Forecasting Automobile Sales using an Ensemble of Methods

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Abstract: - The aim of this paper is to check in practice to what extent an ensemble forecast based on averaging the outcomes of several forecasting methods provides better results than single forecasts. Therefore, we use data of monthly new car registrations in the Netherlands and car sales in the USA. The performances of seven popular forecasting methods are assessed and the results are combined into Ensemble forecasts. Several common performance metrics are applied on the results of the test data and it is shown that the Ensembles perform slightly better than each of the forecasting models separately. This confirms the idea, found in literature, that under certain conditions, a combination of several forecasts leads to more accurate results.

Key-Words: - Automobile Registrations, Automobile Sales, ARIMA, Artificial Neural Network, Ensemble Forecasting, Exponential Smoothing, Generalized Linear Model, Theta, Time Series Forecasting, Random Forest, Vector Auto Regression

1 Introduction

The automotive industry is one of the world's most important economic sectors by revenue. It is characterized by long development and production processes and therefore long-term forecasts for sales of new cars provide valuable information to its many stakeholders. Car developers and manufacturers, but also car dealers, marketeers and national licensing authorities benefit from reliable forecasts of the future demand for new cars. Not surprisingly much effort has been invested in attempts to develop reliable forecasting models.

In a paper by Brühl et al. [1], the time series consisting of the number of newly registered German automobiles in the period 1992 to 2007 was used to train Multilinear Regression and Support Vector Machine forecasting models. Comparing the performance results, they found that the Support Vector Machine model with a Gaussian kernel performed better. Furthermore, it was found that models based on quarterly data were better than those based on monthly or yearly data. In Hülsmann et al. [2] several forecasting models were considered and used to forecast the number of new registered cars in Germany and the USA. It was found that the results of [1] could be further improved by using market-specific absolute, normalized exogenous parameters. In [3] Sa-ngasoonsong et al. found a long-run equilibrium relationship between automobile sales and some economic indicators. They estimated a vector error correction model which outperformed other time series forecasting methods. Fantazzini and Toktamysova [4] found that forecasting models for monthly new registered cars in Germany which included Google search data as an exogenous variable had better performances than other models.

Unlike in the beforementioned papers, the main aim of this contribution is not to consider the performances of models separately but rather to see whether an ensemble of forecasting models is beneficial in this case.

Ensemble forecasting is a method to combine the results of forecasts of several models into one in order to achieve a better accuracy [5]. Usually, different models capture different aspects of reality and therefore they have different biases. When the forecast errors are not positively correlated the errors of these models will, to a certain extent, cancel out when the individual forecasts are averaged. This results in better forecasts.

In this paper 7 forecasting methods are applied on the time series consisting of two data sets. Its accuracy is evaluated and compared with the individual model results and that of a naive method based on the seasonal component.

2 Data Pre-processing

In this study two data sets are used. The first set is new car registrations in the Netherlands during the period 2012 to 2017. The second set is the set of total vehicle sales in the USA between 1992 and 2017.

2.1 Dutch New Car Registrations

New cars have to be registered with the national authorities before they may be sold on the Dutch automobile market. For this research, monthly data of new car registrations ranging from January 2007 until April 2017 were used.

Visual inspection of the original time series (dashed line) in Fig. 1 reveals a seasonal component and several notable spikes in June 2012 and in December of the years 2012 to 2016. These special effects are followed in the next months by remarkable low figures. This unusual pattern in the time series is the result of government intervention. In the months after each of these spikes, previously announced tax measures were enforced by the Dutch government in an attempt to discourage the use of certain types of cars and/or to encourage others.

The spikes are therefore regarded as a result of stocking-up behavior by car dealers in order to avoid the new tax measures coming into force the next month.

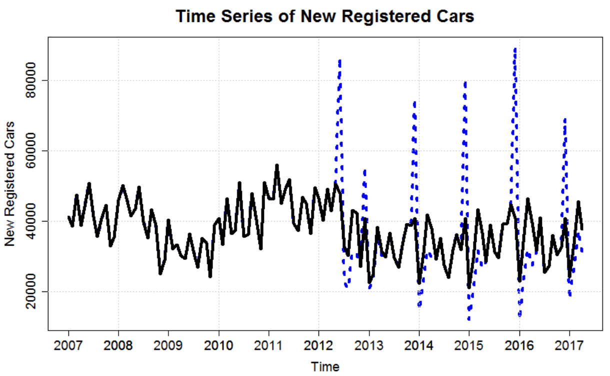

Figure 1: In the original time series of new registered cars (blue dashed line) large spikes as a result of government tax measures have been removed and spread out over the following 4 months resulting in the solid black line.

