

Fuzzy Hammerstein Model Based States Space Identification Approach of Nonlinear Dynamics Systems

JÉSSICA A. SANTOS

Federal University of Maranhão
Department of Electrical Engineering
Av. dos Portugueses, S/N, Bacanga
São Luís - MA, BRAZIL
jessica.almeida.santos@hotmail.com

GINALBER L. O. SERRA

Federal Institute of Maranhão
Department of Electrotechnical and Electronic
Av. Getúlio Vargas, 04, Monte Castelo
São Luís - MA, BRAZIL
ginalber@ifma.edu.br

Abstract: This paper presents a novel methodology for evolving fuzzy identification of nonlinear systems in state space based on Hammerstein models. The nonlinear static characteristic is approximated by an evolving Takagi-Sugeno fuzzy model and the linear dynamics by a state space model. The recursive estimation of the linear model in state space is performed based on the system Markov parameters applied to the algorithm of minimum realization ERA. Computational results illustrate the effectiveness of the proposed method in the online identification of nonlinear systems.

Key-Words: Evolving fuzzy systems, Hammerstein models, state space systems, Markov parameters.

1 Introduction

Modern systems are each day more complex and demand more accurate of the models. New development of modeling techniques increase its effort to incorporate issues like nonlinearities, uncertainties and temporal variation. However, there is not standard and unified mathematical method to describe the complex behavior of processes [1] [2] [3].

Several methods to identify systems and their nonlinearities are currently studied [6] [13]. Block-oriented models are widely used to describe nonlinear systems, and are successfully applied to various problems [5] [7]. Hammerstein model, a typical block oriented model, consists of a static nonlinear block cascading with a linear dynamic block [8]. This type of model has been proven as an efficient tool in the modeling of biological, chemical and electrical nonlinear systems [9].

Takagi-Sugeno (TS) fuzzy systems have been shown to be efficient in modeling the complexities of many systems. In [4], a new modeling methodology based on evolving Takagi-Sugeno neuro-fuzzy network used to forecast seasonal time series, is proposed. In [10], an algorithm for obtaining TS fuzzy models based on ant colony optimization algorithm, is presented. In [11], an extension of the FLEXFIS algorithm is proposed for the construction of more generalized, but less complex, fuzzy models, is proposed

A wide variety of methods have been developed to identify Hammerstein model. In [12] a methodology for identification of Hammerstein models in state

space is presented. The static nonlinearity is modeled by polynomial parameterization. This approach requires *a priori* knowledge of the polynomial order. Moreover, the signal applied for identification is difficult to reproduce experimentally. In [13], a batch identification method of Hammerstein models based on correlation analysis, is proposed. This method uses a neuro-fuzzy network to identify the nonlinear static characteristic and the linear part is approximated by an autoregressive model. For the implementation of this technique is also necessary to apply two separate special signals.

This paper presents a novel methodology for evolving identification of nonlinear systems in state space based in Hammerstein models. The main contributions of the proposed methodology as the following:

- Is does not depend on the use of specific signals for static nonlinearity estimation and linear dynamics as in others methodologies from the literature;
- The application of an evolving Takagi-Sugeno fuzzy system. These models are able to adapt their structure online according to data;
- The new evolving fuzzy modeling algorithm produces models in state space of minimum order, based on system Markov parameters.

2 State Space Hammerstein Modeling: Formulation

The state space Hammerstein model, as shown in Fig. 1, consist of the cascade connection of two blocks: the first describes a nonlinear static characteristic and the second describes the state space linear dynamics [18].

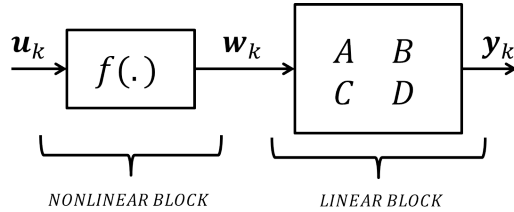


Figure 1: Hammerstein model in state space.

This type of block-oriented model is formulated as follow:

$$\mathbf{w}_k = f(\mathbf{u}_k) \quad (1)$$

$$\begin{aligned} \mathbf{x}_{k+1} &= A\mathbf{x}_k + B\mathbf{w}_k \\ \mathbf{y}_k &= C\mathbf{x}_k + D\mathbf{w}_k \end{aligned} \quad (2)$$

where $\mathbf{u}_k \in \mathbb{R}^m$ is the input vector, $\mathbf{y}_k \in \mathbb{R}^p$ is the output vector and $\mathbf{x}_k \in \mathbb{R}^n$ is the state vector. $\mathbf{w}_k \in \mathbb{R}^m$ is the vector that represents the unknown internal variable and $f(\mathbf{u}_k)$ is the static nonlinearity of the Hammerstein model. The parameters of the linear dynamic block in state space are described by matrices $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{p \times n}$ and $D \in \mathbb{R}^{p \times m}$.

