Wavelet Decomposition for Denoising GPS/INS Outputs in Vehicular Navigation System

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Abstract: - This paper aims to bring together Global Positioning System (GPS) and Inertial Navigation System (INS) integration approach for data fusing using wavelet analysis. The wavelet is employed to compare the sensors outputs at different resolution levels and for several types of errors in INS and GPS as well as to smooth and predict the INS errors. Simulation results show that, the types of filters which are used in multi-resolution algorithm have large effects in reducing the INS/GPS error in position and especially in velocity components. Furthermore, soft thresholding technique is introduced in multi-resolution algorithm, where six rules are proposed to select the threshold value. Real GPS and INS measurements were used in the Matlab simulation to illustrate the effectiveness of the proposed method.

Key-Words: - Vehicular navigation, Inertial navigation system, GPS, Wavelet Analysis, Denoising.

1 Introduction

Strapdown Inertial Navigation System (SINS) technologies are based on the principle of integrating specific forces and rates measured by accelerometers and rate gyros of an Inertial Measurement Unit (IMU) fixed to the navigating body. Given the initial conditions of position, velocity and attitude, accurate real time integration of IMU output will produce position and attitude information in some given navigation coordinate system [1]-[2]. On the other hand, the GPS relies on the technique of comparing signals from orbiting satellites to calculate position (and possibly attitude) at regular time intervals [3]. Nevertheless, being dependent on the satellites signals makes GPS less reliable than self-contained INS due to the possibility of drop-outs or jamming.

Typically, the dynamic error model for a terrestrial INS algorithm requires three position errors, three velocity errors and three attitude errors in an INS (i.e. the system error states). These errors are also augmented by some sensor error states such as accelerometer biases and gyroscope drifts, which are modeled as stochastic processes. In fact, there are several random errors associated with each inertial sensor. Therefore, it is usually difficult to set a certain stochastic model for each inertial sensor capable of working efficiently in all environments and reflecting the long-term behavior of sensor errors [4]. Hence the introduction of wavelet algorithm to perform the self-following of the vehicle under all-conditions maneuvering will be not only beneficial but also required.

In this work, the effect of wavelet filter type on the INS/GPS error will be discussed and implemented on all INS and GPS data components. Thresholding technique with many rules for selecting the threshold value and its effect on the performance of INS/GPS error will be also discussed and examined. Figure 1 is a general block diagram for the wavelet multi-resolution denoising for GPS/INS outputs.

2 Proposed Method

This section describes the algorithm of Multi-Resolution Analysis (MRA), the selection of wavelet decomposition level as well as wavelet filter and the calculation of threshold value.

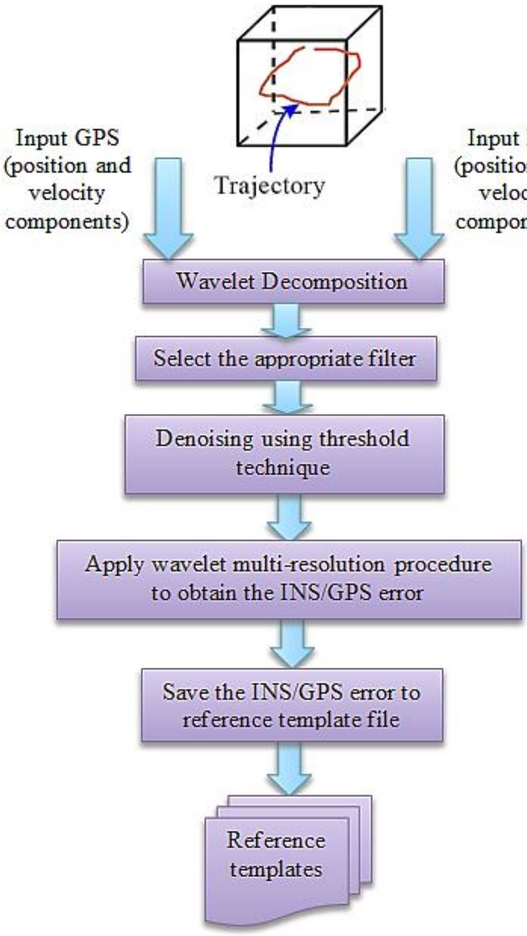

Fig. 1. Main block diagram of wavelet decomposition using GPS/ INS outputs.

