# Shifting Method for Relay Feedback Identification and Control of Delayed Systems

MILAN HOFREITER
Department of Instrumentation and Control Engineering
Czech Technical University in Prague, Faculty of Mechanical Engineering
Technická 4, 166 07 Prague 6
CZECH REPUBLIC
milan.hofreiter@fs.cvut.cz

*Abstract:* - The paper deals with identification and control of time delay stable plants. A single relay feedback test and the new method called "shifting method" are used for plant identification. For this purpose an asymmetric relay is used. This approach enables to estimate three points on a plant frequency response for three important frequencies from sustained oscillations obtained by a single relay test without using the FFT algorithm. This technique extends possibilities of relay feedback identification and allows fitting up to five parameters of plant transfer functions. This identification method is used for estimating five parameters of the selected anisochronic model that is very universal and convenient for modelling time delay stable systems. This model is then used for anisochronic controller design. The anisochronic controller is proposed by the Desired Model Method. The shifting method and the anisochronic controller design are demonstrated on one simulation example and on the laboratory apparatus called "Air Aggregate". These experiments were performed using Matlab/Simulink programing environment. The introduced approach provides an automatic tuning tool for a large class of common control problems.

*Key-Words:* -Anisochronic control, estimation parameter, frequency response, relay feedback identification, time delay

## 1 Introduction

The purpose of control oriented system identification is to obtain information for a feedback controller design. The aim is to obtain a model that reflects the dynamic response relationship among manipulated variables and controlled variables of a system. System identification from the relay feedback belongs among the most popular methods applied in industrial and chemical practice [2], [9], [11], [13], [17], [21]. An important merit of using a relay test is that the process will not drift too far away from the set-point of system operation and no prior knowledge of the closed-loop structure is required. The main technical routes of the existing relay identification methods are categorized into three groups, one is the describing function method, another is the curve fitting approach and the third is the use of frequency response estimation for model fitting [12]. The presented relay identification method belongs to the third group of methods.

A plant under relay feedback is shown in Fig. 1, where $w$ denotes the desired variable, $y$ the controlled variable, $u$ the manipulated variable, $d$ the disturbance variable and $e$ the control error. Let us consider a time invariant plant described by the plant frequency response function $G_P(j\omega)$, where $\omega$ is the angular frequency. Then we can determine one point $P_u=G_P(j\omega_u)$ on the Nyquist curve by relay feedback test, where $\omega_u$ is the angular frequency of the limit cycle that can be computed from the steady oscillation period $T_u$ according to

$$\omega_u = \frac{2\cdot\pi}{T_u} \tag{1}$$

It was derived, e.g. [1], [2] that it holds for a plant under a symmetrical relay without hysteresis

$$P_u = G_P\left(j\omega_u\right) = -\frac{\pi\cdot y_A}{4\cdot u_A}, \tag{2}$$

where $y_A$ is the harmonic oscillation amplitude of the plant output and $u_A$ is the relay amplitude. This approach was also generalized for a relay with hysteresis, where by choosing the relation between the relay amplitude and the hysteresis width it is possible to determine a point on the Nyquist curve with a specified imaginary part, e.g. [4], [18].

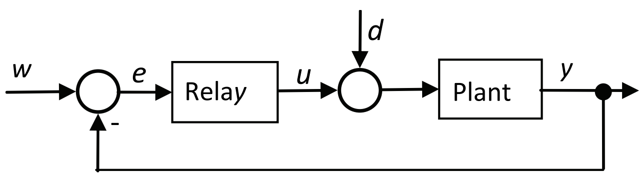


Fig. 1 Block diagram of a plant under relay feedback

If a plant model has more than two unknown parameters then it is necessary for system identification to find out more points on a frequency response function or to add some other information. This is solved, for example using more tests with relay feedback, where a known linearity (e.g. an integrator, a time delay) is connected in series with the plant [6], [18] or by repeating the experiment with different relations between the relay amplitude and the hysteresis [16]. However, the use of an additional relay test is tedious and time consuming. There are also modified relay methods, e.g. with a parasitic relay and using the FFT algorithm [3], with suitable heuristics [10] or it is assumed that the plant is initially in a steady state [8].

The paper is organized as follows: The next chapter presents the new technique enabling to estimate up to five parameters from sustained oscillations of the controlled variable observed during only a single relay feedback experiment. In Chapter 3 the anisochronic controller is designed using the Desired Model Method and the selected anisochronic model. The shifting method and the anisochronic controller design are demonstrated on one simulation example and on the laboratory apparatus called "Air Aggregate" in Chapter 4. Chapter 5 summarizes the results and conclusions of the work.

## 2 Relay Feedback Identification by Shifting Technique

The block diagram of the relay feedback experiment is shown in Fig. 1, where the plant is describable by the frequency response function $G_P(j\omega)$ and a biased (asymmetric) relay with hysteresis is used for this purpose, see Fig. 2. Let the stable oscillation is reached after the time $t_L$. The time courses of the biased relay output $u$ and the plant output $y$ after the time $t_L$ are shown in Fig. 3.

