An Experimental Study of a Lane Departure Warning System Based on the Optical Flow and Hough Transform Methods

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Abstract: - The use of rumble strips on roads can provide drivers lane departure warning (LDW). However, rumble strips require an infrastructure and do not exist on a majority of roadways. Therefore, it is very desirable to have an effective in-vehicle LDW system to detect when the driver is in danger of departing the road and then triggers an alarm to warn the driver early enough to take corrective action. This paper presents the development of an image-based LDW system using the Lucas-Kanade (L-K) optical flow and the Hough transform methods. Our approach integrates both techniques to establish an operation algorithm to determine whether a warning signal should be issued based on the status of the vehicle deviating from its heading lane. The L-K optical flow tracking is used when the lane boundaries cannot be detected, while the lane detection technique is used when they become available. Even though both techniques are used in the system, only one method is activated at any given time because each technique has its own advantages and also disadvantages. The developed LDW system was road tested on several rural highways and also one section of the interstate I-35 freeway. Overall, the system operates correctly as expected with a false alarm occurred only roughly about 1.18% of the operation time. This paper presents the system implementation together with our findings.

Key-Words: - Lane departure warning, Lucas-Kanade optical flow, Hough transform.

1 Introduction

Roadway departure fatalities including run-off-the-road (ROR) and head-on fatalities are a series problem in the United States. According to the National Motor Vehicle Crash Causation Survey (NMVCCS) data [1], ROR crashes contribute to a large portion of fatalities and serious injuries to motor vehicle occupants and over 95% of the critical reasons for single-vehicle ROR crashes were driver-related. Moreover, statistics data indicated that 70% of ROR fatalities occur on rural highways and about 90% occur on two-lane roads [2]. The most common approach to prevent single vehicle lane departure is the use of rumble strips [3, 4] on road shoulders. Actually, the use of rumble strips on roads has proven to be an effective means of providing drivers departure warning [5]. But rumble strips require an infrastructure and are not available on all roadways. Development of techniques such as lane departure warning (LDW) systems can improve traffic safety significantly. A lane departure warning system should be able to detect when the driver is in danger of departing the road and then trigger an alarm to warn the driver early enough to take corrective action.

Various LDW techniques can be found in the literature (e.g., [6-13]). General working principle of camera-based LDW systems is to track lane markers on the road to see if the vehicle is straying outside the lane. But there is no guarantee that the lane markers will always be present on the road. Various environmental factors such as rain and heat might wear out the lane markers and the presence of snow on the road might also affect the visibility of road markers. Therefore, it is highly desirable to develop a system that can not only make use of lane markers when they are available, but can also work when lane markers become invisible for a short period of time. This paper presents the development of an image-based LDW system that integrates two techniques, i.e., the Lucas-Kanade (L-K) optical flow method [14, 15] and the Hough transform-based lane detection method [15-19], into its operation and implementation algorithm to determine the vehicle's lateral status and then issue a warning to the driver, if necessary. The L-K optical flow point tracking is used when the lane boundaries cannot be detected, while the Hough-transform based lane detection technique, via the Gaussian filtering/smoothing and the Hough transform, is used when the lane markers become available. The front-view images captured by an in-vehicle camera were converted to their corresponding top-view images via a homography. Based on these top-view images, the L-K optical flow is then used to track points (i.e., the Harris "corners") from frame to frame to find the vehicle's heading angle and thus its lateral position by evaluating the relationship between these tracked features. The lane detection method looks for the lane markers painted on each side of the road and if available, the system calculates the distance to it and compares that to a past value to determine where the vehicle is in the lane and in which direction it's headed.

In this paper, we present our developed system, which includes the lane detection method, signal flow block diagrams, operation algorithm, and hardware implementation of an alarm warning mechanism. Even though both the L-K optical flow and lane detection techniques are integrated into this system, only one method is activated at any given time because each technique has its own advantages and disadvantages. The paper is organized as follows. Section 2 briefly describes the L-K optical flow method which was used in determining the vehicle's heading angle and lateral speed. Section 3 describes the lane detection technique and an explanation of the image processing via the Gaussian filtering/smoothing and the Hough transformation is given. Section 4 discusses the system implementation which includes the integration of these two techniques, operation algorithm and hardware implementation of an alarm warning mechanism. Section 5 presents the road test results and findings. Finally, Section 6 gives the conclusion.

2 The Optical Flow Method

Optical flow is an approximation of the image motion based on local derivatives in a given sequence of images. It is the distribution of apparent velocities of movement of brightness patterns in an image. In a two-dimensional (2D) plane it specifies how much each image pixel moves between adjacent images. Sequences of ordered images allow the estimation of motion as either instantaneous image velocities or discrete image displacements. Consequently, optical flow can give important information about the spatial arrangement of the objects viewed and the rate of change of this arrangement. There are several methods available for determining optical flow including the phase correlation method [20], the block-based method [21], the differential methods [14, 22] and discrete optimization method [23], etc. The differential methods are based on partial derivatives of the image signal (e.g., the Lucas-Kanade method [14], the Horn-Schunck method [22]).

