

# Study on base-core Mode Coupling Networks

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*Abstract:* - There exist a kind of networks composed of two coupled network which are a base network and a core network respectively. For instance, an urban public transport network consists of a bus network (base network) and subway network (core network), the two networks are coupled together. We proposed a base-core coupling network model to study the performance of the kind of coupling network. In the model, the base network is generated first, then the nodes of the core network are selected from the base network according to a preferential selection scheme based on  $k^\alpha$  and  $b^\alpha$ , where  $k$  and  $b$  represent the node degree and betweenness respectively,  $\alpha$  represents the preferential selection exponent. Under different value of  $\alpha$ , we investigate the performance of the coupling network in different network structure and different network size collocations. Simulation result shows: the greater value of  $\alpha$ , the higher of the use rate of the core network and the smaller of the weighted average shortest path length of the coupling network. Namely, a higher preferential selection exponent corresponds to better coupling network performance.

*Key-Words:* - base-core coupling network, preferential selection exponent, the weighted average shortest path length, the use rate of the core network

## 1 Introduction

Recent years, researches in the field of network science, which dedicates to study the features of complex systems by abstracting these systems into networks, have attracted huge attention from a variety of science fields, such as prediction of protein function [1-4], detection of community structure in social networks [5-8], control of wireless sensor networks [9-12], and so forth. At present, most of the researches about complex networks are limited to single networks. But in fact a single network in many real circumstances cannot sufficiently represent a complex system composed of sub-systems with different structure and function. For these systems, coupled, interdependent or layered networks are better carriers to depict them [13-19]. The properties of each network in a coupling network may be quite different from each other, but they generally have some coupling on or mutual dependence with others, such as the activation and inhibition relationship between protein and gene with each other in a biological

network [1-2], the dependence of social networks on the advanced Internet technology [20-21], the reliance of the internet on energy system [22-24], etc. The coupling relationship between coupled networks is important, therefore, scholars have done several researches on it. For example, Kurant et. al. proposed a logical-physical layer coupled network model to study the transport traffic features in real networks [25], Daren et. al. put forward a research on the robustness in coupled networks[26], Morris et. al. researched the transport on coupled spatial networks[27], etc.

Among many coupled networks, a common genre is the base-core mode coupled network which consists of two networks while the main functions of the two networks are differently oriented [28,29]. We call one of the network the base network and the other the core network; see Fig. 1 for an illustration. Usually, the main function of the core network is to undertake some important tasks in a system which has an important influence on the performance of the whole coupled network, such as the subway network in the public transport system [30], but due to its expensive constructional or operational cost,

its cover range tend to be limited in the real world [31, 32]. By contrast, the base network can be built with a relatively lower cost such that it can be deployed in a wide range, such as the bus transport network in the public transport system. However, the base network has a much weaker networking function. Coupling of the two kinds of networks can offer us a satisfactory balance or tradeoff between network efficiency and cost in real world situations. For example, in a public transportation network of a metropolis, the subway network play the role of the core network to undertake the majority passenger flow within the major regions of the city, on the other hand, the bus network serves as the base network assisting the subway system, responsible for local passenger transportation and would be especially useful for people in the suburb [31,32]. The two networks complement each other have ensured a fast, convenient and cheap traveling for people in the city.

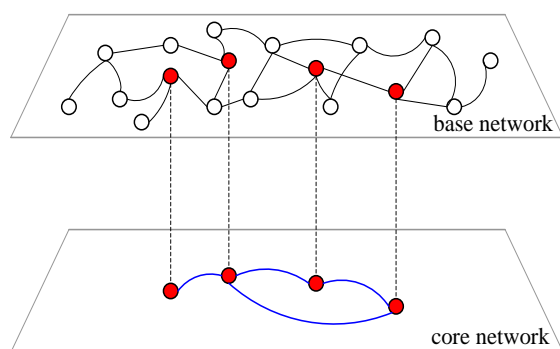


Fig.1 A simple illustration of the base-core mode coupling network. A system made of two coupled networks, where the nodes of the base network coupled with the core network are selected according to a given strategy. The nodes colored in red are considered to be coupled. Edges of base network are shown in black, while edges of core network are shown in blue.

