

# Mathematical Model for Motion Control of Redundant Robot Arms

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*Abstract:* - Motion planning is an essential activity in executing manipulation tasks with robot arms like welding, painting or simple "pick-and-place" operations. A typical requirement for such tasks is to achieve flexibility of the robot arm motion similar to that of a human hand. The objective of the here proposed mathematical model is to satisfy this requirement in path planning a robot arm motion with redundant degrees of freedom. This model provides an efficient procedure for the computation of the motion in the joints that makes the end-effector motion to trace a given geometrical curve with prescribed linear and angular velocity. The novelty of the mathematical model is the clear separation of concerns related to planning the geometrical path in task space on the one hand and on the other hand, the control of motion along this path. This guiding design principle is implemented through vector space methods. It enables a new insight about identifying and processing Jacobian singularities by means of the Continuity principle. The application of the model is enabled through a detailed algorithm for computing the Jacobian, equations for kinematic path control and a heuristic procedure for handling the singularities of the Jacobian. The obtained results can be extended to non- redundant robot arms.

*Key-Words:* - robot motion, kinematics, separation of concerns, redundant robot arm, continuity principle

## 1 Introduction

Robot arms cover a broad range of applications. The list of applications is constantly growing and it ranges from assembly operations in outer space and industry to executing highly specialized activities in surgery [1, 2], for instance. In most cases the robot arm operates as a replacement in an activity usually done by means of a human hand. Research in biomechanics and bionics motivates engineers to provide more degrees of freedom than the dimension of the task space in the design of modern robot arms. Such robot arms are referred to as redundant robot arm.

A robot arm is useful in executing repetitive work tasks. Therefore it is common to preplan and control the motion of its end effector along a path in the task space. The dimension of the task space depends on the specific task. It is represented in terms of a set of parameters specifying the desired position and orientation of the end- effector along the specified path. The solution of the inverse kinematic problem allows to find the motion in the joints that results in a desired motion of the end- effector.

This paper presents a Jacobian based model for path planning the motion of robot arm with redundant degrees of freedom. It is common for this kind

of models to consider motion control implicitly coupled with geometrical characteristics of motion. Such design approach increases the complexity of the model and requires powerful numerical methods and optimization techniques, typically, Singular Value Decomposition or Damped Square Methods [3]. Most often the procedures for the computation of the Jacobian are valid only for a selected robot arm and rely on the symbolic representation of this robot arm model. Heavy numerical methods are employed to identify and avoid robot arm configurations, where the rank of the Jacobian is not full [4].

The novelty of the here proposed model is the clear separation of concerns related to planning the geometrical path in task space on the one hand and on the other hand, the control of motion along this path [5]. This reduces the usual complexity of the Jacobian- based models and allows its implementation in modern computational environments with multithreading. The here proposed model provides computationally efficient procedures for computing the Jacobian and resolving the singularities in motion planning making use of vector space methods.

The paper is organized as follows. The Problem statement and the major mathematical notations are introduced in Section 2. Section 3 presents a computationally efficient algorithm for computing the Ja-

cobian of a robot arm. The major results in this paper are provided in Sections 4 and 5. In the last section we summarize the obtained results.

## 2 Problem Formulation

The robot arm is assembled by  $n$  links connected serially with revolute or prismatic joints in an open loop kinematic scheme (Fig. 1). The last link is the end-effector. Usually it is of type gripper holding a tool that is selected in accordance with a work task. The work task is assigned with respect to the base frame  $O_o(x_o, y_o, z_o)$  in terms of geometrical and motion parameters. The geometrical parameters comprise the orientation of the tool in the end-effector and the position of a characteristic point H of the tool along a geometrical curve. The motion parameters determine the linear speed of a characteristic point H of the tool and the angular speed of rotation of the tool in each point of the geometrical curve. The dimension of the task space is determined by the number  $m$  ( $m \leq 6$ ) of geometrical parameters used to describe the position and the orientation during the execution of the work task.

