Passive Control by Pendulum TMD

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Abstract: The theory behind small amplitude movement of a single pendulum is stated, followed by the construction of a simple example of a coupled pendulum with two degrees of freedom including the unidirectional movement of the structure and the pendulum angle in relation to vertical. These models and the finite elements software SAP 2000 are also applied to a single and to a multiple degree of freedom one bay-shear frame building. The application of the pendulum TMD to control the building vibrations is validated by numerical modelling. An additional case with a more complex model was also studied, comprising the secondary reaction wall of a transmission tower testing station whose dynamic behavior under wind action is benefited from placement of a pendulum TMD. The numerical modelling of the studied cases indicates that the application of pendulum TMD's is worthwhile in civil engineering structures.

Key-Words: TMD's, Pendulum, Vibration Control, Transmission Tower Testing

1 Introduction

The interest to control the dynamic responses of civil engineering structures is found in various aspects, from the rehabilitation of existing buildings enabling them to withstand new levels of demand in service limit states under wind action, to the guarantee not to collapse of fundamental structures operating in a city under the occurrence of major earthquakes.

Continental Portugal is a medium range seismic risk region with higher risk in the Lisbon metropolitan area and in southern region of Algarve. It is known that the potential consequences of a single major earthquake in the Lisbon zone, would cause thousands of human deaths and human injured casualties; as well as property damage in housing and in part of lifeline infrastructure, with property damage in the order of 60% of internal gross revenue.

It is therefore of paramount importance to develop all the effort to minimize the human and property damage caused by earthquakes in a society. In the building sector such developments would lead to less human casualties and to inherent material savings in the construction of new structures or in the rehabilitation of old buildings or existing structures (even if guaranteeing the resistance and the stability of such structural systems by devices not included in the original classical design of building structures (Sousa [1]).

Such control of the dynamic responses can be embodied in various ways including acting on the characteristics of the structure, changing its natural frequency and rigidity, although this change is often difficult to achieve due to architectural and material constraints. Another current and feasible way to achieve structural vibration control, is the introduction of devices forming external control systems under two main types of systems: passive control systems and active control systems; other variations of these are the semi-active systems and the hybrid systems.

Passive control systems and techniques use devices that do not require external energy source to operate them. Current examples of these passive systems are the base isolation devices, tuned mass dampers (TMD) and tuned liquid dampers (TLD).

The base isolation devices are directed only to control of dynamic response in the event of an earthquake, and the operating principle of these devices is based upon a frequency shift. In fact the fundamental frequency of structure vibration is increased through the introduction of blocks of large vertical stiffness and reduced horizontal stiffness, at the connections of the structure to the foundation.

In the case of TMDs typically is used a secondary mass (which may be a concrete block or even a water tank) attached to the main frame by means of springs and small additional buffers or dampers, giving the overall system the required dissipative capacity for the structure to have a satisfactory response when subjected to dynamic actions.

The TLDs are a particular case of the TMDs in which the liquid is used to conceal all the features necessary for the secondary structure, such as mass and damping capacity through viscous damping action.

The active damping systems seeks to impose forces or displacements in strategic points of the structure according to measurements made in real time or according to a history of measurements. This control system re-quires a system of sensors, signal processing, control strategy and a system that can trigger responses in real time, that is requires a major source of external power. The semi-active control systems combine the main advantages of active and passive systems but require less amount of outside energy, and can even run on batteries resources, which makes them very effective in the event of general power distribution faults and cut-offs. Hybrid systems consist in most cases on the simultaneous operation of passive and active systems.

So this work focuses primarily on the operation of TMDs, and in particular in the case of pendulum TMDs, wherein the TMD is materialized in a body (usually spherical) attached to the end of a wire, forming a flywheel pendulum fixed to the structure. The main feature of the pendulum is its oscillation frequency that depends entirely on the length of wire (when the wire is of negligible weight and without rotational stiffness), which often requires excessively large lengths of wire. If it is impossible to implement the required length leading to an optimal tuning, it is possible to achieve positive results for some cases, although the pendulum is not working at its possible full efficiency.

2 Theoretical background and pendulum equations

2.1 Background on TMDs for SDOF systems

Den Hartog [2] was among the first mechanical engineering researchers that ad-dressed the control of vibrations of a primary major structure by using a secondary structure (TMD), whose existence would affect and reduce the dynamic amplification factor of the primary structure. When the primary system has no damping, the optimal parameters of the secondary TMD structure are such that the optimal frequency ratio αopt between the natural frequency of the secondary structure (TMD) and the natural frequency of the primary structure is given by

$$\alpha_{opt} = \frac{1}{1+\mu} \quad (1)$$

where μ is the mass ratio between the mass of the secondary structure (TMD) and the primary structure to be controlled.

Accordingly, a good estimate of the optimal critical damping factor ζ_{opt} required for the secondary structure (TMD) also based upon the philosophy of minimizing the dynamic amplification factor of the primary structure, is given by

$$\zeta_{opt} = \sqrt{\frac{3\mu}{8(1+\mu)^3}} \quad (2)$$

The structures in reality always have some damping, however small it may be. In the case of buildings, bridges and large structures, it is desirable that they should have a considerable damping, since the dynamic loads that they are subjected to usually vary significantly in the range of excitation frequencies, and there could be cases of quite disturbing resonances.