We note that, there are two common problems in the base-core mode coupling networks. The first one is that how the core network couples with the base network, namely which type of nodes in the base network should be chosen to be coupled with the nodes of the core network, to make the whole network achieve a relatively good network performance. The second is how to decide the network size for the core network and the base network. As we all know that, the construction costs of core networks often tend to be higher than base networks. So how to choose the coupling strategy and the size of the two networks to insure a balance

between the network cost and network performances is worth exploring.

It is known that, the importance and features of nodes in a network can usually be characterized by indices like node degree [33], betweenness, clustering coefficient [34,35] and so forth. So the nodes coupling strategies can have a variety of options. For example, the nodes of the base network may be selected as arbitrary nodes to couple with the core network, or be selected as the nodes that have particular node properties. As a result, it is important to assess the impacts of different coupling strategies on the whole network performances, because it can help us to understand what kind of coupling strategy is good or not good or even bad, and it can further help us in the network design and optimization.

Based on these questions, in this paper we designed a base-core coupling network model to study the performance of the coupling network which is composed of a base network and a core network. In the model, the base network is generated first, then the nodes of the core network are selected from the base network according to a preferential selection scheme based on node degree or betweenness. Under different value of preferential selection exponent  $\alpha$ , we investigate the performance of the coupling network in different network structure and different network size collocations. As we will show in the following content that, these factors do influence the network performance. The result may provide some guidance in the design or optimization of real-world networks.

The remainder of this paper is organized as follows. In section 2, the base-core mode coupling network model and the preferential selection scheme strategies are introduced. In section 3, we conduct a simulation based on the model and the coupling scheme to investigate the network performance of the whole network under different circumstances. Then the conclusion and some discussion are listed in Section 4.

## 2. Base-core mode coupling network model and its coupling strategies

### 2.1 Base-core mode coupling network model

In this section, we first present the base-core coupling network model. In this model, each of the nodes in the core network is coupled with one of the nodes in the base network. Besides, the base network and the core network have different edge

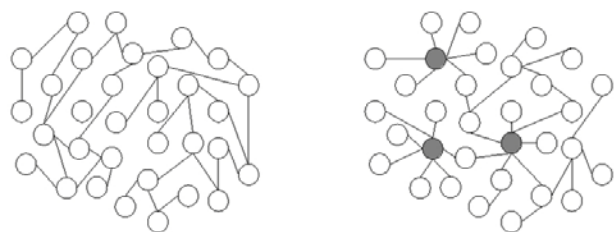
cost. The cost associated with each edge is the length of that edge multiplied by a factor  $\lambda$ . Worth noting that, in this paper we define all the edge length for unweighed networks to be 1, such as ER networks and BA networks [33]. In graph theory, the Erdős–Rényi model is either of two closely related models for generating random graphs, including one that sets an edge between each pair of nodes with equal probability, independently of the other edges. They are named for Paul Erdős and Alfréd Rényi, who first introduced one of the two models in 1959; the other model was introduced independently and contemporaneously by Edgar Gilbert. These models can be used in the probabilistic method to prove the existence of graphs satisfying various properties, or to provide a rigorous definition of what it means for a property to hold for almost all graphs. A typical drawing is displayed in Fig. 2(a). A BA scale-free network is a network whose degree distribution follows a power law, at least asymptotically. That is, the fraction  $p(k)$  of nodes in the network having  $k$  connections to other nodes goes for large values of  $k$  as  $p(k) \sim k^{-\gamma}$  where  $\gamma$  is a parameter whose value is typically in the range  $2 < \gamma < 3$ , although occasionally it may lie outside these bounds. Many networks are conjectured to be scale-free, including World Wide Web links, biological networks, and social networks, although the scientific community is still discussing these claims as more sophisticated data analysis techniques become available. Preferential attachment and the fitness model have been proposed as mechanisms to explain conjectured power law degree distributions in real networks[29]. A typical drawing is displayed in Fig. 2(b). For the base and core networks, the factors mentioned above is denoted by  $\lambda_b$  and  $\lambda_c$ , respectively. Without loss of generality, we set  $\lambda_b=1$  and  $\lambda_c \in [0,1]$  in this paper. Note that, the smaller the factor is, in physical meanings the faster or with fewer cost material can be transferred.