In [24, 25], we used the L-K optical flow method to determine the vehicle's heading angle via the top-view image processing. The movement obtained from each frame is accumulated to obtain the lateral position of the camera and thus, the vehicle's lateral position relative to its previous position. Instead of using a hypothesis of guessing the best candidate among several possible candidate heading angles with calculated sum of absolute differences (SAD) at each frame [26, 27], we used the feature selection and optical flow tracking of the consecutive top-view images to determine the heading angle. Note that other potential application techniques/methods to predict the vehicle's heading angle based on its previous data and/or partial knowledge such as the Kalman filtering [28], the software agent based approach [29], the autoregressive integrated moving averaging approach [30], etc. can be found in the literature. A point of our interest in an image is a point which has a well-defined position and can be robustly detected and tracked. The feature selection/detection and tracking are based on the top-view images. At each time frame, a set of "good" features (or "corners") are determined. Trackable (and reliable) features are points that contain enough information to be picked from the current frame to the next frame. They should have brightness constancy, sufficient texture variations, and do not deform much over time [15]. Trackable points are called "corners", and the words "corner" and "feature" are sometimes used interchangeable in literature. If we choose a point that is unique (i.e., a "good" feature) in an image frame then we have a pretty good chance of finding that point again in the next image frame. In practice, the point or feature we select should be unique, or nearly unique, and should be parameterizable in such a way that it can be compared to other points in another image. The most commonly used definition of a corner was provided by Harris [31] based on the second-order derivatives of the image intensities. However, instead of using the Harris's method, we used the Shi and Tomasi's method [32] to determine good corners (features). For details about the vehicle's heading angle determination, please refer to [25].

3. Lane Detection via the Hough Transform

The lane marker detection is based on the Gaussian filtering and smoothing of images via an in-vehicle camera and also the Hough transformation [15-19]. The Hough transformation is used in lane marker detection, which forms a subset of our entire LDW system. In order to explain the lane detection signal processing, in the following we use a sample front-view image shown in Fig. 1 as an example to illustrate our approach.

Fig. 1 A snapshot of a front-view image

Consecutive front-view images taken by the camera in real time are first converted to their corresponding top-view images via the homography transformation [15]. Fig. 2 shows the corresponding top-view image of Fig. 1 after this transformation.

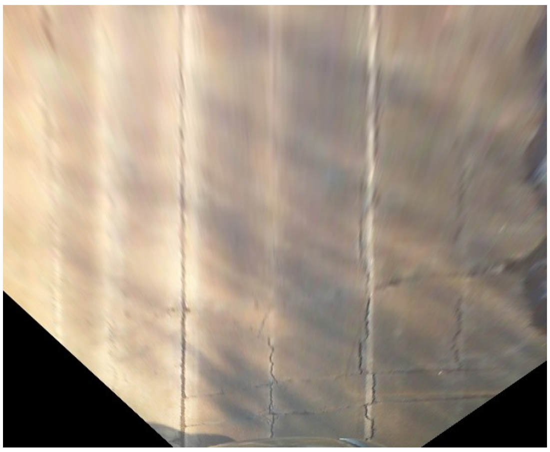

Fig. 2 The front view image of Fig. 1 after the homography.

The conversion to Fig. 2 involves removing the perspective effect in Fig. 1, the front-view image. That is, re-sample the incoming image and re-mapping each pixel in the captured image toward a different position and producing a new two-dimensional array of pixels. The resulting image represents a top view (or the bird's eye view) of the road region in front of the vehicle as it was observed from a significant height. To reduce computational time, a portion of the image, called the Region of Interest (ROI), instead of the whole region in a top-view image is then chosen. For instance, for the same top-view image shown in Fig. 3, we choose its ROI to be a rectangular box image, a 100 pixel × 320 pixel sample of the image of Fig. 2, centered to be straight in front of the vehicle.

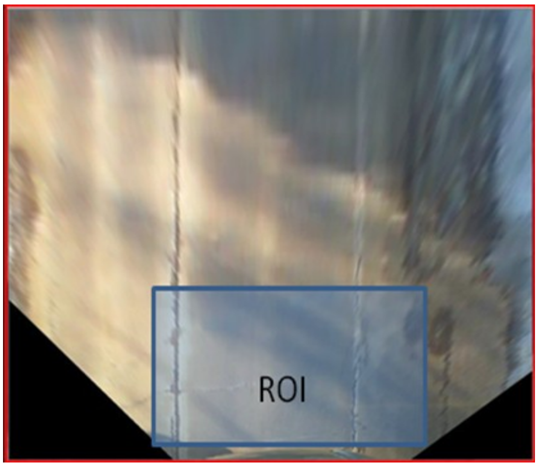

Fig. 3 The front-view image showing the ROI to be processed.