For the cases where the primary systems have some damping, and providing that the critical damping factor of the primary structure is below 1%, solutions for the optimal secondary structure (TMD) parameters are still possible based upon the philosophy of minimizing the dynamic amplification factor of the primary structure.

In the general case when the critical damping factor of the primary structure is above 1%, either empirical formulations should be used with minor approximation errors, or alternatively use the following non-linear graphs or abacus (Figures 1 and 2) of the numerical solutions that were obtained with expression of the dynamic amplification factor of the primary structure (Barros et al. [3]).

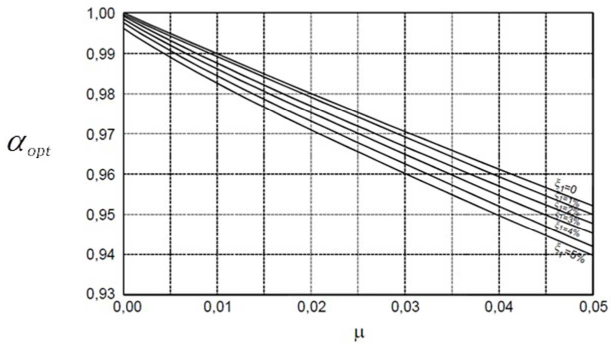


Fig. 1. Non-linear curves for obtaining the optimal frequency ratio (Barros et al. [3])

The later non-linear abacus constitute the most adequate and practical method for determining the optimal parameters of the secondary structure (TMD) from the selected mass ratio μ and the given critical damping factor of the primary structure ζ_1.

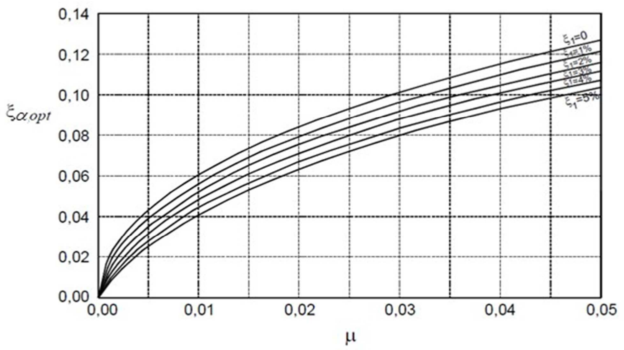


Fig. 2. Curves for obtaining the optimal critical damping factor (Barros et al. [3])

It should be beard in mind that the tuning sensitivity of the frequency ratio values is much higher than the one for the critical damping factor values; this implies that a greater care is needed in determining the value of the stiffness of the secondary system.

2.2 Background on TMDs for MDOF systems, with one degree equivalencies

The tuning of a TMD according to the process described above is only valid for single degree of freedom systems (SDOF); but as it is known, most civil engineering structures have multiple degrees of freedom (MDOF). Through a modal analysis it is possible to obtain various systems of one degree of freedom and therefore it is possible to tune a TMD to each of these systems SDOF; on doing so, it would be necessary to determine the values of mass, stiffness and damping corresponding to each mode.

But to evaluate these modal quantities for each vibration mode, it is needed to determine the configuration of the vibration mode which is possible only arbitrating a component of the vibration mode to which correspond infinite configurations with same proportionalities between components. This myriad of possible solutions leads also to infinite number of possible values for the modal mass, modal stiffness and modal damping. Therefore it should be adopted as a criterion for standardization of these modal settings, by normalizing the modes by setting a unit value in the component for the degree of freedom corresponding to the implementation of the TMD [1] [3].

Taking the example of a multi-story building wherein the first mode is dominant in the wind response to lateral actions, studies by Kwok and Samali [4] showed that the vibration modes controlled by TMDs experience significant decreases in their response contributions and that the vibration of higher order modes are not affected.

Using the same example of a multi-story building now under the influence of seismic action, studies by Wirsching and Campbell [5] (to minimize the response to the first mode through the design of the optimal parameters of TMDs for a building of one, five and ten stories) showed that: (i) the optimal value of the stiffness of the TMD becomes less and less sensitive to changes in the structural damping and in the mass ratio, as the number of stories increases; (ii) the optimum value of the critical damping factor for the TMD, is not sensitive to the value of structural damping of the primary structure, even in case of buildings with just one floor [4].

2.3 Background on Pendulum TMDs for MDOF systems

Pendulum TMD are quite specific in relation to the classic common TMD, since they are not connected by springs to the primary main system; therefore the pendulum stiffness cannot be determined in a classical way, as this depends on the length of wire or bars that bind the secondary mass (the pendulum TMD) to the main primary structure.