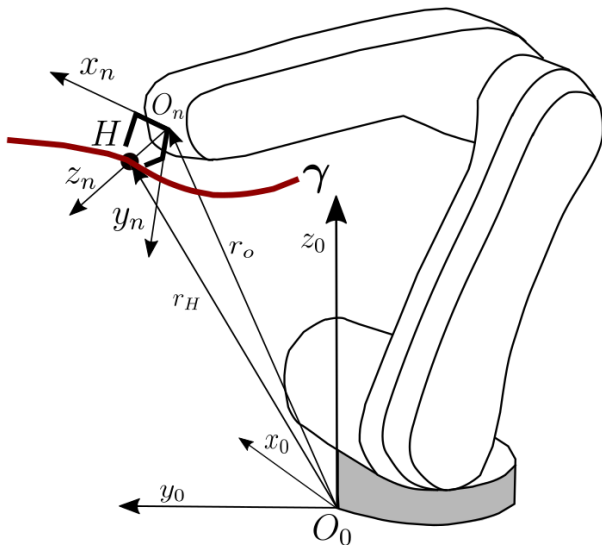


Fig.1 Motion planning with a robot arm.

Denote by  $Int Q_M$  the interior of the set  $Q_M = \{q = (q_1, \dots, q_n): q_i \in [a_i, b_i] \text{ for } i = 1, 2, 3, \dots, n\}$  of configurations of the joint variables. The mapping  $F: Int Q_M \rightarrow W$  defines the forward kinematics of a robot arm with workspace  $W \subset R^m$ . Note, that the workspace is a subset of the task space  $R^m$ .

For clarity, let the geometrical parameters of the work task be represented in  $W$  by a smooth geometrical curve  $\gamma$ , defined parametrically in the interval  $[\lambda_1, \lambda_2]$ . Without loss of generality we assume that the points of  $\gamma$  belong to the workspace  $W$  of the robot arm or in other words,  $\gamma: [\lambda_1, \lambda_2] \rightarrow$

$F(Int Q_M)$ . The motion parameters of the work task are given with respect to the points of  $\gamma$ . The objective is to find laws governing the motion in the joints such that the end-effector motion satisfies the geometrical and motion parameters of the work task. In the beginning we note the existing clear distinction between geometrical and motion parameters in task space. Obviously, the motion parameters can change without the need of changing  $\gamma$ . Once  $\gamma$  is given, the end-effector can execute a variety of different motions along this path. These two groups of parameters reflect two different concerns in executing the work task. It is natural to retain this kind of separation of concerns in the space of configurations  $Int Q_M$  of the robot arm. Then it would be possible to control the motion in the joints in a way similar to controlling motion over a geometric curve. For this reason, we add a  $n + 1$  dimension to the space of configurations, where the parameter  $q_{n+1}$  represents parametrically  $\gamma = \gamma(q_{n+1}), q_{n+1} \in [\lambda_1, \lambda_2]$ .

**Definition.** The set  $Q = Int Q_M \times R$  we refer to as extended space of configurations

The introduction of  $Q$  provides a new interpretation for the inverse kinematic equation. Consider the smooth manifold

$$\mathcal{B} = \{q^* = (q, q_{n+1}): F(q) - \gamma(q_{n+1}) = 0, q^* \in Q\} \quad (1)$$

defined by means of the inverse kinematics equation. It is  $n - m + 1$  dimensional in the general case. Hence, all the motions  $q^*(t) = (q(t), q_{n+1}(t)), t \in [t_1, t_2]$  of the robot arm that satisfy the assigned geometrical parameters of the work task, belong to  $\mathcal{B}$ . Among these motions are also motions that satisfy the motion parameters of the assigned work task.

**Definition.** A smooth motion  $q^*(t) \in \mathcal{B}$  that satisfies given in advance motion parameters of the end-effector motion we refer to as admissible motion of the robot arm.

It is noteworthy, that the introduction of the extended space of configurations enables the separation of concerns in the inverse kinematic problem solution. Similarly to the curve  $\gamma$ , the manifold  $\mathcal{B}$  is a fixed geometrical structure. It is determined by the geometrical structure of the robot arm and the assigned geometrical parameters of the work task. On the other hand, the trajectory of the required robot motion in  $Q$  lies on  $\mathcal{B}$  for any given values of the motion parameters of the work task. This representation allows to control the robot motion over  $\mathcal{B}$  in a similar way we control the motion over  $\gamma$  in task space.