Specifically, a base-core coupling network can be generated in the following steps:

1). Base network: an ER graph or a scale free network with  $n_1$  nodes.

2). Core network: an ER graph or a scale free network with  $n_2$  nodes which are selected from the base network according to specific coupling strategies in section 2.2. The base network and the core network couple with each other due to sharing the same nodes selected from this step.

After the above steps, a base-core mode coupling network has been established. An illustration of a generated coupling network is displayed in Fig. 1.



(a) Random network (b) Scale-free network

Fig. 2 A simple illustration of ER random network and scale-free network.

## 2.2 The preferential selection scheme based on $k^\alpha$ and $b^\alpha$

The importance and features of nodes in a network can usually be characterized by node degree or node betweenness, clustering coefficient and so forth. The degree feature of complex network is usually first studied. In network theory, the degree of a node is defined as the number of its neighbor nodes. The degree reflects the importance of a node as a hub. In directed networks, a degree can be divided into two categories: out-degree and in-degree. The out-degree of a node is defined as the number of nodes that a node can reach through an edge, whereas the in-degree is defined as the number of nodes reaching the node through an edge. Specifically, we have

$$k_i^{\text{out}} = \sum_j a_{ij}; \quad k_i^{\text{in}} = \sum_j a_{ji} \quad (1)$$

where  $k_i^{\text{out}}$  and  $k_i^{\text{in}}$  denote the out-degree and the in-degree of node  $i$ , respectively;  $a_{ij}$  denotes the existence of the edge  $i \rightarrow j$  ( $a_{ij}=1$  when edge  $i \rightarrow j$  exists; otherwise,  $a_{ij} = 0$ ). But for undirected network, We mainly focus on the total node degree of a node in the network, in this paper, the total node degree of a node in the network is studied. Betweenness is usually divided into two kinds of types, which are edge betweenness and node betweenness. Betweenness is a measure of how often a node or vertex is located on the shortest path or geodesic between other nodes in the network. It thus measures the degree to which the node under study can function as a point of control in the communication. If a node with a high level of betweenness were to be deleted from a network, the network would fall apart into otherwise coherent clusters. Unlike degree, which is a count,

betweenness is normalized by definition as the proportion of all geodesics that include the vertex under study. Betweenness is a relational measure. One can expect that a journal which is “between” will load on different factors because it does not belong to one of the dense groups, but relates them. The factor loadings of such journals may depend heavily on the factor-analytic model (e.g., the number of factors to be extracted by the analyst). For example, one might expect inter-factorial complexity among the factor loadings in the case of inter- or multidisciplinary journals (Van den Besselaar & Heimeriks, 2001; Leydesdorff, 2004). Closeness is less dependent on relations between individual vertices because a vertex can be close to two (or more) densely connected clusters. Closeness can thus be expected to provide us with a measure of “multidisciplinarity” within a set while betweenness may provide us with a measure of specific “interdisciplinarity” at interfaces. In network theory, the clustering coefficient of a node measures the tightness of the connections among nodes and their neighbors. The clustering coefficient is defined as the number of directed link triangles that exist among nodes and their neighbors against the total number of triangles that can exist among nodes (Fagiolo, 2007). The definition for the clustering coefficient is written as

$$c_i = \frac{\sum_{jk} (a_{ij} + a_{ji})(a_{ik} + a_{ki})(a_{jk} + a_{kj})}{2[d_i^{\text{tot}}(d_i^{\text{tot}} - 1) - 2d_i^{\leftrightarrow}]} \quad (2)$$

where  $c_i$  denotes the clustering coefficient of node  $i$ ; and  $d_i^{\text{tot}} = \sum_j (a_{ij} + a_{ji}) = k_i^{\text{out}} + k_i^{\text{in}}$  and  $d_i^{\leftrightarrow} = \sum_j a_{ij} a_{ji}$  are the total degree and the number of bi-directed edges of node  $i$ , respectively.

In this paper, we mainly study two parameters which are degree and betweenness. So we define preferential selection schemes according to node degree and node betweenness for the model. They are detailed as below.

1). Preferential selection according to node degree: the probability  $P_k(i)$  of node  $i$  in the base network that is selected to be the coupled node is proportional to  $k_i^\alpha$ , namely

$$P_k(i) = \frac{k_i^\alpha}{\sum_j k_j^\alpha} \quad (3)$$

where  $k_i$  is the node degree of node  $i$ .