The biased relay output $u$ oscillating with the period $T_p$ can be expanded in a Fourier series. If all higher harmonics are neglected and only the first harmonic (the fundamental frequency) is considered then the relay output $u$ can be approximated by the function $u_{ap}(t)$ for $t \geq t_L$

$$u(t) \approx u_{ap}(t) = a_u + A_u \cos(\omega_u (t - t_L) - \phi_u), \quad (3)$$

where

$$a_u = \frac{u_A \cdot T_1 + u_B \cdot T_2}{T_p}, \quad (4)$$

$$\omega_u = \frac{2\pi}{T_p}, \quad (5)$$

$$\phi_u = \frac{\omega_u \cdot T_1}{2} = \frac{\pi \cdot T_1}{T_p}, \quad (6)$$

$$H = u_A + |u_B|, \quad (7)$$

$$A_u = \frac{2H}{\pi} \cdot \sin(\phi_u). \quad (8)$$

The time courses $u(t)$ and $u_{ap}(t)$ are illustrated in Fig. 3 where the parameters $t_L$, $T_1$, $T_2$, $T_p$, $H$, $u_A$ and $u_B$ are also depicted.

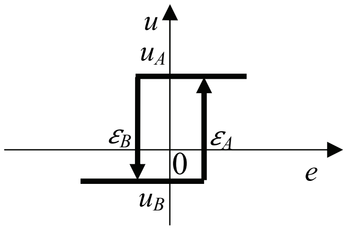


Fig. 2 The characteristic of a biased (asymmetric) relay with hysteresis

For many systems, where the manipulated signal is a square wave, the plant output is close to a sinusoid, which means, that the plant attenuates higher harmonics effectively. Therefore higher harmonics in $y$ can be neglected. The point $G(j\omega_u)$ on the Nyquist curve of the plant can be estimated using the fundamental harmonics of the signal $u(t)$ and $y(t)$. This point is given by

$$G(j\omega_u) = \frac{y_A}{A_u} \cdot e^{j\phi_{uy}} \quad (9)$$

where $y_A$ is an amplitude of the output $y$, $\phi_{uy}$ is a phase shift between $u$ and $y$ and

$$\phi_{uy} = -\omega_u \cdot t_{uy}, \quad (10)$$

see Fig. 3.

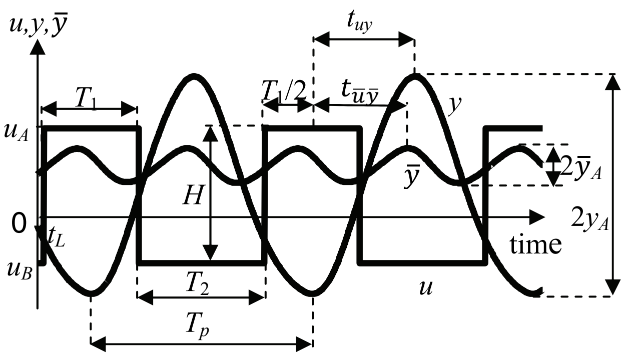


Fig. 3. The time courses $u$, $y$ and $\bar{y}$

In the next step the simple technique called "shifting" is used. It is based on the assumption that the identified plant is time invariant. Then the auxiliary variables $\bar{u}$ and $\bar{y}$ are calculated using (11) and (12) to obtain a rectangular waveform of the variable $\bar{u}$ and a sinusoidal form of the variable $\bar{y}$.

$$\bar{u}(t) = u(t) + u\left(t - \frac{T_p}{2}\right), \tag{11}$$

$$\bar{y}(t) = y(t) + y\left(t - \frac{T_p}{2}\right). \tag{12}$$

The time course $\bar{y}$ is depicted in Fig. 3. Since the variable $\bar{y}$ is close to a sinusoidal form, higher harmonics of $\bar{y}$ can be neglected and the next point on the Nyquist curve of the plant can be estimated using the fundamental harmonics of the signal $\bar{u}(t)$ and $\bar{y}(t)$.

The signal $\bar{u}$ oscillating with the period $T_p/2$ can be expanded in a Fourier series. If all higher harmonics are neglected and only the first harmonic (the fundamental frequency) is considered then the signal $\bar{u}(t)$ can be approximated by the function $\bar{u}_{ap}(t)$ for $t \geq t_L$.