2.1 Static Nonlinearity Parameterization

The nonlinear static function, $f(\mathbf{u}_k)$, is approximated by a TS fuzzy inference system, characterized by a set of fuzzy rules of type IF-THEN, whose structure is given by:

$$\begin{aligned} R^i : & \text{IF } \tilde{x}_{1,k} \text{ IS } Q_{j|\tilde{x}_{1,k}}^i \text{ AND...} \\ & \dots \text{ AND } \tilde{x}_{l,k} \text{ IS } Q_{j|\tilde{x}_{l,k}}^i \\ \text{THEN } & \tilde{\mathbf{w}}_k^i = (E^i(I - G^i)^{-1}H^i + F^i)\mathbf{u}_k \end{aligned} \quad (3)$$

with $i = [1, L]$, where L is the number of rules, $\tilde{\mathbf{x}}_k = [\tilde{x}_{1,k} \ \tilde{x}_{2,k} \ \dots \ \tilde{x}_{l,k}] \in \mathbb{R}^l$ is the vector that contains the antecedent linguistic variables at the k -th sampling time. The linguistic variable $\tilde{x}_{t,k}$, with $t = [1, l]$, belongs to the fuzzy set $Q_{j|\tilde{x}_{t,k}}^i$ with a

truth value $\mu_{Q_{j|\tilde{x}_{t,k}}^i}^i$ defined by a membership function $\mu_{\tilde{x}_{t,k}}^i : \mathbb{R} \rightarrow [0, 1]$. $G^i \in \mathbb{R}^{s \times s}$, $H^i \in \mathbb{R}^{s \times m}$, $E^i \in \mathbb{R}^{q \times s}$ and $F^i \in \mathbb{R}^{p \times m}$ are the parameters of the i -th local linear model of the consequent, $\mathbf{u}_k \in \mathbb{R}^m$ is the input vector and the $\tilde{\mathbf{w}}_k^i \in \mathbb{R}^m$ is the output vector of the i -th local linear model.

The truth value of a certain point for the i -th fuzzy set in the j -th universe of discourse can be described by a gaussian, given by:

$$\mu_{Q_{j|\tilde{x}_{j,k}}^i}^i(\tilde{x}_{j,k}) = e^{-\frac{(\tilde{x}_{j,k} - \tilde{x}_{j,k}^{i*})^2}{2(\sigma_{j,k}^i)^2}} \quad (4)$$

where $\tilde{x}_{j,k}^{i*}$ is the focal point that represents the i -th fuzzy set and $\sigma_{j,k}^i$ is the gaussian variance of the j -th input variable of the i -th rule.

The truth value h^i for the complete rule i is given by:

$$h^i(\tilde{\mathbf{x}}_k) = \mu_{Q_{j|\tilde{x}_{1,k}}^i}^i \star \mu_{Q_{j|\tilde{x}_{2,k}}^i}^i \star \dots \star \mu_{Q_{j|\tilde{x}_{l,k}}^i}^i \quad (5)$$

where (\star) is the operator that represents the product t -norm. The normalized truth value for the rule i is defined by

$$\gamma^i(\tilde{\mathbf{x}}_k) = \frac{h^i(\tilde{\mathbf{x}}_k)}{\sum_{r=1}^L h^r(\tilde{\mathbf{x}}_k)}, \quad \sum_{i=1}^L \gamma^i(\tilde{\mathbf{x}}_k) = 1 \quad (6)$$

The response of the TS fuzzy model consists in a weighted sum of the consequent functions, i.e., a convex combination of the local functions $\tilde{\mathbf{w}}_k^i$

$$\mathbf{w}_k = \sum_{i=1}^L \gamma^i(\tilde{\mathbf{x}}_k) \tilde{\mathbf{w}}_k^i \quad (7)$$

2.1.1 Evolving Approach for Antecedent Estimation

The evolving algorithm applied in the fuzzy rules estimation is based on the FLEXFIS algorithm [14]. The recursive update of the *clusters* centers, is defined by:

$$\mathbf{c}_v^{new} = \mathbf{c}_v^{old} + \eta_{v,k}(\tilde{\mathbf{x}}_k - \mathbf{c}_v^{old}) \quad (8)$$

where \mathbf{c}_v , called winning cluster, is the cluster closest to the sample $\tilde{\mathbf{x}}_k$ according to Euclidean norm, $\eta_{v,k}$ is the learning gain at the k -th sampling time. The recursive computation of $\eta_{v,k}$, is given by:

$$\eta_{v,k} = \frac{g_s}{S_{v,k}} \quad (9)$$

where the $S_{v,k}$ is the support (number of data points belonging to the winning cluster, at the k -th sampling time) and g_s is a constant parameter. A new rule is created when the following condition is true:

$$\|\tilde{\mathbf{x}}_k - \mathbf{c}_v\|_A \geq \rho \mathbf{E} \tilde{\mathbf{x}}_k \text{ is not faulty} \quad (10)$$

where A represents the Euclidean norm. The monitoring parameter ρ , that decides between creating of a new rule and updating an existing rule, is given by:

$$\rho = fac * \frac{\sqrt{l}}{\sqrt{2}} \quad (11)$$

where $fac \in [0, 1]$ and l is the $\tilde{\mathbf{x}}_k$ vector size. The parameter ρ also ensures that a cluster center is never initialized too far from a new data clustering.

To avoid clusters being kept in regions of low representativity and minimizing the effects of outliers, the following condition is used to eliminate rules with low support value:

$$\begin{aligned} \text{IF } S_k^i < 5 \text{ AND } k > I^{i*} + 15 \\ \text{THEN } L = L - 1 \end{aligned} \quad (12)$$

where $i = [1, L]$, S_k^i is the support of the i -th rule in the k -th sampling time, I^{i*} is the creation instant of rule. The influence zone of each cluster is defined by the gaussian variance in (4). The recursive updating of the variance, is given by:

$$\sigma_{v,j}^{new} = \sqrt{\frac{(S_{v,k}-1)(\sigma_{v,j}^{old})^2 + S_{v,k}(\Delta c_{v,j})^2 + (c_{v,j}^{new} - \tilde{x}_j)^2}{S_{v,k}}} \quad (13)$$

where $j = [1, l]$, l is the $\tilde{\mathbf{x}}_k$ vector size, $S_{v,k}$ is the support of the winning cluster, $\Delta c_{v,j}$ is the distance between the last position of the winning cluster and the current position in the j -th dimension.

2.1.2 Recursive Approach Consequent Estimation

The antecedent evolving estimation causes continuous changes in the model structure according to the arrival of the data. Thus, a recursive strategy of consequent parameters estimation is implemented.

