Friedmann-Robertson-Walker with Categorization of Scale Factor by Noether's Method

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Abstract: Friedmann-Robertson-Walker (FRW) models represent behavior of universe. Expansion of universe could be better understood by considering Robertson-Walker scale factor m and $m(t)$ where t represents time . Here FRW spacetime was classified by using Noether's guage symmetries.The behavior of spacetime was different for different kinds of universe i.e. closed, open, and flat. These types are defined according to curvature parameter c. For closed universe c=-1, for open universe c=1, and for flat universe c=0 . We obtained nontrivial symmetries for distinct values of scale factor . With the help of Noether equation and Perturbed Lagrangian a system of partial differential equations was acquired . For the universe as defined above, largest set and least set of Noether operators were obtained. Every operator has integral of motion.

Key–Words: FRW spacetime, Symmetry operators, Conserved quantities.

1 Introduction

FRW models are established on the basis of assumptions that the universe is homogeneous and isotropic in all epochs. The FRW metric is

$$ds^2 = dt^2 - m^2\left[\frac{dr^2}{1-cr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \quad (1)$$

Sharif et al. [1] used ansatz technique to solve Bianchi Type I and V and also classify scale factor. In 1918, Emmy Noether investigated a method to find the conserved quantities [2, 3] of the different problems. She gave the idea about connection between symmetries and conservation laws [3]. Each Noether symmetry correlates to a conserved quantity.

Solutions of many differential equations can be found by conservation laws.These laws can also be to determine the undetermined exponent in the similarity solution[4]. In the field of PDEs these conserved quantities are useful to handle errors in numerical calculations. In general relativity, solutions of field equations possess some symmetry [5].

Noether symmetries paly vital role in updated theories related to gravity. Kucukakca and Camci [6] worked in the field of $f(R)$ and scale factor by Noether's method. Jamil et a1. [7] explored the function $f(R)$ for model of Tachyon. Using NGS [8] technique Bianchi type I cosmology in generalized Saez-Ballester theory has also been studied. Hussain et al. [9] investigated Noether symmetries of flat universe FRW model with the help of gauge term in metric $f(R)$ gravity.

Noether symmetries are supportive in recuperate missing laws of conservation. [10]. In [11], for spherical symmetry a different form of Noether symmetries has been pr . Paledesentiathanasis et al. [12] Lie theory and Noether approach has been used to modified $f(R)$ gravity models.

2 Basic operators

$$ds^2 = g_{pq}dx^p dx^q, \quad (2)$$

then Y a vector filed for above metric is

$$Y = \eta(s, x^p)\frac{\partial}{\partial s} + \xi^q(s, x^p)\frac{\partial}{\partial x^q}\ . \quad (3)$$

The Lagrangian l for above metric is [13, 14]

$$l = \frac{1}{2}g_{pq}\dot{x}^p\dot{x}^q, \quad (4)$$

dot is the derivative w.r.t s. The Noether equation is given by:

$$Y^{[1]}(l) + lD_s(\eta) = D_s(G), \quad (5)$$

G=gauge term, $Y^{[1]}$=1st order prolongation and D_s=total derivative which is

$$D_s = \frac{\partial}{\partial s} + \dot{x}^p\frac{\partial}{\partial x^q}\ . \quad (6)$$

Conserved quantity related to Noether operator Y is given by

$$E = \eta l + (\xi^p - \eta\dot{x}^p)\frac{\partial l}{\partial \dot{x}^p} - G. \quad (7)$$

The Lagrangian for (1) is given by

$$l = \dot{t}^2 - \left(\frac{m^2\dot{r}^2}{1 - cr^2}\right) - m^2r^2\dot{\theta}^2 - m^2r^2\dot{\phi}^2\sin^2\theta. \quad (8)$$

vector field Y is

$$Y = \eta(s,t,r,\theta,\phi)\frac{\partial}{\partial s} + \xi^1(s,t,r,\theta,\phi)\frac{\partial}{\partial t} + \xi^2(s,t,r,\theta,\phi)\frac{\partial}{\partial r} + \xi^3(s,t,r,\theta,\phi)\frac{\partial}{\partial \theta} + \xi^4(s,t,r,\theta,\phi)\frac{\partial}{\partial \phi}.$$

3 Case 1

When universe is flat ($c = 0$) then Lagrangian is

$$l = \dot{t}^2 - m^2\dot{r}^2 - m^2r^2\dot{\theta}^2 - m^2r^2\dot{\phi}^2\sin^2\theta. \quad (9)$$