The occurrences of these spikes are easy to predict due to the fact that tax measures are announced in advance. However, they do not fit the time series models considered here. Because spikes are a temporarily disturbance of the dynamics of the time series and because they can be simply forecasted by hand, these contaminating effects are eliminated from the time series.

It is reasonable to assume that the extra car registrations at the time of these spikes would have occurred in later months in case there had been no change in the tax regime. The extra number of registrations as compared to the average of that particular month are spread out equally over the 4 months following the peak. The analysis done in this paper concentrates on the resulting corrected time series.

2.2 USA Vehicle Sales

The second data set is the total vehicle sales in the USA in the period between January 1992 until August 2017 (see Fig. 2). Contrary to the Dutch data set this time series is analyzed in its original form without further corrections.

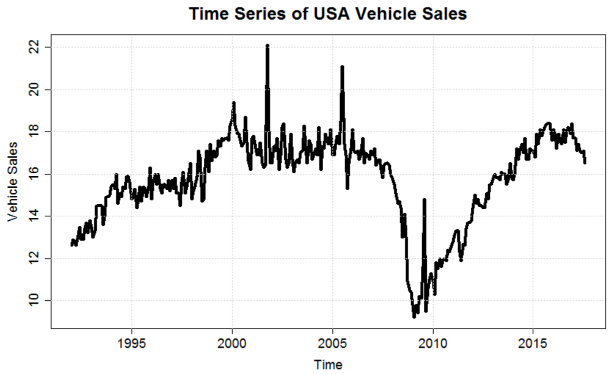

Figure 2: Monthly total vehicle sales in the USA in millions of units in the period 1992 until 2017. The effect of the global financial crisis of 2007-2008 on this time series is clearly visible.

3 Forecasting Methods

In this study the following forecasting models were applied on the two data sets.

3.1 Exponential Smoothing

Exponential smoothing (ETS) is a popular forecasting method in business because of its simplicity and relative good performance. In the ETS-model future values of a time series are weighted averages of all past values. The weights decrease exponentially into the past so that more weight is given to the most recent values.

Because our time series has an additive seasonal component we use the Holt-Winters exponential smoothing method also called triple exponential smoothing [6]. The prediction formula is as follows.

$$\hat{Y}_{t+h} = l_t + hb_t + s_{[t-m]^*} \quad (1)$$

with $\hat{Y}_{t+h}$ the estimation of the h step ahead value of the time series under consideration and level $l_t = \alpha(y_t - s_{t-12}) + (1-\alpha)(l_{t-1} + b_{t-1})$, trend $b_t = \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1}$, seasonal component $s_t = \gamma(y_t - l_{t-1} - b_{t-1}) + (1-\gamma)s_{t-12}$ and where $[.]^*$ indicates that from the data the most recent corresponding month should to be taken. Optimal values for the parameters α, β and γ are estimated during the training process.

3.2 Autoregressive Integrated Moving Average

Autoregressive Integrated Moving Average (ARIMA) models are generalizations of the simple AR model. The AR part of ARIMA indicates that the dependent variable is regressed on its own lagged values. The "I" indicates that the dependent variable may be differenced (once or more times) in the case the time series of the dependent variable is not stationary. Finally, the MA part indicates the option that the regression errors may be a linear combination of past values of errors. An ARIMA(p,d,q) model for a time series of Y_t may be written as

$$\hat{Y}'_t = \sum_{i=1}^{p} \alpha_i Y'_{t-i} + \sum_{i=1}^{q} \beta_i \epsilon_{t-i} + \epsilon_t \qquad (2)$$

where $Y'_t = \Delta^d Y_t$, the difference of Y_t of order d. Furthermore, p and q denote the number of maximal lags in Y_t and ϵ_t respectively and ϵ_t is the regression error.

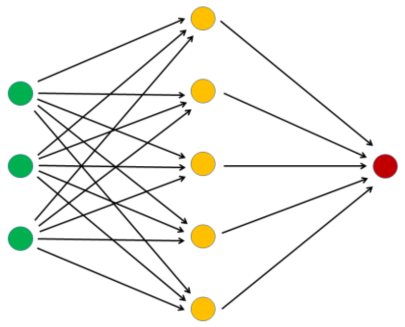

Figure 3: Schematic view of an Artificial Neural Network consisting of 3 input cells, 5 cells in the hidden layer and one output cell. During the training of the network the weights of the connections between the cells are optimized.