According to the definition,  $\alpha$  is the preferential selection exponent. If  $\alpha > 0$ , the selection scheme will be priority to select the nodes with higher degree. Specially, if  $\alpha \rightarrow +\infty$ , the selection scheme

will select all the nodes with the highest degree. On the contrary, if  $\alpha < 0$ , the result is opposite, it will be priority to select the nodes with lower degree. Specially, if  $\alpha \rightarrow -\infty$ , the selection scheme will select all the nodes with the lowest degree. Besides, if  $\alpha = 0$ , the selection scheme will randomly select the nodes with uniform probability.

2). Preferential selection according to node betweenness: the probability  $P_b(i)$  of node  $i$  in the base network that is selected to be the coupled node is proportional to  $b_i^\alpha$ , namely

$$P_b(i) = \frac{b_i^\alpha}{\sum_j b_j^\alpha} \quad (4)$$

where  $b_i$  is the node betweenness of node  $i$ .

According to the definition,  $\alpha$  also is the preferential selection exponent. If  $\alpha > 0$ , the selection scheme will be priority to select the nodes with higher betweenness. Specially, if  $\alpha \rightarrow +\infty$ , the selection scheme will select all the nodes with the highest betweenness. On the contrary, if  $\alpha < 0$ , the result is opposite, it will be priority to select the nodes with lower betweenness. Specially, if  $\alpha \rightarrow -\infty$ , the selection scheme will select all the nodes with the lowest betweenness. Besides, if  $\alpha = 0$ , the selection scheme will randomly select the nodes with uniform probability.

### 2.3 The performance indicators of the coupling network

In the study, we determine two indicators, the weighted average shortest path length and the use rate of the core network respectively, as the measurements of the network performance. The weighted average shortest path length is defined as

$$d = \frac{\sum_{i \neq j} d_{ij}}{N(N-1)} \quad (5)$$

where  $d_{ij}$  is the shortest path length between nodes  $i$  and  $j$ ,  $N$  is the total number of nodes in the network. For a well-designed real-world network,  $d$  is usually found to be small, such as the water supply networks, the Internet or transportation networks, etc, which we call the “small-world” phenomenon[35]. Another indicator is the use rate of the core network  $\eta$  which ranges in  $[0,1]$ . The use rate of the core network was used to characterize the proportion of flows on the core network, We define the use rate of the core subnet  $\eta$  as

$$\eta = \frac{\sum_i \delta_i}{\sum_j \zeta_j} \times 100\% \quad (6)$$

where  $\sum_i \delta_i$  is the number of the shortest paths which pass through the core subnet in the coupling network, while  $\sum_j \zeta_j$  is the total number of the shortest paths in the coupling network.

In the next section, we will explore the impacts of different coupling strategies with different preferential selection exponent  $\alpha$  on the network performances of different structured base-core mode coupling networks based on our model.

### 3. Numerical simulation

To investigate the problems mentioned above, we conduct a numerical simulation. In utilization, the ER random network model and the BA scale-free network model[33-35] are used for the generation processes (step 1 and step 2) of the base network and core network. It results in 4 different assemblies of the final coupling networks, which are ER base network with ER core network (ER-ER), ER base network with BA core network (ER-BA), BA base network with ER core network (BA-ER), and BA base network with BA core network (BA-BA), respectively. Of all these networks, specifically we set up the number  $n_1$  of nodes in the base network to 1000, the number  $n_2$  of nodes in the core network to 100, 200 and 300 respectively, the average node degree  $\bar{k}$  for both the base network and the core network to 6, and the edge cost factor  $\lambda_c$  for the core networks to 0.1.

We simulated under different sizes of the core networks and different settings of preferential selection exponent  $\alpha$ . The weighted average shortest path length of the base networks and the whole coupling network for each simulation are recorded, for which we denote as  $d_0$  and  $d_1$ , respectively.

Fig. 3 shows the weighted average shortest path length ratio  $\langle d \rangle$  ( $d = d_1/d_0$ ,  $\langle d \rangle$  represents an ensemble average.) decreases when the node degree or betweenness preferential selection exponent  $\alpha$  increases in ER-ER coupling networks. Namely, a higher value of preferential selection exponent  $\alpha$  corresponds to a smaller weighted average shortest path length of the coupling network. In addition, the weighted average shortest path length decreases with the increasing of the proportion of

core network nodes. Namely, the greater  $n_2/n_1$  is, the smaller  $\langle d \rangle$  is.