The proposed mathematical model allows to resolve the inverse kinematic problem into two separate types of concerns. The first type of concerns involves modelling the immutable geometrical artefacts in motion control. The second part of the model deals with constructs for motion control. In particular, the separation of concerns guarantees that cyclic motions in task space map cyclic motions in the extended space of configurations.

In the following section we present a uniform model of the robot arm geometrical structure and develop a detailed numerical algorithm for computing the Jacobian on this basis. Next, we make use of the here introduced separation of concerns to build a model for motion control of redundant robot arms subject to given motion parameters. The thus obtained results allow us to address from a new point of view the important issue of singularities in the space of configurations. In contrast to existing approaches, this mathematical model allows to interpret the singularity problem in terms of the Continuity principle [6]. As a result we propose an efficient heuristic procedure for identifying and “passing – through” singularities of the Jacobian during motion control with prescribed geometrical and motion parameters of the work task. This procedure relies on.

### 3 Computation of the Jacobian

The following equation represents the Jacobian-based model for solving the inverse kinematic problem in the extended space of configurations:

$$J(q)\dot{q} - \frac{\partial y}{\partial q_{n+1}}(q_{n+1})\dot{q}_{n+1} = 0 \quad (2)$$

where  $J(q) = D(F)/D(q)$  is the Jacobian matrix of the forward kinematics  $F(q)$  and  $\dot{q}_{n+1}$  is the time derivative of  $q_{n+1}(t)$ . It is common to present the computation of the Jacobian employing the symbolic representation of the forward kinematics  $F(q)$ . The derivation of the symbolic formulas for  $F(q)$  and differentiation of the thus obtained expressions is time-consuming in most cases and it is justified in only in some special cases of robot arm investigation. Here we will develop an algorithm for computing the Jacobian without the knowledge for the forward kinematics  $F(q)$  in symbolic form.

The investigation of a robot arm starts by attaching a frame  $O_i(x_i, y_i, z_i)$ ,  $i = 1, 2, \dots, n$  to each one of the  $n$  links, where the coordinate axes are fixed according to the Denavit- Hartenberg (DH) rules [7]. The mutual disposition of the local frames is defined by a set of fixed geometrical parameters and the values of the joint variables  $q_i$ . This set of fixed geometrical parameters we define in a slightly dif-

ferent way than the one used by the DH notation. It will enable us to introduce a common representation for the displacements in the joints [8].

First we note that axes  $z_i$  and  $z_{i+1}$  are fixed in links  $i$  and  $i + 1$ , respectively (Fig. 2). The fixed geometrical parameters that define the mutual disposition between frames  $i$  and  $i + 1$  for  $i = 1, 2, \dots, n - 1$  are as follows:

- the distance  $l_i$  between axes  $z_i$  and  $z_{i+1}$  measured along the axis  $x_i$ . Without loss of generality we assume  $l_n = 0$ . Corresponds to link length  $a_i$  in the DH notation.
- the distance  $d_{i+1}$  between points  $O_{i,i+1}$  and  $O_{i+1}$  measured along the axis  $z_{i+1}$ . Without loss of generality we assume  $d_n = 0$ . Corresponds to link twist  $d_{i+2}$  in the DH notation.
- the angle  $\alpha_{i+1}$  between axes  $z_{i+1}$  and  $z_i$  measured in the positive direction of axis  $x_i$ . Corresponds to link twist  $\alpha_i$  in the DH notation.
- the angle  $\beta_{i+1}$  between axes  $x_{i+1}$  and  $x_i$  measured in the positive direction of axis  $z_{i+1}$ . In case joint  $i + 1$  is revolute, then angle  $\beta_{i+1}$  coincides with the joint variable  $q_{i+1}$ . Corresponds to link angle  $\theta_{i+2}$  in the DH notation.

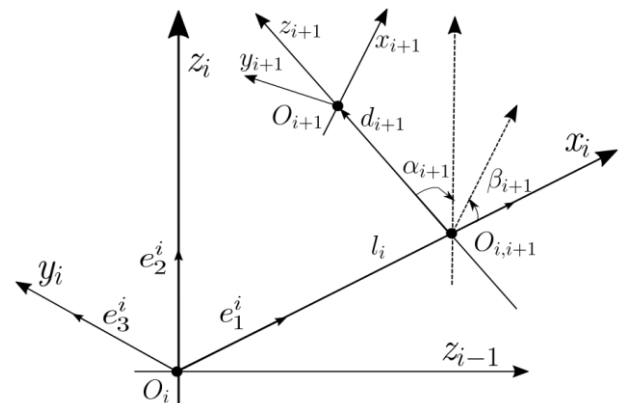


Fig.2 Structure of the local frames in the links.