3.3 Artificial Neural Network

Artificial Neural Networks (ANN) are nonlinear autoregression models, biomimetically inspired by the neurons in the biological brain. An ANN consists of a number of artificial neurons that can pass signals of varying strength to each other (see Fig. 3). If the combined incoming signals are strong enough, the neuron becomes activated and the signal travels to other neurons connected to it.

ANN's have to be trained from examples, and cannot be explicitly programmed. That is why this model is often applied to problems where the solution is difficult to express in a traditional computer programming language.

3.4 Vector Auto Regression

The Vector Auto Regression (VAR) model is a multivariate generalization of the AR model. The VAR-model allows the inclusion of time series which are expected to be linearly interdependent of each other. Each variable has its own equation containing its own lagged values and those of the other variables in the model and is therefore explained by its own history and that of the other variables.
An example of a VAR-model based on 3 variables $Y_{1,t}$, $Y_{2,t}$ and $Y_{3,t}$ is shown in (3).

$$\begin{aligned} \hat{Y}_{1,t} &= c_1 + \sum_{i=1}^{p} (\alpha_{1,i} Y_{1,t-i} + \beta_{1,i} Y_{1,t-i} + \gamma_{1,i} Y_{1,t-i}) + \epsilon_{1,t} \\ \hat{Y}_{2,t} &= c_2 + \sum_{i=1}^{p} (\alpha_{2,i} Y_{2,t-i} + \beta_{2,i} Y_{2,t-i} + \gamma_{2,i} Y_{2,t-i}) + \epsilon_{2,t} \\ \hat{Y}_{3,t} &= c_3 + \sum_{i=1}^{p} (\alpha_{3,i} Y_{3,t-i} + \beta_{3,i} Y_{3,t-i} + \gamma_{3,i} Y_{3,t-i}) + \epsilon_{3,t} \end{aligned} \qquad (3)$$

In [3] and [4] economic variables were used as extra variables. Economic variables reflect the state of the economy and it is assumed that this influences potential customers in their decision whether or not to purchase a new car. For this research several combinations of economic time series were considered to be included in the model.

In the case of the Dutch data set Job Vacancies Index and Car Prices Index were selected as explanatory variables in this model whereas in the case of the USA data set Unemployment Rate, GDP index and Consumer Price Index were chosen.

3.5 Theta

The theta method [7] has caught interest in academic circles and among forecast practitioners due to its remarkable good performance for monthly series at the M3-forecasting competition [8]. The original description of this univariate model is rather involved. It is based on decomposition of the time series through second order differences into so-called Theta-lines to capture long-term behavior and short-term features separately. Hyndman and Bilah [9] however, found that for a large training set the Theta method is equivalent to simple exponential smoothing with drift.

3.6 Random Forest

Random Forest [10] regression (RF) is a tree-base supervised learning algorithm that is often applied for solving classification problems and non-linear regression. The method operates by constructing an ensemble of decision trees whose individual results are combined with each other so that outliers and missing values have less influence on the outcome.

3.7 Generalized Linear Model

A generalization of the well-known simple linear regression model is the generalized linear model (GLM) [11] which is suitable for using several explanatory variables and which allows non-normal error distributions. The GLM is often used for the study of the underlying structure of data in terms of its explanatory variables but it can also be used for time series forecasting. The GLM formula can be written as

$$EY = g^{-1}(X\beta) \quad (4)$$

Where the expected value of the outcome EY is modeled as a linear combination of to be estimated parameters β and the independent explanatory variables X. The link function g provides the relationship between the linear predictor and the mean of the chosen distribution function.

3.8 Naive Seasonal

In the Dutch data set there is clearly a seasonal effect. Therefore, as a benchmark to evaluate the performance of the prediction models, the average monthly figures are used as a naive forecast.

In an activity as future prediction it is recommendable to check if a sophisticated model is indeed an improvement with respect to simpler methods because, not rarely, simple models perform better than complicated ones.

Several common performance metrics (see Section 5) for the results of the models described above are calculated and compared to those of the naive seasonal model.

4 Results

Both data sets were split into a training set and a test set. The Dutch data set was divided in a training set of monthly data from January 2007 to April 2016. This time series contains 112 data points. The above explained forecasting methods were applied to the training set to estimate the parameters of these models. After this, the trained models were used to create forecasts for the test set, the 12 months period from May 2016 to April 2017.

