# Spectrum Shaping based Control Strategy for Unstable TDS

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*Abstract:* - Two possible controller parameters tuning methodologies based on the computational shaping of zeros/poles spectrum of unstable time delay systems (TDS) are presented. Namely, the spectral abscissa minimization based on Quasi-Continuous Shifting Algorithm (QCSA) followed by an advanced iterative optimization algorithm, and the Pole Placement Shifting based Algorithm (PPSA) moving the dominant poles and/or zeros of infinite spectra to the prescribed positions as close as possible step by step are introduced. These desired loci can be determined by a finite-dimensional model matching problem. In contrast to works we extend and to our previous works, the paper concerns unstable TDS in the sense of exponential stability and, moreover, both retarded and neutral TDS are considered. As presented by numerical examples, both the methodologies are closely related to each other and the latter one can collapse close to the former one. The reader is also provided with potential ideas of the future research within this task.

*Key-Words:* - Model matching, Optimization, Root locus, Spectrum shaping, Unstable system, Time-delay systems

## 1 Introduction

A suitable control system selection, a mathematical formulation of the controller structure and tuning of controller parameters are usually three essential steps in the control design procedure. Whereas for standard controllers (such as proportional-integral-derivative (PID) ones) there have been derived and proposed a plenty of controller parameters setting strategies, procedures and algorithms so far [1]-[5], many design ideas yield an unusual control law for which a known tuning method can not be directly applied. Such controllers are mostly obtained by the use of some advanced (e.g. optimization) algorithms [6] or when dealing with non-standard plant or models; for instance, nonlinear, hybrid or time delay systems - the control of which is attacked in this paper.

Linear time-invariant time delay systems (TDS) belonging to the more general class of infinite-dimensional systems have been intensively studied during past decades, see e.g. [7]-[9], to name just a few of the most important recent publications dealing with TDS. Many various control design algorithms in the time- or frequency domain have been proposed and derived since then; however, the most of them are far from to be practically applicable [10]. Nevertheless, some others provide rather simple engineeringly applicable procedures giving rise to the linear controller structure; for instance, elegant and attractive frequency-domain based algebraic control methods [11]-[13].

Even if the suitable controller law for a TDS plant is found, it is mostly parameterized by the set of unknown bounded or unbounded parameters which have to be appropriately set. Unfortunately again, there is a lack of engineeringly effortless methods for this class of systems and controllers, and the tuning of the final control laws is widely neglected; if exists any, usually suffers from an excessive mathematics [10] or the controller model is altered using a rationalization to enable the implementation of standard procedures and algorithms from the finite-dimensional systems theory [14].

As indicated above, one of crucial features of TDS is that the spectrum of such systems is infinite. Regarding spectral properties, there exist three basic types of TDS: advanced, retarded and neutral [7]. Whereas the two former ones have been quite well investigated, the latter one owns a rather complex properties and it is a challenging task to analyze and control such systems. The class of pole placement (assignment) controller parameters tuning principles is aimed at the setting of the finite number of undetermined controller parameters to shape the infinite spectrum as close to the desired form as possible by shifting or placing the dominant system eigenvalues [15]-[19].

Unstable TDS constitute a very delicate subclass of systems to be attacked by means of control theory. Note that the notion of stability of TDS, however, can not be comprehended in the simple and cohesive meaning as for finite-dimensional systems since conditions for asymptotical, exponential, $H_\infty$, bounded-input bounded-output (BIBO), stability etc. are not often straightforward [7], [8], [10], [20]-[23] and they are not given solely by algebraic roots loci. Most of control design studies, so far, deal with input-output delays of a finite-dimensional plant yet with an infinite-dimensional feedback only by means of several stabilization and control techniques [24]-[27] including also the plant rationalization [28], internal model control (IMC) [29] or the Smith predictor control system structure [30], etc. Solutions of various stabilization and control tasks of unstable TDS with state (or internal) delays, however, can also be found in the literature [22], [31]-[34].

This contribution is focused on two possible quasioptimal controller parameters tuning techniques for unstable TDS (retarded or neutral ones with internal delays) that are closely related to each other, so that one may turn to another in some cases as shown herein. It extends the results of Michiels et al. [18], [19] by means of the well-known Nelder-Mead minimization algorithm [35]. In this sense, the work resembles the spectral abscissa optimizations e.g. in [36], [37]; however a different (easier and better applicable) algorithm is used. But we are going further this problem – not only poles but zeros loci as well are optimized by placing the dominant ones to desired positions as close as possible – we called it as the PPSA (Pole-Placement Shifting based controller tuning Algorithm) [38]. Independently, this task has been solved in [39]; however, the authors have used different tools and strategies. The step of selection and prescription process of a demanded finite-dimensional model enables to view the methodology as a model-matching problem.

Basic ideas and practically computational aspects, that we see as useful for the reader, are concerned rather then rigorous mathematical formulations which will be the content of some of our future contributions. We highlight some of our recent results and add new ones related to a complex class of neutral TDS. Because of a wide range of possibilities for the improvement of the methodology, some proposals are suggested and provided to the reader as well.

The research contribution of this paper consists in the presentation of two easy-handling computing-based optimization algorithm for a very intricate family of delayed systems.

The paper is organized as follows. Section 2 provides the reader with the preliminaries of TDS description in the state as well as input-output domain, basic spectral properties and stability notions of both types of systems. The problem formulation and an introductory concise overview and insight into the topics of the spectral abscissa minimization, quasi-continuous root shifting and the PPSA technique can be found in Section 3. Numerical examples supporting the preceding section and giving rise to some novel findings and results are the contents of Section 4. Section 5 includes the discussion on the computational and numeric aspects of the algorithms, matching model selection, etc. for the future research. Section 6 concludes and summarizes the paper.

## 2 Preliminaries

Mathematical background of time-delay systems and models, and their properties including stability can be found in many sources, e.g. in [7]-[9]; hence, the necessary basic facts are presented only.