2.1. Multi-Resolution Analysis Algorithm (MRA)

In order to determine the INS/GPS error that can be used to model the INS position and velocity error, a wavelet multi-level decomposition must be performed for each component of the INS and GPS output signals. The following steps describe the mathematical wavelet decomposition procedure:

Step1: For each one of INS and GPS outputs signals, calculate the approximation coefficient at Sth resolution level using [4]:

$$C_{s,k} = 2^{(s/2)} \sum_{n} x(n)\Phi(2^s n-k) \quad (1)$$

Where $\Phi(n)$ is the wavelet function (the basis function utilized in the wavelet transform) and $\Psi(2^s n-k)$ are scaled and shifted versions of $\Phi(n)$ based on the values of s (scaling coefficient) and k (shifting coefficient). $C_{s,k}$ are the corresponding wavelet coefficients. *x (n) is* the original signal.

This operation is equivalent to low pass filtering.

Step2: calculate the approximation using the coefficient obtained in step1 above as following [5]:

$$x_s(n) = \sum_{k=-\infty}^{\infty} C_{s,k}\Phi_{s,k}(n) \quad (2)$$

Step3: Calculate the details coefficient at s^{th} resolution level using:

$$d_{s,k} = \sum_{n} x(n)\Psi_{s,k}(n) \quad (3)$$

This operation is equivalent to high pass filtering.

Step4: Find the detail using the result of step3 and the following equation:

$$g_s(n) = \sum_{k=-\infty}^{\infty} d_{s,k}\Psi_{s,k}(n) \quad (4)$$

Step5: Return to step one and continue the wavelet decomposition process until appropriate level of decomposition (LOD) is reached which is different from one IMU to another (the appropriate LOD selection will be described in the next sub section). It must be noted that the next wavelet decomposition process must be performed on the approximation obtained from the previous wavelet decomposition process and so on.

Step6: Denoising the details of the INS and GPS signals by applying the thresholding technique which is described later.

Step7: Compare the INS and GPS position and velocity components at several resolution levels (by subtracting the wavelet coefficients of each of the GPS outputs from the corresponding wavelet coefficients of each of the INS outputs) as follows:

$$\begin{aligned}
&A_{E1^{st}} = A_{GPS1^{st}} - A_{INS1^{st}}; \quad D_{E1^{st}} = D_{GPS1^{st}} - D_{INS1^{st}};\\
&A_{E2^{nd}} = A_{GPS2^{nd}} - A_{INS2^{nd}}; \quad D_{E2^{nd}} = D_{GPS2^{nd}} - D_{INS2^{nd}};\\
&A_{E3^{rd}} = A_{GPS3^{rd}} - A_{INS3^{rd}}; \quad D_{E3^{rd}} = D_{GPS3^{rd}} - D_{INS3^{rd}};\\
&\quad\vdots \qquad\qquad\qquad\qquad \vdots\\
&A_{E(s-1)^{th}} = A_{GPS(s-1)^{th}} - A_{INS(s-1)^{th}}; \quad D_{E(s-1)^{th}} = D_{GPS(s-1)^{th}} - D_{INS(s-1)^{th}};\\
&A_{Es^{th}} = A_{GPSs^{th}} - A_{INSs^{th}}; \quad D_{Es^{th}} = D_{GPSs^{th}} - D_{INSs^{th}};
\end{aligned} \quad (5)$$

where:

A_{GPS}, A_{INS} are GPS and INS approximations respectively.

D_{GPS}, D_{INS} are GPS and INS details respectively.

A_E Difference between the GPS and INS approximations for different levels.

D_E Difference between the GPS and INS details for different levels.

Step8: Reconstruct the INS/GPS error signal from the wavelet coefficients differences found in step7 as follows:

$$\begin{aligned}
&s_{1^{st}} = D_{E1^{st}};\\
&s_{2^{nd}} = D_{E2^{nd}} + s_{1^{st}};\\
&s_{3^{rd}} = D_{E3^{rd}} + s_{2^{nd}};\\
&\quad\vdots\\
&s_{s^{th}} = A_{E(s-1)^{th}} + s_{(s-1)^{th}};
\end{aligned} \tag{6}$$

where:

$s_{1^{st}}$ Difference between the GPS and INS details of the first LOD.

