Robust Control of a Seismic Excited Building Facing to Structured Uncertainties

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Abstract: Robust active controllers, designed to seismic excited building structure facing to parametric uncertainties (variations in mass, stiffness, damping coefficients…etc) were studied these last years with recent focus to non parametric ones (time delay, actuator saturation…etc). This study presents an evaluation of the robustness to variations in the model parameters of a three floors seismic exited structure (stiffness and damping coefficients) and modelling errors in the actuator dynamics of a robust controller designed on the base of the μ-synthesis approach chosen for its ability to directly incorporate performance and robustness objectives into multivariable control design. To further check the controller designed, we perform simulations using state feedback control and a seismic excitation source modelled by Kanai Tajimi filter attacked by a white noise. The resulting controller achieves closely similar performances (level of vibrations attenuation) in nominal and worst case of uncertainties variation while accounting for actuator limit and sensor noise considerations and presents a great benefit of costing low energy. At last, as the μ-synthesis generates controllers with too high order, a balanced realization method has been used to reduce the designed controller order without degrading its performance.

Key-Words: Robust active control, Seismic excited building, μ-Synthesis, Structured uncertainties,

1 Introduction

The research in structural robust active control field considerably increased these last two decades since the earthquake protection of structures represents a serious problem that looks for a way more and more efficient to dissipate the energy due to the seismic load for raising their safety and performance [1-2]. These last years, several papers investigate this domain employing number of robust control strategies (LQG, H_∞, adaptive control,...etc) and provided potentially significant reduction of the structure response to a seismic load in presence of uncertainties [3-5]. This being, the major concern of researchers is to insure that the designed controller will really be efficient in the real conditions of implementation. That implies to take into account in the control design procedure maximum of practical considerations which often occur as modelling errors (in coefficients of structure model), sensors noise, time delay [4-5] induced by the application of the actuator force (mechanical, hydraulic), physical actuator limits...etc. In fact, the modelling errors resulting from the evaluation of the parameters of the structural model (parametric uncertainties) or errors in the dynamics of the actuator and structure models (non parametric uncertainties) should be considered simultaneously. Incorporate all these aspects in the robustness and performance objectives design help to well evaluate the usefulness of a controller in a practical point of view [1]. Hence, the motivation of this work is to design a robust controller based on the structured singular value μ technique to a three-degree of freedom structure as its control law allow to account explicitly of robustness to dynamic and parametric uncertainties, in the present case modelling errors in the actuator dynamic and variations in stiffness and damping coefficients in the structure model. In addition to robustness considerations, the μ-synthesis problem formulation poses the performance objectives as minimizing the norm of weight transfer functions [6]. The quantification of these uncertainties as well as the interpretation of the performance requirements in appropriate weighting functions represent the critical step in the problem formulation of this multivariable control technique [7]. To evaluate the designed controller, we perform simulations on a three floors building where an active bracing system (ABS) actuator is attached to the first floor [3] to attenuate the seismic excitation modelled by Kanai Tajimi filter attacked by a white noise [8]. The resulting controller achieves closely similar performance in nominal and worst case of uncertainties variation (vibrations attenuation) while accounting for actuator limit and sensor noise considerations and present a great benefit of costing low energy. However, the μ-synthesis technique presents the convenient of generating too high order controllers. By using a model reduction method as the balanced realisation algorithm, a lower order controller can be provided to achieve the same level of robust performance.

The rest of this paper is organized as follows: Section 2 presents the basis of the μ-synthesis control approach in designing a controller by the mean of the H_∞ norm computation of certain transfer matrices in one hand and a μ-analysis, a very useful tool to evaluate the robustness in performance and stability in another hand. Section 3 gives the building motion equation under a state space formulation. Section 4 illustrates the formulation of a civil engineering problem to achieve robust stability and performance as considering the μ-synthesis control approach. An illustrative example is simulated and discussed in section 5 to provide the applicability of the designed μ-controller in terms of achieving good response (vibration attenuation) facing to a seismic load simulated by Kanai Tajimi filter attacked by a white noise, in presence of parametric and dynamic uncertainties (real and complex) acting simultaneously at different points of the structure control loop. At last, as the μ-synthesis generates controllers with too high order, a balanced realisation method is used to find a lower order controller without degrading the performance of the initial one. Finally, conclusions are presented in section 6 concerning the good results obtained in the nominal and worst case variations of the uncertainties proving the robustness of the designed μ-controller and the usefulness of the chosen control approach to face to structured uncertainties.

