Numerical Iteration Scheme to Generate a Response-Spectrum-Compatible Accelerogram

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Abstract: - The input motion for seismic analysis of building structures is generally specified in design codes in terms of response spectrum. Therefore, real or simulated time histories for ground motions whose response spectrum matches approximately the design spectrum is desired. Among the various techniques currently available for generating spectrum compatible time histories it is possible to distinguish two groups. One group consists of those methods in which the Fourier spectrum of a trial time history is modified to produce desired modifications in the response spectrum. This group could be referred as frequency domain iteration methods. The second group corresponds to those methods in which the solution proceeds from an initial time history approximation, which is modified in successive steps resulting in uniform convergence to the target spectrum. This second group could be called time domain iteration methods. In this research, iteration in time domain method is used to generate a spectrum compatible time history record. This method has application criteria that must be satisfied, and this fact limits its application in some way. In this research, those difficulties are minimized by a proper implementation of an iteration scheme which ensures stability and convergence of the numerical calculation. The modified time histories closely resemble characteristics and appearance of the corresponding unmodified earthquake time histories and, at the same time, their spectrum closely match the target spectrum.

Key-Words: - Simulated earthquake record, response spectrum, vibration frequency

1 Introduction

For time history analysis on earthquake response of building structures real recorded earthquake signal are used. However, there are many types of earthquakes like near source earthquakes and far-source earthquakes which produce earthquakes record of different characteristics. On the other hand, in the seismic analysis of building structures, the characteristics of input motion are specified in terms of a design response spectrum. In this case is desirable to have an earthquake record which response spectrum resembles the design spectrum. Therefore, real or simulated time histories for ground motions whose response spectrum matches approximately the design spectrum is desired.

Among the various techniques currently available for generating spectrum compatible time histories it is possible to distinguish two groups. One group consists of those methods in which the Fourier spectrum of a trial time history is modified to produce desired modifications in the response spectrum. This group could be referred as frequency domain iteration methods. In this case the iteration for approximation is done considering that velocity response spectrum is similar as the Fourier spectrum of velocity.

The second group for generating spectrum compatible time histories corresponds to those methods in which the solution proceeds from an initial time history approximation, which is modified in successive steps resulting in uniform convergence to the target spectrum. This second group could be called time domain iteration methods.

In this research, iteration in time domain method is used to generate a spectrum compatible time history. Real earthquake records are used as seed to initiate the iteration and the record is modified in successively steps until error fall under some limits of tolerance. Therefore, this method has some application criteria that must be satisfied, and this fact limits its application in some way. In this research, those difficulties are minimized by a proper implementation of an iteration scheme which ensures stability and convergence of the numerical calculation.

2 Basic Theory

To evaluate the performance of building structures during earthquakes, response spectrum represents an important tool. A response spectrum is a plot of the maximum responses or peaks (acceleration, velocity or displacement) considering that buildings are single degree of freedom systems subjected to an input motion. These maximum responses are plotted versus the variable frequencies of the one degree of freedom system. Response spectrum is calculated for a given viscous damping. Then, using a response spectrum for a specify earthquake it is possible to estimate the maximum response of a structure according its natural frequency of vibration. Response spectrum are also used in assessing the response of multi-degree of freedom systems. In this case, modal analysis is performed to identify the modes, and the response in each mode can be picked from the response spectrum. Then, these peak responses are combined to estimate a total response. On the other hand, in case of seismic design a design spectrum is specify to estimate maximum responses of building. To perform a time-history analysis instead of use an arbitrary earthquake record, a spectrum-compatible input motion is desired. Therefore, it is necessary to generate a simulated earthquake to satisfy that condition.

To adjust a spectrum to a target one, iterations in time domain or iterations in frequency domain are used. However, the records generated using the frequency domain approach exhibit unrealistic strong motion characteristics and a tendency to induce less inelastic demand in the structures. In the case of methodologies based on modification of actual earthquake records, it was found that when the seed records are selected based on their initial compatibility with the target spectrum, the resultant compatible records not only better retain the original characteristic of the records but the variability in the structural response is reduced.