### 2.1 TDS model

Consider the following single-input single-output (SISO) TDS model

$$\begin{aligned} \mathrm{d}\dot{\mathbf{x}}(t) &= \sum_{i=1}^{n_H} \mathbf{H}_i \mathrm{d}\dot{\mathbf{x}}(t-\vartheta_i) + \sum_{i=0}^{n_A} \mathbf{A}_i \mathbf{x}(t-\vartheta_i) \\ &\quad + \sum_{i=0}^{n_B} \mathbf{B}_i u(t-\tau_i) \\ y(t) &= \mathbf{C}\mathbf{x}(t) \end{aligned} \tag{1}$$

where $0=\vartheta_0<\vartheta_1<\ldots<\vartheta$ stand for state (internal) and input-output delays, respectively, $u, y \in \mathbb{R}$, expresses input and output, respectively, $\mathbf{x}, \dot{\mathbf{x}} \in \mathbb{R}^n$ means the state vector and its derivative, respectively, $\mathbf{A}_i$, $\mathbf{B}_i$, $\mathbf{C}$, $\mathbf{H}_i$ are matrices of compatible dimensions. Note that whenever $\exists i : \mathbf{H}_i \neq \mathbf{0}$, the system is of neutral type, while the retarded model is obtained if $\mathbf{H}_i = \mathbf{0}, \forall i \in 1,2,...n_H$ [7]. By means of the Laplace transform, model (1) directly gives rise to the following transfer function

$$G(s) = \frac{b(s)}{a(s)} = \mathbf{C}(s - \mathbf{A}_H(s))^{-1}\mathbf{B}(s) \tag{2}$$

where

$$\mathbf{A}_H(s)=s\sum_{i=1}^{n_H}\mathbf{H}_i\exp(-s\vartheta_i)+\sum_{i=0}^{n_A}\mathbf{A}_i\exp(-s\vartheta_i)$$
$$\mathbf{B}(s)=\mathbf{B}_0+\sum_{i=1}^{n_B}\mathbf{B}_i\exp(-s\tau_i)$$

and $a(s)$, $b(s)$ are quasipolynomials in $s\in\mathbb{C}$ as

$$x(s)=s^n+\sum_{i=0}^{n}\sum_{j=1}^{h_i}x_{ij}s^i\exp(-s\eta_{ij}) \quad (3)$$

where $\eta_{ij}\geq 0$ are delays (linear combinations of state and/or input delays), and $x_{ij}\in\mathbb{R}$. A retarded system has $a_{nj}=0$ for all $\eta_{nj}>0$; whereas, it holds that $a_{nj}\neq 0$ for some $\eta_{nj}>0$ for a neutral TDS.

## 2.2 TDS spectral properties

The two considered system types, retarded and neutral, has significantly different spectral features.

**Assumption 1**. There are no common roots of $a(s)$ and $b(s)$. ■

**Remark 1**. Under Assumption 1,

i) $a(s)$ coincides with the characteristic quasipolynomial of (1),

ii) roots of $a(s)$ are system poles, $\sigma_i$, and

iii) roots of $b(s)$ are system zeros, $\zeta_i$. ■

**Definition 1**. The *associate characteristic exponential polynomial* of a neutral system (1) is

$$a_n(s)=1+\sum_{j=1}^{h_n}a_{nj}\exp(-s\eta_{nj}) \quad ■$$

*2.2.1 Retarded system*

**Remark 2**. Consider system (1) of retarded type. Then

i) the system has only isolatedly distributed poles,

ii) there is only a finite number of poles in any vertical strip $\alpha<\operatorname{Re}(s)<\beta$, $\alpha,\beta\in\mathbb{R}$, $s\in\mathbb{C}$,

iii) for any $\gamma\in\mathbb{R}$ with $|\gamma|<\infty$, only finitely many poles are in the half-plane $\operatorname{Re}(s)>\gamma$ while infinitely many are located in $\operatorname{Re}(s)<\gamma$,

iv) poles are continuous w.r.t. coefficients and delays of $a(s)$ [7]. ■

*2.2.2 Neutral system*

**Definition 2.** Define sets of roots

$$\Omega_P:=\{\sigma : a(\sigma)=0\}$$
$$\Omega_a:=\{\operatorname{Re}(\sigma_a) : a_n(\sigma_a)=0\} \quad ■$$

**Remark 3**. For a TDS (1) of neutral type, we can claim:

i) all system poles are isolated,

ii) for the closure $\overline{\Omega}_a$ of $\Omega_a$ it holds that if $\gamma\in\overline{\Omega}_a$, then there is an infinite sequence of poles $\{\sigma_{a,i}\}_{i=1}^{\infty}$ such that $\lim_{i\to\infty}\operatorname{Re}(\sigma_{a,i})=\gamma$, $\lim_{i\to\infty}\operatorname{Im}(\sigma_i)=\infty$,

iii) if there are poles of (1) in bounded vertical strip $\alpha<\operatorname{Re}(s)<\beta$ on the complex plane, these poles converge to the infinite sequence defined in ii) as $|\sigma_i|\to\infty$,

iv) for any $\varepsilon>0$, the system has only a finite number of poles in the right half-plane $\operatorname{Re}(s)>\sup(\Omega_a)+\varepsilon$

v) poles are continuous w.r.t. coefficients of $a(s)$ but not w.r.t. small delay perturbations [7]. ■

**Corollary 1**. The rightmost infinite vertical chain of poles of a neutral TDS (1) approaches the vertical line

$$\operatorname{Re}(s)=\sup(\Omega_a):=C \quad ■$$

A detailed analysis of asymptotic behavior of chains of neutral-type poles for TDS with commensurate delays can be found e.g. in [40].

*2.2.3 Spectral abscissa*

One of crucial notions for this paper is the spectral abscissa.