2 µ-Synthesis Control Theory

The μ-synthesis is a robust control technique since the formulation of its law explicitly takes into account the uncertainties in the system. It is based on the combination of the application of the μ-analysis which is in fact a stability robustness criterion, in one hand and H_∞ bound computations of some weight functions representing the performance specifications and limitations in another hand. The μ-synthesis is derived so that the algorithm called "*D-K* iterations" constitutes the effective mean to the computation of the controller. To truly model uncertainties, one should distinguish them from their origins. This leads to classify them into: parametric (real) uncertainty and dynamic (frequency dependent) uncertainty [9-10]. The first class encompasses the uncertainties in system physical parameters such as: time constant, natural frequency...etc, in the structure case it is question of the mass, stiffness and damper. While the second one refers to those met by simplifying a complex model or in system neglected high frequency dynamics. There is an interesting formulation of uncertainties called structured uncertainties and formed by the combination of a block of real uncertainties and another of dynamic uncertainties. This formulation is particularly adopted in μ-synthesis theory because it is possible to make a link between the uncertainties and the physical system by means of the computation of a very useful tool called structured singular value μ which can be used in both stability analysis and synthesis of the control law [10].

Most uncertain systems are represented in the Linear Fractional Transformation (LFT) representation to extract explicitly the structured uncertainties (parametric and dynamic) from the system in a set called $\Delta(s)$ block. The LFT representation of figure (1) is adopted because it offers a prior knowledge of the studied process, indeed it reflects how the block $\Delta(s)$ of structured uncertainties affects the transfer matrix $M(s)$ representing the feedback structure of the system.

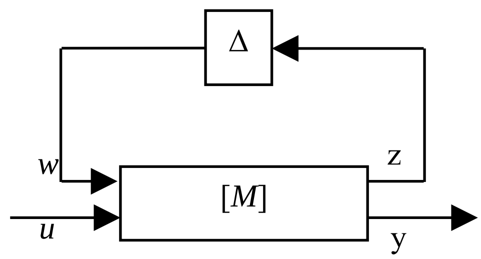

Fig. 1 Upper LFT representation performance

An efficient way to study the robustness in stability and performance of any uncertain system consists to calculate its structured singular value (ssv) μ defined as

$$\mu_\Delta(M) = \frac{1}{\min_{\Delta\in\boldsymbol{\Delta}}\{\bar{\sigma}(\Delta(s)) : \det(I - M(s)\Delta(s)) = 0\}} \qquad (1)$$

where,

$\Delta(s)$ represents the structured uncertainties block; $\bar{\sigma}$is the maximum singular value; $M(s)$ is the transfer matrix of the feedback structure system and $s(=j\omega)$ is the Laplace variable in the frequency domain.

This is called μ-analysis and in fact, is just a Multi Input Multi Output (MIMO) extension of the Nyquist Stability Criterion in a Robust Stability Condition [11]. Hence, the system of figure (1) is internally stable for any structured Δ , with $\|\Delta\|_\infty \leq 1$, if and only if

$$\det\left(I - M(j\omega)\Delta(j\omega)\right) \neq 0, \forall\omega\forall\Delta\in \Delta, \|\Delta\|_\infty \leq 1 \Leftrightarrow \mu\left(M(j\omega)\right) := \frac{1}{\underset{\Delta\in\Delta}{min}\{\bar{\sigma}\left(\Delta(j\omega)\right):\det\left(I-M(j\omega)\Delta(j\omega)\right)=0\}} < 1, \forall\omega \quad (2)$$

This being, the concept of the μ-synthesis technique tends to use the ssv μ_Δ to design a controller for the feedback uncertain system of figure (2) below, by combining the H_∞ control design and the diagonal scaling techniques from the ssv μ. Therefore, the parametric and non parametric uncertainties, modelled by the Δ block, are shaped under a specific form called *structured uncertainties* and are hence bounded as H_∞ norm of uncertain gains representing different perturbations acting in one or more inputs or outputs of the controlled system. Though the μ *analysis* procedure, the robustness (both stability and performance) of the system with the uncertainties hence structured are analyzed using the ssv μ. The control design will directly seek to minimize the μ value which represents the principal actor in the μ-*synthesis approach* Eq.3 [9-10].