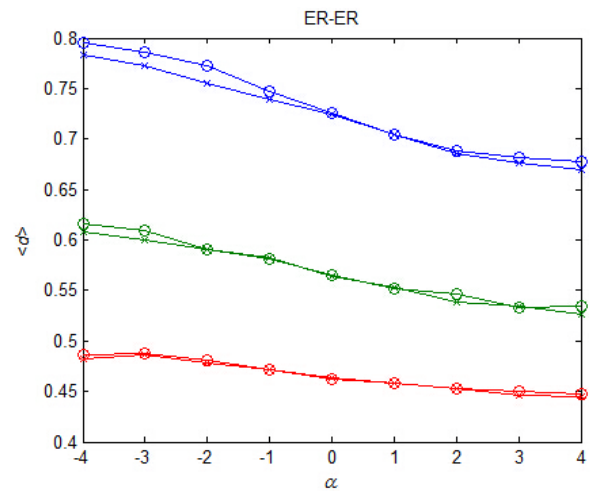


Fig. 3 Simulation results for the weighted average shortest path length ratio  $\langle d \rangle$  in ER-ER coupling networks. The number  $n_1$  of nodes in the base network is set to be 1000 while the number  $n_2$  of nodes in the core network is set to be 100(blue color), 200(green color), 300(red color) respectively. The circular line represents the nodes of the core network are selected according to degree preferential selection scheme while the cross mark line represents the nodes of the core network are selected according to betweenness preferential selection scheme.

Fig. 4 shows the weighted average shortest path length ratio  $\langle d \rangle$  ( $d = d_1/d_0$ ,  $\langle d \rangle$  represents an ensemble average.) decreases when the node degree or betweenness preferential selection exponent  $\alpha$  increases in ER-BA coupling networks respectively. Namely, a higher preferential selection exponent corresponds to a smaller weighted average shortest path length of the coupling network. In addition, the weighted average shortest path length decreases with the increasing of the proportion of core network nodes. Namely, the greater  $n_2/n_1$  is, the smaller  $\langle d \rangle$  is.

Fig. 5 shows the weighted average shortest path length ratio  $\langle d \rangle$  ( $d = d_1/d_0$ ,  $\langle d \rangle$  represents an ensemble average.) decreases when the node degree or betweenness preferential selection exponent  $\alpha$  increases in BA-ER coupling networks respectively. Namely, a higher value of preferential selection exponent  $\alpha$  corresponds to a smaller weighted average shortest path length of the coupling network. In addition, the weighted average shortest path length decreases with the increasing of the proportion of core network nodes. Namely, the greater  $n_2/n_1$  is, the smaller  $\langle d \rangle$  is.

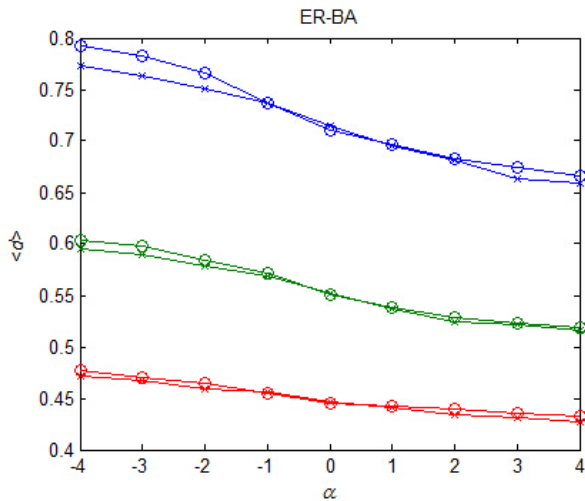


Fig. 4 Simulation results for the weighted average shortest path length ratio  $\langle d \rangle$  in ER-BA coupling networks. The number  $n_1$  of nodes in the base network is set to be 1000 while the number  $n_2$  of nodes in the core network is set to be 100(blue color), 200(green color), 300(red color) respectively. The circular line represents the nodes of the core network are selected according to degree preferential selection scheme while the cross mark line represents the nodes of the core network are selected according to betweenness preferential selection scheme.