**Definition 3**. The spectral abscissa $\alpha_P\in\mathbb{R}$ (of poles) is defined as

$$\alpha_P:=\sup\operatorname{Re}\Omega_P \quad ■$$

The spectral abscissa may be nonsmooth, nonconvex or non-Lipschitz [36], especially for neutral delayed systems and, moreover, according to v) of Remark 3, this function can not be continuous due to small delay changes in general [19].

## 2.3 TDS stability

Several stabilities can be established for TDS, namely, asymptotic, exponential, strong, BIBO, $H_\infty$, and others. Let us briefly provide the reader with basic stability properties useful for this contribution.

Asymptotic and exponential stabilities are defined as for finite-dimensional systems, that is:

**Definition 4**.

i) System (1) is *asymptotically stable* if

$$\forall \varepsilon > 0, \exists \delta > 0, \phi(t) \in C([-L,0], \mathbb{R}^n), \|\phi([-L,0])\| < \delta :$$
$$\lim_{t\to 0}\|x(t)\| < \varepsilon$$

where $x(t)$ stands for the solution of (1) with the initial condition $\phi(t) = x(t)$ from the Banach space of real valued continuous functions on $t \in [-L,0]$ and $L$ means the sum of all system delays.

ii) System (1) is *exponentially stable* if

$$\exists \alpha, \beta > 0, \forall \phi(t) \in C([-L,0], \mathbb{R}^n):$$
$$\|x(t)\| \le \alpha \exp(-\beta t)\|\phi([-L,0])\|, t \ge 0$$ ■

Note that exponential and asymptotic stability of retarded are equivalent; while for neutral TDS, exponential stability implies the asymptotic one but the converse does not hold. For exponential stability, the following statements can be claimed [7].

**Proposition 1**. If Assumption 1 is satisfied, it holds that:

i) System (1) (or (2)) of retarded type is exponentially stable iff

$$\forall \operatorname{Re}(\sigma_i) < 0, \sigma_i \in \Omega_P$$

where $\Omega_P$ is defined in Definition 2.

ii) System (1) (or (2)) of neutral type is exponentially stable iff

$$\exists \varepsilon > 0 : \forall \operatorname{Re}(\sigma_i) < -\varepsilon, \sigma_i \in \Omega_P$$ ■

Considering asymptotic stability, a TDS with commensurate delays may or may not be stable in the case of chains of poles asymptotic to the imaginary axis [23]. Analogously, $H_\infty$ or oven the stronger notion of BIBO stability are not possible to be defined in terms of poles loci, in particular, in case of chains of poles asymptotic to the imaginary axis [22], [40]. For instance, a system governed by the transfer function with poles tending to the imaginary axis $G(s) = 1/\left((s+1)^k (s+1+s\exp(-s))\right)$ is BIBO stable for $k \ge 4$.

Strong stability is connected solely with neutral TDS and expresses the sensitivity of $\sup(\Omega_a)$ to infinitesimal perturbations in internal delays; namely, the system is strongly stable if

$$\overline{C} := \lim_{\delta\to 0^+} \sup \Omega_{a,\Delta\tau} < -\lambda \tag{4}$$

for some $\lambda > 0$ where $\Omega_{a,\Delta\tau}$ stands for roots of $a_n(s)$ under a delay perturbation $\Delta\boldsymbol{\tau} = \boldsymbol{\tau} + \delta\boldsymbol{\tau}$, $\|\delta\boldsymbol{\tau}\| < \varepsilon$ in which $\boldsymbol{\tau}$ means the vector of all system delays.

**Proposition 2** [19]. A system (2) is strongly stable iff

$$\sum_{j=1}^{h_n} |a_{nj}| < 1 \tag{5}$$

where $a_{nj}$, $j = 1,2,...h_n$ are real coefficients of the exponential polynomial defined in Definition 1. ■

Notice that if none of $a_{nj}$ can be adjusted by selectable controller parameters in the designed feedback structure, a strongly unstable plant can not be stabilized.

# 3 Problem Formulation and Main Results

Let us concisely introduce and summarize the formulation of the problem solved herein and corresponding basic developed algorithms. The reader is referred to literature for further details if necessary. Two controller design tasks are attacked.

## 3.1 Spectral abscissa minimization

As first, once the controller structure is determined, the spectrum (of feedback poles) is to be shaped so that the "maximum possible" exponential stability is reached. Rephrasingly, the objective is to solve the optimization problem

$$\min_{\mathbf{K}} \Phi(\mathbf{K}) = \min_{\mathbf{K}} \alpha_P(\mathbf{K}) = \min_{\mathbf{K}} \sup \operatorname{Re} \Omega_P$$

where $\mathbf{K} = [K_1, K_2, ..., K_r]^T \neq \mathbf{0} \in \mathbb{R}^r$ represents the vector of tunable controller parameters, taking account of limitations on $\mathbf{K}$ and specific features of neutral TDS, namely, strong stability condition (5). For instance, the use of the penalty function $\pi(\mathbf{K})$ as

$$\Phi(\mathbf{K}) = \alpha_P(\mathbf{K}) + \lambda\pi(\mathbf{K}), \lambda > 0, \pi(\cdot) \ge 0 \tag{6}$$

In [41], the option $\pi(\mathbf{K}) = \left(\sum_{j=1}^{h_N} |a_{nj}(\mathbf{K})|\right)^2$ due to strong stability of the feedback system was suggested; however, it does not guarantee that the constrain (5) holds. A rather more suitable option would be e.g.

$$\pi(\mathbf{K}) = \lambda\left(\max\left(0, \varepsilon - 1 + \sum_{j=1}^{h_N} \left|a_{nj}(\mathbf{K})\right|\right)\right)^2, 0 < \varepsilon << 1$$

yielding $\sum_{j=1}^{h_N} \left|a_{nj}(\mathbf{K})\right| \to 1 - \varepsilon$.