$s_{2^{nd}}$ Summation of the difference between the GPS and INS details of the second LOD with the difference of previous LOD (s_{1st}).

$s_{s^{th}}$ Summation of difference between the GPS and INS approximation at s^{th} level and the difference between the GPS and INS details of the previous LOD ($S_{(s-1)^{th}}$).

2.2. Selection of the Appropriate Wavelet Level of Decomposition (LOD)

To select an appropriate LOD in this case, several decomposition levels are applied and the Standard Deviation (STD) is computed for each approximation difference components (INS/GPS error) and compared with the real INS error. The proper LOD will be the one having the minimum difference between these two errors.

Kinematic inertial data (Real Data) denoising the output of the sensors contains both effects of the actual vehicle motion dynamics and the sensor noise as well as some other undesirable effects (e.g. vehicle engine vibrations). Therefore, the criterion for the selection of the appropriate LOD will be different from the static data case. Before applying the wavelet multi-resolution analysis on kinematic SINS data, it should be ensured that the decomposition or denoising process does not remove any actual motion information [5].

2.3. Threshold Algorithm Analysis

Thresholding operations are applied on the coefficients of the wavelet and wavelet packet transforms, and generally can be classified into Hard-thresholding and Soft-thresholding as described in [6].

The choice of threshold is crucial to the quality of the denoising process and should be made carefully. In thresholding process, the coefficients smaller than threshold value (Thrv) are judged negligible or noise.

In this work six methods are used to select the value of Thrv. These methods are:

- **First Method**:One possibility of selecting the threshold by estimating the standard deviation σ_x of the noise at each scale. We take into account that threshold values have to be different on each scale. The threshold in this case can be calculated as [7]:

$$Thrv = \frac{\sigma^2}{\sigma_x} \tag{7}$$

where:

σ^2 : Noise Power for noisy signal.

σ_x : deviation for the detail coefficients.

- **Second Method:** here, we select the threshold value Thrv by estimating the standard deviation σ_x of the noise at each scale. We take into account that the threshold value is different on each scale. In this case, the value can be calculated as [8]-[9]:

$$Thrv = \frac{\sqrt{2\sigma_x^2 \ln N}}{2} \tag{8}$$

where:

σ_x : Standard deviation for each detail coefficient

N : the sequence length.

- **Third Method:** Selection using principle of stein's unbiased risk estimate (SURE) (MatLab code "rigrsure").
- **Fourth Method:** Selection using fixed form threshold (MatLab code "sqtwolog").

- **Fifth Method:** Selection using a mixture of the third and fourth selection rules (MatLab code "heursure").

- **Sixth Method:** Selection using minimax principle (MatLab code "minimaxi").

2.4. Selection of the Appropriate Filter

The wavelet transform has a flexible feature of using a variety of filters that differ by their coefficients. In this work, all types of the wavelet filters will be applied to original GPS/INS data in order to show the best possible filter for each component of position and velocity.

3 Results and analysis

In this section we present and discuss the results obtain using the proposed method.

Table 1. Multi-resolution algorithm applied to obtain INS/GPS standard deviation error.

Types of data	Components	Direction	INS Error	Estimated INS /GPS error	LOD
Best INS data with original GPS data	Position (m)	X-axis	1.3394	1.3960	10
		Y-axis	1.2884	1.3561	11
		Z-axis	0.0418	0.0659	15
	Velocity (m/s)	North	0.0023	0.0032	19
		East	0.0618	0.0863	17
		Down	0.0016	0.0020	22
Worst INS data with original GPS data	Position (m)	X-axis	92.2495	91.5469	2
		Y-axis	238.8248	229.4452	1
		Z-axis	113.4915	115.2453	1
	Velocity (m/s)	North	2.8079	4.0708	10
		East	7.8505	10.7992	11
		Down	4.1089	5.8583	11

Table 1 illustrates the results of applying MRA algorithm to original GPS measurements. It reflects also, two cases of INS data (best and worst case) selected from the eight types of IMUs errors described in [6]. The purpose of this is to investigate and study various IMUs specification where the accuracy level of IMUs can be categorized as high (strategic grade), medium (navigation grade), and low (tactical grade) at the end of the vehicle's journey.

