Sequential use of ordinal multicriteria methods to obtain a ranking for the 2012 Summer Olympic Games

SILVIO FIGUEIREDO GOMES JÚNIOR
Centro UniversitárioEstadual da ZonaOeste
Avenida Manuel Caldeira de Alvarenga, 1203, Rio de Janeiro, RJ
BRAZIL
silviogomes@uezo.rj.gov.br

JOÃO CARLOS CORREIA BAPTISTA SOARES DE MELLO
Production Engineering Department – Fluminense Federal University,
RuaPasso da Pátria 156, Niterói, RJ
BRAZIL
jccbsmello@id.uff.br

LIDIA ANGULO MEZA
Production Engineering Department – Fluminense Federal University
Av. dos Trabalhadores, 420, Volta Redonda, RJ
BRAZIL
lidia@metal.eeimvr.uff.br

*Abstract.*There is no official method to establish a final ranking for the Olympic Games. It is usual to rank the participant countries in these games in accordance with the number of medals they have won using a lexicographic multicriteria method. However, this does not take into account the fact that the various sports may be of different importance. This work proposes a ranking model to eliminate those drawbacks. Using the results of the 2012 Summer Olympic Games for the model, firstly, we use a Lexicographic multicriteria method in each sport. After obtaining a ranking for each and all sports, we build a general ranking by aggregating all the sports using a Bordamulticriteria method.

1. Introduction

As pointed out in [1, 2], the modern Olympic Games, initiated in 1896 by Baron Coubertin, tried to keep the initial spirit of individual competition. That purpose clearly failed. Ever since the very first modern Games, it has become usual to play the national anthem of the winner's country. However, despite the importance of an Olympic Ranking for national pride and for the perception of national success [3, 4], the Olympic Committee has never issued an official ranking to pick an overall Olympic winner country.

The mass media, however, has issued a ranking, which has become the quasi-official ranking. It is based on the Lexicographic Multicriteria Method[5]. This method rranks the countries taking into account the total number of gold medals, silver medals and bronze medals won by each country, withthe gold medal being the most important. The number of

silver medals is only considered when draws exists in the number of gold medals. The number of bronze medals is only used when two or more countries simultaneously have the same number of gold medals and the same number of silver medals.Because of these properties, this ranking does not deal properly with the possible existence of countries that have won a large number of silver and bronze medals but no gold medal [6], asthis method over-values the gold medal.

The Lexicographic Method is not the sole method used by the media to rank countries in the Olympic Games. Some newspapers produce a ranking determined by the total number of medals earned by each country. They simply add up bronze, silver and gold medals. This method was widely used by American newspapers during the 2008 Beijing Olympic Games. This was done because when using the Lexicographic Method China was in

thefirst position and using the total number of medalsthe USA was in the first place. The obvious disadvantage of this method is to under-value gold medals.

An alternative approach is to make an arbitrary evaluation of each medal, for instance, 1 point for bronze, 2 for silver and 3 for gold. This is a very unsophisticated approach, as it assumes that there is no difference between the attractiveness ofgold and silver medals and between the silver and bronze medals.

Many alternative Olympic Rankings have been proposed taking into account the resources used to obtain the medals and others take into account only the number of medals. In the first approach, we can cite [7],who used statistics, and [1, 8-15], who used standard and advanced Data Envelopment Analysis Models with multiple inputs and multiple outputs. Data Envelopment Analysis models have also been used by [16, 17] to determine the efficiency of each country in the use of its delegation to the Summer and Winter Olympic Games, respectively.

In the second approach, using only the number of medals we can cite [6, 18,19]. Those authors used a unitary input Data Envelopment Analysis (DEA) approach. All the works considered that a gold medal is more important than a silver medal, a silver medal is more important than a bronze medal, and the difference between a gold and silver medal is greater than the difference between a silver and bronze medal. However, these studies are rather complex to be understood by the general public, despite the fact that they can usefull for other purposes, for instance the allocation of financial resources for different Olympic sports [20].Other methods, despite their mathematical sophistication, produce a ranking which is easy to understand because they attribute a fixed value to each medal. For instance, in [21] it was used the incenter of a convex cone to obtain a system of points for medals in Olympic ranking and to obtain an alternative ranking for Formula 1 motor races. In addition, in[22]it was used the weighted mean value and another method which used volume-based sensitivity analysis. Another advantage presented in [22]is that they use a Multicriteria based approach instead of Data Envelopment Analysis. As a consequence there is no need for sophisticated models to avoid a large number of draws in the final rank. Other rankings based on a Multicriteria approach use the Analytical Hierarchic Process[23], the Copeland Method[24] and a method based on an optimistic evaluation, in that case using a database from the Panamerican Games instead of the

Olympic Games [25]. Other multicriteria methods for rankings in sport are presented in [26].

The aforementioned papers aim to override the distortions caused by the use of the Lexicographic Method. Another distortion pointed out [2]is that medals won in different competitions do not have the same value. As a matter of fact, the existing rankings do not take into account that in some sports there are more events than in others, and so there are more possibilities of winning a medal. For instance, in gymnastics there are a lot of gold medals to be won and in football there are only two possibilities for a country to win a gold medal (one for men, the other for women). In [2]is proposed a methodology to deal with this problem. To take into account the difference in winning values for different sports, they aggregated competitions into clusters. The first clustering was obtained directly from the IOC (www.olympic.org), where each sport is a cluster. They also tried to use a second clustering based on the aggregation of the Olympic Sports performed[27], however with disappointing results.