Another possibility is to introduce a barrier function $\varphi(\mathbf{K})$ instead of $\pi(\mathbf{K})$ in (6), e.g. as

$$\varphi(\mathbf{K}) = -\log\left(1 - \varepsilon + \sum_{j=1}^{h_N} \left|a_{nj}(\mathbf{K})\right|\right)$$

as utilized in [37] yet for a state feedback controller and the spectral abscissa minimization only.

Significant contributions to this first problem were given e.g. in [18], [19], [36], [37], [41]. In this paper, we particularly follow in [18], [19] so that the Quasi-Continuous Shifting Algorithm (QCSA) derived therein is extended by an advanced iterative direct-search algorithm to improve the (sub)optimal solution. We have coped with that in [13], [42] where also an example of the algebraic control of an unstable retarded TDS in a special ring of meromorphic functions [43] has been presented. Conclusions of the example are briefly summarized in Section 4 for the reader.

The algorithm solving the first problem in the form of a meta-optimization framework procedure can be summarized as follows.

**Algorithm 1**.

*Input*: Objective function $\Phi(\mathbf{K})$ reflecting (5).

*Step 1*: Set the number $m = 1$ of moved (controlled) poles arbitrarily giving rise to the initial set $\mathbf{K}_0$, and termination parameters for the QCSA [18], [19].

*Step 2*: Move $m \le r$ dominant poles to the left-half plane by applying small changes in $\mathbf{K}$ using the QCSA. If necessary, increase or decrease $m$. Stop when the available degrees of freedom in the controller do not allow to further reduce $\Phi(\mathbf{K})$.

*Step 3*: Select an advanced iterative optimization algorithm minimizing $\Phi(\mathbf{K})$, its control and termination parameters.

*Step 4*: Use the algorithm from Step 3 starting with $\mathbf{K}$ from Step 2.

*Output*: Values of $\mathbf{K}_{opt}$. ■

**Remark 4**. It must be noted here that the root dominancy can be viewed in a variety of ways. The most of approaches adopt the idea that the dominant pole (or a pair) satisfies

$$\sigma_{dom} := \left\{\sigma_i \in \Omega_P : \operatorname{Re}(\sigma_i) \ge \operatorname{Re}(\sigma_j), \forall j \ne i\right\} \tag{7}$$

i.e. it is the rightmost one. This concept seems to be acceptable for retarded TDS. However, some authors hold another definition, which might be useful for neutral TDS due to ii) and iii) of Remark 3, that the dominancy depends on the distance form the origin of the complex plane, i.e.

$$\sigma_{dom} := \left\{\sigma_i \in \Omega_P : \left|\sigma_i\right| \le \left|\sigma_j\right|, \forall j \ne i\right\} \tag{8}$$

The second comment on Algorithm 1 concerns the variations in $m$. The original conception of the QCSA allows only increasing the number; whereas we observed that the decrease can be useful mainly in the case of a clump of the controlled poles. ■

During numerical experiment, the following substantial observation we made. A sketch of the proof of its legitimacy is attached.

**Observation 1**. Although it is stated in [19] that for a neutral TDS there is no reason to deal with poles $\sigma < \overline{C}$ where $\overline{C}$ is defined in (4), we have observed that it is desirable to control also poles left from this vertical line with a sufficiently small modulus. □

*Proof*. Consider the pole dominancy according to (8). Because of ii) – iv) or Remark 3, the rightmost vertical strip for a (exponentially) stable neutral TDS given by Corollary 1 goes from $(-\infty, 0\mathrm{j}) \in \mathbb{C}_0^-$ and tends to $(C, \infty\mathrm{j}) \in \mathbb{C}_0^-$, and only a finite number of isolated poles lies in $s > C$ (or $s > \overline{C}$ - under delay perturbations). The essential part of the system dynamics is determined by a small number of poles with a small modulus in the subset

$$\Omega_P \supset \Omega_{P,dom} := \left\{\begin{matrix}\sigma_i \in \Omega_P : \left|\sigma_i\right| < \left|\sigma_j\right|, \\ i = 1,2,\ldots,i_{\max}, i \ne j\end{matrix}\right\}$$

However, such poles must be located mostly right from poles within the rightmost strip but left from $C$ or $\overline{C}$, i.e. in the region $\mathcal{D}$ given by the contour

$$\overline{\mathcal{D}} := \left\{\begin{matrix}\left\{(\alpha, 0\mathrm{j}), \alpha \in \left[-\alpha_{\max}, C\right]\right\} \\ \cup \left\{(C, \beta\mathrm{j}), \beta \in \left[0, \beta_{\max}\right]\right\} \cup S_{[-\alpha_{\max}, C] \times [0, \beta_{\max}]}\end{matrix}\right\}$$

for sufficiently high $\alpha_{\max}$, $\beta_{\max}$ > 0 where $S_{[-\alpha_{\max}, C] \times [0, \beta_{\max}]}$ expresses the continuous curve joining poles within the neutral strip in the region $\left[-\alpha_{\max}, C\right] \times \left[0, \beta_{\max}\right]\mathrm{j}$ of the complex plane. ■

### 3.2 Finite-dimensional model matching

The second task goes beyond the spectral abscissa minimization and it based on the endeavour to place the dominant poles and/or zeros to prescribed loci. The algorithm framework stems from a finite-dimensional model with the desired behaviour (e.g. a suitable maximum relative step response overshoot, the relative dumping factor and the relative time-to-overshoot), the poles and/or zeros of which represent the required goal to be matched. The procedure has been called the PPSA [38] – in the cited source, three subalgorithms are provided to the reader; however, only retarded systems are considered therein and many statements have to be reformulated or completed.