For $M(s) = \mathcal{F}_\ell(N, K)$ we can find a matrix D such as

$$\mu_\Delta(M) \leq \min_{D\in\mathcal{D}} \bar{\sigma}(DMD^{-1}) \quad (3)$$

with

$$\mathcal{D} = \left\{ \begin{bmatrix} d_1 I_{r1} & & & & \\ & \ddots & & & \\ & & d_s\, I_{rs} & & \\ & & & \mathcal{D}_1 & & \\ & & & & \ddots & \\ & & & & & \mathcal{D}_F \end{bmatrix} \begin{matrix} D_i \in \mathbb{C}^{r_i \times r_i} \\ D_i = D_i^* > 0 \\ d_j \in \mathbb{R} \\ d_j > 0 \end{matrix} \right\},$$

N is the uncertain system, and K is the controller.

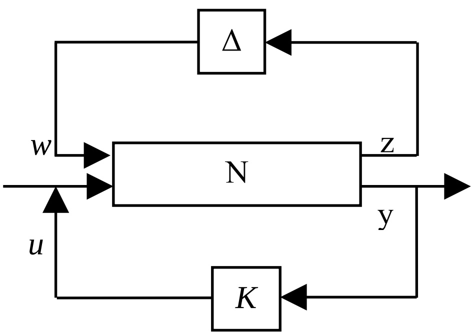

Fig. 2 Structure for controller synthesis

The μ-synthesis problem consists on designing a nominally stabilizing controller K solving the following minimization problem, which represents the μ upper bound over frequencies

$$\min_K \sup_\omega \min_{D(j\omega)\in\mathcal{D}} \bar{\sigma}'\left(D\mathcal{F}_\ell(N, K)(j\omega)D^{-1}\right) \quad (4)$$

This problem is equivalent to a minimizing scaled H_∞ norm and can be expressed as

$$\min_K \min_{D, D^{-1}\epsilon\mathcal{R}H_\infty, D(j\omega)\epsilon\mathcal{D}} \|D\mathcal{F}_\ell(N, K)D^{-1}\|_\infty \quad (5)$$

The Eq.(5) is no convex in regard to D and K, and the μ-*synthesis* is executed by means of an alternating optimization algorithm to find a local minimum. Thus, if D is fixed we are in face of an H_∞ optimal controller design. However, if the controller K is fixed, the Eq.5 becomes convex problem and the μ-*analysis* procedure can be derived. The computation of the described controller is conducted by executing the so called *D-K iteration* algorithm using the MATLAB toolbox [11]. If the obtained minimized μ-value is less than one, then K is a *robust stabilizing controller*.

3 Formulation of the Building Design

In the present study, the system in question consists on a linear building structure subject to horizontal seismic excitation, governed by the equation of motion that can be written as

$$M\ddot{y}(t) + C\dot{y}(t) + \dot{K}y(t) = Hu(t) + E\ddot{u}_g(t) \quad (6)$$

where,

$y(t) = [x_1(t)\, x_2(t) \ldots x_i(t)]^T$ with $x_i(t)$ is the n[th] floor relative displacement with respect to ground; $\dot{y}(t)$ and $\ddot{y}(t)$ are the velocity and acceleration respectively;

$u(t) = [u_1(t)\, u_2(t) \ldots u_i(t)]^T$, $u_i(t)$ is the r^{th} control force at the r^{th} floor, $\ddot{u_g}(t)$ is the seismic acceleration; $M, C, K \in \mathbb{R}^{nxn}$ are the mass, damping, and stiffness matrices of the structure, respectively. $H \in \mathbb{R}^{nxr}$ gives the location of the r controllers, and E is a vector denoting the influence of the external excitation namely the seismic solicitation.