Three distinct pendulum configurations cases were considered by the second author [1] and their corresponding mathematical equations of motion were obtained using the Euler-Lagrange formulation of classical mechanics. Such three pendulum cases were respectively: (i) simple pendulum (Fig. 3 (i)); (ii) coupled pendulum harmonically forced (Fig. 3 (ii)); (iii) coupled pendulum with a base motion excitation (Fig. 3 (iii)).

When an isolated pendulum (Fig. 3 (i)), with a possible rotation spring (of stiffness k_m) at the suspension point, is subjected to an exterior horizontal force F_a *(t)*, the known equation of motion of such simple pendulum is given by:

$$m l^2 \ddot{\theta} + \left(k_m + m g l\right)\theta = F_a(t)\, l \qquad (3)$$

The natural frequency is expressed by one of the following expressions: the former, for a pendulum with a rotation spring at the suspension point; the latter, for a simple pendulum (when such spring does not exist).

$$\omega = \sqrt{\frac{k_m + m g l}{m l^2}} \quad or \quad \omega = \sqrt{\frac{g}{l}} \qquad (4)$$

Figure 3 (ii) represents a SDOF system with a coupled pendulum, constituting a MDOF system with 2 degrees of freedom. The pendulum is a rigid bar with uniformly distributed mass of total value m_b, also with a concentrated mass m_p at its bottom tip materialized as a sphere of radius R_p.

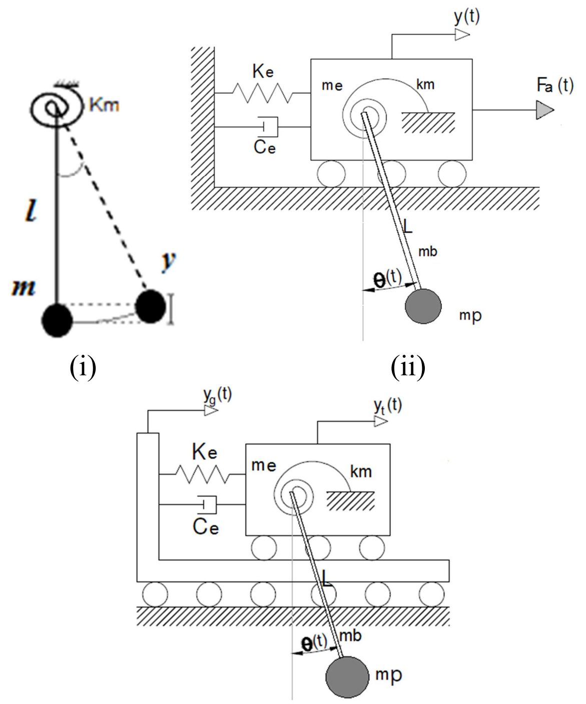


Fig. 3. Pendulum models: (i) simple pendulum; (ii) coupled pendulum acted by force; (iii) coupled pendulum with base motion of the support

This coupled system is subjected to a horizontal harmonic force $F_a(t)$, acting on the primary system whose displacement is $y(t)$.

The equations of motion of this coupled pendulum with harmonic force are given in equations (5) and (6). Notice that the coupled pendulum with base-motion excitation has distinct equations of motion (Sousa [1]), but equation (7) is general in calculating the natural frequency of the coupled pendulum secondary structure.

$$\left(m_e + m_b + m_p\right)\ddot{y} + \\ + \left(m_p\left(R_p + l_b\right) + \frac{1}{2}m_b l_b\right)\ddot{\theta} + \\ + C_e\dot{y} + K_e y = F_a(t) \tag{5}$$

$$\left(\frac{1}{3}m_b l_b^2 + \frac{2}{5}m_p R_p^2 + m_p\left(R_p + l_b\right)^2\right)\ddot{\theta} + \\ + \left(m_p\left(R_p + l_b\right) + \frac{1}{2}m_b l_b\right)\ddot{y} + \\ + k_m\theta + \left(m_p\left(R_p + l_b\right) + \frac{1}{2}m_b l_b\right)g\,\theta = 0 \tag{6}$$

$$\omega = \sqrt{\frac{k_m + \left(m_p\left(R_p + l_b\right) + \frac{1}{2}m_b l_b\right)g}{\frac{1}{3}m_b l_b^2 + \frac{2}{5}m_p R_p^2 + m_p\left(R_p + l_b\right)^2}} \tag{7}$$

The natural frequency of the pendulum is invariant with respect to changes of the type of actions that excite or force to vibrate the primary structural system (Sousa [1]).

3 Application of a Pendulum TMD to Calibration Frames and Real Structure

3.1 Pendulum TMD for SDOF Shear Frame

In order to demonstrate the applicability of this type of pendulum TMD to some didactic hypothetical frame for numerical calibration tests, a SDOF shear frame 2 m high and 2 m wide span was considered made of concrete (mass density 2500 kg/m3); the floor beam is a 12 cm x 26 cm rectangular section, and the two equal square columns (built-in at the base and at the floor level) are 3 cm wide. The columns total stiffness is 4050 N, and the total mass for sway dynamic analysis (considering only half of the column's masses concentrated or lumped at the floor) is 160.5 kg. With these data, the natural frequency of the primary shear frame is 0.7995 Hz. This same natural frequency was obtained when modelling the frame by the finite element analysis software SAP 2000 (Figure 4).