$$\bar{u}(t) \approx \bar{u}_{ap}(t) = a_{\bar{u}} + A_{\bar{u}} \cdot \cos(\omega_{\bar{u}} \cdot (t - t_L) - \phi_{\bar{u}}), \tag{13}$$

where

$$a_{\bar{u}} = \frac{2(u_A \cdot T_1 + u_B \cdot T_2)}{T_p}, \tag{14}$$

$$\omega_{\bar{u}} = 2 \cdot \omega_u, \tag{15}$$

$$\phi_{\bar{u}} = 2 \cdot \phi_u = \omega_u \cdot T_1 = \frac{\omega_{\bar{u}} \cdot T_1}{2} = \frac{2\pi \cdot T_1}{T_p}, \tag{16}$$

$$A_{\bar{u}} = \frac{2H}{\pi} \cdot \sin(\phi_{\bar{u}}). \tag{17}$$

The point $G(j\omega_{\bar{u}})$ on the Nyquist curve of the plant can be estimated using the fundamental harmonics of the signals $\bar{u}(t)$ and $\bar{y}(t)$. This point is given by

$$G(j\omega_{\bar{u}}) = \frac{\bar{y}_A}{A_{\bar{u}}} \cdot e^{j\phi_{\bar{u}\bar{y}}}, \tag{18}$$

where $\bar{y}_A$ is an amplitude of the signal $\bar{y}$; $\phi_{\bar{u}\bar{y}}$ is a phase shift between $\bar{u}$ and $\bar{y}$ and

$$\phi_{\bar{u}\bar{y}} = -\omega_{\bar{u}} \cdot t_{\bar{u}\bar{y}}, \tag{19}$$

see Fig. 3.

The asymmetric relay enables to calculate the plant static gain $K$ from

$$K = G(0) = \frac{\int_t^{t+T_p} y(\tau)d\tau}{\int_t^{t+T_p} u(\tau)d\tau}, t \geq t_L, \tag{20}$$

see [3], [15].

Summing up the previous results, the shifting method enables to determine the static gain $K$ and two points $G(j\omega_u)$, $G(j\omega_{\bar{u}})$ of the Nyquist curve from one relay feedback experiment without utilising the Fourier transform. For this purpose it is sufficient to use only the relay output $u$, the output $y$ and the sum signal $\bar{y}$. The parameters, necessary for this goal, can be read out from the time courses of these signals, see Fig. 3.

The shifting method can be described by the following steps:

a) Control a plant by a biased relay.

b) Determine the period $T_p$ of the stable oscillation.

c) Compute the plant static gain $K=G(0)$ using (20) and the angular frequency $\omega_u$ using (5).

d) Calculate the point $G(j\omega_u)$ from

$$G(j\omega_u) = \frac{y_A}{\frac{2H}{\pi} \cdot \sin\left(\frac{\pi \cdot T_1}{T_p}\right)} \cdot e^{-j\omega_u \cdot t_{uy}} \tag{21}$$

where $y_A$, $T_1$, $u_A$, $u_B$, $t_{uy}$ can be determined directly from the time courses $u(t)$ and $y(t)$, see Fig. 3.

e) Calculate the course of the auxiliary variable $\bar{y}(t)$ using (12). (We need not calculate the auxiliary variable $\bar{u}(t)$).

f) Calculate the point $G(j \cdot \bar{\omega}_u) = G(2j\omega_u)$ according to

$$G(2j\omega_u) = \frac{\bar{y}_A}{\frac{2H}{\pi} \cdot \sin\left(\frac{2\pi \cdot T_1}{T_p}\right)} \cdot e^{-2j\omega_u \cdot t_{\bar{u}\bar{y}}} \tag{22}$$

where $\bar{y}_A$, $t_{\bar{u}\bar{y}}$ can be determined directly from the time courses $u(t)$ and $\bar{y}(t)$, see Fig. 3.

g) Estimate up to five parameters of mathematical models from the three points $G(0)$, $G(j\omega_u)$ and $G(2j\omega_u)$ of the Nyquist curve.

*Remark #1*

*From relations (21) and (22) it follows*

$$\frac{G(2j\omega_u)}{G(j\omega_u)}=\frac{\overline{y}_A\cdot e^{j\omega_u\left(t_{uy}-t_{\overline{u}\overline{y}}\right)}}{y_A\cdot\cos\left(\dfrac{\pi\cdot T_1}{T_p}\right)}. \tag{23}$$

*Remark #2*
*The shifting method described by relations (11) and (12) corresponds to use of the filter with the frequency transfer function*

$$G_F(j\omega)=1+e^{-j\omega\frac{T_p}{2}}. \tag{24}$$

*Therefore the filter filters out all odd harmonic frequencies including the fundamental harmonic frequency $\omega_u$ and two times amplifies even harmonic frequencies including $\omega_{\overline{u}}$, see Fig. 4.*

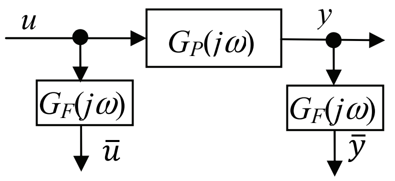


Fig. 4 The block scheme of the shifting method

*Remark #3*
*The points $G(j\omega_u)$ and $G(j\omega_{\overline{u}})$ on the Nyquist plot can also be determined using relationships (23) and (24).*

$$G(j\omega_u)=\frac{\int_t^{t+T_p} y(\tau)e^{-j\omega_u\tau}d\tau}{\int_t^{t+T_p} u(\tau)e^{-j\omega_u\tau}d\tau}, t\ge t_L, \tag{25}$$

$$G(j\omega_{\overline{u}})=\frac{\int_t^{t+T_p} \overline{y}(\tau)e^{-j\omega_{\overline{u}}\tau}d\tau}{\int_t^{t+T_p} \overline{u}(\tau)e^{-j\omega_{\overline{u}}\tau}d\tau}, t\ge t_L. \tag{26}$$