The method is similar to the one independently derived in [39]; however, there are some crucial differences between them. Namely, the PPSA uses the input-output frequency space of meromorphic Laplace transfer functions, whereas the one in [39] operates purely with poles in the state space domain. The PPSA takes both poles and zeros into account; moreover, they are initially placed in desired positions unambiguously according to the desired dynamical properties; however, they can leave their loci during the shifting. In [39], poles can not leave the prescribed positions and the unrestrained rest of the spectrum is pushed to the left, which may results in a lengthy trial-and-reset placing procedure. Last but not least, a different optimization algorithm is used in this paper.

The concept and framework of the algorithm followed by remarks on two subalgorithms is being introduced now. Recall that they are to be revised in the future; nevertheless, useful detail are provided to the reader in [38].

**Algorithm 2**.

*Input*: The closed-loop reference-to-output transfer function $G_{WY}(s)$.

*Step 1*: Select a suitable desired finite-dimensional model $G_{WY,m}(s)$ structure of the feedback relation, and prescribe its poles and/or zeros.

*Step 2*: Place infinite-dimensional model roots into desired positions, e.g. using the technique introduced in [15].

*Step 3*: If there roots are dominant, terminate the algorithm; else shift the dominant roots to the desired ones, e.g. using the QCSA [18], [19], and push the rest of both spectra to the left as far as possible, i.e. solve Algorithm 1 for this subset.

*Step 4*: If the shifting is successful, terminate the algorithm; otherwise, minimize the cost function $\Phi(\mathbf{K})$ reflecting the distance of dominant from prescribed roots and the spectral abscissa of the rest of both spectra, using advanced iterative algorithms, e.g. from the realm of artificial intelligence.

*Output*: Values of $\mathbf{K}_{opt}$. ■

Note that the selected matching model $G_{WY,m}(s)$ has to be of the structure that guarantees requirements on the poles (zeros) placement problem solvability, model feasibility and a necessary number of degrees of freedom that conditions will be specified in the future research. This may cause the necessity to reset the selection of $G_{WY,m}(s)$.

**Remark 4**. Within the framework of Algorithm 2, there it is possible to develop several subalgorithms. Let us briefly introduce ideas of two of them: The substrategy called "Poles first independently" can be used only if there exist unspecified numerator parameters in $G_{WY}(s)$ not included in the denominator. Poles are shifted first, and afterwards, the spectrum of zeros can be adjusted by the mentioned parameters in the numerator. In some cases, it is not possible to use this strategy even if the parameters exist, see Section 4 for the example. The second subalgorithm attempts to shift poles and zeros together. ■

## 4 Numerical Examples

Both the spectrum shaping strategies briefly discussed in this paper together with the highlighting of some specific troubles are supported by two following numerical examples. As mentioned above, the spectral abscissa minimization of an unstable retarded TDS is the matter of the first one, and the second example is aimed to demonstrate that the PPSA can collapse to a simplified form and to indicate the legitimacy of Observation 1.

**Example 1** [13]. Consider the model

$$y^{(4)}(t) - \ddot{y}(t-0.2) = 0.1u(t-0.4)$$

of unstable retarded TDS describing a skater on the swaying bow where $u(t)$ is the input power and $y(t)$ stands for the output angle deviation. By means of an algebraic controller design method, the following reference-to-output transfer function can be obtained

$$
\begin{aligned}
G_{WY}(s) &= \frac{0.2b_q(s)\exp(-0.4s)}{(s+m_0)^4 a_q(s)} \\
b_q(s) &= 0.2(s+m_0)^4 \textstyle\sum_{i=0}^{3} q_i s^i \\
&\quad + p_0 m_0^4 s^2 \left(s^2 - \exp(-\vartheta s)\right) \\
a_q(s) &= s^2\left(s^2 - \exp(-\vartheta s)\right)\left(s^3 + \textstyle\sum_{i=0}^{2} p_i s^i\right) \\
&\quad + 0.2\exp(-0.4s)\textstyle\sum_{i=0}^{3} q_i s^i
\end{aligned} \tag{9}
$$

where the denominator expresses the characteristic quasipolynomial which has obviously two factors, a polynomial and a quasipolynomial one. Since the spectral assignment for the polynomial factor is trivial, the goal is to find seven unknown parameters of the quasipolynomial factor.

The optimization task can be characterized as follows

$$
\begin{aligned}
\mathbf{K}_{opt} &= [p_2, p_1, p_0, q_3, q_2, q_1, q_0]_{opt}^T = \arg\min_{\mathbf{K}} \alpha_P(\mathbf{K}) \\
&= \arg\min_{\mathbf{K}} \alpha(\mathbf{K}) = \arg\min_{\mathbf{K}} \left\{ \operatorname{Re} s_i : \left[a_q(s)\right]_{s=\sigma_i} = 0 \right\}
\end{aligned}
$$

The initial unstable setting $\mathbf{K}_0 = [1,1,1,1,1,1,1]^T$ with $\alpha(\mathbf{K}_0) = 0.8492$ can be stabilized by the use of the QCSA as $\alpha(\mathbf{K}_{3305}) = -1.4454$ giving rise to the controller parameters

$$
\begin{aligned}
\mathbf{K}_{3305} = [&469418.2, 640264.2, 10560107, 8222650, \\
&106523133, 26247749, 5617613]^T
\end{aligned}
$$

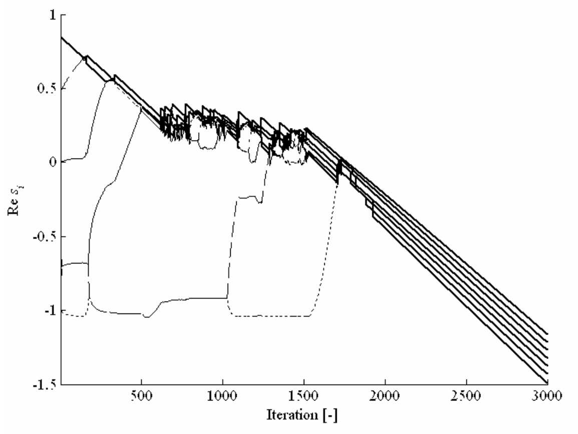
The corresponding evolution of real parts of the dominant part of the spectrum is displayed in Fig. 1. Note that the bold lines mean controlled poles.