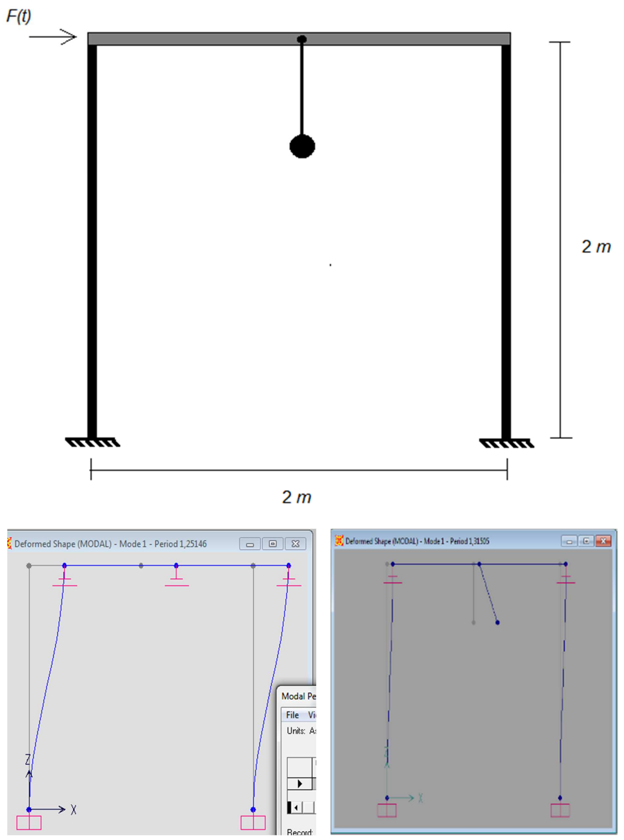


Fig. 4. SDOF shear frame with pendulum TMD and its modeling using SAP2000

This figure shows the SDOF shear frame acted at the floor level by a horizontal harmonic force, almost resonance with the primary frame and expressed by $F(t) = 10 \sin(2\pi(0.8)\,t)$. For numerical purposes, a damping factor of 0.01 (or 1% of critical) was used for this test demonstration frame.

Designing a simple pendulum TMD with mass ratio μ=0.01 for optimal tuning of this test frame led to the use of a wire length of 1.091 m as well as a rotation spring stiffness of 30 Nm/rad at the hanging point. Other non-optimal configurations can also be used, with different efficiencies, whose results are graphically shown in Figure 5 (Sousa [1]).

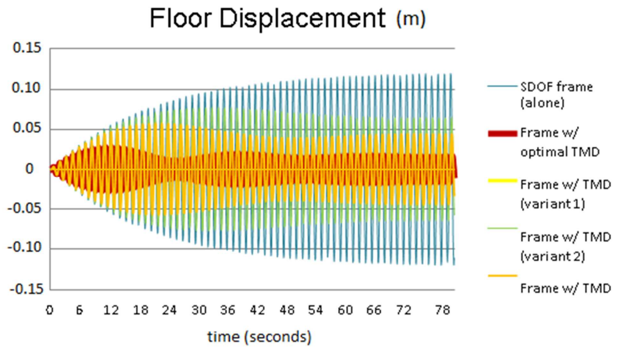


Fig. 5. Floor displacements for optimal pendulum TMD and some variants of optimal case

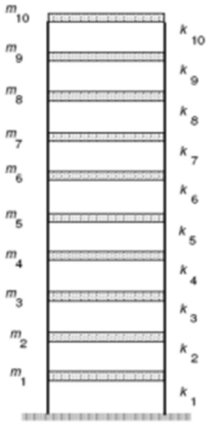
3.2 Pendulum TMD for MDOF Shear Frame

A second example considers the use of one or two pendulum TMD to control the lateral vibrations of a 10 floor MDOF shear building, acted upon by harmonic forces in every floor almost resonant with the fundamental frequency of the building, with decreasing amplitudes from 3.5 kN in top floor to 0.35 kN in first floor. The MDOF building data is detailed by Sousa [1], also with the 10 natural frequencies and modes shapes.

A schematic view of the 10 floor MDOF shear-frame as well as of the building inter-story characteristics are given in Figure 6. Also Figure 7 details part of the output of SAP 2000 software, concerning the values of the 10 first natural frequencies (Hz); while Figure 8 details the first four vibration modes of such shear building frame.

For such considered shear frame, the resonant forces are expressed by $F(t) = A \sin(2\pi\,(0.48817)\,t)$ where A is the floor force amplitude, linearly varying along the height as mentioned above.

For the dynamic analysis, it was considered a constant modal critical damping ratio of 1%. The top floor resonant displacement and resonant acceleration responses, for the set of harmonic resonant forces considered, are shown in Figures 9 and 10.