From Eq. (9) and Eq. (5) a system of linear PDEs was obtained

$$\eta_\phi = 0, \quad \eta_\theta = 0, \quad \eta_r = 0, \quad \eta_t = 0, \quad G_s = 0, \quad (10)$$

$$2\xi^1_s = G_t, \quad -2m^2\xi^2_s = G_r, \quad -2m^2r^2\xi^3_s = G_\theta, \quad (11)$$

$$-2m^2r^2\sin^2\theta\xi^4_s = G_\phi, \quad \xi^1_r - m^2\xi^2_t = 0, \quad (12)$$

$$\xi^1_\theta - m^2r^2\xi^3_t = 0, \quad \xi^1_\phi - m^2r^2\sin^2\theta\xi^4_t = 0, \quad (13)$$

$$m^2r^2\xi^3_r + m^2\xi^2_\theta = 0, \quad m^2\xi^2_\phi + m^2r^2\sin^2\theta\xi^4_r = 0, \quad (14)$$

$$m^2r^2\xi^3_\phi + m^2r^2\sin^2\theta\xi^4_\theta = 0, \quad 2\xi^1_t - \eta_s = 0, \quad (15)$$

$$m^2\eta_s - 2m^2\xi^2_r - 2mm_t\xi^1 = 0, \quad (16)$$

$$-2m^2r\xi^2 + m^2r^2\eta_s - 2m^2r^2\xi^3_\theta - 2mm_tr^2\xi^1 = 0, \quad (17)$$

$$m^2r^2\sin^2\theta\eta_s - 2m^2r^2\sin^2\theta\xi^4_\phi - 2m^2r\xi^2\sin^2\theta - 2m^2r^2\sin\theta\cos\theta\xi^3 - 2mm_tr^2\sin^2\theta\xi^1 = 0. \quad (18)$$

3.1 Least set of Noether operators

Least Noether symmetries are obtaines for arbitrary m

$$\mathbf{Y_{01}} = \frac{\partial}{\partial \mathbf{s}}, \quad \mathbf{Y_{02}} = \frac{\partial}{\partial \phi}$$

integral of motions related to $\mathbf{Y_{01}}$ and $\mathbf{Y_{02}}$ are:

$$\mathbf{E_{01}} = -\dot{\mathbf{t}}^2 + \mathbf{m^2\dot{r}^2} + \mathbf{m^2r^2}\dot{\theta} + \mathbf{m^2r^2}\sin^2\theta\dot{\phi}^2,$$

$$E_{02} = -2m^2r^2\sin^2\theta\dot{\phi}.$$

3.2 5 Noether operators

For some other values of m having five Noether operators presented in Table given below and $\mathbf{Y_{01}}$, $\mathbf{Y_{02}}$

$$\mathbf{Y_{03}} = -\cos\theta\frac{\partial}{\partial \mathbf{r}} + \frac{\sin\theta}{\mathbf{r}}\frac{\partial}{\partial \theta}$$

$$\mathbf{Y_{04}} = -\cos\phi\sin\theta\frac{\partial}{\partial \mathbf{r}} - \frac{\cos\phi\cos\theta}{\mathbf{r}}\frac{\partial}{\partial \theta} + \frac{\sin\phi}{\mathbf{r}\sin\theta}\frac{\partial}{\partial \phi}$$

and

$$\mathbf{Y_{05}} = \sin\phi\sin\theta\frac{\partial}{\partial \mathbf{r}} + \frac{\sin\phi\cos\theta}{\mathbf{r}}\frac{\partial}{\partial \theta} + \frac{\cos\phi}{\mathbf{r}\sin\theta}\frac{\partial}{\partial \phi}.$$

Integral of motions related to $\mathbf{Y_{03}}$, $\mathbf{Y_{04}}$ and $\mathbf{Y_{05}}$ are:

$$\mathbf{E_{03}} = -\mathbf{2rm^2}\sin\theta\dot{\theta} + \mathbf{2}\cos\theta\mathbf{m^2\dot{r}}$$

$$\mathbf{E_{04}} = \mathbf{2}\sin\theta\cos\phi\mathbf{m^2\dot{r}} + \mathbf{2r}\cos\theta\cos\phi\mathbf{m^2}\dot{\theta} - 2r\sin\theta\sin\phi m^2\dot{\phi},$$

$$\mathbf{E_{05}} = -\mathbf{2}\sin\theta\sin\phi\mathbf{m^2\dot{r}} - \mathbf{2r}\cos\theta\sin\phi\mathbf{m^2}\dot{\theta} - 2r\sin\theta\cos\phi m^2\dot{\phi}.$$