The method for generating synthetic time histories matching response spectrum is based on the observation that the time at which the spectral response of a time history occurs, is not perturbed by a small adjustment made on the time history. Based on this observation, the small change in the response acceleration value, $\delta R(\omega_i)$, at spectral frequency ω_i can be related to the small adjustment $\delta a(t)$ on the initial input acceleration time history, $a(t)$, through the Duhamel's integral:

$$\delta R_i = \delta R(\omega_i) = \int_0^{t_i} \delta a(\tau)\, h_i(t_i - \tau)\, d\tau \qquad (1)$$

where $h_i(t)$ is the acceleration impulse response function for a single-degree-of-freedom oscillator with frequency ω_i, and specified damping ratio; t_i is the time at which the spectral response occurs; and τ is the time lag. The task of adjusting a time history to match the target spectrum is to solve equation (1) for $\delta a(t)$, given $\delta R(\omega_i)$.

For matching a set of target spectral values at N spectral frequencies, the solution of equation (1) can be transformed into the solution of a set of N linear algebraic equations by letting $\delta a(t)$ be a linear combination of a set of N prescribed linearly independent functions, $f_j(t)$, as follows:

$$\delta a(t) = \sum_{j=1}^{N} b_j f_j(t) \qquad (2)$$

in which b_j are unknown constant coefficients to be determined. From equations (1) and (2), the set of linear algebraic equations become

$$\delta R_i = \sum_{j=1}^{N} C_{ij} b_j\, ; \quad C_{ij} = \int_0^{t_i} h_i(t_i - \tau)\, f_j(\tau)\, d\tau \qquad (3)$$

It is obvious from equation (3) that in order to be efficient in computing C_{ij}, the function $f_j(\tau)$ should be prescribed as:

$$f_j(\tau) = h_j(t_j - \tau)\, S(\tau) \qquad (4)$$

in which

$$S(t) = 1 - e^{-ct}\, ; \quad c \gg 0 \qquad (5)$$

The time function, $S(t)$, is introduced in equation (4) for satisfying the requirement that at t = 0 the seismic acceleration input is zero and therefore the correction must also be zero. Then the coefficients C_{ij} are symmetric and can be computed by:

$$C_{ij} = \int_0^{t_i} h_i(t_i - \tau)\, h_j(t_j - \tau)\, S(\tau)\, d\tau\, ; \quad t_i \le t_j \qquad (6)$$

Having computed the coefficient matrix, equation (3) can be solved for b_j by standard linear equation solver; then, the small adjustment $\delta a(t)$ can be obtained from equation (2). The adjusted time history for each iteration, $a_{n+1}(t)$, can be obtained from the time history of previous iteration, $a_n(t)$, by

$$a_{n+1}(t) = a_n(t) + \delta a_n(t) \qquad (7)$$

By repeatedly applying the above iteration scheme, the desired accuracy of matching between the time history spectrum and target spectrum can be achieved.

This basic theory has been presented using acceleration response spectrum, however, similar equations can be deduced using velocity or displacement response spectrum.

2.1 Algorithm Implementation

A computer program was developed and implemented for the application of the described method. To minimize the difficulties in the application of the method that are pointed out by previous researchers, the iteration scheme was improved to ensure convergence and accuracy of results.

The first application criterion is that differences of target and actual responses at given frequencies must be small, since it is the basic assumption of the method. This criterion is established to ensure that the time of occurrence of the spectral response is not perturbed. Therefore, the initial time history should have a spectrum which does not differ so much from the target spectrum. In this research this problem is solved, as a first step, by a proper scaling of the initial time history in such a way that the response spectrum of this scaled time history is closer to the target spectrum. Then instead of correcting the total difference between the actual and target spectra, only half of the difference is corrected in each iteration step, ensuring the convergence of the method.