A. Recursive Estimation of the System Fuzzy Markov Parameters

Consider a TS fuzzy system in state space, given by:

$$\begin{aligned} R^i : \text{IF } \tilde{x}_{1,k} \text{ IS } Q_{j|\tilde{x}_{1,k}}^i \text{ AND...} \\ \dots \text{ AND } \tilde{x}_{l,k} \text{ IS } Q_{j|\tilde{x}_{l,k}}^i \\ \text{THEN } \tilde{\mathbf{y}}_k^i = \begin{cases} \mathbf{z}_{k+1}^i = G^i \mathbf{z}_k^i + H^i \mathbf{u}_k \\ \tilde{\mathbf{y}}_k^i = E^i \mathbf{z}_k^i + F^i \mathbf{u}_k \end{cases} \end{aligned} \quad (14)$$

where $\mathbf{z}_k^i \in \mathbb{R}^s$ is the state vector. The recursive estimation of G^i , H^i , E^i and F^i is performed by the minimum realization fuzzy algorithm based on the system fuzzy Markov parameters for each rule [16].

It is possible to obtain the system fuzzy Markov parameters from the observer Markov parameters [19]. By inserting a state observer in the consequent of (14) [16], the submodel can be written as:

$$\begin{aligned} \mathbf{z}_{k+1}^i &= \bar{G}^i \mathbf{z}_k^i + \bar{H}^i \mathbf{v}_k \\ \tilde{\mathbf{y}}_k^i &= E^i \mathbf{z}_k^i + F^i \mathbf{v}_k \end{aligned} \quad (15)$$

where

$$\bar{G}^i = G^i + \Omega^i E^i \quad (16)$$

$$\bar{H}^i = [H^i + \Omega^i F^i, -\Omega^i] \quad (17)$$

$$\mathbf{v}_k^i = [\mathbf{u}_k \quad \tilde{\mathbf{y}}_k^i]^T \quad (18)$$

and $\Omega^i \in \mathbb{R}^{p \times m}$ is the observer gain of i -th local linear model.

Solving the equation (15) in function of matrices \bar{G}^i , \bar{H}^i , E^i and F^i and of the vectors \mathbf{u}_k and \mathbf{y}_k , where k indicates that are used samples from instant 0 until k , and $\mathbf{z}_0^i = 0$, obtain the following relation:

$$\tilde{\mathbf{y}}_k^i = \sum_{j=1}^k E^i (\bar{G}^i)^{j-1} \bar{H}^i \mathbf{v}_{k-j}^i + F^i \mathbf{u}_k \quad (19)$$

Because the state observer presence, it has $(\bar{G}^i)^{q_f} \approx 0$. Thus, (19) can be rewritten as:

$$\tilde{\mathbf{y}}_k^i = \sum_{j=1}^{q_f} \bar{Y}_{k,j}^i \mathbf{v}_{k-j}^i + F^i \mathbf{u}_k \quad (20)$$

where $\bar{Y}_{k,j}^i = E^i (\bar{G}^i)^{j-1} \bar{H}^i$ is the j -th observer Markov parameter and $\bar{Y}_{k,0}^i = F^i$.

Rewritten (20) as a matrix operation, results:

$$\tilde{\mathbf{Y}}_k^i = \Theta_k^i \Phi_k^i \quad (21)$$

where $\Theta_k^i = [F_k^i \quad \bar{Y}_{k,1}^i \quad \dots \quad \bar{Y}_{k,q_f}^i]$ is the observer Markov parameters vector of the i -th rule. The regressors matrix is given by:

$$\Phi_k^i = \begin{pmatrix} (\phi^i(1))^T \sqrt{\gamma^i(\tilde{\mathbf{x}}(1))} \\ (\phi^i(2))^T \sqrt{\gamma^i(\tilde{\mathbf{x}}(2))} \\ \vdots \\ (\phi^i(N))^T \sqrt{\gamma^i(\tilde{\mathbf{x}}(N))} \end{pmatrix} \quad (22)$$

with $\phi_k^i = [\mathbf{u}_k^T \quad \mathbf{v}_{k-1}^T \quad \cdots \quad \mathbf{v}_{k-q_f}^T]^T$, and the vector output is given by:

$$\tilde{\mathbf{Y}}_k^i = \begin{pmatrix} \tilde{\mathbf{y}}^i(1) \sqrt{\gamma^i(\tilde{\mathbf{x}}(1))} \\ \tilde{\mathbf{y}}^i(2) \sqrt{\gamma^i(\tilde{\mathbf{x}}(2))} \\ \vdots \\ \tilde{\mathbf{y}}^i(N) \sqrt{\gamma^i(\tilde{\mathbf{x}}(N))} \end{pmatrix} \quad (23)$$

where N is the number of samples until the instant k .

Applying the local learning approach [15] and solving the equation (21) by the batch weighted least squares to generate the initialization for recursive approach, results:

$$(\Theta_k^i)^T = (\Phi^i(\Phi^i)^T)^{-1}(\Phi^i)(\tilde{\mathbf{Y}}_k^i)^T \quad (24)$$

The recursive estimation of the observer Markov parameters, based in input-output data, is done by the local learning approach of the recursive fuzzily weighted least squares algorithm [15], given by:

$$(\Theta_{k+1}^i)^T = (\Theta_k^i)^T + (\Gamma_{k+1})^T((\mathbf{y}_{k+1})^T - (\phi_{k+1}^i)(\Theta_k^i)^T) \quad (25)$$

$$(\Gamma_{k+1})^T = \frac{(P_k^i)^T (\phi_{k+1}^i)^T}{\frac{1}{\gamma^i(\tilde{\mathbf{x}}_{k+1})} + (\phi_{k+1}^i)(P_k^i)^T (\phi_{k+1}^i)^T} \quad (26)$$

$$(P_{k+1}^i)^T = (I - (\Gamma_{k+1})^T (\phi_{k+1}^i))(P_k^i)^T \quad (27)$$

where P_k^i is the covariance matrix.