We have concluded that, it is unnecessary to increase the order of LOD because the features of the INS/GPS-error will disappear. In other words, it can't be used to model the INS error because the resulting error (INS/GPS error) will not equal the desired INS-error. On the other hand it must be mentioned that the main GPS errors can be denoised by wavelet unlike the INS error where some of the error can be eliminated by wavelet denoising (optimal low pass filtering).

Such error is called short-term error while the other part of the INS error is called long-term error. The latter is reduced by GPS/INS integration, which is accomplished by the multi-resolution algorithm described previously.

The output of the multi-resolution for the GPS and INS is subtracted to obtain the INS/GPS error which can’t be eliminated by the denoising algorithm.

After using all types of the wavelet filters, Table 2 shows the best possible filters considered for each component of position and velocity using original GPS measurements (with best and worst INS data).

In this work, soft thresholding technique was adopted to remove some of the details part noise while keeping the original features of the signal and improving the signal to noise ratio (SNR). In Table 3, we illustrate the SNR and Root Mean Square Error (RMSE) for the two cases of INS data (worst and best) and GPS data before applying thresholding. To compare with the results obtained after using thresholding, we consider Table 4. It illustrates the SNR and RMSE after applying thresholding using the six methods mentioned previously for the INS and GPS position and velocity components where the labeled values in Tables II which represent best results, used to determine the performance of thresholding selection rule.

Table 2. Results of using different types of wavelet filters for best and worst INS data

Filter	Standard Deviation for 1st LOD of INS/GPS Error					
	Position (m)			Velocity (m/s)		
	X-axis	Y-axis	Z-axis	North	East	Down
Best	Db4	Db9	Db6	Bior5.5	Bior2.2	Coif2
Worst	Db10	Bior2.2	Db4	Bior5.5	Bior2.2	Coif2

The Root Mean Square Error (RMSE) and Signal to Noise Ratio (SNR) can be evaluated using equation (9) and (10), respectively [5]:

- Root Mean Square Error (RMSE).

$$RMSE = \sqrt{\frac{1}{N}\sum_{n=0}^{N-1}\left[\hat{X} - X(n)\right]^2} \tag{9}$$

- Signal to Noise Ratio (SNR) in decibels.

$$SNR(X,\hat{X}) = 10\,Log_{10}\frac{\sum_{n=0}^{N-1}\left|X(n)\right|^2}{\sum_{n=0}^{N-1}\left|\hat{X} - X(n)\right|^2} \tag{10}$$

where

$\hat{X}(n)$: the processed signal after removing noise.

$X(n)$: the signal without any noise.

From these results it is conclude that:

1. The fourth method of rule selection is better for all the data types of position components.
2. The first or third selection rule is very efficient for all data types of velocity components.
3. The effect of using optimum selection rule to specify threshold value is very important for position and velocity. Moreover, it has a great impact on velocity denoising performance.

Table 3. Performance comparison before using thresholding technique

			Types of data				
			Best INS data	Worst INS data	Original GPS data	Newton GPS data	Spline GPS data
Position	X-axis	SNR (dB)	131.9482	96.2806	104.2492	20.4457	100.5992
		RMSE (m)	0.0530	3.2307	0.2706	1.3801e+004	0.5458
	Y-axis	SNR (dB)	132.1107	86.6690	104.0634	22.8794	100.4140
		RMSE (m)	0.0509	9.5015	0.2706	1.0278e+004	0.5458
	Z-axis	SNR (dB)	160.8400	92.1371	103.4611	90.1873	99.8115
		RMSE (m)	0.0017	4.5635	0.2706	2.5311	0.5458
Velocity	North	SNR (dB)	81.2745	26.0203	-27.4347	-13.0925	-34.0355
		RMSE (m/s)	4.621e-005	0.0190	16.4407	134.8124	58.7688
	East	SNR (dB)	67.8059	25.9519	-10.4431	-13.1550	-15.9587
		RMSE (m/s)	0.0025	0.3094	16.4407	129.1399	58.7688
	Down	SNR (dB)	116.1809	43.4004	2.5397	-12.6000	-3.1375
		RMSE (m/s)	9.4905e-006	0.1108	16.4407	163.4179	58.7688