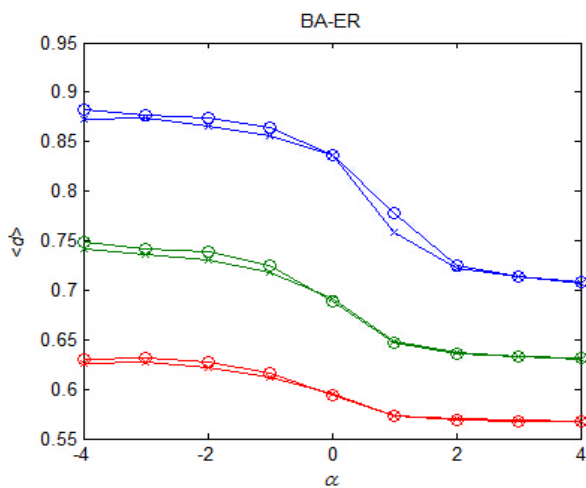


Fig. 5 Simulation results for the weighted average shortest path length ratio  $\langle d \rangle$  in BA-ER coupling networks. The number  $n_1$  of nodes in the base network is set to be 1000 while the number  $n_2$  of nodes in the core network is set to be 100(blue color), 200(green color), 300(red color) respectively. The circular line represents the nodes of the core network are selected according to degree preferential selection scheme while the cross mark line represents the nodes of the core network are selected according to betweenness preferential

selection scheme.

Fig. 6 shows the weighted average shortest path length ratio  $\langle d \rangle$  ( $d = d_1/d_0$ ,  $\langle d \rangle$  represents an ensemble average.) decreases when the node degree or betweenness preferential selection exponent  $\alpha$  increases in BA-BA coupling networks. Namely, preferential selection exponent corresponds to a smaller weighted average shortest path length of the coupling network. In addition, the weighted average shortest path length decreases with the increasing of the proportion of core network nodes. Namely, the greater  $n_2/n_1$  is, the smaller  $\langle d \rangle$  is.

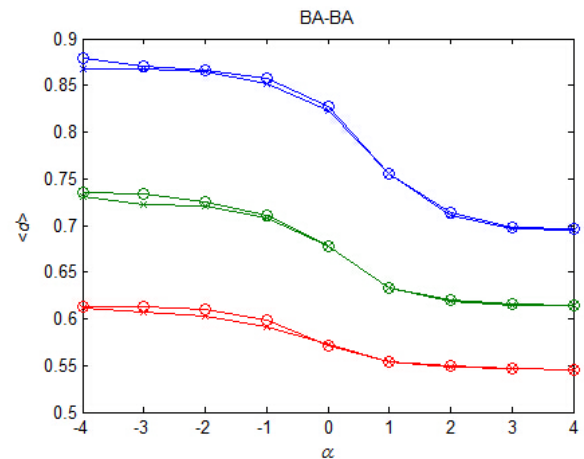


Fig. 6 Simulation results for the weighted average shortest path length ratio  $\langle d \rangle$  in BA-BA coupling networks. The number  $n_1$  of nodes in the base network is set to be 1000 while the number  $n_2$  of nodes in the core network is set to be 100(blue color), 200(green color), 300(red color) respectively. The circular line represents the nodes of the core network are selected according to degree preferential selection scheme while the cross mark line represents the nodes of the core network are selected according to betweenness preferential selection scheme.

The curve of the two strategies for  $\langle d \rangle$  is roughly the same. The reason is that researches have indicated that for networks like ER and BA networks there exists a linear correlation of the node degree with the node betweenness [36].

Besides, the use rate of the core network is also recorded, for which we denote as  $\eta$ . We detail the result as Fig. 7~ Fig. 10. Figure 7 shows that the use rate of the core network  $\eta$  decreases when the node degree or betweenness preferential selection exponent  $\alpha$  increases in ER-ER coupling networks. Namely, a higher preferential selection exponent corresponds to a higher use rate of the core network. In addition, the use rate of the core network

increases with the increasing of the proportion of core network nodes. Namely, the greater  $n_2/n_1$  is, the bigger  $\eta$  is.