Rearranging the r -th observer Markov parameter of the i -th rule in the k -th sampling time [19], results:

$$\bar{Y}_{k,r}^i = [\bar{Y}_{k,r}^{i,1}, -\bar{Y}_{k,r}^{i,2}] \quad (28)$$

The system Markov parameters can be recursively estimated using the following equations [16]:

$$Y_{k,0}^i = \bar{Y}_{k,0}^i \quad (29)$$

$$Y_{k,r}^i = \bar{Y}_{k,r}^{i,1} - \sum_{j=1}^r \bar{Y}_{k,j}^{i,2} Y_{k,r-1}^i, \quad r = [1, q_f] \quad (30)$$

$$Y_{k,r}^i = - \sum_{j=1}^{q_f} \bar{Y}_{k,j}^{i,2} Y_{k-j}^i, \quad r > q_f \quad (31)$$

where $Y_{k,r}^i$ is the r -th system fuzzy Markov parameter of the i -th rule in the k -th sampling time.

B. Recursive Estimation of the Consequent Submodels Matrices.

Since the observer Markov parameters are estimated recursively, the system fuzzy Markov parameters are also computed in (29), (30) and (31). The algorithm to find the matrices G^i , H^i , E^i and F^i begin with the construction of de fuzzy Hankel matrices for each local model [17], given by

$$H_0^i = \begin{bmatrix} Y_1^i & Y_2^i & \cdots & Y_{\beta_f}^i \\ Y_2^i & Y_3^i & \cdots & Y_{\beta_f+1}^i \\ \vdots & \vdots & \ddots & \vdots \\ Y_{\alpha_f}^i & Y_{\alpha_f+1}^i & \cdots & Y_{\alpha_f+\beta_f-1}^i \end{bmatrix} \quad (32)$$

and

$$H_1^i = \begin{bmatrix} Y_2^i & Y_3^i & \cdots & Y_{\beta_f+1}^i \\ Y_3^i & Y_4^i & \cdots & Y_{\beta_f+2}^i \\ \vdots & \vdots & \ddots & \vdots \\ Y_{\alpha_f+1}^i & Y_{\alpha_f+2}^i & \cdots & Y_{\alpha_f+\beta_f}^i \end{bmatrix} \quad (33)$$

where α_f and β_f are integers restricted by the condition $\alpha_f m \leq \beta_f p$, m and p is the number of inputs and outputs, respectively.

The order of submodels is defined by the rank of their respective Hankel matrices, that is, the significant singular values number of H_0^i . The next step to obtain of the parameters is the application of the singular value decomposition (SVD) in (32), given by:

$$H_0^i = R^i \Sigma^i (S^i)^T \quad (34)$$

where

$$\Sigma^i = \begin{bmatrix} \Sigma_s^i & 0 \\ 0 & 0 \end{bmatrix} \quad (35)$$

and

$$\Sigma_s^i = \text{diag} [\delta_1^i, \delta_2^i, \dots, \delta_s^i] \quad (36)$$

is a diagonal matrix composed of the most significant singular values. The matrices R_s^i and S_s^i are formed by the first s columns of the matrices R and S , respectively. Thus, the Hankel matrix H_0^i can be rewritten as:

$$H_0^i = R_s^i \Sigma_s^i (S_s^i)^T \quad (37)$$

The local submodels parameters are obtained as follows:

$$G^i = (\Sigma_s^i)^{-1/2} (R_s^i)^T (H_1^i) (S_s^i) (\Sigma_s^i)^{-1/2} \quad (38)$$

$$H^i = \text{first } s \text{ collumns of } (\Sigma_s^i)^{1/2} (S_s^i)^T \quad (39)$$

$$E^i = \text{first } s \text{ rows of } (R_s^i) (\Sigma_s^i)^{1/2} \quad (40)$$

$$F^i = Y_0^i \quad (41)$$

Finally, the approximation of the static nonlinearity is given by the following fuzzy model resulted from the evolving clustering of experimental data:

$$\mathbf{w}_k = \sum_{i=1}^L \gamma^i(\tilde{\mathbf{x}}_k) (E^i (I - G^i) H^i + F^i) \mathbf{u}_k \quad (42)$$

2.2 Linear Dynamics Parameterization

Since the \mathbf{w}_k is estimated in the parameters of the linear dynamic model can be estimated recursively as described in the sequel.

A. Recursive Estimation of the Linear System Markov Parameters

Considering the model presented in (2) and adding a state observer, results:

$$\begin{aligned} \tilde{\mathbf{x}}_{k+1} &= \bar{A} \tilde{\mathbf{x}}_k + \bar{B}^i \mathbf{w}_k \\ \mathbf{y}_k &= C \tilde{\mathbf{x}}_k + D \mathbf{w}_k \end{aligned} \quad (43)$$

where

$$\bar{A} = A + MC \quad (44)$$

$$\bar{B} = [B + MD, -M] \quad (45)$$

$$\tau_k = [\mathbf{w}_k \quad \mathbf{y}_k]^T \quad (46)$$

and $M \in \mathbb{R}^{p \times m}$ is the observer gain.