This ROI image, as shown in Fig. 4, starts right after the vehicle's hood on the bottom of Fig. 1 and goes up to show about 10 feet in front of the vehicle.

Fig. 4 The ROI image.

The ROI is chosen just wide enough to fit the width of a normal lane on a road or highway. We use the ROI to greatly speed up the computations, allowing the computer program to run in real time. In addition, the possible distortions around the edges of the original image (the so-called "fisheye" effect) caused by converting to its top-view image can also be avoided. The Hough transform is then applied to the Gaussian filtered and smoothed ROI image to find all vertical lines on the image and draws them as red lines as shown in Fig. 5.

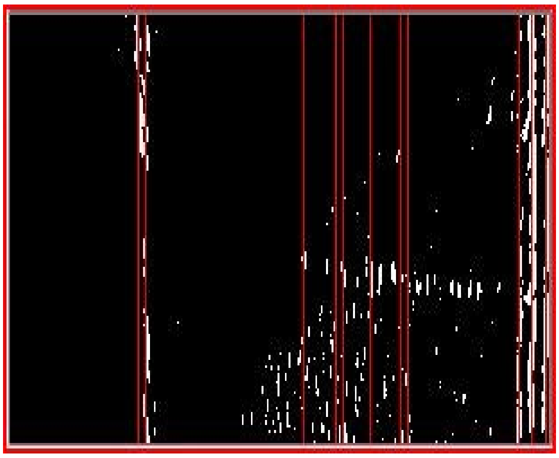

Fig. 5 Vertical lines (in red) indicate possible lane edges.

The Hough transform is a feature extraction technique used in image analysis and computer vision. It original version is to detect straight lines and later the transform has been extended to identify arbitrary shapes, most commonly circles. We use this transform [15-19] to detect straight lines and the vertical lines have to have a specific score in order to be counted. This is known as the line score. To find the line score the image is split into columns and the pixel values are averaged in this column. This average is the line's line score. The Hough transform basically uses a voting scheme to detect the most likely edge of the lane. If the line with the most votes is higher than the threshold level (i.e., meets a certain score requirement) then that line is said to be the lane marker. This is done on the left half of the image to get the left lane marker and then performed again on the right half of the image to get the right lane marker as shown in Fig. 6.

4 The LDW System Integration and Implementation

In this section, we briefly describe the system we developed. To improve the overall system performance and reduce the possibility of false alarms, our system operation takes a mixed approach by using the Lucas-Kanade (L-K) optical flow technique described in [14, 24] and the Hough transform-based lane detection method given in the previous section. Even though both the L-K optical flow and lane detection techniques are used in our system, only one method is activated at any given time because each technique has its own advantages and also disadvantages. The L-K method is used by tracking points from frame to frame to determine where the vehicle is located with respect to the center of the lane and in which direction the vehicle is headed. The schematic diagram of the data flow and signal processing in [24, 25] to determine the vehicle's heading angle and thus, its lateral position is shown in Fig. 7. In this flow chart, θ_i represents the vehicle's heading angle at time instant i, that is, the angle between the vehicle moving direction and its longitudinal axis at time i. Note that in Fig. 7 the vehicle "initial" lateral position will be reset as often as possible once the lane markers become available. In our system implementation, the OpenCV software package [15] is used to process the image conversion and transformation. As described in Section 3, the Hough transform-based lane detection method looks for the lane boundaries painted on each side of the road and if available, then the system calculates the distance to it, and compares that to a past value to determine where the vehicle is in the lane and in which direction it's headed. Combining both methods, the overall system has been implemented based on the signal flow chart given in Fig. 8.

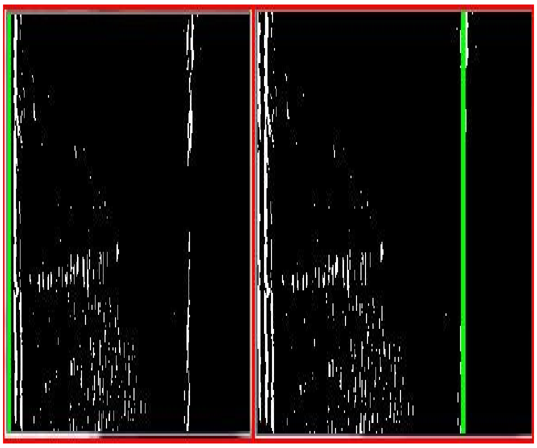

Fig. 6. Lane markers (in green) after performing the Hough transform.

