi_2^k\right)\frac{\alpha}{1-\alpha} Var\left(p_{2,t}\right) \end{aligned} \tag{A.4}$$

$$\begin{aligned} & Cov\left(D_{1,t-1}, D_{2,t-1}\right) \\ & = Cov\left(\begin{array}{l} \mu_1 - \rho_{11}P_{1,t} + \rho_{12}P_{2,t} + \eta_{1,t}, \\ \mu_2 - \rho_{21}P_{2,t} + \rho_{22}P_{1,t} + \eta_{2,t} \end{array}\right) \\ & = -\rho_{11}\rho_{22}Var\left(P_{1,t}\right) - \rho_{12}\rho_{21}Var\left(P_{2,t}\right) \\ & +(\rho_{11}\rho_{21} + \rho_{12}\rho_{22})\tfrac{\alpha}{1-\alpha}Var\left(P_{2,t}\right) + \gamma_{12}^2 \end{aligned} \tag{A.5}$$

$$\begin{aligned} & Cov\left(D_{1,t-1}, D_{2,t-k-1}\right) \\ & = Cov\left(\begin{array}{l} \mu_1 - \rho_{11}P_{1,t-1} + \rho_{12}P_{2,t-1} \\ +\eta_{1,t-1}, \mu_2 - \rho_{21}P_{2,t-k-1} \\ +\rho_{22}P_{1,t-k-1} + \eta_{2,t-k-1} \end{array}\right) \\ & = -\rho_{11}\rho_{22}\phi_1^k Var\left(P_{1,t}\right) - \rho_{12}\rho_{21}\phi_2^k Var\left(P_{2,t}\right) \\ & + \left(\rho_{11}\rho_{21}\phi_1^k + \rho_{12}\rho_{22}\phi_2^k\right)\frac{\alpha}{1-\alpha}Var\left(P_{2,t}\right) \end{aligned} \tag{A.6}$$

$$
\begin{aligned}
&Cov\left(D_{1,t-k-1}, D_{2,t-1}\right)\\
&= Cov\begin{pmatrix} \mu_1 - \rho_{11}P_{1,t-k-1} + \rho_{12}P_{2,t-k-1} \\ +\eta_{1,t-k-1}, \mu_2 - \rho_{21}P_{2,t-1} \\ +\rho_{22}P_{1,t-1} + \eta_{2,t-1} \end{pmatrix}\\
&= \left[\left(\rho_{11}\rho_{21}\phi_2^k + \rho_{12}\rho_{22}\phi_1^k\right)\frac{\alpha}{1-\alpha} - \rho_{12}\rho_{21}\phi_2^k\right]\\
&Var\left(P_{2,t}\right) - \rho_{11}\rho_{22}\phi_1^k Var\left(P_{1,t}\right)
\end{aligned} \tag{A.7}
$$

$$
\begin{aligned}
&Cov\left(D_{1,t-k-1}, D_{2,t-k-1}\right)\\
&= Cov\begin{pmatrix} \mu_1 - \rho_{11}P_{1t-k-1} + \rho_{12}P_{2t-k-1} \\ +\eta_{1,t-k-1}, \mu_2 - \rho_{21}P_{2t-k-1} \\ +\rho_{22}P_{1t-k-1} + \eta_{2,t-k-1} \end{pmatrix}\\
&= \left(\rho_{11}\rho_{21} + \rho_{12}\rho_{22}\right)\frac{\alpha}{1-\alpha}Var\left(P_{2,t}\right)\\
&- \rho_{11}\rho_{22}Var\left(P_{1,t}\right) - \rho_{12}\rho_{21}Var\left(P_{2,t}\right) + \gamma_{12}^2
\end{aligned} \tag{A.8}
$$

$$
\begin{aligned}
&Cov\left(D_{2,t-1}, D_{2,t-k-1}\right)\\
&= Cov\begin{pmatrix} \mu_2 - \rho_{21}P_{2t-1} + \rho_{22}P_{1t-1} \\ +\eta_{2,t-1}, \mu_2 - \rho_{21}P_{2t-k-1} \\ +\rho_{22}P_{1t-k-1} + \eta_{2,t-k-1} \end{pmatrix} \quad \text{(A.9)}\\
&= \left[\rho_{21}^2\phi_2^k - \left(\rho_{21}\rho_{22}\phi_2^k\right)\frac{\alpha}{1-\alpha}\right]\\
&Var\left(p_{2,t}\right) + \rho_{22}^2\phi_1^k Var\left(p_{1,t}\right)
\end{aligned}
$$

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