Then the transformation matrix from frame  $i$  into frame  $i - 1$  can be represented in terms of these notations as follows.

$$G_{i,i-1} = \begin{bmatrix} \cos\beta_i & -\sin\beta_i & 0 \\ \sin\beta_i\cos\alpha_i & \cos\beta_i\cos\alpha_i & -\sin\alpha_i \\ \sin\beta_i\sin\alpha_i & \cos\beta_i\sin\alpha_i & \cos\alpha_i \end{bmatrix} \quad (3)$$

Accordingly, the matrix product

$$T_i = G_{1,0}G_{2,1} \dots G_{i,i-1} \quad (4)$$

defines the transformation of the unit vectors  $e_1^i, e_2^i, e_3^i$  from frame  $O_i(x_i, y_i, z_i)$ ,  $i = 1, 2, \dots, n$  into the base frame  $O_o(x_o, y_o, z_o)$ . The columns of matrix  $T_i$  are the coordinates of the unit vectors  $e_1^i, e_2^i, e_3^i$  of  $O_i(x_i, y_i, z_i)$  in frame  $O_o(x_o, y_o, z_o)$ . It allows us to express the rotations and the transla-

tions that occur along the  $z_i$  axis in case of revolute and prismatic joint in terms of a common notation.

Let  $\mu_i, i = 1, 2, \dots, n$  takes values 0 or 1 for revolute and prismatic joints, correspondingly. Then depending on  $\mu_i$  the following expression

$$z_i = [(1 - \mu_i)d_i + \mu_i q_i]e_3^i \quad (5)$$

determines the distance between points  $O_{i,i+1}$  and  $O_{i+1}$  in case of a revolute and in case of prismatic joint, as well. Similarly, we represent the vector of the angular velocity  $\omega_i$  in joint  $i = 1, 2, \dots, n$  as follows:

$$\omega_i = (1 - \mu_i)\dot{q}_i e_3^i \quad (6)$$

where the scalar function  $\dot{q}_i$  denotes a generalized joint velocity  $i = 1, 2, \dots, n$ .

Denote by  $v$  and  $\omega$  the given motion parameters of the work task motion defined along the path  $\gamma$ . Here  $v$  is the velocity vector of the characteristic point H of the end-effector and  $\omega$  is the angular velocity vector of the tool in the end effector. Using the notations on Figure 1 we obtain the following representation for  $v$

$$v = \omega \times \rho + \dot{r}_o \quad (7)$$

For shortness  $\rho$  denotes  $O_n H$  and  $\dot{r}_o$  is the velocity vector of point  $O_n$ . Here we note that  $\dot{r}_o$  is the sum of the velocities in the prismatic joints along their unit vector  $e_3^i$ . Hence, from (5) we obtain:

$$\dot{r}_o = \sum_{i=1}^n \mu_i \dot{q}_i e_3^i \quad (8)$$

Similarly, vector  $\omega$  is the vector sum of the angular velocities  $\omega_i$  (6):

$$\omega = \sum_{i=1}^n (1 - \mu_i)\dot{q}_i e_3^i \quad (9)$$

Replacing (8) and (9) in (7) yields the following general expression for the vector  $v$

$$v = \sum_{i=1}^n [(1 - \mu_i)e_3^i \times \rho + \mu_i e_3^i] \dot{q}_i \quad (10)$$

Consider the general case when the task space has the maximum dimension of 6 (three parameters for position and three parameters for orientation). Then  $\frac{\partial \gamma}{\partial q_{n+1}}(q_{n+1}) \dot{q}_{n+1} = (v, \omega)^T$  and from (2), (9) and (10) it follows that column  $i = 1, 2, 3, \dots, n$  of the Jacobian can be represented by the following vector row

$$\left[ \left( (1 - \mu_i)e_3^i \times \rho + \mu_i e_3^i \right)^T, \left( (1 - \mu_i)e_3^i \right)^T \right] \quad (11)$$

Note, that vectors  $e_3^i$  and  $\rho$  must be in the same frame in order to compute  $e_3^i \times \rho, i = 1, 2, 3, \dots, n$ . Therefore, the coordinates of vector  $\rho$  must be transformed from  $O_n(x_n, y_n, z_n)$  down to frame  $O_i(x_i, y_i, z_i)$ , before computing the vector product  $e_3^i \times \rho$ .