The second difficulty of the method is the interference of the corrections between two adjacent or consecutive matching frequencies. Therefore, it is recommended that this spacing must be sufficiently large. However, large spacing leads to poor accuracy in the response spectra, since the number of matching points is reduced. To solve this problem, previous researchers recommend an optimal spacing or minimum frequency spacing, which is product of their experiences. In the present research this problem is solved by dividing the number of matching points (N) in m groups, and each group consists of n = N/m frequencies with large spacing, as is shown in Fig. 1. These groups overlapped each other such that the whole range of the matching points are covered. This scheme ensures the stability of the method and the computational effort is also reduced since the number of operations to solve m times a system of n×n equations is smaller than the number of operation to solve a system of N×N equations.

The compatibility criteria, recommended in the "Guideline for Evaluating Design Earthquake Ground Motion for Buildings" (Building Research Institute and the Building Center of Japan) are as follows.

1) Minimum spectral ratio

$$\varepsilon_{mim} = \left|\frac{S_{psv}(T_i, 0.05)}{DS_{psv}(T_i, 0.05)}\right| \geq 0.85 \qquad (8)$$

where S_{psv} is the calculated response spectrum and DS_{psv} is the design response spectrum (target spectrum) and T_i is the period at which compatibility is calculated. The value of 0.05 represents the damping ratio for which spectrum is calculated, in this case 5 %.

2) Coefficient of variation

$$\text{v} = \sqrt{\frac{\sum(\varepsilon_i - 1)^2}{N}} \leq 0.05 \qquad (9)$$

where

$$\varepsilon_i = \frac{S_{psv}(T_i, 0.05)}{DS_{psv}(T_i, 0.05)} \qquad (10)$$

3) Deviation of average spectral ratio from 1.0

$$|1 - \varepsilon_{ave}| \leq 0.02 \qquad (11)$$

Where

$$\varepsilon_{ave} = \frac{\sum \varepsilon_i}{N} \qquad (12)$$

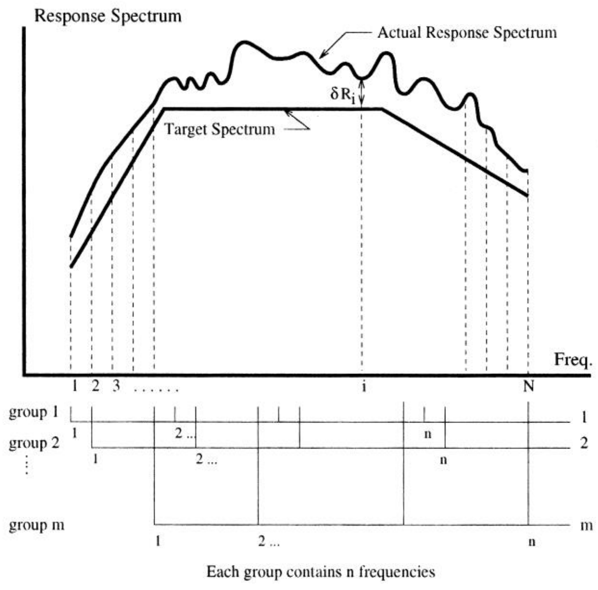

Fig. 1 Grouping of matching frequencies

3 Simulated Earthquake

The simulated earthquakes were obtained using El Centro Earthquake record, modifying this record by successive iterations to match its response spectrum with the corresponding design spectrum. The 1940 El Centro earthquake (or 1940 Imperial Valley earthquake) occurred at 21:35 Pacific Standard Time on May 18 (05:35 UTC on May 19) in the Imperial Valley in southeastern Southern California, USA. It was the first major earthquake to be recorded by a strong-motion seismograph located next to a fault rupture. The original acceleration time histories (for EW and NS directions) which were used as the initial time history for modification are shown in Fig. 2.