The location of the device control and the seismic influence are represented by $H = [1\ 0\ 0]^T$ and $E = [1\ 1\ 1]^T$.

To express the equation (6) into the state space we use the state vector $x(t) = [y(t) \quad \dot{y}(t)]$, [11-7].

$$\dot{x}(t) = Ax(t) + Bu(t) + B_w w(t) \quad (7)$$

where, A, B and B_w are defined as follows

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, B_w = -\begin{bmatrix} 0 \\ M^{-1}H \end{bmatrix} ,$$

and,

$$B_w = -\begin{bmatrix} 0 \\ M^{-1}E \end{bmatrix},$$

For the numerical simulations, we consider a three-story building structure of one degree of freedom per floor, resulting in a total of a three degree of freedom as it is represented in figure (4). This model is chosen because it was the subject of an experimental verification and active control has been applied to in [3]. The structure is subjected to horizontal seismic excitation producing a maximum energy deformation at the first floor, where an active bracing system is connected between the ground and this floor to procure the active force assuring seismic protection. The structure is a building of rectangular shape with a floor area of 4.5m x 3m resulting in a total height of 9 m (3m for each story), the masses from the bottom to top are 1000 kg, the stiffness are also identical for all floors and are equal to 1407 KN /m. The damping in the isolation system is so chosen to provide a damping ratio about 1.5% in each floor; this corresponds to damping coefficients of 1470 N.m/s^2.The three natural frequencies are 7.5, 22.5 and 37.5 rd/s for each mode respectively.

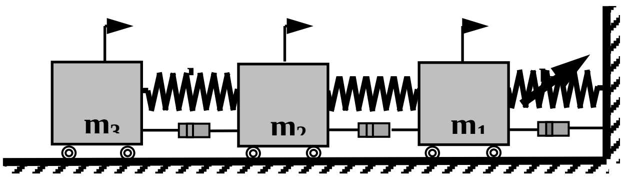

Fig. 3 A three mass-spring-damper system

4 Formulation of the Synthesis Design Problem

The example consists of three floors building simulated by a three mass-spring-damper system. The parametric uncertainties concern the coefficients of mass (20%), damper (30%) and stiffness (40%) of the second floor of the structure model. Ever variation on these parameters affects directly the two other floors and may produce an inaccurate control force. This force is provided by the actuator and represents the active component which consists on an Active Bracing System (*ABS*) applied between the building base and the first floor.

As the displacement measurements of the three floors are not so expensive to achieve than in the last years, they are used in this example for the control feedback.

The control objectives can be summarized as shown in figure (4), with two external sources of disturbances:

- W_{dist}: seismic excitation filter, so called Kanai Tajimi [11-12] whose output has the frequency peak corresponding to the maximum energy in a set of near fault earthquakes, and input is white noise excitation. Hence f_2 represents the disturbance signal modelled as a normalized signal d shaped by a weighting transfer function W_{dist} [8].

$$W_{dist} = \frac{\sqrt{S_0}.(2.z_g.s+w_g^2)}{s^2+2.z_g.s+w_g^2} \qquad (8)$$

where,

w_g = 37.3 rd/s and z_g,= 0.3 and $S_0 = \frac{0.03*z_g}{\left(\pi*w_g*\left(4*z_g^2+1\right)\right)}$

with w_g is the natural frequency, z_g is the damping ratio, and S_0 is a gain coefficient of the filter

- W_n *: s*ensor noise on displacement measurements (control feedback) W_n. This noise is modelled as constant of 0.01. That reflects a reduction of the effect of the sensor noise on the measurements by a rate of 1/100. In a more realistic design with practical considerations, W_n would be frequency dependent to model the noise spectrum of the displacement sensors; however constant weight has the advantage of reducing the number of states in the weighted open loop model of the system.

$$W_n = 0.01 \qquad (9)$$

The model inputs are the seismic disturbance, f_2 and actuator force, f_1. The model outputs are the displacements of the three floors z_i avec i =1:3. The objective is to regulate the fundamental mode response by trying to use weight transfer functions with lowest order because it increases directly the order of the controller. A first order filter W_p is chosen to weight the outputs (displacements z_i of the floors).

$$W_p = \alpha.\frac{a}{s+a} = 80.\frac{8.5}{s+8.5} \qquad (10)$$

with a = 8.5 rd/s determines the roll of frequency.