A. Conditions for Issuing an Alarm

Since the U.S. Interstate Highway System uses a 12-foot standard for lane width, we followed this standard width to implement our LDW operation algorithm. If the vehicle is moving along the center of the lane, then there is roughly about 6 feet distance to both the right and left edges of the road.

Initialization (Initial vehicle lateral position, heading angle (θ_o), camera intrinsic parameters, etc.)

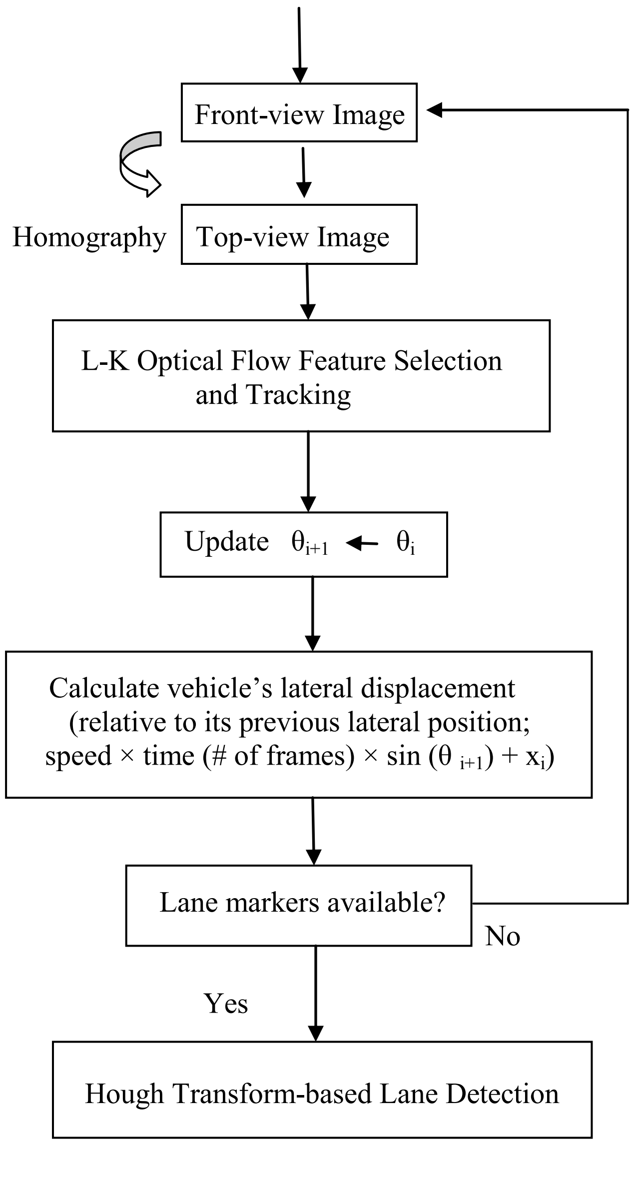

Fig. 7 The L-K optical flow implementation flow chart.

The conditional statement for the alarm is established as follows: if the vehicle is within 3 feet of the lane boundary (edge) and it's current distance to this lane boundary is smaller than it was in the previous frame (i.e., the vehicle is moving toward the lane boundary), then the alarm will sound, otherwise no alarm. The sequence of this operation is: (1) load frame from camera, (2) find any lane boundaries, (3) find the distance to the detected boundaries, (4) compare lane distance to previous frame, (5) sound alarm if conditions are met, and (6) load next frame. The steps (1) to (6) keep repeating as the vehicle is moving. Note that if the lane boundaries cannot be detected then the optical flow point tracking operation is activated to determine the vehicle's lateral position.