In the USA case, the training set consists of the monthly data from January 1992 until September 2014. The remaining 36 months were used as test set.

The results were calculated using the statistical open source language R. The univariate Holt-Winters ETS, ARIMA, NNET, Theta models and the GLM model have been estimated using the forecast package [12] while the RF and VAR models were established using randomforest [13] and vars [14] package functions respectively.

In the following section the model checks in the case of the Dutch data is shown. Similar checks have been done with the models based on the USA data.

4.1 Check of the Model Output Properties

To see whether the predictive models could be improved a few checks have been conducted on its residuals.

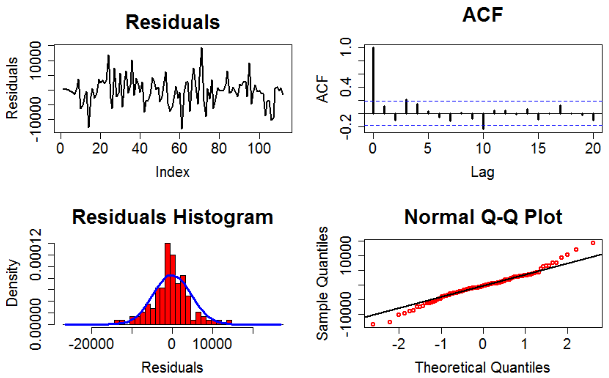

Figure 4: The residuals of the Holt-Winters model have more or less a constant variance, no significant autocorrelations and approximately a Normal distribution with mean zero.

In the top left of Fig.4 the in-sample residuals of the Holt-Winters model forecast are displayed. One can visually establish that the variance is more or less constant over time. Furthermore, in the top right of Fig. 4 one can see that no autocorrelations at lags 1-20 of the in-sample forecast errors greatly exceed the significance bounds at the dotted lines. This indicates that there is little evidence of non-zero autocorrelations at lags 1-20.

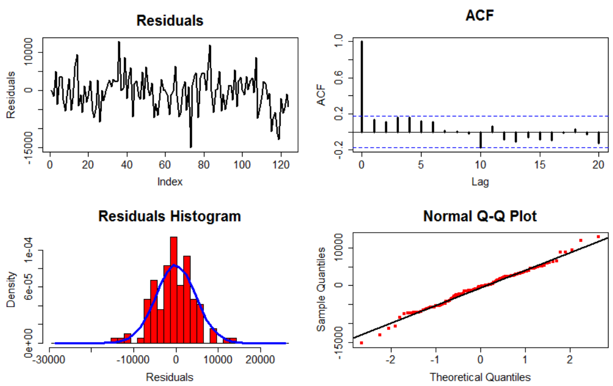

Figure 5: The ARIMA model residuals have more or less a constant variance, no autocorrelations and approximately a Normal distribution with mean zero.

In the lower part of the figure a histogram of the residuals with overlaid normal curve and a QQ-plot is displayed.

From these pictures it seems plausible that the forecast errors are normally distributed with mean zero. We may conclude that the Holt-Winters model fits the new car registrations appropriately well and that it provides a forecast that probably cannot be improved.

The same analysis as above has been conducted with respect to the other models.

For establishing the ARIMA model, the Box-Jenkins methodology [15] was applied which lead to an ARIMA(2,1,0)(1,0,0)$_{12}$ model i.e. a differenced second order autoregressive model with a first order seasonal component of 12 months. The in-sample residuals and its properties are displayed in Fig.5.

A neural autoregressive network was estimated with 4 hidden nodes, 7 time lags and a seasonal component. See Fig.5 for its residual properties. Again the conclusion can be drawn that the model fits the data quite well and probably cannot be further improved without changing the model itself.

The VAR model used for the Dutch data set contains 2 explanatory economic variables, namely Job Vacancies Index (JVI) and Car Prices Index (CPI). The rationale behind this choice is that demand for new cars tends to raise when more people acquire a (better) job and/or when car prices are low.

Several conditions are necessary to be fulfilled when creating a VAR model. First, it was checked whether these three time series are stationary. Using the Augmented Dickey Fuller test it was found that differencing was required in the case of JVI and CPI to acquire stationarity.