The bracing actuator used for active reinforcing of the structure is represented by G_{act}, a first order transfer function modelling the nominal actuator dynamic.

$$G_{act} = \frac{K_{act}}{T_{act}.s+1} \qquad (11)$$

with

$K_{act} = a_2$, $T_{act} = c/a_1.k$ and $a_1 = 10$, $a_2 = 20$, $k = 1500$, $c = 30000$.

G_{act} approximates the physical actuator dynamic and the variations between this model and the physical device can be represented as a family of actuator models by considering respectively 10% and 20% errors on K_{act} and T_{act}. These uncertainties are treated as neglected dynamic and modelled as multiplicative uncertainty shape by the weighting function W_{unc} and is obtained by a graphical trial/error approximation.

$$W_{unc} = \frac{0.38s - 0.5475}{s + 5.475} \qquad (12)$$

To capture the limit on the actuator command magnitude we pick the constant weighting function W_{act}, corresponding to (+/-1000N) actuator force limit

$$W_{act} = 0.001 \qquad (13)$$

The control objectives can then be reinterpreted as a disturbance rejection goal, where the impact of the signals of the seism and noise on the structure is to be minimized. Then, the actual controller problem design that corresponds to the diagram of figure 2 can be formulated as:

- The controller measures the noisy displacements z_{1n}, z_{2n}, z_{3n} of the three floors and applies the control force f_1 via the actuator of G_{act} transfer function
- The actuator command is penalized by a factor of 0.001 at all frequencies through the constant weight $W_{act}= 10^{-3}$.
- The seismic disturbance is modelled by Kanai Tajimi second order filter attacked by a white noise dist and shaped by W_{dist} [15]. The normalised disturbance signal f_2 is applied at the system input.
- The performance objective is to attenuate the disturbances by a factor of 80 below the frequency 8.5 rd/s and resumed in W_p.

5 Simulations and results

Using the Matlab software computing [13], we developed a controller on the base of μ synthesis theory with displacement measurements feedback to actively control the described structure. This technique is able to treat the real case of a structure model affected by parametric and dynamic uncertainties commonly named structured uncertainties.

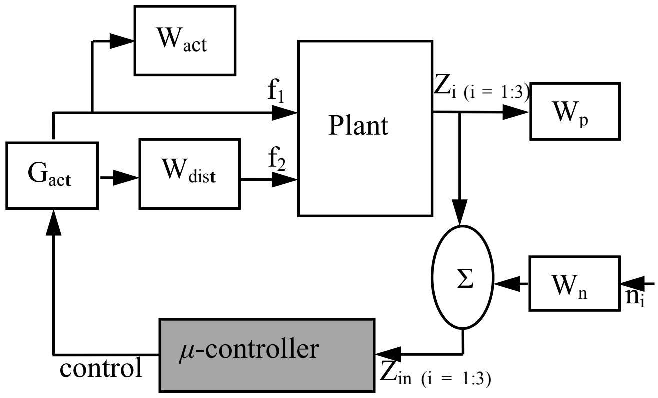

Fig. 4 A diagram of the structure model interconnection with the weighted functions (μ- control design objectives)

Thus, in our example we treat particularly the parametric uncertainties in the coefficients k_2, c_2 and m_2. The second type of uncertainty is inherent to the nature of the actuator. The ABS actuator produces a mechanical force that introduces a non negligible dynamic, which is included in our design to better perform the control of the structure. This being, the dynamic uncertainty is coming from the possible variations in the actuator model which is treated as input multiplicative error modelled by W_{unc}.