## 3 Anisochronic Control

The shifting technique enables to estimate up to five model parameters by a single relay feedback experiment. Therefore this approach can be used for the parameter estimation of the anisochronic model described by the transfer function

$$G_a(s)=\frac{K\cdot e^{-s\tau_u}}{(\tau_1 s+1)\left(\tau_2 s+e^{-s\tau_y}\right)}. \tag{27}$$

This linear anisochronic model is the second order and it contains only five parameters, where $K$ is the plant static gain, $\tau_u$ is the pure input time delay, $\tau_1$ and $\tau_2$ are the time constants and $\tau_y$ is the feedback time delay (the internal delay in state). The variable $s$ represents the complex argument defined by the Laplace transform.

The delay $\tau_y$ appears in the denominator of transfer function (27). Consequently characteristic equation (28) becomes transcendental in $s$.

$$(\tau_1\cdot s+1)\left(\tau_2\cdot s+e^{-s\cdot\tau_y}\right)=0 \tag{28}$$

The characteristic quasipolynomial has an infinite set of roots. It may also lead to an alternative interpretation of model (27) as an infinite dimensional model. Therefore it may serve as a good approximation for general higher-order delay free systems.

In the time domain, model (27) is described by equation (29).

$$\begin{aligned}\tau_1\tau_2 y''(t)+\tau_2 y'(t)+\tau_1 y'(t-\tau_y)\\ +y(t-\tau_y)=Ku(t-\tau_u)\end{aligned} \tag{29}$$

The parameters $K$, $\tau_u$, $\tau_1$, $\tau_2$ and $\tau_y$ are depicted in Fig. 5 on the unit step response of model (27), where $I$ represents the position of the inflection point, $p$ is the tangent at the inflection point $I$, see [7], [14], [22]. Model (27) is stable if $\tau_y/\tau_2<\pi/2$, over-damped if $\tau_y/\tau_2<1/e$, critically damped if $\tau_y/\tau_2=1/e$ and under-damped if $\tau_y/\tau_2>1/e$, where $e$ is Euler's number, see [5], [20].

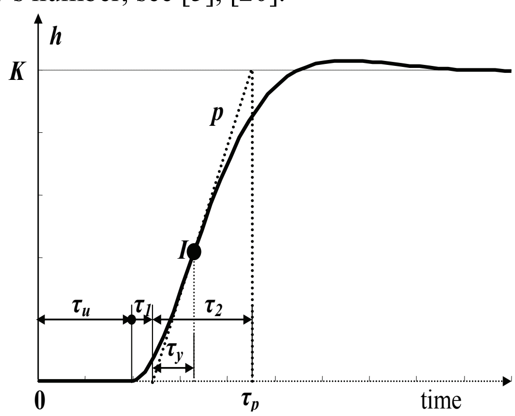


Fig. 5 The unit step response $h(t)$ of model (27)

Although anisochronic model (27) has only five parameters it is very universal and convenient for modeling time delay plants and for description dynamics of high-order systems. It may be used for both oscillatory and nonoscillatory plants, see [8], [22], [23].

The great universality of model (27) and the possibility to estimate by the shifting technique all the model parameters can be used for an anisochronic control design. A suitable method for direct synthesis using model (27) is the Desired Model Method (DMM) [19].

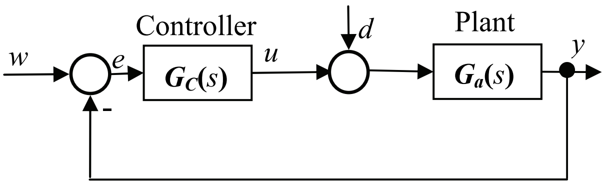


Fig. 6 Closed-loop system

The DMM uses the formula for direct synthesis (see Fig. 6)

$$G_C(s)=\frac{G_{wy}(s)}{G_a(s)\cdot\left(1-G_{wy}(s)\right)}, \tag{30}$$

where $G_C(s)$ is the controller transfer function, $G_a(s)$ is the plant transfer function, $G_{wy}(s)$ is the desired control system transfer function and it is selected in the form

$$G_{wy}(s)=\frac{k_0}{s+k_0\cdot e^{-\tau_u\cdot s}}e^{-\tau_u s}, \tag{31}$$

$k_0$ is the open-loop gain. The open-loop transfer function

$$G_0(s)=G_C(s)\cdot G_a(s)=\frac{k_0}{s}e^{-\tau_u s} \tag{32}$$

corresponds to desired control system transfer function (30).