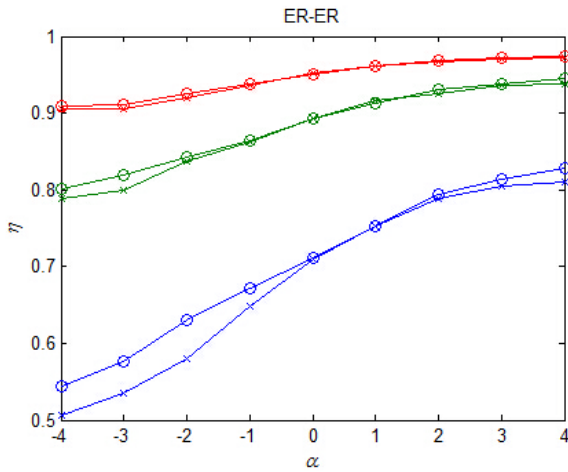


Fig. 7 Simulation results for the use rate  $\eta$  of the core network in ER-ER coupling networks. The number  $n_1$  of nodes in the base network is also set to be 1000 while the number  $n_2$  of nodes in the core network is set to be 100(blue color), 200(green color), 300(red color) respectively. The circular line represents the nodes of the core network are selected according to degree preferential selection scheme while the cross mark line represents the nodes of the core network are selected according to betweenness preferential selection scheme.

Figure 8 shows that the use rate of the core network  $\eta$  decreases when the node degree or betweenness preferential selection exponent  $\alpha$  increases in ER-BA coupling networks. Namely, a higher preferential selection exponent corresponds to a higher use rate of the core network. In addition, the use rate of the core network increases with the increasing of the proportion of core network nodes. Namely, the greater  $n_2/n_1$  is, the bigger  $\eta$  is.

Figure 9 shows that the use rate of the core network  $\eta$  decreases when the node degree or betweenness preferential selection exponent  $\alpha$  increases in BA-ER coupling networks. Namely, a higher preferential selection exponent corresponds to a higher use rate of the core network. In addition, the use rate of the core network increases with the increasing of the proportion of core network nodes. Namely, the greater  $n_2/n_1$  is, the bigger  $\eta$  is.

Figure 10 shows that the use rate of the core network  $\eta$  decreases when the node degree or betweenness preferential selection exponent  $\alpha$  increases in BA-BA coupling networks. Namely, a higher preferential selection exponent corresponds

to a higher use rate of the core network. In addition, the use rate of the core network increases with the increasing of the proportion of core network nodes. Namely, the greater  $n_2/n_1$  is, the bigger  $\eta$  is.

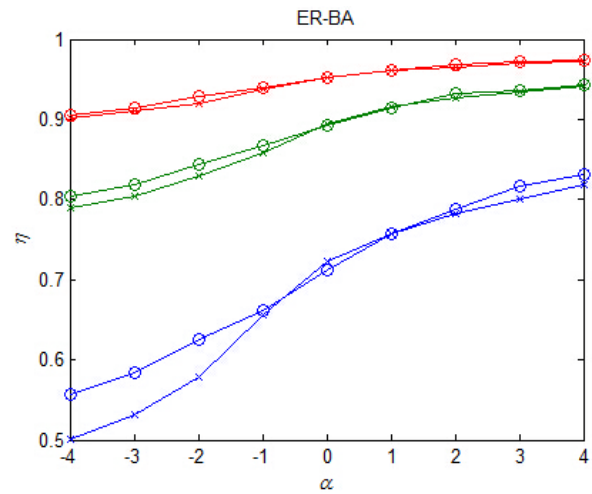


Fig. 8 Simulation results for the use rate  $\eta$  of the core network in ER-BA coupling networks. The number  $n_1$  of nodes in the base network is also set to be 1000 while the number  $n_2$  of nodes in the core network is set to be 100(blue color), 200(green color), 300(red color) respectively. The circular line represents the nodes of the core network are selected according to degree preferential selection scheme while the cross mark line represents the nodes of the core network are selected according to betweenness preferential selection scheme.