Table:

No.	$m(t)$
i	$\sin t, \cos t, \tan t$
ii	$\sinh t, \cosh t, \tanh t$
iii	$(\sin t)^{\frac{1}{2}}, (\cos t)^{\frac{1}{2}}, (\tan t)^{\frac{1}{2}}$
iv	$(\sinh t)^{\frac{1}{2}}, (\cosh t)^{\frac{1}{2}}, (\tanh t)^{\frac{1}{2}}$
v	$(\ln t)^{\frac{1}{2}}$
vi	$(kt+b)^{\frac{1}{2}}$
vii	$t + \frac{1}{t}$
$viii$	$k^2 b^2 \cosh(\frac{t}{b})^2$

3.3 6 Noether operators

some others forms of m having six Noether operators.

(1): $m(t) = t^n$
When n having even values with $n \geq 2$ and $\mathbf{Y_{01}} - \mathbf{Y_{05}}$ We have

$$Y_{06} = s\frac{\partial}{\partial s} + \frac{t}{2}\frac{\partial}{\partial t} - \frac{1}{2}(n-1)r\frac{\partial}{\partial r}$$

We have

$$E_{06} = -s\dot{t}^2 + sm^2\dot{r}^2 + sm^2r^2\dot{\theta}^2 + sm^2r^2\sin^2\theta\dot{\phi}\dot{\phi}$$
$$+t\dot{t} + (n-1)m^2r\dot{r}.$$

When n having odd values with $n \geq 3$ and $n = 3+2b$ and $\mathbf{Y_{01}} - \mathbf{Y_{05}}$ and

$$Y_{06} = s\frac{\partial}{\partial s} + \frac{t}{2}\frac{\partial}{\partial t} - (b+1)r\frac{\partial}{\partial r}.$$

We have

$$E_{06} = -s\dot{t}^2 + sm^2\dot{r}^2 + sm^2r^2\dot{\theta}^2 + sm^2r^2\sin^2\theta\dot{\phi}\dot{\phi}$$
$$+t\dot{t} + 2(b+1)a^2r\dot{r}.$$

(2): $m(t) = (t)^{\frac{n}{2}}$
When n possess odd values with $n \geq 1$ we have $\mathbf{Y_{01}} - \mathbf{Y_{05}}$ with

$$Y_{06} = s\frac{\partial}{\partial s} + \frac{1}{2}t\frac{\partial}{\partial t} - \frac{1}{4}(n-2)r\frac{\partial}{\partial r}$$

The integral of motion

$$E_{06} = -s\dot{t}^2 + sm^2\dot{r}^2 + sm^2r^2\dot{\theta}^2 + sm^2r^2\sin^2\theta\dot{\phi}\dot{\phi}$$
$$+t\dot{t} + \frac{n}{2} - m^2r\dot{r}$$

Here $n = 4, 8, 12, 16, ...$ taking n= $4+4a$ where $a = 0, 1, 2, 3.....$
we obtained:
$\mathbf{Y_{01}} - \mathbf{Y_{05}}$ and

$$Y_{06} = s\frac{\partial}{\partial s} + \frac{1}{2}t\frac{\partial}{\partial t} - \frac{(2a+1)r}{2}\frac{\partial}{\partial r}$$

We have

$$E_{06} = -s\dot{t}^2 + sm^2\dot{r}^2 + sm^2r^2\dot{\theta}^2 + sm^2r^2\sin^2\theta\dot{\phi}^2$$
$$+t\dot{t} + 2am^2r\dot{r} + m^2r\dot{r}$$

Now $n = 6, 10, 14, 18, 20, ...$ with n= $6+4a$ where $a = 0, 1, 2, 3, ...$
we have $\mathbf{Y_{01}} - \mathbf{Y_{05}}$ and

$$Y_{06} = s\frac{\partial}{\partial s} + \frac{1}{2}t\frac{\partial}{\partial t} - (ar+r)\frac{\partial}{\partial r}$$

we have

$$E_{06} = -s\dot{t}^2 + sm^2\dot{r}^2 + sm^2r^2\dot{\theta}^2 + sm^2r^2\sin^2\theta\dot{\phi}^2$$
$$+t\dot{t} + m^2ar\dot{r} + m^2r\dot{r}$$

3.4 7 Noether operators

Seven Noether operators are given below.