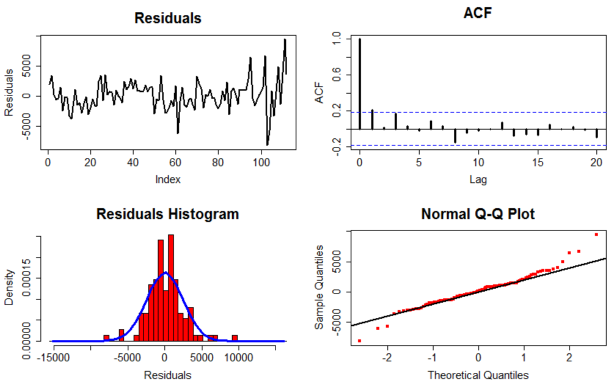

Figure 6: In-sample residuals plots of the ANN model.

Secondly, the autocorrelations of each of the time series were checked. See Fig. 7 for the autocorrelation plots of the new car registrations. It shows that 3 lags seem appropriate for the AR part of the model.

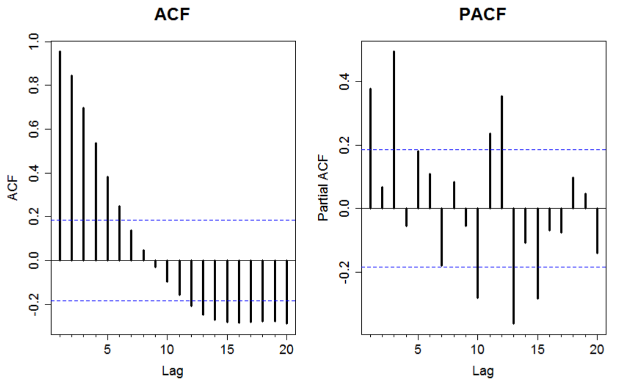

Figure 7: Plots of the autocorrelations and partial autocorrelations of the deseasonalized time series of the number of new car registrations.

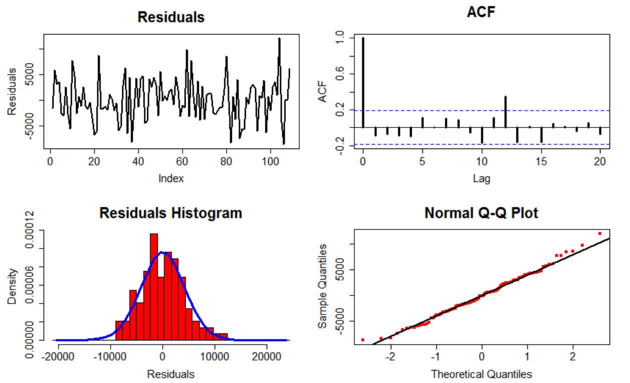

Figure 8: The residuals of the VAR model have more or less a constant variance, no autocorrelations and approximately a Normal distribution with mean zero.

This choice was confirmed by Akaike's Information Criterion (AIC). Finally, the model was estimated using the training data and its residuals were analyzed (see Fig. 8). From the figure the residual properties are deemed satisfactory.

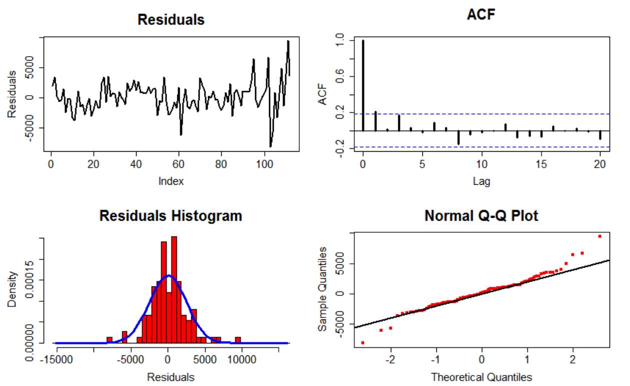

Figure 9: Several displays of the residuals of the Theta model to show its properties. The residuals seem more or less to have the necessary properties.

In the case of the USA data the same procedure was followed using the explanatory variables Unemployment Rate, GDP index and Consumer Price Index. A model with 4 lags gave the best AIC value.

Then the Theta model was estimated. As with the other models the residuals were checked graphically (see Fig. 9) and it was concluded the model cannot be further improved.

As shown in Fig. 10 the Random Forest model residuals gives less favorable diagnostics. The residuals are symmetrical around zero but despite fine tuning efforts, it is unlikely that they are normally distributed. As can be seen in the results section the model provides not the most accurate forecasts in comparison with most of the other models considered here.