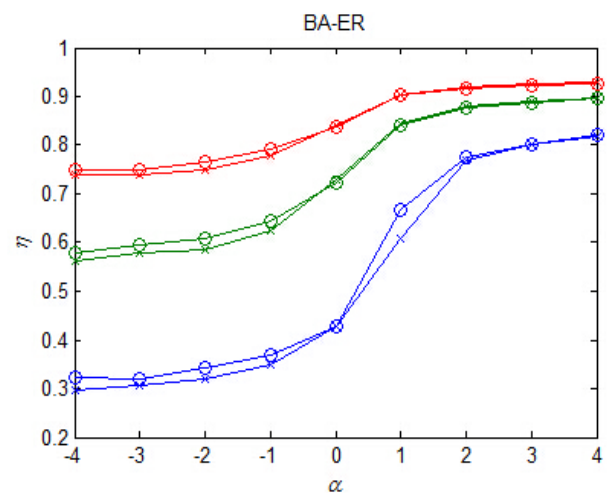


Fig. 9 Simulation results for the use rate  $\eta$  of the core network in BA-ER coupling networks. The number  $n_1$  of nodes in the base network is also set to be 1000 while the number  $n_2$  of nodes in the core network is set to be 100(blue color), 200(green color), 300(red color) respectively. The circular line represents the nodes of the core network are selected

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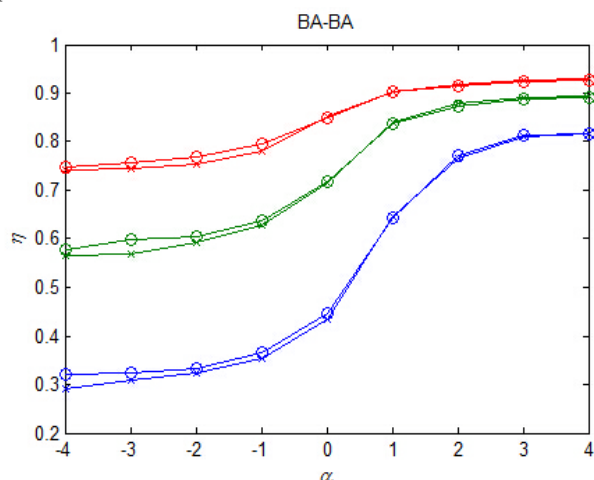


Fig. 10 Simulation results for the use rate  $\eta$  of the core network in BA-BA coupling networks. The number  $n_1$  of nodes in the base network is also set to be 1000 while the number  $n_2$  of nodes in the core network is set to be 100(blue color), 200(green color), 300(red color) respectively. The circular line represents the nodes of the core network are selected according to degree preferential selection scheme while the cross mark line represents the nodes of the core network are selected according to betweenness preferential selection scheme.

The curve of the two strategies for  $\eta$  is roughly the same. The reason is that researches have indicated that for networks like ER and BA networks there exists a linear correlation of the node degree with the node betweenness [36].

## 4. Conclusions

In this paper, we proposed a base-core coupling network model composed by a base network and a core network in accordance with many real-world networks. We conduct the investigation with four different assemblies of common structure coupling networks, which are ER base network with ER core network (ER-ER), ER base network with BA core network (ER-BA), BA base network with ER core network (BA-ER), BA base network with BA core network (BA-BA) respectively. In the model, the base network is generated first, then the nodes of the core network are selected from the base network according to a preferential selection scheme based on node degree or betweenness. Under different preferential selection exponent, we investigate the

performance of the coupling network in different network structure and different network size collocations. Simulation result shows that: the greater of the preferential selection exponent is, the smaller of the weighted average shortest path length of the coupling network is and the higher of the use rate of the core network is. Namely, a higher preferential selection exponent corresponds to better network performance.

Our studies can provide some guidance in real world situations on how to choose the nodes of base networks as the coupling nodes with the core network to achieve a good coupling network performance. For example, it can be applied to choose the location of subway stations in public transport networks, making the whole network to obtain a good network performance while low down the construction fee. More importantly, we should take spatial factors into account when design and optimize the real-world networks. And we should say that there are still lots of problems left for us. For example, there should be some better selection strategies if other factors are concerned. Besides, the multi-layered complex networks, the coupling network traffic flow characteristics considering unevenly traffic demand and different node capacity, and the dynamical behaviors on coupling networks are also worth studying. As coupling and interdependent networks have attracted more and more attentions, the research on this field should be more delighting in the further.

## 5. Acknowledgements

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