Floor	*Mass (kg)*	*Stiffness (kN)*
1	179060	62470
2	170060	59260
3	161060	56140
4	152040	53020
5	143040	49910
6	134040	46790
7	125040	43670
8	116040	40550
9	107040	37430
10	97540	34310
Total	1384960	

Fig. 6. Detail view and mechanical properties of the 10-floor MDOF shear building

Modal Periods And Frequencies

File View Format-Filter-Sort Select Options

Units: As Noted — Modal Periods And Frequencies

OutputCase Text	StepType Text	StepNum Unitless	Period Sec	Frequency Cyc/sec	CircFreq rad/sec	Eigenvalue rad2/sec2
MODAL	Mode	1	2,04848	0,48817	3,0672	9,408
MODAL	Mode	2	0,773037	1,2936	8,1279	66,063
MODAL	Mode	3	0,477123	2,0959	13,169	173,42
MODAL	Mode	4	0,349566	2,8607	17,974	323,07
MODAL	Mode	5	0,280263	3,5681	22,419	502,61
MODAL	Mode	6	0,238448	4,1938	26,35	694,34
MODAL	Mode	7	0,21196	4,7179	29,643	878,73
MODAL	Mode	8	0,194729	5,1353	32,266	1041,1
MODAL	Mode	9	0,183545	5,4483	34,232	1171,9
MODAL	Mode	10	0,177063	5,6477	35,486	1259,2

Fig. 7. SAP 2000 results for the 10 natural periods (s) and frequencies (Hz)

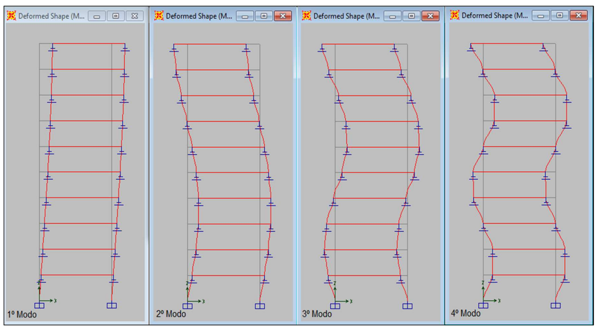


Fig. 8. First four vibration modes of the 10-floor shear-frame building

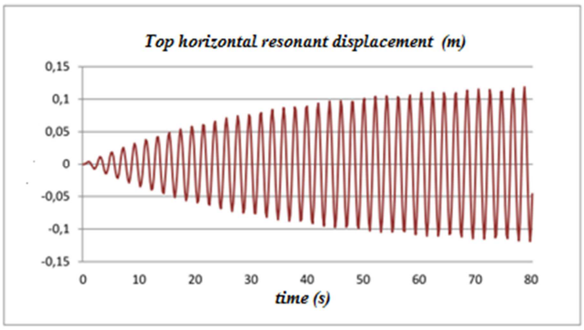


Fig. 9. Top floor horizontal resonant displacements during 80 seconds (without pendulum TMD)

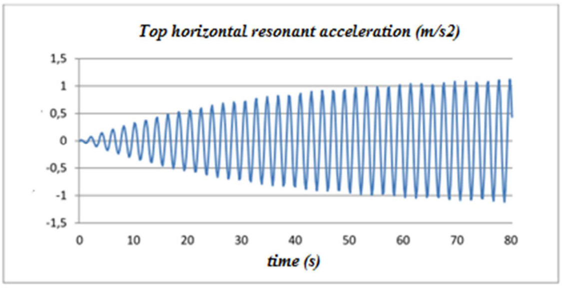


Fig. 10. Top floor horizontal resonant accelerations during 80 seconds (without pendulum TMD)

The maximum top floor displacement and acceleration responses, for the set of harmonic forces considered, will be reduced by using a pendulum TMD at top floor. With the building data the first modal mass is M1=589083 kg; using a mass ratio of 1%, it was possible to find the frequency of the secondary system from equation (1) and to select the pendulum properties by equation (4): with a rotational spring of 50 Nm/rad, the optimal pendulum length was found to be 1.069 m (Sousa [1]).

Figures 11 and 12 show the reduced and modulated building top floor displacement and acceleration responses, when the building is controlled solely by such pendulum TMD suspended from top floor: a displacement reduction of 52.8% was observed from 0.1277 m to 0.0603 m; an acceleration reduction of 49% was also observed from 1.209 m/s^2 to 0.611 m/s^2.

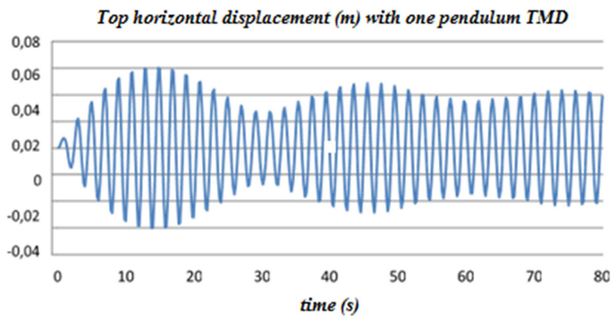


Fig. 11. Time-history of top displacements (m) for MDOF building with optimum single pendulum