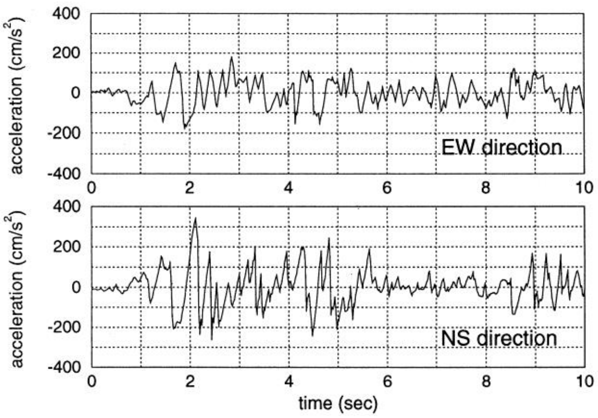

Fig. 2 Original El Centro earthquake waves

The target spectrum is that corresponding to the basic spectrum for design level 1 and for design level 2 which are recommended in the "Guideline for Evaluating Design Earthquake Ground Motion for Buildings" (Building Research Institute and the Building Center of Japan). The recommended damping factor for both levels is 5 %.

The acceleration response spectra for 5% damping are shown and compared with the target spectrum in Fig. 3.

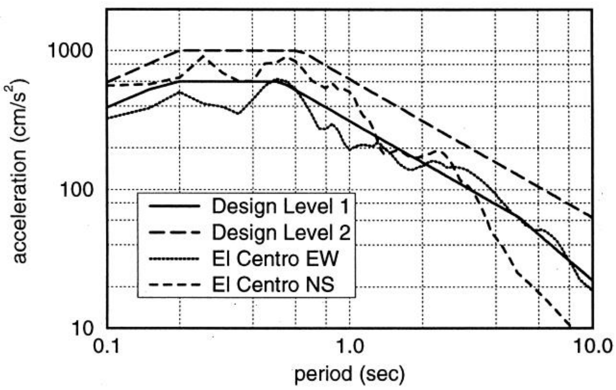

Fig. 3 Target and original response spectra

Using the proposed iteration scheme, modified spectrum-compatible time histories are obtained and they are show in Fig. 4. The original records for EW direction and NS direction show in Fig. 2 have maximum acceleration of 190 cm/s^2 and 342 cm/s^2 respectively. The shape of modified signals, that are show in Figure 4, resembles well the shape of the original records. These modified records present maximum acceleration of the order for 200 cm/s^2 and 400 cm/s^2 for level 1 and level 2.

It can be also observed that the modified earthquakes records have zero values at the beginning and at the end of the wave. This condition is obtained by using a simple trapezoidal envelope function which ensures the zero condition at both extremes of the record.