This result then can be slightly enhanced e.g. by the well-known Nelder-Mead iterative simplex method (see Fig. 2); however the adjustable controller parameters almost do not differ from $\mathbf{K}_{3305}$.

**Example 2**. Let a non-minimum phase unstable TDS plant be modeled by the transfer function

$$
G(s) = \frac{(s-4)\exp(-s)}{s+1-2\exp(-0.4s)} = \frac{b(s)}{a(s)} \tag{10}
$$

Although system (10) is of a retarded type, whenever it is controlled by any feasible controller, the feedback loop contains neutral delays.



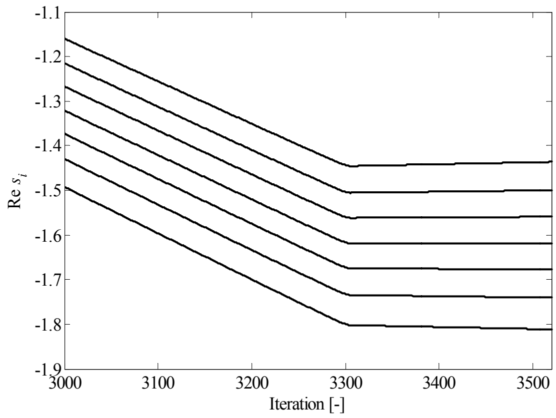




Fig. 1. Evolution of real parts of the dominant poles of (9) using the QCSA

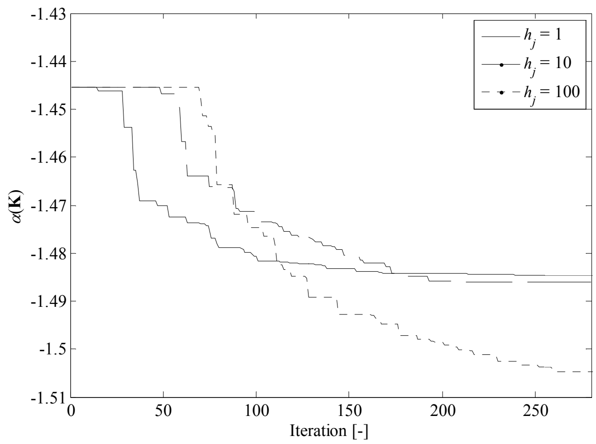



Fig. 2. Evolution of $\alpha(\mathbf{K}) = \alpha_P(\mathbf{K})$ using the Nelder-Mead algorithm for three different initial simplex edge sizes, $h_j$, starting from $i$ = 3305 of the QCSA

For instance, using the Two-Degrees-of-Freedom (2DoF) control system depicted in Fig. 3 (note that $d(t)$ expresses the load disturbance), the simple quasipolynomial approach yields the following reference-to-output transfer function

$$G_{WY}(s)=\frac{b(s)\left(\sum_{i=1}^{2}r_i s^i+q_0+p_0/4\right)}{b(s)\left(s^2+\sum_{i=0}^{1}p_i s^i\right)+a(s)\sum_{i=0}^{1}q_i s^i} \quad (11)$$

where $\mathbf{K}=[p_1,p_0,q_1,q_0,q_2,r_2,r_1]^T$ stand for suitable free real controller parameters which influence the feedback zero-pole distribution; however, the spectrum of zeros can not be pushed to the stable left half-plane in any way. Thus, apply poles shifting without any attempt to adjust zeros.

As first, let us find $C$ defined in Corollary 1. Since $a_n(s)=a_3(s)=1+q_2\exp(-s)$, we have $C=-\ln|-1/q_2|$ that must be strictly negative. This condition yields

$$|q_2|<1 \quad (12)$$

Note that a technique described in [19] enables to verify that $C\approx\overline{C}$, i.e. condition (12) guarantees also the strong stability of the feedback system.

Select a pair of prescribed poles $s_{1,2}=-0.1\pm0.2\mathrm{j}$. To obtain the vertical strip of neutral poles left from the desired pair, the inequality $-\ln|-1/q_2|<-0.1$ must hold, the solution of which reads $|q_2|<0.9048$. Hence set, for instance, $q_2=0.6$.

By placing poles of (11) directly to the desired loci using for $\mathbf{K}_D=[p_1,p_0,q_1,q_0]^T$ in the denominator, the initial feedback spectrum computed e.g. via [44] is

$$\Omega_{P,0}=\left\{\begin{matrix}0.15403\pm3.9438\mathrm{j},-0.1\pm0.2\mathrm{j},\\ -0.4612\pm22.1148\mathrm{j},-0.4618\pm16.016\mathrm{j},...\end{matrix}\right\}$$

Apparently, such a setting results in the asymptotically unstable control system; hence, use the QCSA for shifting the dominant pair of poles towards $s_{1,2}$, and simultaneously push the rest of the spectrum to the left half-plane. In Fig. 4, the distance of the dominant pair, $\sigma_{1,2}$, from the desired one, $|\sigma_1-s_1|$, the overall spectral abscissa, $\alpha(\mathbf{K}_D)$, and the spectral abscissa of the rest of the spectrum, $\alpha_r(\mathbf{K}_D)$, are displayed. The spectral abscissa has been pushed to the left half-plane and the system is thus stabilized while the imaginary part of the dominant pair has increased such that the distance $|\sigma_1-s_1|$ is unacceptable.

Once the QCSA is finished, an iterative optimization algorithm initialized by eventual values of $\mathbf{K}_D$ from the shifting is used to improve the values of $|\sigma_1-s_1|$ and $\alpha_r(\mathbf{K}_D)$.