After substitution of plant transfer function (27) to relationship (32) one obtains

$$G_0(s)=G_C(s)\cdot\frac{K\cdot e^{-s\tau_u}}{(\tau_1 s+1)\left(\tau_2 s+e^{-s\tau_y}\right)} = \frac{k_0}{s}e^{-\tau_u s} \tag{33}$$

hence

$$G_C(s)=\frac{k_0}{K}\cdot\frac{(\tau_1 s+1)\left(\tau_2 s+e^{-s\tau_y}\right)}{s}. \tag{34}$$

The open-loop gain $k_0$ can be easily determined analytically [19] assuming that the non-dominant poles and zeros of the control system have a negligible influence on its behaviour. The value of the open-loop gain $k_0$ can be decided according to

$$k_0=\frac{1}{\beta\cdot\tau_u} \tag{35}$$

where $\beta$ is the coefficient depending on the relative overshoot $\kappa$, see Table I (copy from [19]).

Table 1 Values of coefficients $\beta$ for given relative overshoot $\kappa$

| $\kappa$ | 0 | 0.05 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|---|---|---|---|---|---|---|---|
| $\beta$ | 2.718 | 1.944 | 1.720 | 1.437 | 1.248 | 1.104 | 0.992 |

Transfer function (34) is completed by a low-pass filter with a steady-state gain of one to guarantee the physical realizable controller. The transfer function of the controller is then

$$G_C(s)=\frac{k_0}{K}\cdot\frac{(\tau_1 s+1)\left(\tau_2 s+e^{-s\tau_y}\right)}{\left(\tau_f s+1\right)^r s}, \tag{36}$$

where $\tau_f$ is the time constant of the filter and the natural number $r$ can be chosen so that the order of the denominator is at least the same order as the numerator. The value of the time constant $\tau_f$ also allows restricting actions of the manipulated variable $u$.

# 4 Examples

The extended relay feedback identification and the anisochronic control are demonstrated by the following practical examples.

*Example #1*

A plant with the transfer function

$$G_P(s)=\frac{2e^{-8s}}{(5s+1)^6} \tag{37}$$

is connected with the relay controller in the closed-loop. The biased relay with hysteresis has following parameters, see Fig. 2:

$$u_A=2, u_B=-1, \varepsilon_A=0.5, \varepsilon_B=-0.5\,. \tag{38}$$

The time courses of the biased relay output $u$ and the plant output $y$ are shown in Fig. 7 provided that the system was initially in a steady state, plant transfer function (37) is not known and anisochronic model (27) is used for plant description.

The task is to estimate

a) the plant static gain $K$ and two points $G(j\omega_u)$, $G(j2\omega_u)$ of the Nyquist curve from a limit cycle oscillation obtained from one relay feedback experiment depicted in Fig. 7;
b) the parameters $\tau_u$, $\tau_1$, $\tau_2$ and $\tau_y$ of anisochronic model (27) using the two points $G(j\omega_u)$, $G(2j\omega_u)$ on the Nyquist plot.

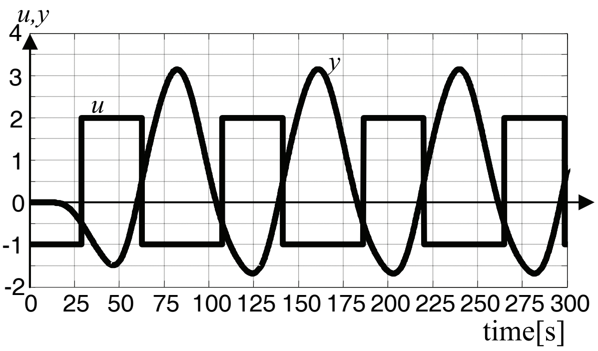


Fig. 7. The time courses $u$ and $y$

*Solution:*
a) The stable oscillation is reached after the time $t_L$=186.3s. The following parameters are estimated by graphical analysis of the plant output $y$, the relay output $u$ and the sum signal $\bar{y}$, see Fig. 8, where the sum signal $\bar{y}$ was computed by (12).

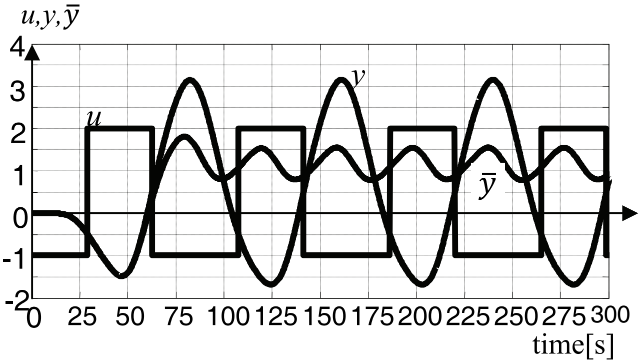


Fig. 8. The time courses $u(t)$, $y(t)$ and $\bar{y}(t)$

$$\begin{aligned} &T_p = 78.7\,\text{s}, T_1 = 33.8\,\text{s}, T_2 = 44.9\,\text{s}, H = 3, \\ &t_{uy} = 36.7\,\text{s}, y_A = 2.42, t_{\overline{uy}} = 34.2\,\text{s}, \bar{y}_A = 0.385. \end{aligned} \quad (39)$$