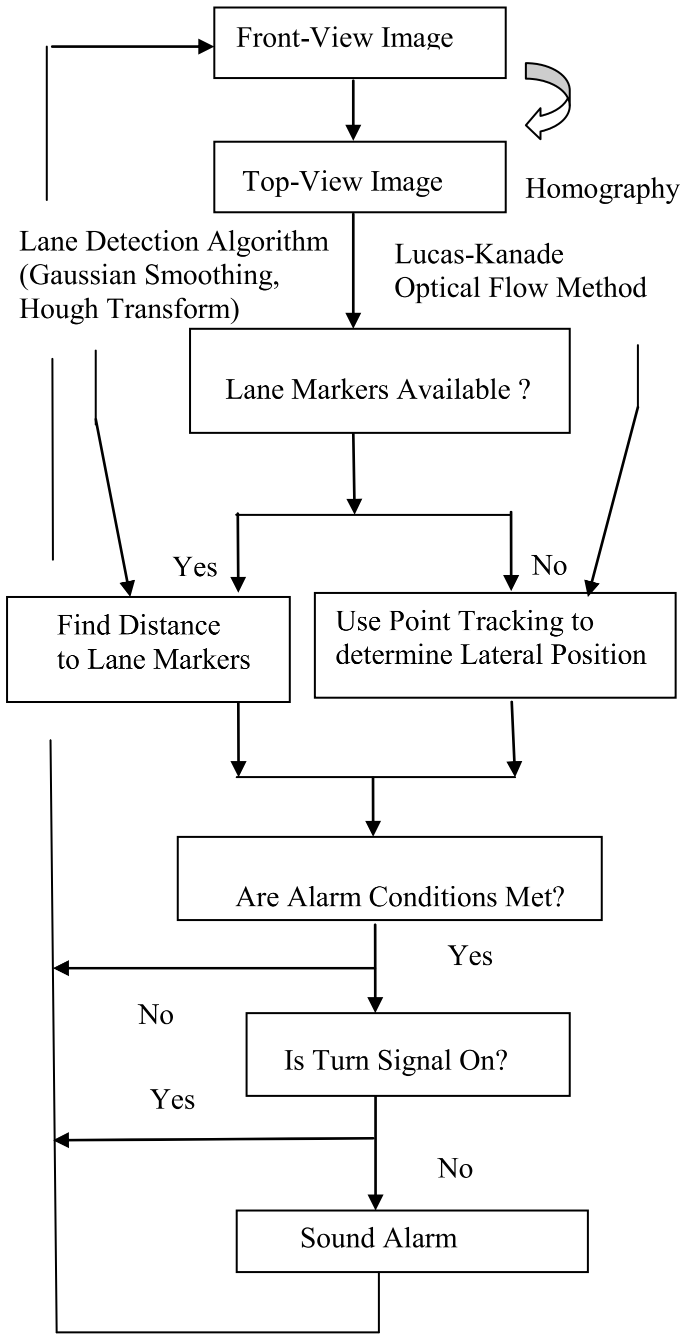

Fig. 8 Flow chart of the developed LDW operation.

B. A Switch Mechanism for Lane Changing

It is possible that the vehicle deviates from its current lane due to the driver changes the lane. If this is the situation, then the alarm should not be issued. To deal with the situation of the driver intentionally changing the lane without issuing an alarm, two conditions need to be satisfied at the same time. That is, the vehicle's later position is within the 3-feet zone of the lane edge and the driver also turns on the turn signal light. To implement this, a two-switch mechanism attached to the signal control lever is used and shown in Fig. 9.

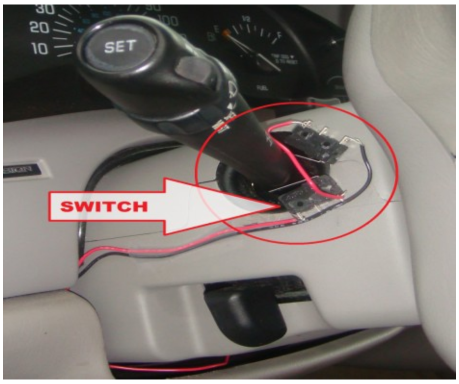

Fig. 9 Part of the LDW system showing the two switches connected with the turn signal control.

The circuit consists of an Altec portable speaker powered by a battery plus two limit switches. The output from the speaker is fed into the computer code run by a laptop computer. The speaker is modified in such a way that we are able to control its operation through two limit switches connected in parallel. These switches are further connected to the vehicle's turn signal light indicator lever such that it works in synchronous with the turn signal control. When the vehicle's deviates from its lane center more than three feet and the turn signal control is triggered, then the alarm system will be switched off, indicating the situation that the driver is changing the lane.

The system showing the alarm (speaker) and camera is given in Fig. 10. Extensive road tests were conducted on I-35, US-53, Minnesota Highway 5 and several other rural highways outside the city of Duluth, Minnesota to test the performance of the developed system. In the next section, we briefly describe our test results and findings together with some discussions. For details about road test results under different weather conditions and factors that could affect the system performance, please refer to [33-35].

Fig. 10 The LDW system showing the in-vehicle camera and the alarm speaker.

5 Results and Findings

During the implementation of the operation algorithms shown in Fig. 8, the OpenCV software was used. Note that OpenCV is an open source computer vision library written in C and C++ and runs under Linux, Windows and Mac OS X. There is active development on interfaces for Matlab, and other languages. This software originally was developed for computational efficiency with a strong focus on real time applications. Since its library contains over 2,500 optimized algorithms that span many areas (e.g., medical imaging, user interface, camera calibration, stereo vision, robotics, etc.), we use the open source library functions OpenCV 2.3.1 [15] to implement our operation algorithms. In this section, we discuss about the results and finding during our road tests.