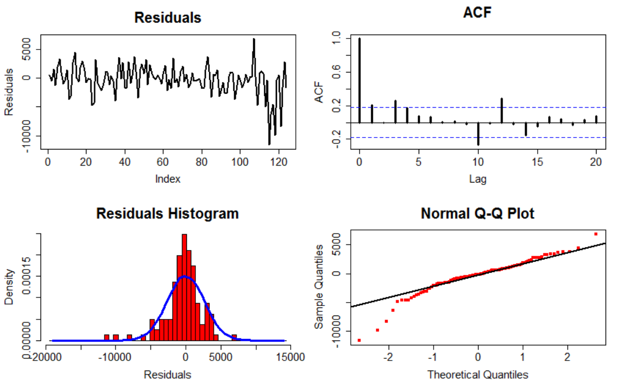

Figure 10: The residuals of the RF model seem more or less symmetrically distributed around 0 but are not normally distributed.

Finally, the Generalized Linear Model residuals are shown in Fig. 11 where it is shown that they behave as required.

In short, the residuals of almost all 7 estimated models have the same desirable properties (zero mean, constant variance, Normally distributed).

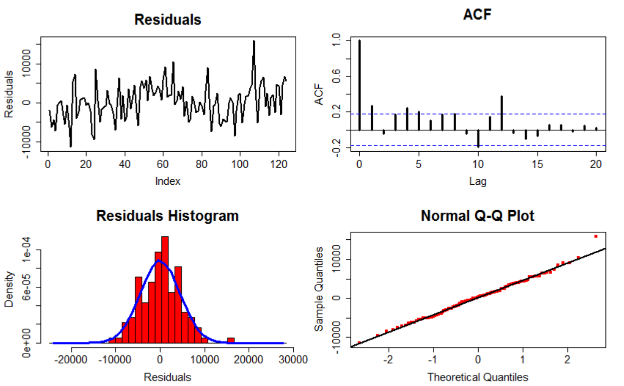

Figure 11: The residuals of the GLM model seem to have the necessary properties.

4.2 Establishment of the Ensembles

In [5] it was recommended to average the results of at least 5 different forecasting models which do not correlate positively. In such a case forecasting errors tend to single each other out which would result in a more accurate forecast. The correlations between the individual model forecasts were calculated and displayed in Fig. 12 and Fig. 13.

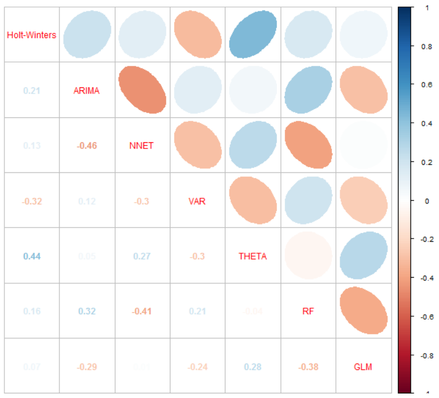

Figure 12: Correlogram of the forecast errors of the models under consideration in the case of the Dutch data set. The graph illustrates that the test set errors of the 7 models are not highly correlated.

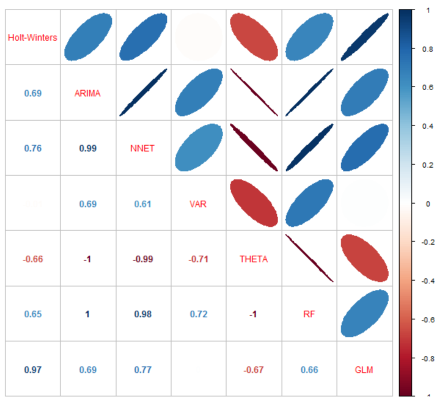

Figure 13: Some of the forecast results of the models under consideration are highly correlated. It was decided to remove the VAR model from the Ensemble forecast.

After checking these correlograms and the performance metrics in Table I and Table II it was decided to leave out the least performing and most sophisticated two models (i.e. ARIMA and VAR) and combine the remaining 5 models into an ensemble forecast for the Dutch time series.

In the case of the USA data, it was decided to discard the VAR model only.

The real data of the test period, the forecasts of the selected models and the Ensembles have been plotted in Fig. 13 and Fig. 14 for visual inspection.

5 Performance Evaluation

In this section the model forecasts and the forecast of the combined models at the test data period are compared with the real outcomes. This gives an impression about the performance capability of these models if they were to be used for real forecasts.