The solving of equation (43) in terms of matrices A , B , C and D , and of the vectors \mathbf{w}_k and \mathbf{y}_k , where k indicates that are used samples from instant 0 until k , and $\mathbf{x}_0 = 0$, results:

$$\mathbf{y}_k = \sum_{j=1}^k C(\bar{A})^{j-1} \bar{B} \tau_{k-j} + D \mathbf{w}_k \quad (47)$$

The presence of state observer causes $(\bar{A})^{q_l} \approx 0$, where q_l is a integer. Thus, (47) can be rewritten as:

$$\mathbf{y}_k = \sum_{j=1}^{q_l} \bar{\psi}_{k,j} \tau_{k-j} + D \mathbf{w}_k \quad (48)$$

where $\bar{\psi}_{k,j} = C(\bar{A})^{j-1} \bar{B}$ is the j -th observer Markov parameter and $\bar{\psi}_{k,0} = D$.

Writing (20) in the matrix form, results:

$$\Upsilon_k = \bar{\Psi}_k W_k \quad (49)$$

where, $\Upsilon_k = [\mathbf{y}_{q_l+1} \quad \mathbf{y}_{q_l+2} \quad \dots \quad \mathbf{y}_k]$, $\bar{\Psi}_k = [D_k \quad \bar{\psi}_{k,1} \quad \dots \quad \bar{\psi}_{k,q_l}]$ is the observer Markov parameters vector and W_k is the regressors matrix, given by:

$$W_k = \begin{bmatrix} \mathbf{w}_{q_l+1} & \mathbf{w}_{q_l+2} & \dots & \mathbf{w}_{k-1} \\ \tau_{q_l} & \tau_{q_l+1} & \dots & \tau_{k-2} \\ \vdots & \vdots & \ddots & \vdots \\ \tau_1 & \tau_2 & \dots & \tau_{k-q_l-1} \end{bmatrix} \quad (50)$$

The initialization of recursive approach is done by least square, as follows [19]:

$$\bar{\Psi}_k = \Upsilon_k W_k^T [W_k W_k^T]^{-1} \quad (51)$$

The recursive estimation of the observer Markov parameters, is done applying the recursive least square [16], as follow:

$$\tilde{Z} = \frac{\pi_{k+1}^T \tilde{P}_k}{\lambda_l + \pi_{k-1}^T \tilde{P}_k \pi_{k+1}} \quad (52)$$

$$\bar{\Psi}_{k+1} = \bar{\Psi}_k + [\mathbf{y}_{k+1} - \bar{\Psi}_k \pi_{k+1}] \tilde{Z} \quad (53)$$

$$\tilde{P}_{k+1} = \lambda_l^{-1} \tilde{P}_k [I - \pi_{k+1} \tilde{Z}] \quad (54)$$

where \tilde{Z} is a gain vector, \tilde{P}_k is the covariance matrix and $\pi_k = [\mathbf{w}_k^T \quad \tau_{k-1}^T \quad \dots \quad \tau_{k-q_l}^T]^T$ is a vector regressors.

Thus, with the recursive observer Markov parameters, the system Markov parameters are given by:

$$\psi_{k,0} = \bar{\psi}_{k,0} \quad (55)$$

$$\psi_{k,r} = \bar{\psi}_{k,r}^{(1)} - \sum_{j=1}^r \bar{\psi}_{k,j}^{(2)} \psi_{k,r-1}, \quad r = [1, q_l] \quad (56)$$

$$\psi_{k,r} = - \sum_{j=1}^{q_l} \bar{\psi}_{k,j}^{(2)} \psi_{k-j}, \quad r > q_l \quad (57)$$

where $\bar{\psi}_{k,j} = [\bar{\psi}_{k,j}^{(1)}, \bar{\psi}_{k,j}^{(2)}]$ [16] and $\psi_{k,r}$ is the r -th linear system Markov parameter in the k -th sampling time.

B. Recursive Estimation of the Linear Block Matrices.

The Hankel matrix to the linear dynamics, is given by:

$$\Pi_0 = \begin{bmatrix} \psi_1 & \psi_2 & \dots & \psi_{\beta_l} \\ \psi_2 & \psi_3 & \dots & \psi_{\beta_l+1} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{\alpha_l} & \psi_{\alpha_l+1} & \dots & \psi_{\alpha_l+\beta_l-1} \end{bmatrix} \quad (58)$$

$$\Pi_1 = \begin{bmatrix} \psi_2 & \psi_3 & \dots & \psi_{\beta_l+1} \\ \psi_3 & \psi_4 & \dots & \psi_{\beta_l+2} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{\alpha_l+1} & \psi_{\alpha_l+2} & \dots & \psi_{\alpha_l+\beta} \end{bmatrix} \quad (59)$$

where α_l and β_l are integers restricts by $\alpha_l m \leq \beta_l p$ [17].

The next step for linear system minimum realization is to apply SVD in (58), given by:

$$\Pi_0 = \Lambda \Delta \Xi^T \quad (60)$$

Choosing the n significant singular values of Π_0 , (60) is rewritten by:

$$\Pi_0 = \Lambda_n \Delta_n \Xi_n^T \quad (61)$$

where $\Delta_n \in \mathbb{R}^{n \times n}$ is a diagonal matrix and the matrices Λ_n and Ξ_n are the n first columns of Λ and Ξ , respectively.

Finally, the state matrices of the linear system are computed recursively, by:

$$A = (\Delta_n)^{-1/2} (\Lambda_n)^T (\Pi_1) (\Xi_n) (\Delta_n)^{-1/2} \quad (62)$$

$$B = \text{first } n \text{ collumns of } (\Delta_n)^{1/2} (\Xi_n)^T \quad (63)$$

$$C = \text{first } n \text{ rows of } (\Lambda_n) (\Delta_n)^{1/2} \quad (64)$$

$$D = \psi_0 \quad (65)$$

3 Computational Results

To demonstrate the efficiency of the proposed methodology, it is considered in a dynamic system with one input and one output, whose static nonlinearity is a complex discontinuous function. The benchmark is applied in [13] and given by:

$$w_k = \begin{cases} 2 \tanh(2u_k) & u_k \leq 1.5 \\ -2 \frac{e^{u_k} - 1}{e^{u_k} + 1} & u_k > 1.5 \end{cases} \quad (66)$$

$$y_k = 0.8y_{k-1} + 0.2w_{k-1} \quad (67)$$

where u_k is the input, w_k is the unknown intermediate signal and y_k is the output.