The following values result from these readings and formulas (5)÷(10) and (15)÷(19)

$$\begin{aligned} &\omega_u = 0.08\text{rad/s}, \\ &\phi_u = 1.35\text{rad}, A_u = 1.86, \phi_{uy} = -2.94\text{rad}, \end{aligned} \quad (40)$$

$$\begin{aligned} &\omega_{\bar{u}} = 0.16\text{rad/s}, \\ &\phi_{\bar{u}} = 2.7\text{rad}, A_{\bar{u}} = 0.82, \phi_{\overline{uy}} = -5.4\text{rad}. \end{aligned} \quad (41)$$

The frequency point $G(j\omega_u)$ at the frequency $\omega_u$=0.08 rad·s$^{-1}$ is then with respect to (9)

$$\begin{aligned} G(0.08j) &= \frac{y_A}{A_u} e^{j\phi_{uy}} = 1.3e^{-2.94j} \\ &= -1.27 - 0.26j \end{aligned} \quad (42)$$

and the frequency point $G(j\omega_{\bar{u}})$ at the frequency $\omega_{\bar{u}}$=0.16 rad·s$^{-1}$ is then with respect to (18)

$$\begin{aligned} G(0.16j) &= \frac{\bar{y}_A}{A_{\bar{u}}} \cdot e^{j\phi_{\overline{uy}}} = 0.47 \cdot e^{-5.4j} \\ &= 0.29 + 0.36j \end{aligned} \quad (43)$$

The asymmetric relay enables to calculate the plant static gain $K$ from (20) where $T_p$ is the period of the stable oscillation and $t \geq t_L$. Integral (20) can be computed by a numerical integration, e.g. a trapezoidal or rectangular integration algorithm. In this case the computed static gain

$$K = G(0) = 2. \quad (44)$$

The Nyquist diagram for plant (37) together with the points $G(0.08j)$, $G(0.16j)$ and $G(0)$ is depicted in Fig. 10.

b) Anisochronic model (27) has five parameters. The parameter $K$ can be estimated by (20) and in the solved example it holds (44). The other parameters $\tau_u$, $\tau_1$, $\tau_2$ and $\tau_y$ of anisochronic model (27) can be estimated from two complex generally nonlinear equations

$$G(0.08j) = G_a(0.08j), \quad (45)$$

$$G(0.16j) = G_a(0.16j). \quad (46)$$

Therefore the time constants $\tau_u$, $\tau_1$, $\tau_2$ and $\tau_y$ can be obtained by numerical fitting transfer function (27) to the points given by relations (42) and (43). The following time constants were estimated this way

$$\tau_u = 16.1\text{s}, \tau_1 = 15.0\text{s}, \tau_2 = 17.1\text{s}, \tau_y = 9.5\text{s}. \quad (47)$$

The frequency transfer function of the anisochronic model is then

$$G_a(j\omega) = \frac{2 \cdot e^{-16.1j\omega}}{(15j\omega + 1)(17.1j\omega + e^{-9.5j\omega})}. \quad (48)$$

The unit step responses of both plant (37) and anisochronic model (48) are displayed in Fig. 9.

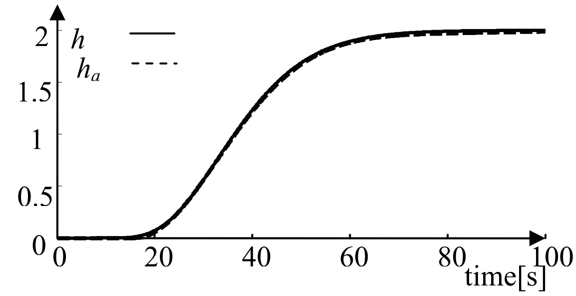


Fig. 9. The unit step response $h$ of plant (37) and the unit step response $h_a$ of anisochronic model (48)

Although there is very good conformity between the unit step response $h$ of identified plant (37) and the unit step response $h_a$ of anisochronic model (48), transfer functions (37) and (48) are different. The Nyquist diagrams of plant (37) and model (48) are depicted in Fig. 10 and they display very good conformity.

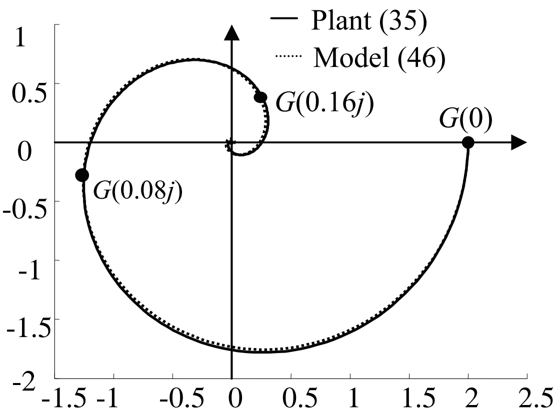


Fig. 10. The Nyquist diagrams for plant (37) and model (48)

*Example #2*

A laboratory model "Air Aggregate" consists of two ventilators located in a tunnel. The ventilators are fed by a controlled supply voltage (Fig. 11). In the tunnel there is also a sensor for measuring air flow. The manipulated variable (power to the primary ventilator) $u$, the disturbance variable (power to the secondary ventilator) $d$ and the controlled variable (air flow) $y$ are provided via unified electrical signals (0-10 V). The task is to estimate by the shifting technique the parameters of model (27) and then to design a controller using the DMM.