5.1 Road Tests

We conducted road tests to check the accuracy of our operation algorithm in issuing a warning sound when the deviation of the vehicle from the center of its moving lane exceed a preset threshold. The developed system was set up in a 2001 Buick Century vehicle. Sets of images were taken on highways surrounding the city of Duluth area. These highways included Interstate I-35, US-53, Minnesota HW 5, Rice Lake Road, Martin Road, and Jean Duluth Road. These roads varied in speed limit, curvature, amount of traffic, number of lanes, and road construction, which gives a richer diversity in the data. Each test contains 200 or 300 consecutive images looking out the front of the test vehicle. This corresponds to about 40 or 60 seconds of driving. In each test the test vehicle maintained a constant speed using the cruise control, speeds ranged from 45 miles per hour (mph) to 70 mph depending on the speed limit of the highway. The tests were taken at different times of day and in different conditions. By purposely departing from the lane we can tell if the system is working or not. If the system is working then a warning is issued to the driver only when the vehicle is about to leave the lane.

5.2 Results and Discussion

Road tests under different weather conditions (e.g., sunny, rain, snow) including night time driving were conducted to evaluate the performance of the developed system. Fig. 11 shows a partial set of consecutive images captured by the camrea while driving north on interstate I-35 under a clear weather condition. And the results showing the vehicle's heading angle and its lateral position versus time (frame) together with other information can be found in Fig. 12.

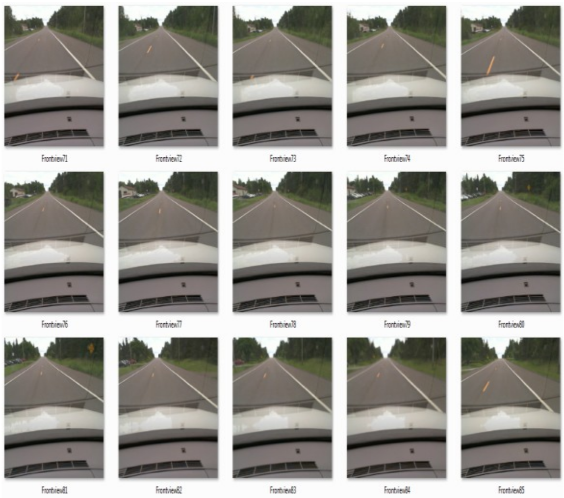

Fig. 11 Partial set of consecutive image frames captured during a road test on interstate I-35.

In the following, we briefly explain the results shown in this figure. Very similar results were found in many other road tests. In Fig. 12, the horizontal axis represents the time (frame), 1 would be the very first frame of the test and 200 would be the very last frame. This corresponds to 40 seconds of driving. The heading angle (shown in yellow bars) in the vertical axis represents the direction the vehicle is going. Travelling straight ahead would give a heading angle of 0 degrees. If the vehicle turned to the left then the heading angle would be negative, and to the right would be positive. Heading angle is given in degrees. It is calculated using the previous frame and the current frame. The

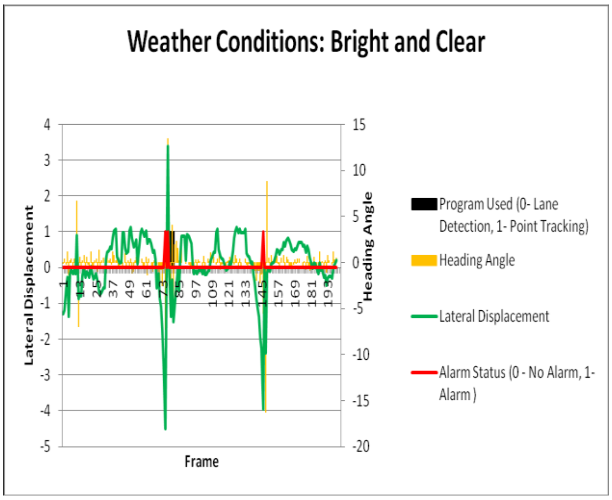

Fig. 12 Road test results north bound on I-35.