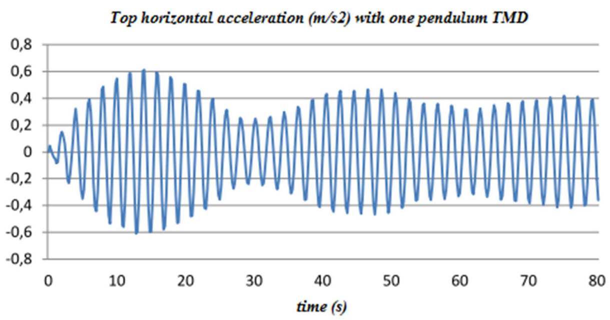


Fig. 12. Time-history of top accelerations (m/s^2) for MDOF building with optimum single pendulum

3.3 Pendulum TMD for a MDOF Industrial Real Tower of a Transmission Tower Test Site

In order to apply the previously described knowledge to a real tall structure subjected to wind actions evaluated according to Eurocode 1, it was decided to use the geometric and mechanical data of the pre-design of a reaction wall of the testing platform of Metalogalva.

This is a large and tall structure that will be built in Trofa (northern Portugal) in the sequel of the VHSSPOLES project [6], and will support the testing of large transmission tower structures or akin telecommunication poles. Through the use of cables connecting an automated system of base-supported hydraulic jacks and the reaction walls, passing by and connecting to the structure by a system of pulleys, are applied to real structural prototypes the respective actions according to the terms of design reception and testing of these structures. The test platform consists of 2 reaction walls, one primary and one secondary, in a disposition quite similar to the following photo in Figure 12 of a test site in Nagpur (India) detailed earlier (Paiva et al. [7, 8]).

Fig. 13. Test site at Nagpur (India), with primary and secondary reaction walls [7, 8]

For this study only pre-design data of the secondary reaction wall will be used, for which the wall is analyzed when no test is being performed but continues to withstand wind or earthquakes. At present the ideas and methodology of vibration control using pendulum TMD are successfully applied (Sousa [1]), simulating the control of wind vibrations of the secondary reaction tower of a transmission tower testing site (Barros [6]).

The final design of the secondary reaction wall developed in the scope of the VHSSPOLES project (Barros [6]) is shown in various perspectives in Figure 14, taken from Autodesk Robot Structural Analysis Professional software. With such structural data, it was possible to construct the two-dimensional model in SAP 2000 software, shown in Figure 15.

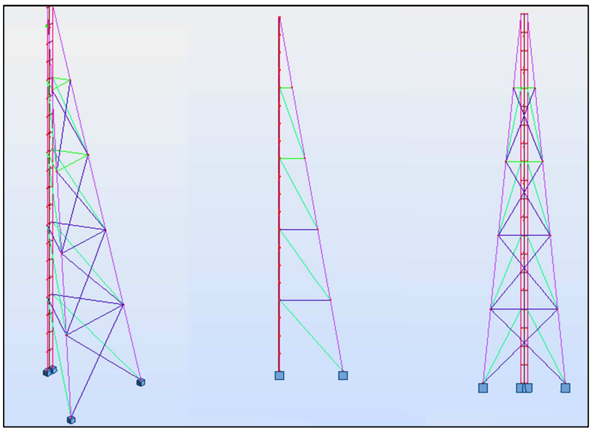


Fig. 14. Perspectives of the secondary reaction wall in the Autodesk Robot Structural Analysis software

A two-dimensional model in SAP 2000 was used in the transverse direction (Figure 15) in order to capture essentially the flexural vibration modes (in Y direction). This was the only direction considered because the direction corresponding to the third image of Figure 14 is extremely rigid and its analysis is therefore not justified as a demonstration of the application of pendulum TMD in this direction (Sousa [1]). It is therefore for the structural model of Figure 15 that the regulatory characterization of wind velocities wind pressures and wind forces (according to Eurocode 1) has to be determined, to evaluate and compare corresponding dynamic responses (without and with an optimal pendulum TMD on the secondary reaction wall).

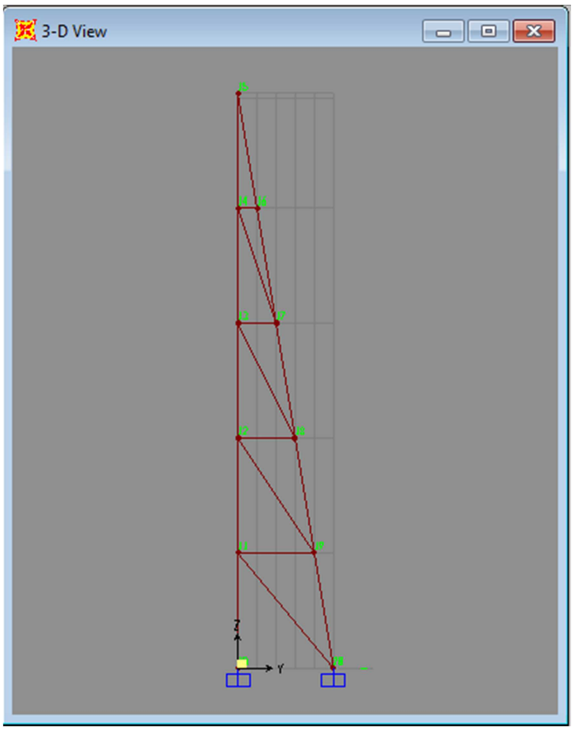


Fig. 15. Two-dimensional model of the secondary reaction wall in SAP 2000 software