(1): $m(t)$=$\gamma t + \zeta$, γ and ζ are constants
Here $\mathbf{Y_{01}} - \mathbf{Y_{05}}$ and other operators are

$$Y_{06} = \frac{s^2}{2}\frac{\partial}{\partial s} + \frac{s}{2}\left(t + \frac{\zeta}{\gamma}\right)\frac{\partial}{\partial t}, \qquad (19)$$

$$Y_{07} = s\frac{\partial}{\partial s} + \frac{1}{2}\left(t + \frac{\zeta}{\gamma}\right)\frac{\partial}{\partial t}, \qquad (20)$$

$$G = \frac{1}{2\gamma}\Big[(k_1t^2 + 2k_4)\gamma + 2k_1\zeta t\Big].$$

Conserved quatities related to Y_{06} and Y_{07} are:

$$E_{06} = \frac{s^2}{2}[-\dot{t}^2 + m^2\dot{r}^2 + m^2r^4\dot{\theta}^2 + m^2r^4\dot{\phi}^2\sin^2\theta]$$
$$+st\dot{t} + \frac{s\zeta\dot{t}}{\gamma} + \frac{1}{2}t^2 + \frac{\zeta}{\gamma},$$

$$E_{07} = -s\dot{t}^2 + sm^2\dot{r}^2 + sm^2r^4\dot{\theta}^2 + sm^2r^4\dot{\phi}^2\sin^2\theta$$
$$+t\dot{t} + \frac{\zeta\dot{t}}{\gamma}.$$

(2): $m(t) = (t^2)^{\frac{1}{2}}$

Here $\mathbf{Y_{01}} - \mathbf{Y_{05}}$,

$$Y_{06} = s\frac{\partial}{\partial s} + \frac{t}{2}\frac{\partial}{\partial t}, \qquad Y_{07} = \frac{s^2}{2}\frac{\partial}{\partial s} + \frac{st}{2}\frac{\partial}{\partial t}, \tag{21}$$

$$G = \frac{k_1 t^2}{2} + k_4.$$

The associated integral of motions will be

$$E_{06} = -s\dot{t}\dot{t} + sm^2\dot{r}\dot{r} + sm^2r^2\dot{\theta}\dot{\theta} + sm^2r^2\sin^2\theta\dot{\phi}\dot{\phi} + t\dot{t},$$

$$E_{07} = -s^2\dot{t}\dot{t} + s^2m^2\dot{r}\dot{r} + s^2m^2r^2\dot{\theta}\dot{\theta} + s^2m^2r^2\sin^2\theta\dot{\phi}\dot{\phi} + 2st\dot{t} - t^2.$$

3.5 Largest set of Noether operators

For constant value of m we have $\mathbf{Y_{01}} - \mathbf{Y_{03}}$ and

$$Y_{04} = \frac{\partial}{\partial t},\ Y_{05} = \frac{1}{2}s\frac{\partial}{\partial t}, \tag{22}$$

$$Y_{06} = -\frac{1}{2}s\cos\theta\frac{\partial}{\partial r} + \frac{1}{2r}s\sin\theta\frac{\partial}{\partial\theta},$$

$$Y_{07} = r\cos\theta\frac{\partial}{\partial t} + t\cos\theta\frac{\partial}{\partial r} - \frac{t}{r}\sin\theta\frac{\partial}{\partial\theta},$$

$$Y_{08} = s\frac{\partial}{\partial s} + \frac{1}{2}t\frac{\partial}{\partial t} + \frac{1}{2}r\frac{\partial}{\partial r},$$

$$Y_{09} = \frac{1}{2}s^2\frac{\partial}{\partial s} + \frac{1}{2}st\frac{\partial}{\partial t} + \frac{1}{2}sr\frac{\partial}{\partial r},$$

$$Y_{10} = -\frac{1}{2}s\sin\theta\sin\phi\frac{\partial}{\partial r} - \frac{1}{2r}s\cos\theta\sin\phi\frac{\partial}{\partial\theta} - \frac{1}{2r}\frac{\sin\theta}{s\cos\phi}\frac{\partial}{\partial\phi}$$