The following pseudocode explains in details how to compute the Jacobian making use of the

expression (11). For clarity of explanation the pseudocode sequentially computes the rows  $J_i, i = 1, 2, 3, \dots, n$  of the transposed Jacobian, starting from row  $n$ .

- S1. Compute the transformation matrixes  $T_i, i = 1, 2, 3, \dots, n$  defined in (4).
- S2. Compute vector  $h = T_n \rho$ , where  $\rho$  is defined in (7).
- S3. Set a counter  $i = n$ .
- S4. Compute the six coordinates of vector

$$jcbRow = \begin{bmatrix} \left( (1 - \mu_i)T_{i,3} \times h + \mu_i T_{i,3} \right)^T \\ \left( (1 - \mu_i)T_{i,3} \right)^T \end{bmatrix}$$

where  $T_{i,3}$  is the third column of matrix  $T_i$ .

- S5. Store  $jcbRow$  as row  $i$  of the transpose of the Jacobian.
- S6. Set  $i = i - 1$ .
- S7. If  $i$  is equal to zero go to Step S11. Else continue with step 8.
- S8. Compute the scalar  $z$
- S9. Compute vector  $h = h + l_i T_{i,1} + z T_{i+1,3}$ , where  $T_{i,1}$  is the first column of matrix  $T_i$  and  $l_i$  is the distance parameter.
- S10. Go to step S4.
- S11. End of algorithm.

This algorithm is applicable for both redundant and non redundant robot arms ( $n = m$ ). It can be easily adapted for computing the Jacobian in cases when the work task is defined in terms of a subset of the coordinates of the motion parameters  $v$  and  $\omega$ .

## 4 Motion Control

The separation of concerns introduced in the problem statement enables the computation of an admissible motion matching given motion criteria. The definition (1) of the smooth manifold  $\mathcal{B}$  implies that the vector  $\dot{q}^* = (\dot{q}^T \dot{q}_{n+1})^T$  belongs to the tangent vector space at an arbitrarily selected point  $q^* = (q, q_{n+1})$  of  $\mathcal{B}$ . Furthermore, an arbitrary vector  $u^* = (u^T, u_{n+1})^T$  that satisfies

$$J(q)u - \frac{\partial \gamma}{\partial q_{n+1}}(q_{n+1})u_{n+1} = 0 \quad (12)$$

also belongs to the tangent vector space at point  $q^* = (q, q_{n+1})$  of  $\mathcal{B}$ . This way we can solve the inverse kinematic problem by controlling the tangent vector field over  $\mathcal{B}$  with respect to one or another quality criterion. The manifold  $\mathcal{B}$  and the function controlling the vector field reflect the geometry concern and the motion concern, respectively.

Equation (12) allows to define explicitly the control function for the vector field of  $\mathcal{B}$ . For a fixed value of  $u_{n+1}$  the solutions  $u$  of this equation represent a linear manifold

$$L_{u_{n+1}} = u^{(0)} + \mathfrak{N}(J) \quad (13)$$

where  $u^{(0)}$  is an arbitrary particular solution of (12) and  $\mathfrak{N}(J)$  is the linear subspace  $\mathfrak{N}(J) = \{x \in R^n : Jx = 0\}$  defined at any point of the smooth manifold

$$\mathcal{B}_{q_{n+1}} = \{q \in \text{Int } Q_M : F(q) - \gamma(q_{n+1}) = 0\} \quad (14)$$

The manifold  $\mathcal{B}_{q_{n+1}}$  (Fig. 3) consists of all the configurations  $q$  of the robot arm for which the end effector position retains position  $\gamma(q_{n+1})$ ,  $q_{n+1} \in [\lambda_1, \lambda_2]$ . Therefore  $\mathcal{B}_{q_{n+1}}$  is known also in the literature as Null- space. This is a special feature of redundant robot arms ( $n > m$ ). In particular,  $\mathfrak{N}(J)$  contains just the zero vector in case of non-redundant robot arms ( $n = m$ ).