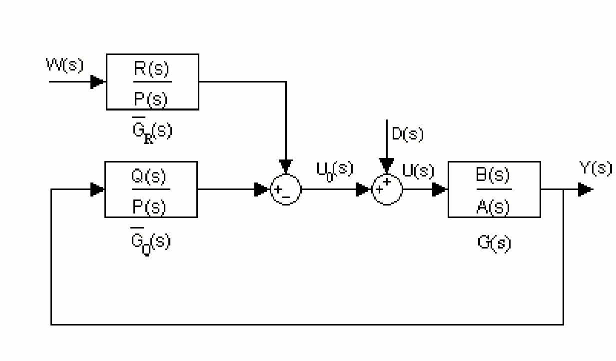



Fig. 3. The TFC control system

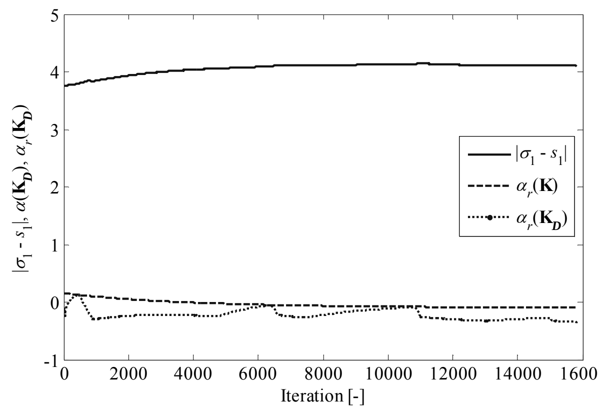



Fig. 4. The evolution of $|\sigma_1-s_1|$, $\alpha(\mathbf{K}_D)$, $\alpha_r(\mathbf{K}_D)$ for (11) using the QCSA with $q_2=0.6$

Consider the following objective function

$$\Phi(\mathbf{K}_D)=|\sigma_1-s_1|+\lambda\alpha_r(\mathbf{K}_D)$$

and solve the problem $\mathbf{K}_{D,opt}=\arg\min_{\mathbf{K}}\Phi(\mathbf{K}_D)$ again by the use the well-know NM algorithm. Initial parameters setting from the QCSA reads

$$\mathbf{K}_{D,opt,0}=\mathbf{K}_{D,15860}=[3.309,0.9853,0.5754,0.1184]^T$$

Note that the edge length of the initial rectangular simplex was set to $h_i=10$. Since the dominant pair of poles is the only pair $\sigma_{i,i+1}$ with $\mathrm{Re}\,\sigma_{i,i+1}>C$ and hence the solution would be trivial while ignoring poles with the real part less than $C=-0.5108$, we have taken the region for the poles seeking as $R=[-6,15]\times[0,40]\mathrm{j}$ in the accordance to Observation 1. Evolutions of $|\sigma_1-s_1|$

and $\alpha_r^R(\mathbf{K}_D)$ for three different values of $\lambda$ are provided in Fig. 5 where $\alpha_r^R(\mathbf{K}_D)$ means the spectral abscissa of the rest of poles in the particular region $R$. Consequently, corresponding step responses compared with the desired model where the transfer function numerator is taken as a constant value (because of unknown zeros) are displayed in Fig. 6.

Finally, verify the Observation 1 by the numerical test. Let $\Omega^i_{P,opt,250}$ be the subset with the number $i$ of the most dominant poles of the spectrum $\Omega_{P,opt,250}$, see (13), in $R=[-3,0]\times[0,200]\mathrm{j}$ covering all low-frequency poles obtained by 250 iteration steps of the NM algorithm for $\lambda = 0.2$.

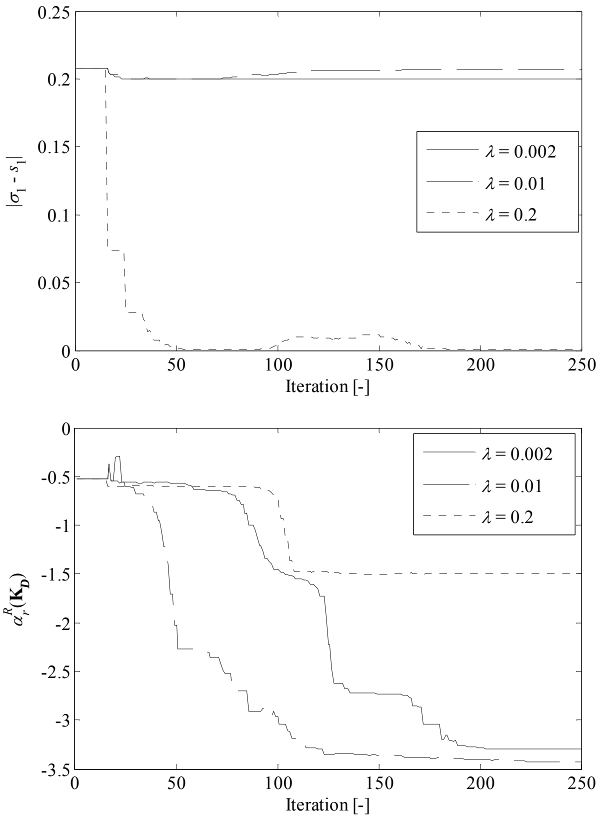



Fig. 5. The evolution of $|\sigma_1 - s_1|$ and $\alpha_r^R(\mathbf{K}_D)$ for (11) with $q_2 = 0.6$ using the NM algorithm for various $\lambda$