$$Y_{11} = -\frac{1}{2}s\sin\theta\cos\phi\frac{\partial}{\partial r} - \frac{1}{2r}s\cos\theta\cos\phi\frac{\partial}{\partial\theta} + \frac{1}{2r}\frac{s\sin\phi}{\sin\theta}\frac{\partial}{\partial\phi},$$

$$Y_{12} = r\sin\phi\sin\theta\frac{\partial}{\partial t} + t\sin\theta\sin\phi\frac{\partial}{\partial r} + \frac{t\sin\phi\cos\theta}{r}\frac{\partial}{\partial\theta} + (\frac{t\cos\phi}{r\sin\theta})\frac{\partial}{\partial\phi},$$

$$Y_{13} = r\cos\phi\sin\theta\frac{\partial}{\partial t} + t\sin\theta\cos\phi\frac{\partial}{\partial r} + \frac{t\cos\theta\cos\phi}{r}\frac{\partial}{\partial\theta} - (\frac{t\sin\phi}{r\sin\theta})\frac{\partial}{\partial\phi}.$$

The conserved quantities related to $Y_{04} - Y_{13}$ are

$$E_{04} = 2\dot{t}, \quad E_{05} = s\dot{t} - t,$$

$$E_{06} = [\frac{1}{2}sm^2\dot{r} - r]\cos\theta - sm^2r^2\sin\theta\dot{\theta},$$

$$E_{07} = 2[r\dot{t} - m^2t\dot{r}]\cos\theta - 2\sin\theta m^2rt\dot{\theta},$$

$$E_{08} = -s\dot{t}\dot{t} + sm^2\dot{r}\dot{r} + sm^2r^2\dot{\theta}\dot{\theta} + sm^2r^2\sin^2\theta\dot{\phi}\dot{\phi} + t\dot{t} - rm^2\dot{r},$$

$$E_{09} = -\frac{1}{2}s^2\dot{t}\dot{t} + \frac{1}{2}s^2m^2\dot{r}\dot{r} + \frac{1}{2}s^2m^2r^2\dot{\theta}\dot{\theta} + \frac{1}{2}s^2m^2r^2\sin^2\theta\dot{\phi}\dot{\phi} + st\dot{t} - srm^2\dot{r} - \frac{t^2}{2} + \frac{r^2}{2},$$

$$E_{10} = m^2s\sin\phi\sin\theta\dot{r} + m^2rs\cos\theta\sin\phi\dot{\theta} + srm^2\sin\theta\cos\phi\dot{\phi} - r\sin\phi\sin\theta,$$

$$E_{11} = m^2s\cos\phi\sin\theta\dot{r} + m^2rs\cos\theta\cos\phi\dot{\theta} - srm^2\sin\theta\sin\phi\dot{\phi} - r\cos\phi\sin\theta,$$

$$E_{12} = 2r\sin\phi\sin\theta\dot{t} - 2tm^2\sin\phi\sin\theta\dot{r} - 2trm^2\sin\phi\cos\theta\dot{\theta} - 2m^2rt\sin\theta\cos\phi\dot{\phi},$$

$$E_{13} = 2r\cos\phi\sin\theta\dot{t} - 2tm^2\cos\phi\sin\theta\dot{r} - 2trm^2\cos\phi\cos\theta\dot{\theta} - 2m^2rt\sin\theta\sin\phi\dot{\phi}.$$

4 Case 2

Taking curvature $c = 1$ i.e pen universe then Lagrangian converts to

$$l = \dot{t}^2 - \left(\frac{m^2\dot{r}^2}{1-r^2}\right) - m^2r^2\dot{\theta}^2 - m^2r^2\dot{\phi}^2\sin^2\theta. \tag{23}$$

After some simplifications a system of linear PDEs is obtained.

4.1 Least set of Noether operators

Least Noether symmetries $\mathbf{Y_{01}} - \mathbf{Y_{02}}$ reported for arbitrary m.