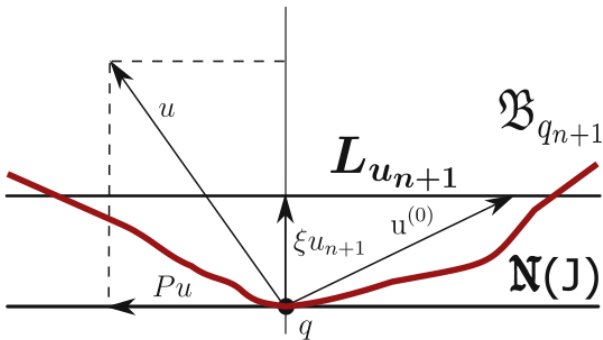


Fig. 3. The null space of a redundant robot arm.

Let us define the operator

$$P = E - J^+ J \quad (15)$$

where  $E$  is the unity  $n \times n$  matrix and  $J^+$  is the Moore- Penrose pseudoinverse matrix of the Jacobian  $J$ . From the properties of the pseudoinverse matrix [9] it follows that for a given  $q$  the operator  $P(q)$  is a symmetric  $n \times n$  matrix projecting  $R^n$  onto  $\mathfrak{N}(J(q))$  in (13) i.e.  $Pu \in \mathfrak{N}(J(q))$  for an arbitrary  $u \in R^n$ .

Let us consider the vector  $\xi$  defined as follows:

$$\xi = J^+ \frac{\partial \gamma}{\partial (q_{n+1})} \quad (16)$$

The definition of the operator  $P$  implies that the product  $P\xi$  is the zero vector. Furthermore, the vector  $u^{(0)} = \xi u_{n+1}$  is a particular nonzero solution of (12). In summary, vector  $\xi u_{n+1}$  is orthogonal to the linear space  $\mathfrak{N}(J)$  and  $\xi u_{n+1} \in L_{u_{n+1}}$ . Moreover,  $\xi u_{n+1}$  is the vector with the minimum norm possessing this property.

Thus, we obtain the following common representation for any element of  $L_{u_{n+1}}$  in an arbitrary point  $q$  of the manifold  $\mathcal{B}_{q_{n+1}}$  (Fig. 3):

$$L_{u_{n+1}} = Pu + \xi u_{n+1} \quad (17)$$

Equation (16) represents an arbitrary solution of the inverse kinematic problem for redundant robot arms (12). Hence, the vector  $\dot{q}^* = (\dot{q}^T \dot{q}_{n+1})^T$  in equation (12) can be represented explicitly in the following concise form

$$\dot{q}^* = K \hat{U}_t, t \in [t_0, t_1] \quad (18)$$

where  $\hat{U}_t = (u^T, u_{n+1})^T : \mathcal{B} \rightarrow R^{n+1}$  is a given continuous, piecewise smooth vector function and  $K$  is a  $(n + 1) \times (n + 1)$  block matrix in the form

$$K = \begin{bmatrix} P & \xi \\ 0 & 1 \end{bmatrix} \quad (19)$$

The vector function  $\hat{U}_t$  is projected on the tangent vector space on  $\mathcal{B}$  by means of matrix  $K$ . Hence, the selection of  $\hat{U}_t$  allows us to control the tangent vector field on  $\mathcal{B}$  and in particular, the robot arm motion. Therefore we refer to  $\hat{U}_t$  as kinematic path control. It is defined by the quality criteria for computing an optimal robot motion. The kinematic path control and the manifold  $\mathcal{B}$  are the analogues for the motion parameters and the geometric path in task space, respectively.

## 5 Processing singularities

The application of Jacobian- based models in inverse kinematics including the here proposed mathematical model relies on the assumption that the rank of the Jacobian is equal to the dimension of the task space for all the configurations of joint variables. In practice, there exist configurations  $q$ , where  $\text{rank } J(q) < m$ .

These so called singularities of the Jacobian appear to be a major obstacle for the wide implementation of this kind of models. Typical approaches for resolving singularities attempt to “avoid” them by employing singularity analysis based on a symbolic representation of the Jacobian [10], Singular Value decomposition [7] or manipulability resolution [11].