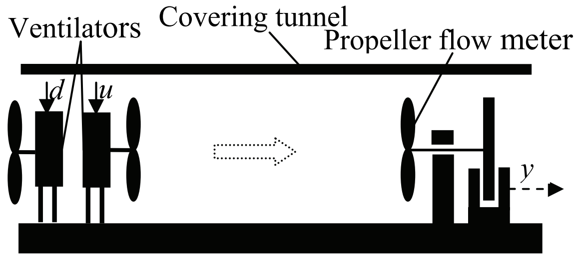


Fig. 11 The laboratory model "Air Aggregate"

*Solution:*

a) Extended relay feedback identification

After a single relay feedback test the parameters of anisochronic model (27) can be estimated from the sustained oscillations $u$ and $y$. This part is denoted by the rectangle in Fig. 12.

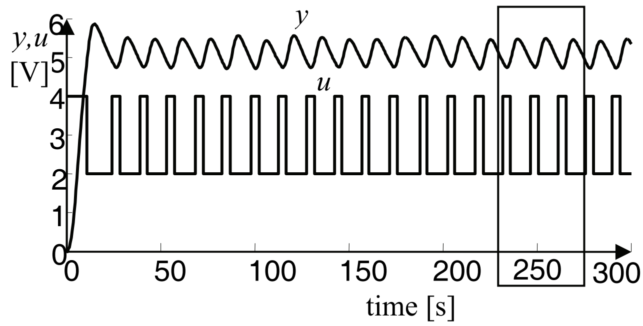


Fig. 12 The observed input $u$ and the observed output $y$

The calculated sum signal $\bar{y}$ is depicted in Fig. 13. The signal $\bar{u}$ is not in Fig. 13 because it does not need to be calculated.

Using the shifting method it was obtained

$$H = 2\ \mathrm{V}, \omega_u = \frac{2\pi}{T_p} = 0.45\ \mathrm{rad \cdot s^{-1}}. \quad (49)$$

$$K = G(0) = 2.02, \quad (50)$$

$$G(0.45j) = 0.35e^{-3.08j} = -0.35 - 0.02j, \quad (51)$$

$$G(0.9j) = 0.1e^{-3.95j} = -0.07 + 0.07j. \quad (52)$$

The verification was performed by sinusoidal inputs with the following results:

$$G(0) = K = 1.99, \quad (53)$$

$$G(0.45j) = -0.38 - 0.01j, \quad (54)$$

$$G(0.9j) = -0.03 + 0.09j. \quad (55)$$

The points of the Nyquist diagram estimated by the relay feedback identification and by the sinosuidal inputs (see Fig. 14) are depicted in the complex plain in Fig. 15.