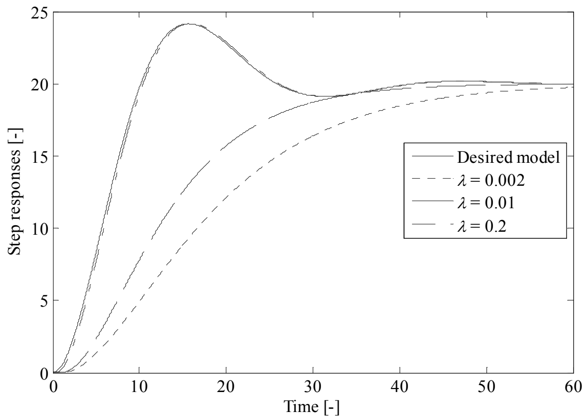



Fig. 6. The comparison of step responses for (1) with the constant numerator and $q_2 = 0.6$ using the NM algorithm for various $\lambda$

$$\Omega_{P,opt,250} = \left\{ \begin{array}{l} -0.1 \pm 0.2\mathrm{j}, -1.4967 \pm 3.7889\mathrm{j}, \\ -2.589 \pm 10.1536\mathrm{j}, -2.2301 \pm 16.7133\mathrm{j}, \\ -2.1565 \pm 22.8112\mathrm{j}, -2.0871 \pm 29.2877\mathrm{j}, \\ -1.8438 \pm 35.5272\mathrm{j}, ... \end{array} \right\} \tag{13}$$

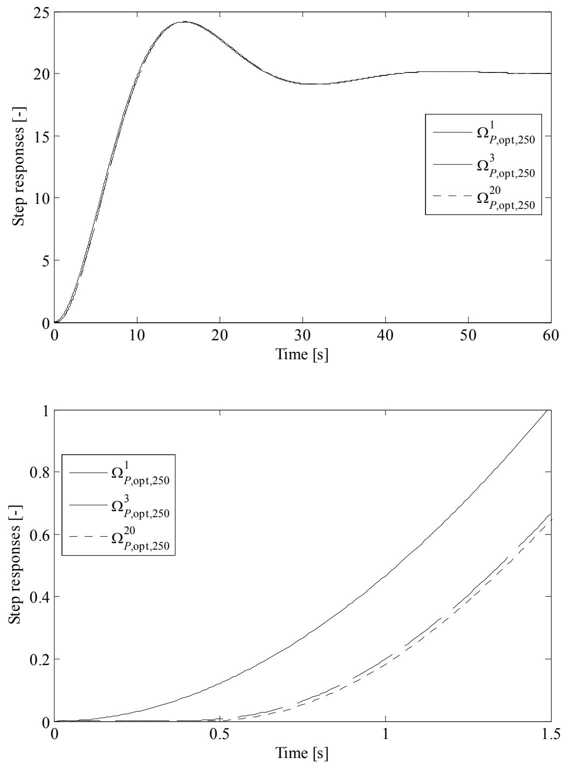



Fig. 7. The comparison of step responses of models with various subspectra with the constant numerator and $q_2 = 0.6$, $\lambda = 0.2$ using the NM algorithm

In Fig. 7, the comparison of step responses for $i = 1,3,20$ as the overall and a detailed view is provided. The substantial difference between $\Omega^1_{P,opt,250}$ and $\Omega^3_{P,opt,250}$ yet a negligible one between $\Omega^3_{P,opt,250}$ and $\Omega^{20}_{P,opt,250}$ verifies Observation 1.

## 5 Future Research

Prior to conclusions of the paper, let us briefly discuss weaknesses of both the algorithms yielding the suggestions to the course of the future research.

Computational aspects of the QCSA have been touched in [42]. Namely, the convergence and speed of the shifting seems to be improved by the strategy that only poles (zeros) of the same type (real, complex) are approaching to each other, or by thorough consideration that a complex conjugate pair means two separate roots instead of one for the course of the QCSA. Moreover, it might be better to consider a multiple root as a nest of single close roots.

The selection of a suitable matching model constitutes another problem of the latter algorithm. So far, the presented idea of the desired maximal step response overshoot, time-to-overshoot and the relative dumping has been adopted only.

Regarding the selection of the minimization method, calculation acceleration and the level of its computational complexity, the substantial question is the number of spectrum determinations per iteration which is the most time consumptive computational operation of the current version of the PPSA. This time can be effectively reduced by a suitable demarcation of the region in which the spectrum is computed. The searching region, ought to be extensive enough providing a sufficient margin for possible abscissa discontinuities. There is naturally a plenty of possible modern evolutionary or genetic algorithms; for instance, the Particle Swarm Optimization (PSO) dosed by means of the deterministic chaos [45] is a possible candidate for the future tests. Moreover, optimization algorithms themselves include many control and weighting parameters which must be set properly, e.g. using trial-to-reset tests.

Last but not least, the cost function has to be subjected a metaoptimization procedure searching its (sub)optimal weighting parameters; for instance, by applying ideas on intelligent multiagent optimization techniques [46]-[48]. We also observed e.g. that the higher parameter $\lambda$ in the function is, the better declination and eventual value of $\alpha_r(\mathbf{K})$ would be obtained with a worse approaching of prescribed and desired roots loci; which, however, does not hold in every cases, see e.g. Fig. 5, partially because of a negative value of $\alpha_r(\mathbf{K})$.

## 6 Conclusions

To summarize this contribution, two ideas of controller parameters tuning strategies for unstable TDS based on the shaping of the spectrum of feedback poles and/or zeros have been outlined. The first one adopts the natural idea of pushing the spectrum to the stable left half-plane as far as possible. The extension of existing algorithms by means of an additional optimization iteration algorithm has been proposed and demonstrated by a concise example of control of an unstable retarded TDS. The second approach (PPSA) goes beyond this and attempts to match the feedback system with the selected desired finite-dimensional model by adjusting tunable controller parameters. This strategy has been shown and verified in a numerical example of shaping the spectrum of an unstable neutral TDS system which has i.a. shown that the latter algorithm can almost collapse to the former one. Moreover, new findings about the course of the PPSA have been obtained. Both the procedures are easy to be practically implemented via general programming languages and thus useful for engineers. The course of the future research has also been provided.

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