A large number of investigations have been made over the last decades in order to develop a spectrum that could adequately predict the wind dynamic characteristics. Since wind action analytically treats a turbulent flow in interaction with immersed bodies, it is of complex calculation and prediction. When wind action over unusual structures needs to be calculated as accurately as possible, physical scale models are built that are tested in wind tunnels simulating flow properties.

As the objective of this section is not the exhaustive calculation of the wind dynamic actions that act on the secondary reaction wall structure but rather their control, some simplifications will be considered during this calculation. The fluid structure interaction will be considered negligible; also, simplified correlations of wind speed variation along the height of the structure are considered.

The instantaneous wind velocity at any point in the structure is given by the sum of a constant mean component, with the fluctuating dynamic component. First, it is synthetically referred the methodology to be used to generate time wind series, which will be later used in the calculation of the instantaneous dynamic pressure, and consequently in the quantification of the acting temporal loads at each bracing level of the reaction wall, that is to say 20 in 20 meters (Figure 15).

The methodology used to obtain a stochastic process described by the spectral density function that characterizes the wind random process (here considered unidimensional and univariate), usually referred to as the Shinozuka Method [9, 10, 11], is based on the generation of time series for the fluctuating dynamic part of the instantaneous wind velocity. The amplitude of each fluctuating velocity component is generated in a random process [9, 10, 11] with an approximately Gaussian distribution of the atmospheric flow wind process, in the wavelength range corresponding to the values of the wind speed fluctuation. The method uses a weighted sum of harmonic functions, adding N waves of the wind spectrum corresponding to N discretization frequencies of the spectrum.

Past research indicates [9, 10, 11] that for the generation of time series corresponding to an ergodic process N must be high enough; herein, N=1000 is a number that shows good results.

According to Eurocode 1 (EC 1), for structures in the form of this study case the parameters of the spectral density function -- for the calculation of the structural factor (used in the calculation of the wind forces) -- should be calculated for a reference height H of approximately 0.6 times the height of the building.

Taking this into account, the height chosen for the generation of different wind series was H=60 meters. It was also considered that the reaction wall was located in a class I terrain category according to EC 1; also the base velocity assumed for a 50-year return period was 30 m/s (zone B in EC 1), for which under these conditions the value of standard deviation is $\sigma_V = 5{,}7$.

With these characteristics inherent to the determination of wind forces on structures and structural members according to EC 1, and with the mechanical properties associated with the SAP 2000 model pictured in Figure 15, a graph of the instantaneous forces for the second level (or, at 40 meters of height) of the reaction wall and corresponding to the time series 1 of the calculation process is shown in Figure 16.

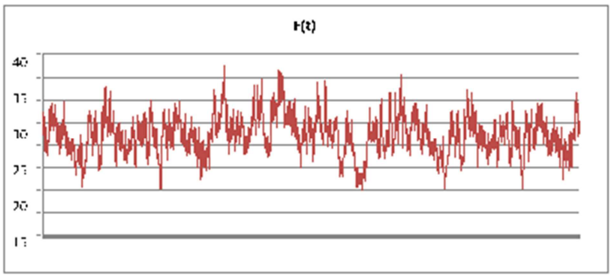


Fig. 16. Instantaneous forces for second level generated in time series 1

Similar calculations are done under wind time series 1, for each height level of the reaction wall (20-40-60-80-100 m), and such wind force time series are entered as instantaneous actions into SAP 2000 software as shown in Figure 17.

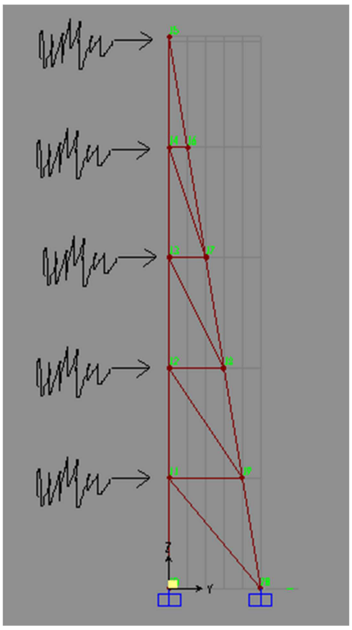


Fig. 17. Instantaneous forces depiction at each level of secondary reaction wall, for wind time series 1

Figures 18 and 19 show the top displacements and top accelerations of the reaction wall, due to regulatory EC 1 wind forces applied at each level under the fluctuating velocities generations associated with wind time series 1.