Table 4. Performance comparison after using thresholding technique

DATA	Threshold selection Method	Position						Velocity					
		X-axis		Y-axis		Z-axis		North		East		Down	
		SNR	RMSE	SNR	RMSE	SNR	RMSE	SNR	RMSE	SNR	RMSE	SNR	RMSE
Best case of INS data	First	124.585	0.116	118.490	0.203	121.027	0.157	103.597	3.75e-6	78.4684	9.06e-5	251.834	2.6e-13
	Second	129.346	0.078	129.076	0.078	129.911	0.076	14.183	0.045	7.5405	0.0011	44.449	0.003
	Third	129.346	0.078	129.076	0.078	129.911	0.076	103.597	3.77e-6	78.4684	9.02e-5	251.834	2.6e-13
	Fourth	136.922	0.044	136.626	0.044	137.876	0.043	18.018	0.0431	10.0063	0.0011	48.203	0.011
	Fifth	124.585	0.116	118.490	0.203	121.027	0.157	13.829	0.0455	-1.3695	0.0011	44.156	0.003
	Sixth	124.585	0.116	118.490	0.203	121.027	0.157	13.829	0.0455	17.1651	0.0011	44.156	0.003
Worst case of INS data	First	124.506	0.118	118.299	0.209	120.948	0.158	110.272	4.5e-7	78.5042	9.36e-5	251.606	1.8e-12
	Second	129.192	0.080	128.926	0.080	128.831	0.083	14.451	0.0387	7.8809	0.0011	44.266	0.031
	Third	129.192	0.080	128.926	0.080	128.831	0.083	110.272	4.5e-7	78.5042	9.36e-5	251.606	1.8e-12
	Fourth	136.635	0.046	136.349	0.046	136.132	0.050	18.449	0.035	10.4677	0.0011	48.098	0.011
	Fifth	124.506	0.118	118.299	0.209	120.948	0.158	13.820	0.038	-1.3703	0.0011	43.684	0.031
	Sixth	124.506	0.118	118.299	0.209	120.948	0.158	13.820	0.038	17.1251	0.0011	43.684	0.031
Original data of GPS	First	106.722	0.188	106.538	0.276	105.951	0.229	47.243	0.001	71.3298	1.16e-4	53.157	0.002
	Second	122.369	0.031	122.183	0.031	121.588	0.031	36.107	0.006	37.0163	0.006	43.291	0.006
	Third	122.369	0.031	122.183	0.031	121.588	0.031	47.243	0.001	71.3298	1.167	53.157	0.002
	Fourth	126.873	0.018	126.687	0.018	126.091	0.018	40.804	0.003	41.7005	0.003	47.996	0.003
	Fifth	106.722	0.188	106.538	0.276	105.951	0.229	3.853	1.672	4.3523	1.66	7.562	1.757
	Sixth	106.722	0.188	106.538	0.276	105.951	0.229	0.275	1.806	0.8894	1.761	4.926	1.757

Also, we have applied many levels of decomposition and it was found that the appropriate LOD varies for each component of position and velocity. This depends on the INS/GPS error, which is nearly equal to the real INS-error. Figure 2 shows respectively the signals of error in X, Y and Z before (left column) and after (right coloumn) thresholding, for worst INS and original GPS data.
The corresponding type of filter to the lowest standard deviation of INS/GPS error value is the perfect filter to be used. It should be mentioned that all calculations in this section we performed for first level of decomposition. Our objective is to choose the best filter to be used for each component of position and velocity

4 Conclusion

The following points summarize the main conclusions of this paper:
1. Wavelet analysis was beneficial in filtering out some of the noise components and disturbances that may exist at the INS and GPS outputs.
2. Wavelet MRA algorithm provides the advantage of comparing the INS and GPS position and velocity components at different levels of resolution.