The criteria used to validate obtained model are the mean square error (MSE), given by:

$$MSE = \frac{1}{N} \sum_{k=1}^N (y_k - \hat{y}_k)^2 \quad (68)$$

where N is the number of samples, y_k and \hat{y}_k denote the real and estimated outputs, respectively, and the best fit (FIT), given by:

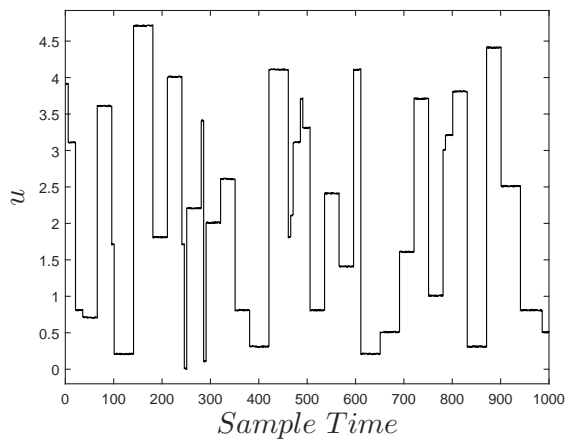
$$FIT = \left(1 - \frac{\|\mathbf{y} - \hat{\mathbf{y}}\|}{\|\mathbf{y} - \mathbf{y}_{mean}\|} \right) \times 100\% \quad (69)$$

where \mathbf{y} and $\hat{\mathbf{y}}$ denote the vectors containing the real and the estimated outputs, \mathbf{y}_{mean} is the vector containing the mean value of \mathbf{y} and $\|\cdot\|$ is the vector norm.

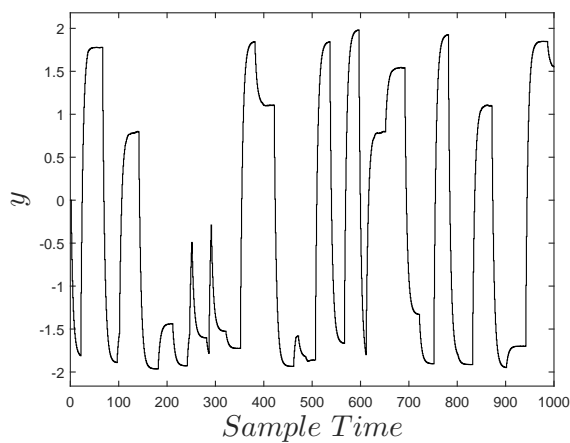
The performance of the proposed methodology is compared with two different methodologies: one approach based on polynomial approximation and another based on a neuro-fuzzy network [13]. For estimation of the evolving fuzzy Hammerstein model, a random multi-step input signal with uniform distribution between $[0, 5]$ is applied. The data set used for training is illustrated in Fig. 2. The first 200 samples of the training data set are used to initiate the algorithm.

3.1 Evolving Parametric Estimation of Static Nonlinearity

The antecedent parameters estimation of the evolving Takagi-Sugeno fuzzy model is computed as formulated in the section 2.1.1. The following design parameters are employed: $fac = 0.03$ (manages the number of clusters evolved), $g_s = 0.04$ (is the initial learning gain that guides the degree of shifting the centers and is responsible for a convergence of the cluster center). The clusters number evolution for the training data set applied is presented in the Fig. 3,



(a)



(b)

Figure 2: Training data set. (a) input data; (b) output data.

where it can be seen that the final model presents 23 rules.

The consequent recursive estimation is performed by applying the algorithm proposed in the section 2.1.2. The design parameters used in the training are the following: $q_f = 1$ (is the number of system Markov parameters), $\alpha_f = 3$ (is the number of rows of the Hankel matrix) and $\beta_f = 7$ (is the number of columns of the Hankel matrix).

The recursively estimated consequent parameters are presented in Fig. 4, Fig. 5, Fig. 6 and Fig. 7.

3.2 Recursive Parametric Estimation of Linear Dynamics

For recursive estimation of the linear model parameters, the design parameters are: $q_l = 2$ (number of linear system Markov parameters), $\alpha_l = 25$ (number of rows of the Hankel matrix of the linear dynamic

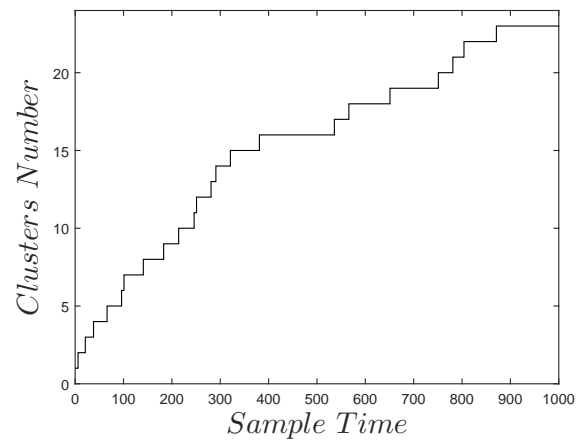


Figure 3: Rules number evolution during training (including the first 200 initialization samples of the algorithm).