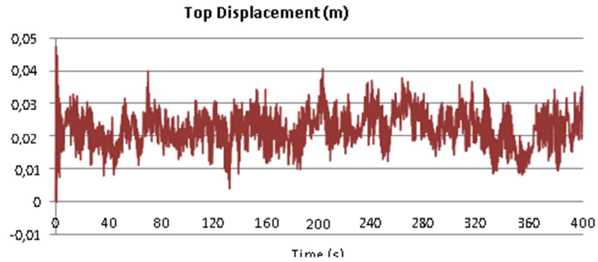


Fig. 18. Top displacements over time, for the reaction wall under wind time series 1 (without pendulum TMD)

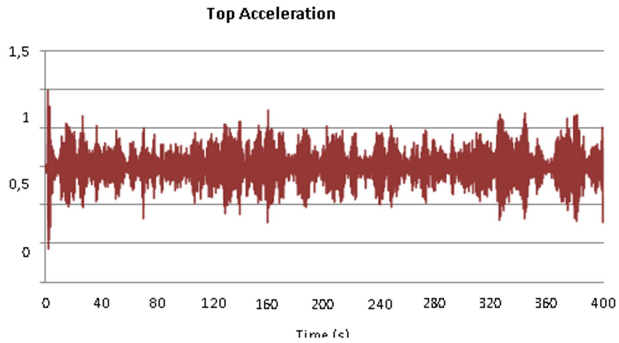


Fig. 19. Top accelerations over time, for the reaction wall under wind time series 1 (without pendulum TMD)

These were results coherently associated with wind time series 1. To obtain reliable results it would be necessary to perform the calculations for at least 4 different wind time series and then proceed to the evaluation of mean values [10, 11]. In this work only the calculations for the first time series will be carried out, due to time constraints but mainly because the methodology for pendulum TMD application in this real structure case is numerically illustrated with the calculations developed for a single time series.

For a very rigid structure, given by the predesign characteristics of the secondary reaction wall, the design of an optimal pendulum TMD will lead to a short length pendulum suspended at the top level.

Since the first modal mass is 19186.3 x 10^3 kg, obtained from the modal analysis of the reaction wall using SAP 2000, now choose: (a) a mass ratio μ of 1% for the TMD; (b) optimal TMD parameters, frequency ratio and damping ratio, given by equations (1) and (2) or taken from Figures 1 and 2; (c) a rotational spring stiffness of 50 Nm/rad.

The optimal frequency of 8,329 rad/s leads to an optimal length of the pendulum TMD of 0,1613 m. Suspending such pendulum TMD from top of the structure leads to a tuned wind response under wind time series 1, given by Figures 20 and 21 respectively for top displacements and top accelerations.

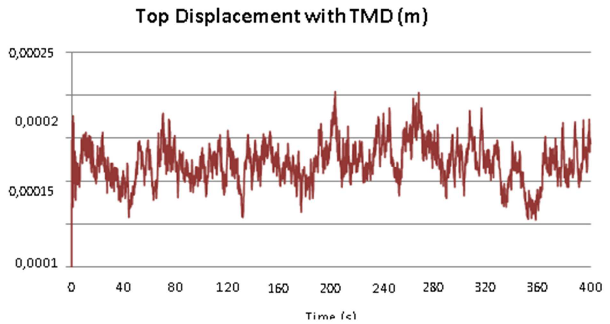


Fig. 20. Top displacements over time, for the reaction wall under wind time series 1 (equipped with optimal pendulum TMD)

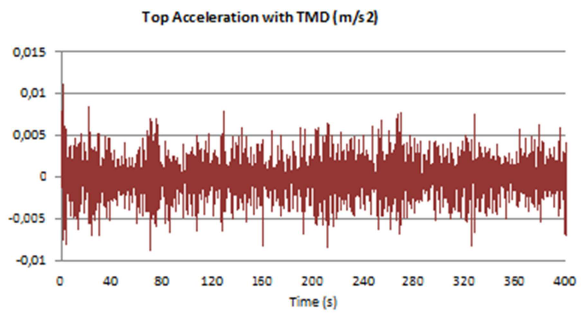


Fig. 21. Top accelerations over time, for the reaction wall under wind time series 1 (equipped with optimal pendulum TMD)

Although the reaction walls are very stiff, the tuning of the secondary reaction wall by a very short (0.1613 m) but heavy (191.86 ton) pendulum TMD theoretically reduced significantly the reaction wall top displacement from 4.8 cm to 0.2 mm. Such fact is compatible with the wall motionless and precision needs of the benchmark test site for transmission towers. Accordingly top accelerations would be reduced from 1 m/s^2 to 1 cm/s^2 (Figures 20 and 21).

4 Conclusions

Some review on the design of optimal TMD was presented in the context of SDOF and MDOF structures, which included the non-linear abacus for selection of the design parameters of the secondary structure. Also fundamental properties of single and coupled pendulum cases are synthesized, to be used as general expressions for selecting the fundamental properties of secondary pendulum TMD systems.

The concepts were numerically applied with efficiency to practical structural cases, including one-bay shear frames and the optimal control of the stiff reaction wall of a transmission tower testing site, showing the validity accuracy and interest of this passive control technique.

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