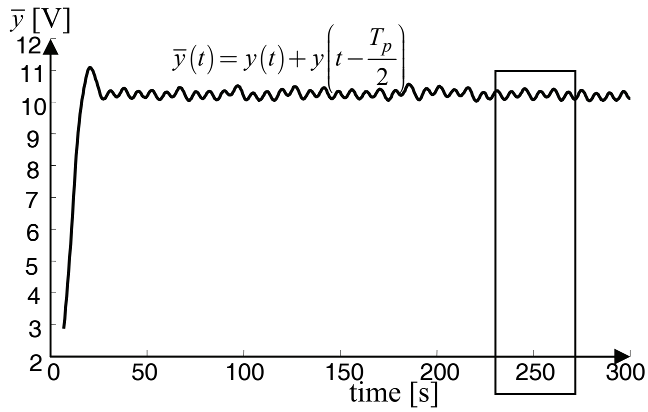


Fig. 13 The calculated output $\bar{y}$

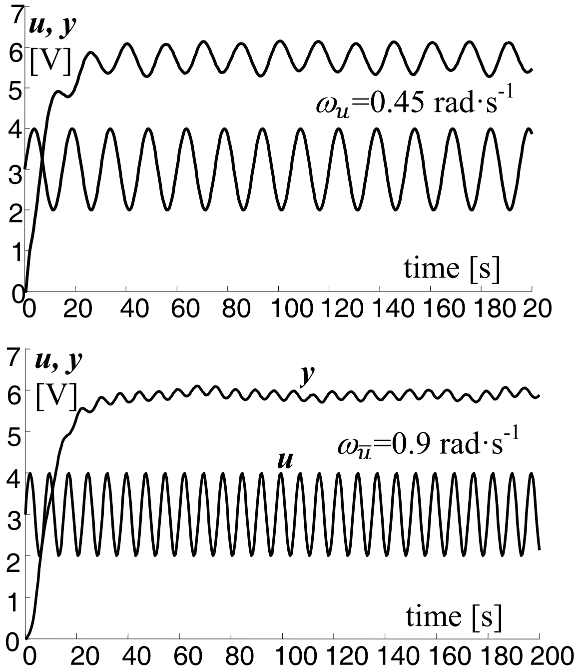


Fig. 14 Sinusoidal inputs and responses for verification

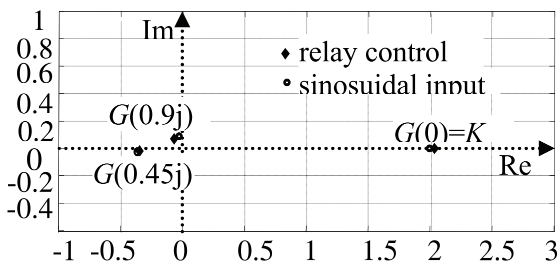


Fig. 15 The points of the Nyquist diagram estimated by the relay feedback identification and by the sinosuidal inputs

The time constants $\tau_u$, $\tau_1$, $\tau_2$ and $\tau_y$ can be obtained by numerical fitting transfer function (27) to the points given by relations (51) and (52). The following time constants were estimated this way

$$\tau_u = 1.56\,\mathrm{s}, \tau_1 = 1.63\,\mathrm{s}, \tau_2 = 11.68\,\mathrm{s}, \tau_y = 5.55\,\mathrm{s}. \quad (56)$$

The frequency transfer function of the anisochronic model is then

$$G_a(j\omega) = \frac{2.02 \cdot e^{-1.56 j\omega}}{(1.63 j\omega + 1)\left(11.68 j\omega + e^{-5.55 j\omega}\right)}. \quad (57)$$

b) Anisochronic controller design

The transfer function $G_C(s)$ of the controller can be selected using the DMM with respect to (57) and (36) in the form

$$G_C(s) = k_0 \cdot \frac{(1.63s+1)\cdot\left(11.68s + e^{-5.55s}\right)}{2.02\cdot(0.2s+1)\cdot s}, \quad (58)$$

where

$$K = 2.02, \tau_1 = 1.63\ \mathrm{s}, \tau_2 = 11.68\ \mathrm{s}, \\ \tau_y = 5.55\ \mathrm{s}, \tau_f = 0.2\ \mathrm{s}, r = 1. \quad (59)$$

Therefore with respect to Tab. 1 and (56) the value of the open-loop gain is

$$k_0 = \frac{1}{\beta \cdot \tau_u} = \frac{1}{1.720 \cdot 1.56} = 0.37 \ \text{for}\ \kappa{=}0.1 \quad (60)$$

Fig. 16 shows the closed loop control of the laboratory model "Air Aggregate" for the step changes of the desired variable $w$ and the disturbance variable $d$ (the required relative overshoot $\kappa$=0.1).

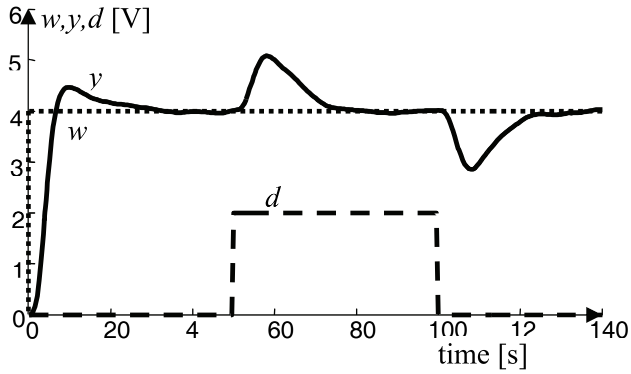


Fig. 16 The closed loop control of the laboratory model "Air Aggregate" for the step changes of the desired variable $w$ and the disturbance variable $d$

## 5 Conclusion

This paper presents the shifting technique that significantly extends the capabilities of the relay feedback identification. No prior knowledge of the closed-loop structure is required to perform a relay feedback test. This technique has been developed for the estimation up to five parameters of mathematical models from a single relay feedback experiment without utilizing the Fourier transform. The method concentrates on specific frequencies that the relay feedback automatically finds them. The estimate up to five model parameters allows achieving a better fit over a wider frequency range. It can be applied for the parameter tuning both isochronic and anisochronic models. For this purpose it uses a biased relay with a hysteresis which also helps to avoid measurement noise causing incorrect relay switching. The necessary computations are simple and can be easily applied in practice. The introduced technique was demonstrated on the simulation example and on the laboratory model called "Air Aggregate" where five parameters of the anisochronic model were estimated. Then, this very versatile model was used to derive also a very universal anisochronic controller. After this step, the anisochronic control was demonstrated on the air flow control using the same laboratory model "Air Aggregate". The introduced algorithm offers the possibilities to provide an automatic tuning tool for a large class of common control problems.

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