lateral displacement (in green line) in the vertical axis represents how far away the vehicle is from the center of the lane (shown in feet). When the center of the vehicle is in the center of the lane, this would correspond to a lateral displacement of 0 feet. If the vehicle moved to the left, the displacement would be negative, and if the vehicle moved to the right it would be a positive displacement. Since the width of a standard (and also typical) U.S. highway is 12 feet, this means that if the lateral displacement was 6 feet, then the center of the vehicle would be on the right lane boundary, and if the lateral displacement was negative 6 feet then the center of the vehicle would be on the left lane boundary. The words "Program Used" (in black bars) in Fig. 12 indicate whether the operation algorithm in Fig. 8 used the lane detection method or the L-K optical flow method to determine the vehicle's position. For example, the black bars shown in Fig. 12 indicated that the optical flow method was used because no lane markers were detected during that period of time. Clearly, it is used much less than the lane detection method. The alarm status (in red line) shows if and when an alarm was sounded. If the alarm spikes up to 1 this means that a warning was issued. We issue an alarm if the lateral displacement is more than ± 3 feet, and the vehicle is moving closer to the edge. We choose 3 feet for our test vehicle. In the beginning of this road test we started with our vehicle in the middle of the road which corresponds to a lateral displacement of 0 feet. From Fig. 12, it can be seen that an alarm sounded when the lateral displacement exceeded the allowable 3 feet margin. During the first 74 frames we do not stray very far from the center of the road. To test if an actual warning would sound we intentionally drove the vehicle towards the edge of the left lane and right lane and indeed an alarm sounded and indicated at frames 75 and 77. Note that in each of these frames, except frames 75-81, the lane detection method was used, which means that the lane boundaries were found and the distance to them was determined. The lateral displacements we found at frames 75 and 77 were - 4.50 feet and 3.40 feet, respectively. This means that the vehicle is about to drive out of the lane, so a warning is issued to the driver. After these warnings were issued we corrected our position in the lane and back towards the center. The program was running well and during the time period frame 75-81 since no lane markers were found, the optical flow point tracking method was used. For the rest of the frames (i.e., 82 to 200), lane markers were found and so the optical flow method was no longer used.

We conducted extensive road tests on I-35, Minnesota Highway 5, US-53, Rice Lake Road, Martin Road, and Jean Duluth Road under different weather conditions (e.g., rain, snow, sunny, snow storm, night time). In addition, during these tests, the vehicle was travelling at different speeds. Figures 13-18 show partial consecutive images taken by the camera during the test and also summarize each of these test results. The detailed road test results and discussion including the effects of image quality, road quality, shade, reflections, etc. can be found in [33]. Overall, the LDW system correctly sounded a warning it should have been given most of the time. False Warnings mean that the LDW system issued a warning when it shouldn't have. Overall, we found that the system sounded a false warning about 1.18% of the time.

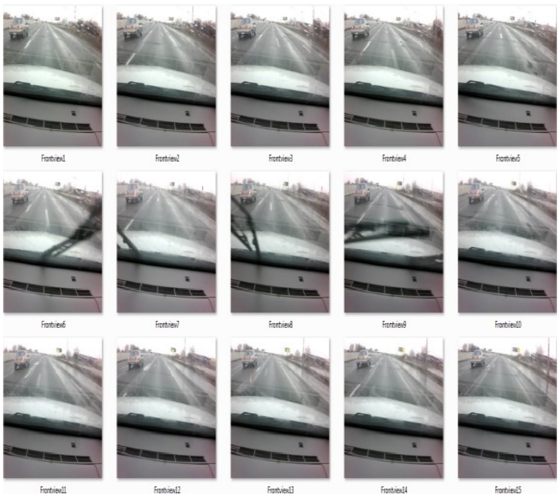

Fig. 13 Partial set of consecutive image frames captured during a road test on HW 5.

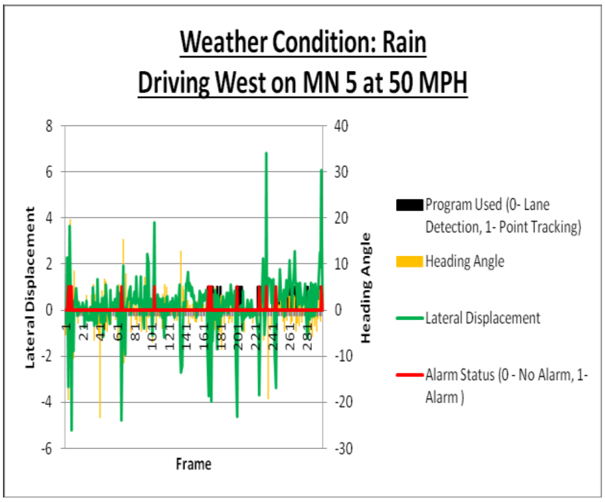

Fig. 14 Road test results north bound on HW 5.