4.2 3 Noether operators

Three Noether operators of some other values of cosmic scale factor are given in Table and Noether symmetries $\mathbf{Y_{01}} - \mathbf{Y_{02}}$ and

$$Y_{03} = -\cos\theta[(r-1)(1+r)]^{\frac{1}{2}}\frac{\partial}{\partial r} + \frac{\sin\theta[(r-1)(1+r)]^{\frac{1}{2}}}{r})\frac{\partial}{\partial\theta}, \qquad (24)$$

conserved quantity related to Y_{08}:

$$E_{03} = \frac{2m^2\dot{r}\cos\theta[(r-1)(1+r)]^{\frac{1}{2}}}{1-r^2} - 2m^2r\dot{\theta}\sin\theta[(r-1)(1+r)]^{\frac{1}{2}}.$$

4.3 5 Noether operators

Five Noether operators for different values of m

(1): $m(t)$=$\gamma t + \zeta$, γ and ζ are constants. Fuether $\mathbf{Y_{01}} - \mathbf{Y_{02}}$, (19), (20) and (24) having

$$G = \frac{1}{2\gamma}\Big[(k_1t^2 + 2k_4)\gamma + 2k_1\zeta t\Big]. \qquad (25)$$

4.4 Largest set of Noether operators

For constants values of m, largest set of Noether operators is
$\mathbf{Y_{01}} - \mathbf{Y_{02}}$,(22) and (24) with $G = k_2t + k_3$.

5 Case 3

When $c = -1$ i.e closed universe then Lagrangian takes the form

$$l = \dot{t}^2 - \left(\frac{m^2\dot{r}^2}{1+r^2}\right) - m^2r^2\dot{\theta}^2 - m^2r^2\dot{\phi}^2\sin^2\theta. \qquad (26)$$

After same simplification we get linear PDEs.

5.1 Least set of Noether operators

Least Noether symmetries $\mathbf{Y_{01}} - \mathbf{Y_{02}}$ are presented for arbitrary value of m.

5.2 5 Noether operators

$\mathbf{Y_{01}} - \mathbf{Y_{02}}$ and

$$Y_{03} = (-\sqrt{r^2+1}\sin\theta\cos\phi)\frac{\partial}{\partial r} - (\frac{\sqrt{r^2+1}\cos\phi\cos\theta}{r})\frac{\partial}{\partial\theta} + (\frac{\sqrt{r^2+1}\sin\phi}{r\sin\theta})\frac{\partial}{\partial\phi}, \qquad (27)$$

$$Y_{04} = (\sqrt{r^2+1}\sin\theta\sin\phi)\frac{\partial}{\partial r} + (\frac{\sqrt{r^2+1}\sin\phi\cos\theta}{r})\frac{\partial}{\partial\theta} + (\frac{\sqrt{r^2+1}\cos\phi}{r\sin\theta})\frac{\partial}{\partial\phi}, \qquad (28)$$

$$Y_{05} = (\frac{\sqrt{r^2+1}\sin\theta}{r})\frac{\partial}{\partial\theta} + (-\sqrt{r^2+1}\cos\theta)\frac{\partial}{\partial r}. \qquad (29)$$

We have

$$E_{03} = \frac{1}{(r^2+1)^{\frac{1}{2}}}2m^2\dot{r}\sin\theta\cos\phi + 2m^2r(r^2+1)^{\frac{1}{2}}\cos\phi\cos\theta\dot{\theta} - 2m^2r(r^2+1)^{\frac{1}{2}}\sin\phi\sin\theta\dot{\phi},$$

$$E_{04} = -\frac{1}{(r^2+1)^{\frac{1}{2}}}2m^2\dot{r}\sin\theta\sin\phi - 2m^2r(r^2+1)^{\frac{1}{2}}\sin\phi\cos\theta\dot{\theta} - 2m^2r(r^2+1)^{\frac{1}{2}}\cos\phi\sin\theta\dot{\phi},$$

$$E_{05} = \frac{1}{(r^2+1)^{\frac{1}{2}}}2m^2\dot{r}\cos\theta - 2m^2r(r^2+1)^{\frac{1}{2}}\sin\theta\dot{\theta}.$$

5.3 Least set of Noether operators

(1): m=$\gamma t + \zeta$, γ and ζ represent constants.
$\mathbf{Y_{01}} - \mathbf{Y_{02}}$, (19), (20), (27),(28) and (29) having

$$G = \frac{1}{2\gamma}\Big[(k_1t^2 + 2k_4)\gamma + 2k_1\zeta t\Big]. \qquad (30)$$

(2): For constant value of m
$\mathbf{Y_{01}} - \mathbf{Y_{02}}$, (22) and (27),(28) and (29) having

$$G = k_2t + k_3.$$

6 Conclusion

FRW was considered to classify Robertson-Walker scale factor for closed, open and flat universe with the help of Noether Gauge symmetries. PDEs were acquire by using Lagrangian. Different solutions were obtained for distinct values of scale factor. For all values of scale factor Noether operators were presented and related integral of motion was obtained.

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