Commonly, the performance objective in the case of structural active control is to ensure a good compromise among reducing the response of the structure while limiting the control effort produced to this aim. The μ-based controller obtained, is checked to verify its efficiency to reach the desired objective. That tends to determine how large the μ value of the closed loop gain of the transfer from disturbance-to-model outputs (earthquake-to-floors displacement and acceleration) can get for the specified structure uncertainties and then, verify that the μ bound is less than one to attest of the controller robustness. The same checking process is applied to the gain of the closed loop transfer from disturbance- to- controller output (earthquake-to-command) which relates to actuator control effort. Figures (6) and (7) show the time domain representation of the first floor inter-story drift and acceleration of the structure responses in the open loop (without μ-control) and closed loop (with μ-control) respectively. We can easily observe from these figures that the structure responses to seismic load are significantly reduced by the μ-controller; it is about 77% for the inter-story drift with a mu value reaching 0.5 a nd 41% for the accelerations with mu equal to 1.02 w hich means that all the performance objectives are achieved. Note that, as a rule of thumb, when μ is near to 1, as it is obtained

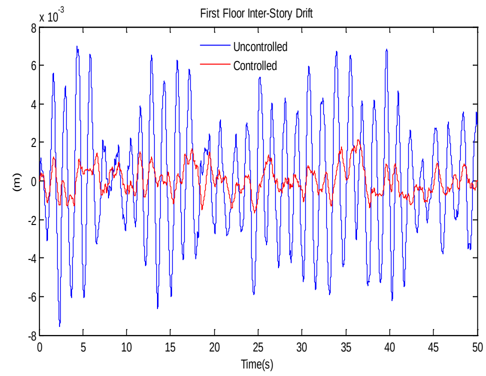

Fig. .5 Time domain representation of the first floor inter-story drift before and after control.

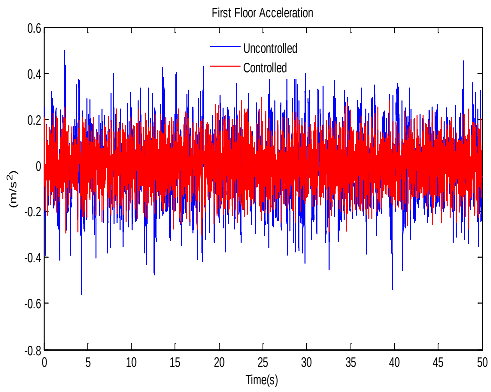

Fig. 6 Time domain representation of the first floor acceleration before and after control.

in the present case, the desired and effective closed loop bandwidths match closed.

For comparison purposes, the structural root mean square (*RMS)* quantities before and after control of the inter-story drifts, and accelerations of the three floors when the system uncertainties are varied to them nominal values are represented in figure (7). We can easily observe from these evaluations (figures of time domain and *RMS* values) that the structure responses to seismic load are significantly attenuated by the μ controller.

The figure (8) shows the time domain response of the first floor inter-story drift in the nominal case and worst case scenario of parameters variation.
The controller achieves closely similar performance in nominal and worst case which traduces a good maintaining of performances even in presence of severe degradation in the structure model.

For more detailed evaluation, the μ-value corresponding to how large the gain from disturbance-to-output norm can get for the specified system uncertainties is computed. As μ is equal to 1.4, it is steel not far from 1.2 that confirms the robustness of the designed controller.

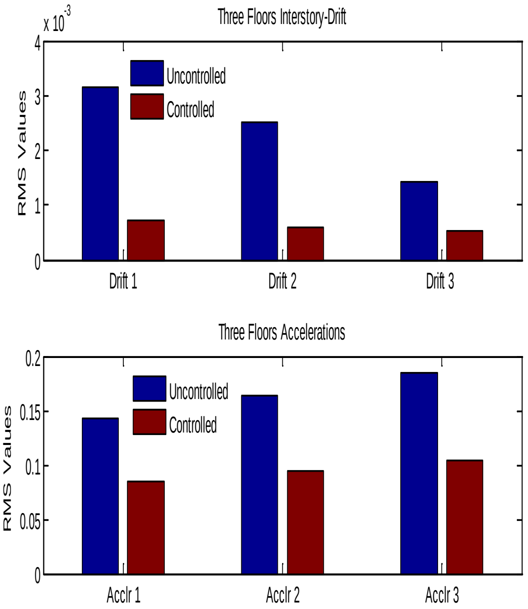

Fig.7 RMS values of the floors inter6story drifts and accelerations before and after control