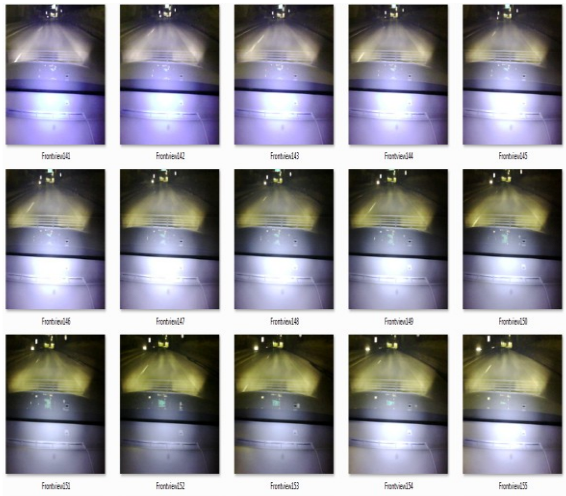

Fig. 15 Partial set of consecutive image frames taken during a nighttime driving on interstate I-35.

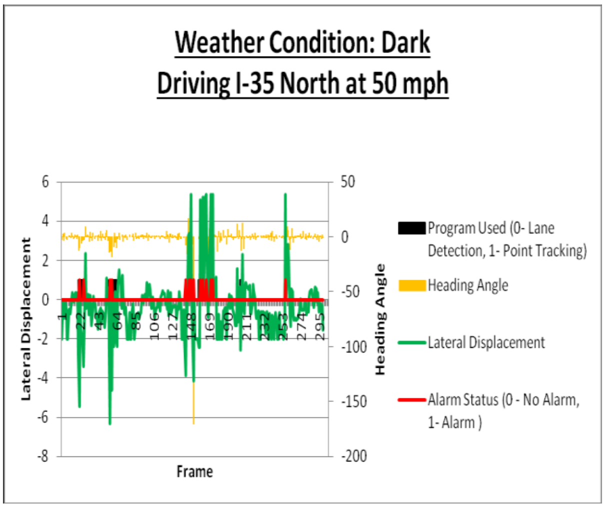

Fig. 16 Road test results north bound on I-35.

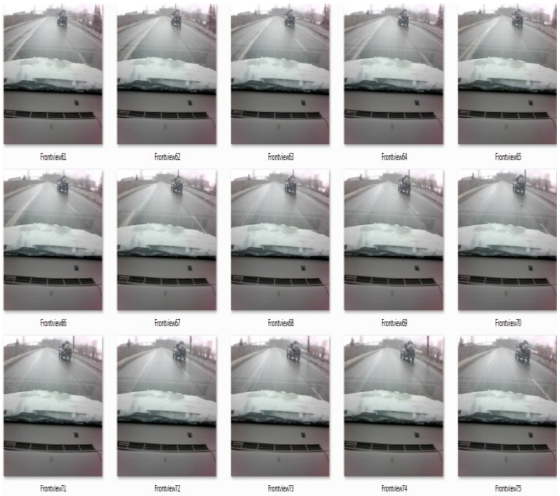

Fig. 17 Partial set of consecutive image frames captured during a road test on I-35.

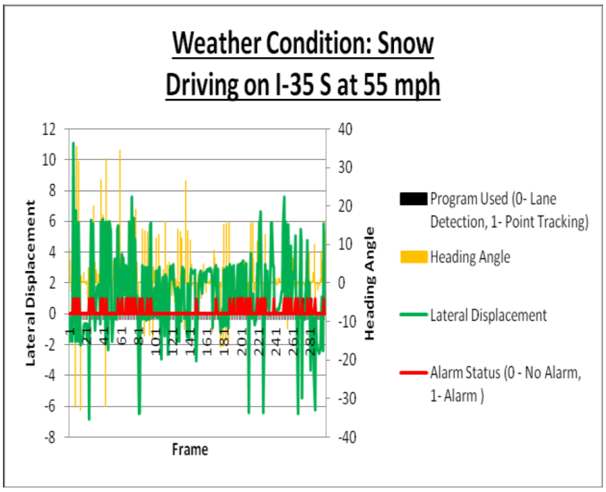

Fig. 18 Road test results on I-35

6 Conclusion

The use of rumble strips on roads has proven to be an effective means of providing drivers lane departure warning (LDW). However, rumble strips require an infrastructure and do not exist on a majority of roadways. Development of various techniques such as LDW systems can improve traffic safety significantly. This paper presents an image-based LDW system by integrating the L-K optical flow method and the Hough transform-based lane detection method. The L-K point tracking is used when the lane boundaries cannot be found, while the lane detection technique is used when they become available. Even though both the L-K optical flow and lane detection techniques are used in our system, only one method is activated at any given time. An operation algorithm via the mixed techniques is implemented with some detailed explanation. Based on the implemented hardware/software system, we conducted extensive road tests. Overall, for the most part our system operates correctly as expected with a false alarm occurred only roughly about 1.18% of the operation time. Factors that could affect the system performance can be found in [33]. Since our approach only needs the minimal set of information to characterize the vehicle lateral characteristics, this makes it more feasible in a vehicle application.

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