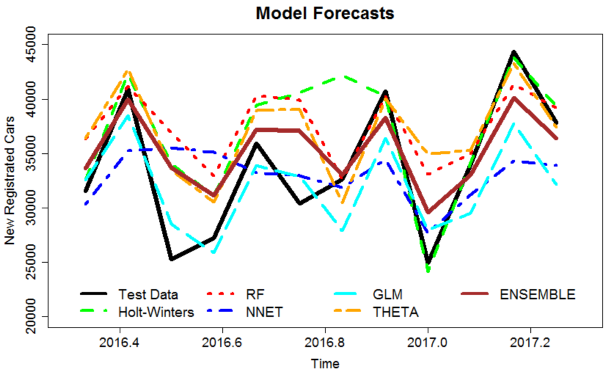

Figure 13: Forecast results of 5 selected models on the Dutch 12 months test set. The thick black line represents the real outcome of the new car registrations time series and the brown line is the Ensemble forecast.

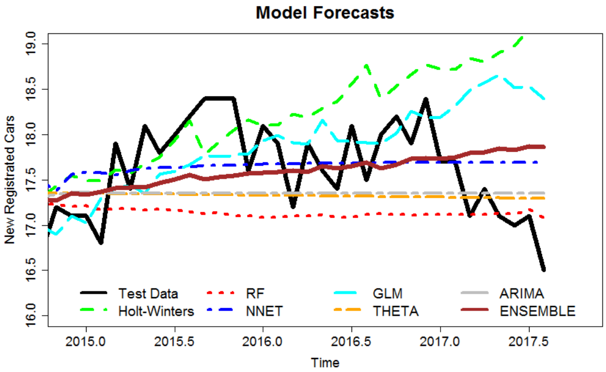

Figure 14: Forecast results of 6 selected models on the 36 months USA test set. The thick black line represents the real outcome of the car sales time series and the brown line is the Ensemble forecast.

The models' performances are compared with each other using several standard evaluation metrics which are based on the forecast errors $e_i = y_i - \hat{y}_i$.

The Mean Error

$$ME = \frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t) \quad (5)$$

The Mean Absolute Error

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|y_t - \hat{y}_t| \quad (6)$$

The Mean Absolute Prediction Error

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\frac{|y_t - \hat{y}_t|}{|y_t|}100\% \quad (7)$$

The Root Mean Square Error

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2} \quad (8)$$

The Maximum Absolute Error

$$MXAE = \max_{t}|y_t - \hat{y}_t| \quad (9)$$

The Mean Directional Accuracy

$$MDA = \frac{1}{n}\sum_{t=1}^{n}\mathbf{1}_{sgn(\Delta y_t)=sgn(\Delta\hat{y}_t)} \quad (10)$$

Where $\mathbf{1}$ *is the indicator function,* $\Delta x_t = x_t - x_{t-1}$ *is the difference of time series* x_t *at time t and t-1, and* $sgn(x)$ *extracts the sign of* x.

Forecasting Performance Metrics Calculated over a 12 Months Test Set

Model \ Metric	ME	MAE	MAPE
ETS	-3188	3580	11.7%
ARIMA*	-5790	5790	19.1%
NNET	*1104*	4215	12.6%
VAR*	-4463	4517	15.6%
THETA	-3080	3796	13.1%
RF	-3730	4257	14.7%
GLM	1799	3431	*10.0%*
NAIVE*	-3706	3722	12.9%
ENSEMBLE	*-1455*	*3117*	10.3%

Model \ Metric	RMSE	MXAE	MDA
ETS	5079	10200	*0.73*
ARIMA*	7018	12994	*0.73*
NNET	5041	9469	0.91
VAR*	5785	9841	0.80
THETA	4975	9989	*0.73*
RF	5630	11530	0.82
GLM	*3810*	*6622*	0.82
NAIVE*	4991	9555	0.82
ENSEMBLE	3944	8426	0.82

Table I: Comparison of the forecasting performances of the 7 models individually, the Naive seasonal model and the Ensemble. Models indicated with an "*" are not included in the Ensemble. The best performance figures are printed in italics.

The lower the outcomes of metrics (5)-(9) the better the performance of the corresponding model. Contrarily, the MDA metric (10) should be as high as possible as it compares the forecasted direction with the actual realized direction of the time series.

The metrics are applied to each model result of both data sets and the outcomes are listed in Table I and Table II for easy comparison.

5.1 Forecasting Performance Dutch Data

Table I shows the outcomes of the performance metrices for all individual models and their combination, the Ensemble. According to two of the metrices the Ensemble performs better than all the other models. In 3 other cases the Ensemble is the second best performer. The relatively good performance can also be seen in Fig. 13.