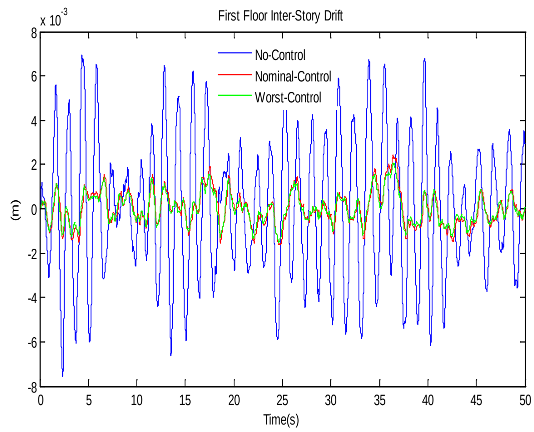

Fig.8 Time domain representation of the first floor inter-story drift response in the nominal case (Nominal-Control) and worst case (Worst-Control) of uncertainties variation

The μ-controller is also checked on the base of the energy produced to reach the desired performance, to this aim in figure (9) we plot the control effort in time domain for nominal uncertainties and we observe a low cost control reaching a maximum of 549 N.

In practical point of view, the μ-synthesis technique presents the convenient of generating too high order controllers that can be avoided by using a model reduction realisation to find a lower order controller that achieves the same level of robust performance. Hence, a balanced realisation method is employed to look for a lower order controller without degrading the performance of the initial one. The figure (10) shows the robust performance as a function of the μ-controller order, where it is easily shown that from order equal to 15 (order 13 provides also closely similar performances), the reduced controller is able to reach the performance objectives.

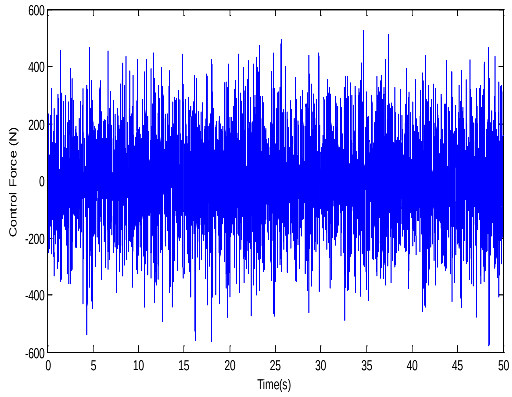

Fig.9 Control effort produced by μ-controller

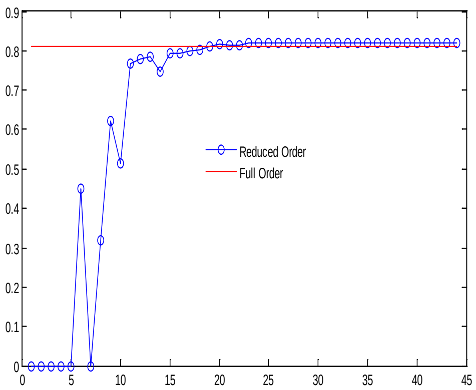

Fig.10 Approximation of the reduced controllers order by comparing their robust performance margin.

6 Conclusion

This article is devoted to the performance evaluation of a robust control law, known as μ-synthesis, to actively control the response of a civil structure consisting on three stories building exposed to seismic effect and submitted to mixed uncertainties (parametric and dynamic). The obtained simulations results, allow emphasizing the following aspects of the μ-synthesis that further its application in active control structure:

- This control is adapted to closely match the desired performances and compromises though adapted weighting functions.
- The parametric and dynamic uncertainties are shaped in a structured form to derive the stability robustness.
- Similar good vibrations attenuation is obtained in the nominal and worst case uncertainties variation.
- This approach shows a great advantage of costing low control energy comparing to others robust control already performed on structures while providing significant vibrations attenuation (70% - 78%).

However, the experimental tests are the principal criterion for evaluating the usefulness of an approach. They shall be carried out in a future work to verify the actual simulations results and the efficiency of the designed µ-controller.

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