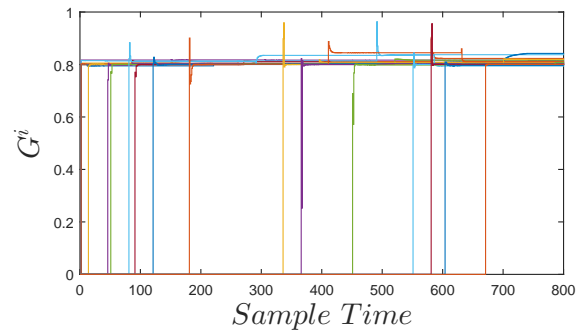


Figure 4: Consequent parameters recursive estimation for static nonlinearity modeling: parameters of matrix $G^i |_{i=1, \dots, 23}$.

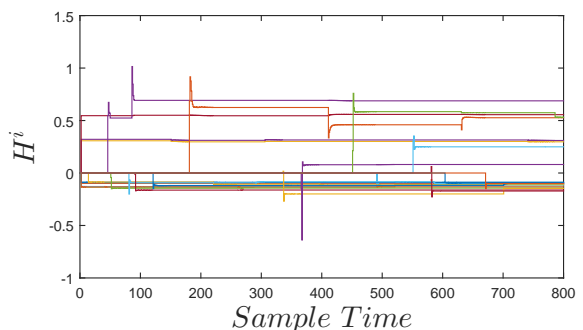


Figure 5: Consequent parameters recursive estimation for static nonlinearity modeling: parameters of matrix $H^i |_{i=1, \dots, 23}$.

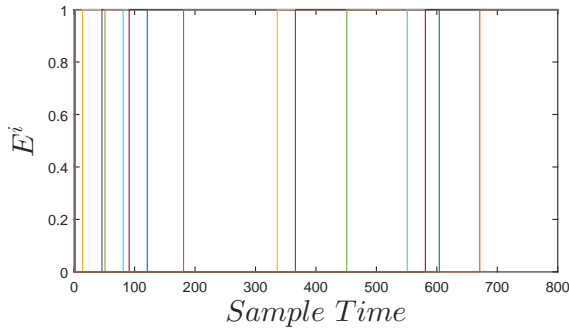


Figure 6: Consequent parameters recursive estimation for static nonlinearity modeling: parameters of matrix $E^i | i=1, \dots, 23$.

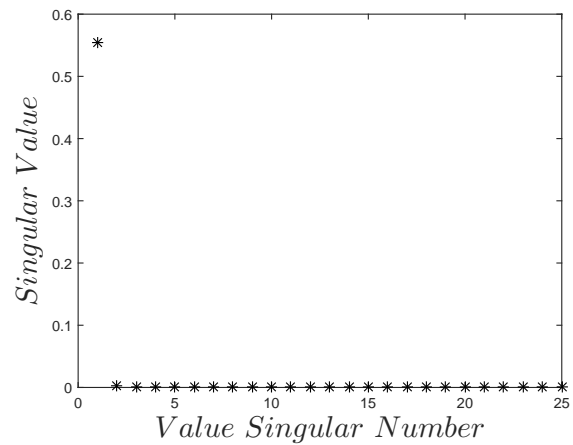


Figure 8: Singular values of the Hankel matrix of the linear dynamics.

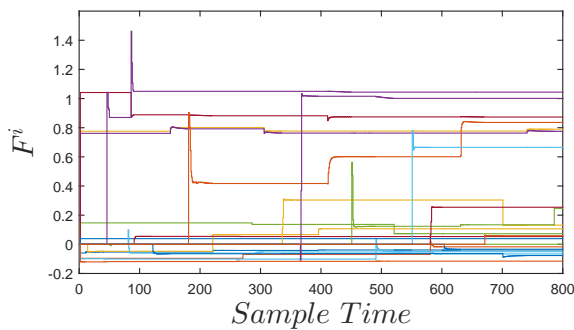


Figure 7: Consequent parameters recursive estimation for static nonlinearity modeling: parameters of matrix $F^i | i=1, \dots, 23$.

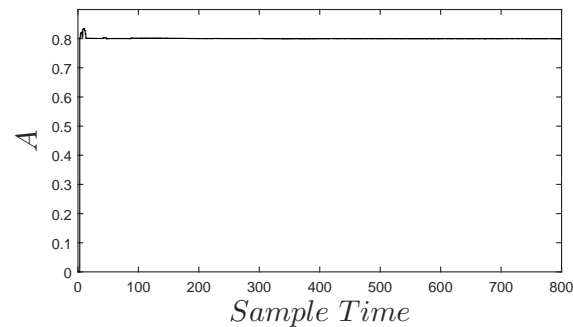


Figure 9: Parameters variation of the linear dynamic model: parameters variation of matrix A .

model), $\beta_l = 50$ (number of columns of the Hankel matrix of the linear dynamic model) and $\lambda_l = 0.98$ (forgetting factor for the recursive least squares). The singular values of the Hankel matrix estimated with the input-output data set, w_k and y_k , referring to the training samples applied in the algorithm initialization, is shown in Fig. 8. The analysis of the Fig. 8 results in a linear model of order $n = 1$.

The recursive estimation of the linear dynamic model parameters, as well as the convergence, the parametric variation of the matrices A , B , C and D , are presented in Fig. 9, Fig. 10, Fig. 11 and Fig. 12.

The validation results of evolving fuzzy Hammerstein model and its respective error is presented in Fig. 13. The validation of the static nonlinearity is illustrated in Fig. 14. The comparison between the MSEs for both training and validation data obtained by the proposed algorithm and by the best models of the other two methods in [13], is presented in Table 1. The FIT performance indice is also applied. During validation a $FIT = 98.8774\%$ is obtained for the

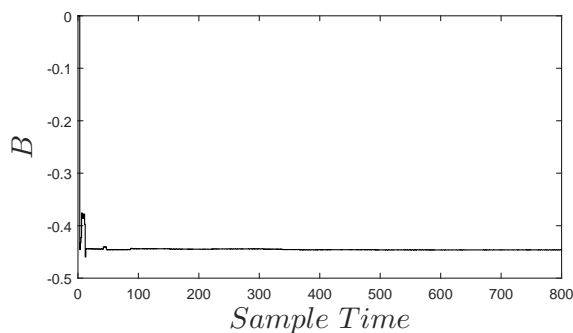


Figure 10: Parameters variation of the linear dynamic model: parameters variation of matrix B .

proposed methodology.