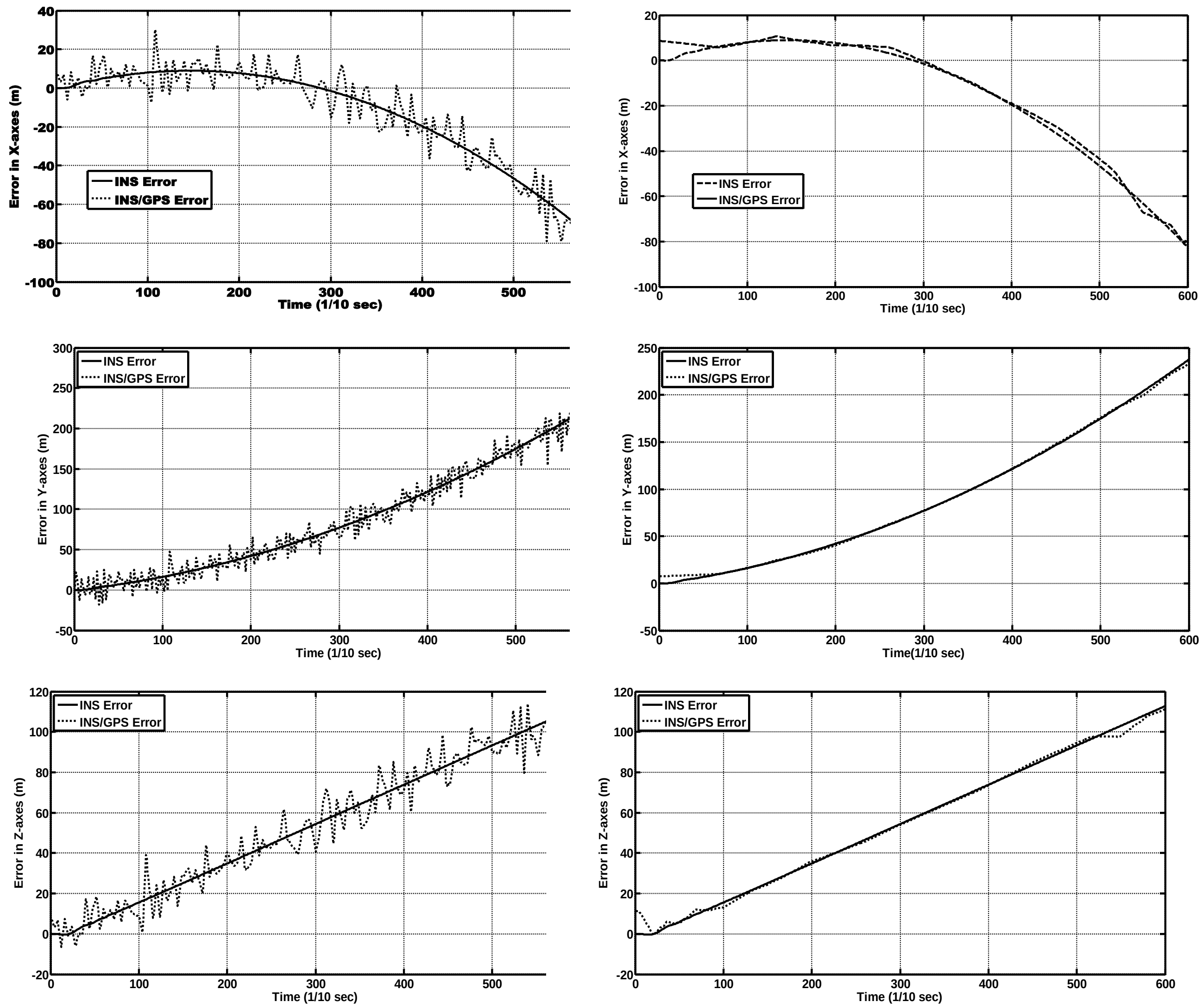

Fig.2. Comparison between INS and INS/GPS Error position, respectively, in X-axis, Y-axis, and Z-axis for worst INS and original GPS data before (left) and after (right) using thresholding.

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