Forecasting Performance Metrics Calculated over a 36 Months Test Set			
Metric / Model	ME	MAE	MAPE
ETS	-0.61	0.74	4.26%
ARIMA	0.24	0.51	2.84%
NNET	-0.045	0.52	2.53%
VAR*	0.54	0.75	4.21%
THETA	0.60	0.67	3.75%
RF	0.46	0.60	3.35%
GLM	-0.23	0.54	3.09%
ENSEMBLE	*0.016*	*0.46*	*2.63%*
Metric / Model	RMSE	MXAE	MDA
ETS	0.99	2.89	0.37
ARIMA	0.59	*1.05*	0.49
NNET	*0.52*	1.19	0.37
VAR*	0.84	1.65	0.46
THETA	0.80	1.09	0.46
RF	0.72	1.29	*0.34*
GLM	0.72	1.89	0.37
ENSEMBLE	0.55	1.37	0.40

Table II: Comparison of the forecasting performances of the 7 models individually and the Ensemble. The best performance figures are printed in italics.

5.2 Forecasting Performance USA Data

In the case of the USA data of total vehicle sales the Ensemble also performs relatively well. In Table II it can be seen that 3 of the 6 performance metrices are much better than those of the other models while remaining metrices are also satisfying.

6 Conclusion

In this paper 7 common forecasting models have been applied on two different data set concerning car sales. Despite their relative simplicity ETS and GLM outperformed more complicated models in the Dutch data set. Furthermore, the naive seasonal model performed better than ARIMA and VAR, which are remarkably the most sophisticated models in this study.

Most importantly, it was shown in this paper that, for these two data sets, Ensemble forecasts based on sets of forecasting models perform consistently better than individual models. This is a confirmation of [5] and shows that with only a few available models, one can improve forecasting accuracy.

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References:

[1] B. Brühl, M. Hülsmann, C.M. Friedrich, D. Reith, A sales forecast model for the German automobile market based on time series analysis and data mining methods, *ICDM 2009: Advances in Data Mining. Applications and Theoretical Aspects*, pp. 146-160.

[2] M. Hülsmann, D. Borscheid, C.M. Friedrich, D. Reith, General sales forecast models for automobile markets based on time series analysis and data mining techniques, *Transactions on Machine Learning and Data Mining,* (5)2, 2012, pp. 65-86.

[3] A. Sa-ngasoonsong, Multi-step sales forecasting in automotive industry based on structural relationship identification, *Int. J. Production Economics* (140), 2012, pp. 875-887.

[4] D. Fantazzini, et al., Forecasting German car sales using Google data and multivariate models, *Int. J. Production Economics*, 170A, pp. 97-135, 2015.

[5] J. Scott Armstrong, *Combining Forecasts Principles of Forecasting: A Handbook for Researchers and Practitioners*, Kluwer Academic Publishers, 2001.

[6] C.C. Holt, Forecasting trends and seasonals by exponentially weighted averages, Carnegie Institute of Technology, Pittsburgh ONR memorandum, no. 52. 1957.

[7] V. Assimakopoulos, K. Nikolopoulos, The theta model: a decomposition approach to forecasting, *Int. J. of Forecasting*, (16), 2000, pp. 521–530.
[8] S. Makridakis, M. Hibon, The M-3 Competition: results, conclusions, and implications, *Int. J. of Forecasting*, (16), 2000, pp. 451–476.
[9] R.J. Hyndman, B. Billah, Unmasking the Theta method, *Int. J. of Forecasting*, (19)2, 2003, pp. 287-290.
[10] L. Breiman, Leo, Random Forests, *Machine Learning*, (45)1, 2001, pp. 5–32.
[11] J. Nelder, R. Wedderburn, Generalized Linear Models, *J. of the Royal Stat. Soc.*, A (135)3, 1972, pp. 370–384.
[12] R.J. Hyndman, Y. Khandakar, Automatic time series forecasting: the forecast package for R, *J. of Stat. Software*, (26)3, 2008, pp. 1–22.
[13] A. Liaw, M. Wiener, Classification and Regression by randomForest, *R News* (2)3, 2002, pp. 18-22.
[14] B. Pfaff, Package Vars, 2007. http://cran.r-project.org/web/packages/vars/index.html.
[15] G.E.P. Box, G.M. Jenkins, G.C. Reinsel, *Time Series Analysis: Forecasting and Control*. 3rd ed. Englewood Cliffs, NJ: Prentice Hall, 1994.