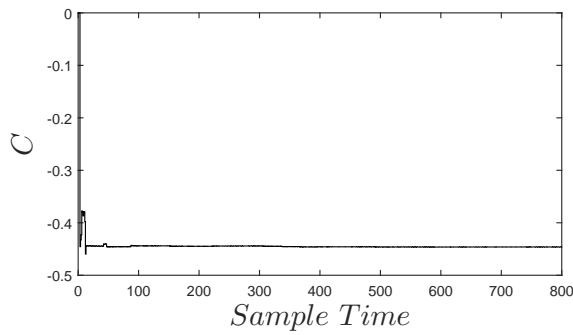


Figure 11: Parameters variation of the linear dynamic model: parameters variation of matrix C .

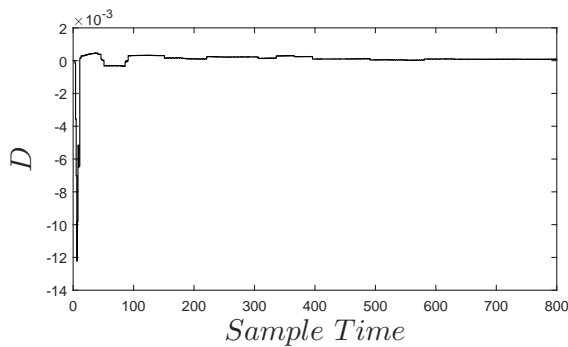


Figure 12: Parameters variation of the linear dynamic model: parameters variation of matrix D .

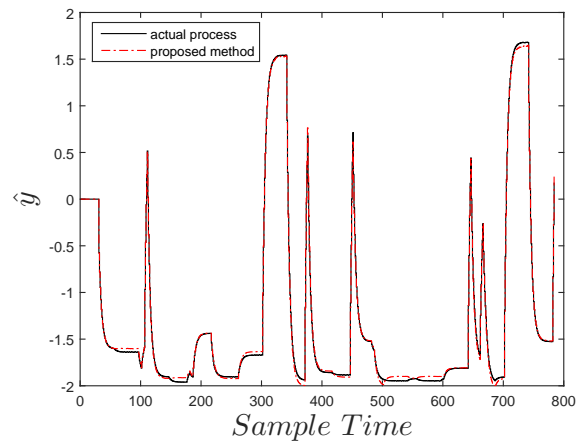
4 Conclusion

The novel methodology proposed in this paper is efficient in identification of systems with complex nonlinearities. The identification of the static nonlinear block by an evolving TS fuzzy model shows to be satisfactory for estimation of the static nonlinear characteristic in all operations regions of the nonlinear system, with a smaller number of rules. Moreover, the proposed method varies the antecedent and consequent structure according experimental data. The re-

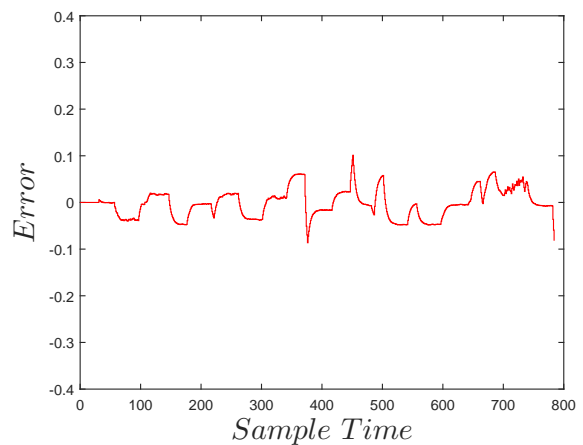
Table 1: Comparative analysis for the proposed methodology.

Model	MSE^*	MSE^+
Proposed	2.7088×10^{-4}	9.1415×10^{-4}
Neuro-fuzzy	5.2323×10^{-4}	9.9665×10^{-4}
Polynomial	4.3000×10^{-3}	7.6000×10^{-3}

MSE^* - training.
 MSE^+ - validation.



(a)



(b)

Figure 13: Validation result. (a) solid black line: actual process output y ; dashed dotted red line: estimated output by proposed method \hat{y} . (b) prediction error.

cursive identification of the linear block, through the ERA algorithm presented good performance. Thus, the results presented in this paper show that both nonlinear block and linear block approximation were considered satisfactory. The comparison analysis the with methods in [13] shows that the proposed methodology presents better results with lower number of rules.

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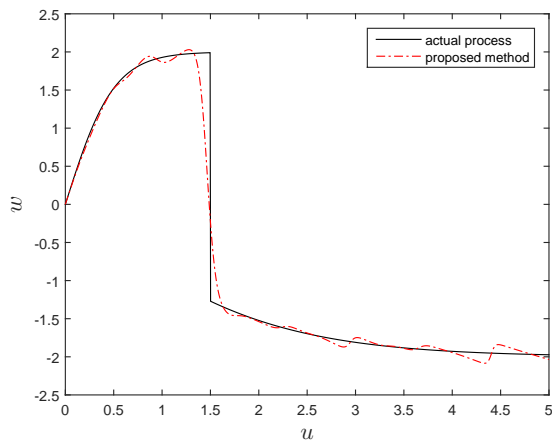


Figure 14: Nonlinear static function. Solid black line: actual process; dashed dotted red line: proposed method.

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