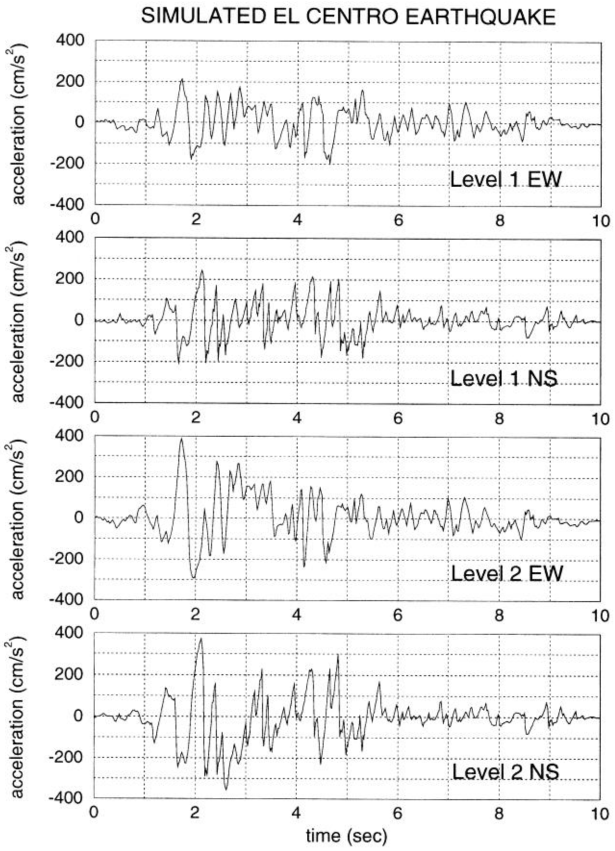

Fig. 4 Modified El Centro waves

Fig. 5 shows the response spectrum of the spectrum-compatible acceleration time history. It can be observed that the modified time histories closely resemble the realistic characteristics and appearance of the corresponding unmodified actual earthquake time histories, at the same time, the modified time history spectrum closely match the target spectrum. The compatibility criteria given by equations (8), (9) and (11) are satisfied. The values of these parameters are also shown in Fig. 5.

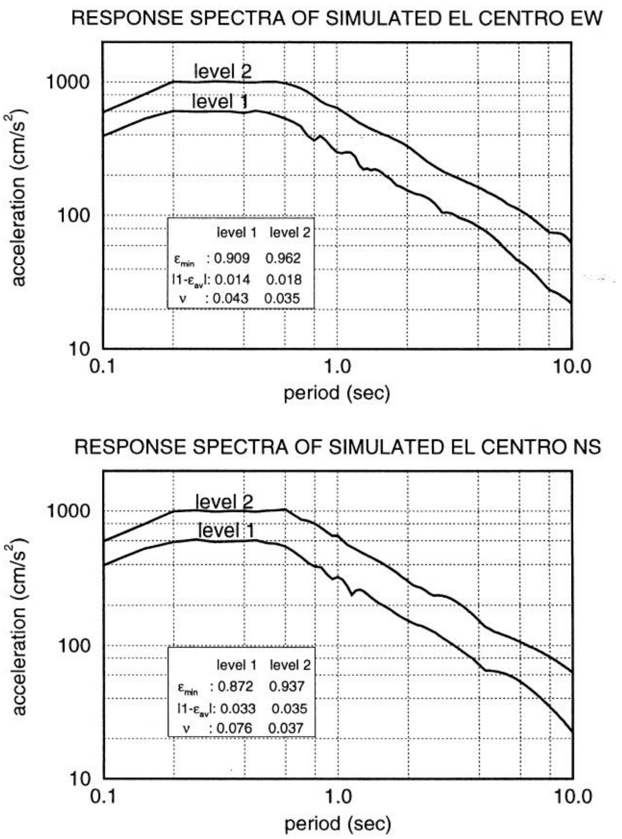

Fig. 5 Response spectra of modified signals

4 Conclusion

Iteration in time domain method is used to generate a spectrum compatible accelerogram. This time history record is generated considering that a small change in the response acceleration value at any spectral frequency can be related to the small adjustment on the initial input acceleration time history through the Duhamel's integral. Then this integral is numerically solved by transforming it into the solution of a set of linear algebraic equations. Since iteration scheme is efficient when the correction in response spectrum is small, it is proposed, as first step, a scaling of the initial time history in such a way that the response spectrum of this scaled time history is closer to the target spectrum. Then instead of correcting the total difference between the actual and target spectrum, only half of the difference is corrected in each iteration step, ensuring the convergence of the method.

To avoid the interference of the corrections between two adjacent or consecutive matching frequencies, matching points are divided into groups, and each group consists of frequencies with large spacing. These groups overlapped each other such that the whole range of the matching points are covered.

The proposed iteration scheme ensures stability and convergence of the numerical calculation to generate an appropriate spectrum compatible acceleration signal.

The proposed methodology was used to generate simulated time history signals using El Centro Earthquake record, modifying this record by successive iterations to match its response spectrum with the corresponding design spectrum. The modified time histories closely resemble characteristics and appearance of the corresponding unmodified earthquake time histories and, at the same time, their spectrum closely match the target spectrum.

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