Here we will present a solution to this problem in the context of the here proposed mathematical model for motion control of redundant robot arms. The first part of this solution provides an efficient criterion to identify a singularity and the second part deals with processing of the identified singularity.

Consider the matrix  $P$  defined in (15). It is a symmetric  $n \times n$  matrix and  $\text{rank } P(q) = n - m$  for all  $q$ , where  $\text{rank } J(q) = m$ . On the other hand, the trace of  $P$  is the sum of its eigenvalues. This trace is invariant with respect to the change of basis. Now from the Spectral theorem in algebra [12] it follows that  $P$  has exactly  $n - m$  orthonormal eigenvectors. The sum of their corresponding eigenvalues is equal to  $n - m$ . This proves the following proposition.

**Proposition.** The Jacobian has full rank, if and only if, the trace of matrix  $P$  is equal to  $n - m$ .

This proposition provides an efficient numerical criterion to identify a singularity of the Jacobian. It is enough to establish when the trace of matrix  $P$  becomes larger than  $n - m$ .

**Corollary.** A configuration  $q$  is a singularity of the Jacobian, if and only if, the trace of matrix  $P$  is larger than  $n - m$ .

Consider now an admissible motion  $q^*(t) = (q(t), q_{n+1}(t))$  of the robot arm. Denote by  $\gamma(q_{n+1})$  the position of the end-effector in work space  $W$ . Then all the configurations  $q$  that allow the end-effector to remain in this position belong to  $\mathcal{B}_{q_{n+1}}$  (Fig. 3). The dimension of manifold  $\mathcal{B}_{q_{n+1}}$  is  $n - m$  for all  $q$  where  $\text{rank } J(q) = m$ . Assume that among these configurations there is a singularity  $q^{(s)}$ . Then  $\text{rank } P(q^{(s)}) > n - m$  and the tangent vector field on  $\mathcal{B}$  at  $q^*(t) = (q^{(s)}, q_{n+1})$  is undefined in a singular configuration. It appears that the singularity  $q^{(s)}$  separates one set of non-singular configurations from another set of non-singular configurations. In terms of the Principle of continuity [6] we can interpret the singularity as a "gate" between sets of non-singular configurations on  $\mathcal{B}_{q_{n+1}}$ . This allows a redundant robot arm to reconfigure itself with continuous internal movements without changing the position and orientation of the end-effector even when "passing through" a singular configuration like  $q^{(s)}$  for instance. Therefore we can replace the tangent vector field at  $(q^{(s)}, q_{n+1})$  in  $\mathcal{B}$  with the tangent vector field at point  $(q, q_{n+1})$  in  $\mathcal{B}$  selected in a sufficiently small neighbourhood of  $(q^{(s)}, q_{n+1})$  such that  $\text{rank } J(q) = m$ .

Consider now a small variation in the configuration  $\delta q^{(s)} = (\delta q^{(s)}, \delta q_{n+1})$ . The end-effector movement has a limited set of independent manipulability directions in a singularity. Recall that  $q^*(t)$  is admissible. Then the variation  $\delta q_{n+1}$  of the parameter of  $\gamma$  produces variations both in task space and in the extended space of configurations, where the manipulability directions satisfy the given geometrical and motion parameters of motion control.

These findings allow us to propose the following heuristic procedure for "passing through" singularities in motion control in the context of the here considered problem statement. When the integration procedure of equation (18) reaches a Jacobian singularity, continue the integration procedure by computing the right side of equation (18) for the last computed non-singular configuration.

## 6 Conclusion

This paper presents a Jacobian-based mathematical model for motion control of redundant robot arms subject to given geometrical and motion parameters. A guiding design principle in this model is the separation of concerns addressing geometry and motion in the inverse kinematic problem. This approach enables to reduce the inherent complexity in existing models and formulate results suitable for software implementation. Such results are efficient procedures for computing the Jacobian and the kinematic control. The Principle of continuity provides a new insight about resolving singularities through a heuristic procedure. In view of this principle non-redundant robot arms are a particular case of redundant robot arms. Therefore the obtained results can be extended for non-redundant robot arms, as well.

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