In this work, we propose a different way to rank the countries taking into account the number of medals available in each sport as explained in section 3.

2. Ordinal MulticriteriaMethods

The so-called ordinal methods have their origins in the eighteenth century with applications to social choice theory. These ordinal methods are considered fairly intuitive and they are not very demanding in terms of the information needed from the decision maker. To use the ordinal methods, the decision maker needs do no more than rank the alternatives according to their preferences in each criterion. Instead of rankings given by the decision makers, in some cases we can use some natural ranking, for instance, incomes earned, Formula One race results or medals won.

According to [28-31], those methods present two main advantages: they are user-friendly and easily understood. In[5]it is stated that these two characteristics are vital for the acceptance of the methods by the users. In [32]it was concluded that when a decision maker finds it difficult to establish a precise cardinal scale, it is preferable to perform the analysis with ordinal judgements and methods.

Despite their simplicity, ordinal methods present an important disadvantage[33], these methods cannot produce just choices. In other words, there is no "perfect" ordinal multicriteria method. That is, there is no method which obeysthe axioms of universality, unanimity, independence in relation to

irrelevant alternatives, transitivity and totality. The Arrow theorem states that, with the exception of dictatorial methods, no choice or decision-aiding method serves all of these axioms simultaneously.

The axioms of independence in relation to irrelevant alternatives of transitivity and of universality are of special interest to this study. The first states that the order of preference between two alternatives must not depend on their preferences in relation to a third alternative. The transitivity axiom states that if one alternative is preferable to a second, and this one to a third, then the first must be preferable to the third (the fact that in the results of football matches this property is not confirmed is the reason for the popular saying that "football has no logic"). The universality axiom, meanwhile, requires the method to function, respecting all the other axioms, for any group of preferences of the decision-makers. Thus, a method that respects the axioms in some particular cases, does not respect universality.

Due to the problems deriving from the Arrow theorem, the use of ordinal methods is quite limited. Its main field of application is in sports rankings, as the information available is mainly of the ordinal kind. In this field of application the two methods most commonly used are the Lexicographic method and the Borda method and its variant.

2.1. The Lexicographic Method

In the Lexicographic Method criteria are ranked in the order of their importance. The alternatives are ranked according to the ranking in the most important criterion. If there are drawsin respect to this criterion, the ties are broken according to the second most important criterion, and so on.

The Lexicographic Method satisfies the Arrow Theorem. In fact, this method is a dictatorial one: the most important criterion acts as a dictator, the other criteria can be considered as less important dictators[5].

2.2. The Borda Method

In the Borda Method each decision-maker must order the alternatives according to their preferences[31]. One point is assigned to the alternative of highest preference; the second receives two points and so on. Then, for each alternative we sum all the points assigned by all the decision makers (or by all criteria) as in (1).

$$P_a = \sum_{i=1}^{n} r_{a_i} \qquad (1)$$

Where $P_a$is the total number of points obtained by alternative a and r_{ai} is the rank of alternative a in criterion$i$.

The alternatives are ranked in increasing order according to this sum, i.e., the fewer the points, the better the alternative is ranked [5]. It is to be noted that if we divide the number of points by the number of criteria we will obtain the average position of the alternative.

In sports, variations of the Borda method are widely used, with each competition considered a decision-maker. It is common to perform an inversion of the method, assigning a greater number of points to the preferred alternative (the winner of the competition). It is interesting to note that one case of using the original Borda method is in the Olympic Games yachting competitions[34].

In spite of its simplicity and the widespread use of its variations, the Borda Method does not respect one of Arrow's axioms, namely that the final classification of two alternatives is not independent in relation to irrelevant alternatives. This fact may create undesirable situations, for instance it may encourage the unsporting inversion of positions in a competition to benefit a given competitor, as often occurs in Formula Onemotor races[34].

3. The Lexicographic-Borda integrated approach

We are going to use the Lexicographic and Borda Methods, applied in two steps,firstly in order to avoid the problem of the over-valuing of the gold medal, and secondly to take into account the differences of medal values among the various Olympic sports. The first step is to rank each sport independently using a Lexicographic Method. We rank all countries, even those that did not participate in a specific sport. Obviously, for such countries the number of medals won is nil. As the position of a country in the ranking of each sport is taken into account, instead of the number of medals, we eliminate distortions caused by the different number of medals available in each sport. In the second step, we aggregate the different rankings obtained in step one using the Borda Method. As we do not use any weighting system our method cannot be classified in the taxonomy found in [35, 36].

Besides eliminating the distortions caused by the different number of medals available in each sport, this methodology also eliminates the distortion caused by the sole use of the Lexicographic method, i.e. the over-valuing of gold medals. In fact, let us consider a country which wins no gold medal in any sport, and winssome silver medals in a large number

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of sports. Using this methodology this country can be better ranked than another country that wins a single gold medal in only one sport. This situation could not happen using the quasi-official Lexicographic Method. We will illustrate this situation using a numerical example.

Table 1 presents the data for a group of 8 countries that won medals in a fictitious Olympic Games with three sports. In this table, countries are presented according to the ranking obtained using the Lexicographic Method

Table 1. Medals obtained in three different sports by a group of 8 countries

Country	Gold	Silver	Bronze
A	3		
B	2	2	1
C	1	1	
D	1		1
E	1		1
F	1		
G	1		
H		4	1

Tables 2, 3 and 4 presents the results for each sport: the number of medals, the Lexicographic Ranking and the Borda points. Table 5 presents the aggregated results using the proposed methodology: the total Borda points (taking into account the three sports) and the Final Ranking.

Table 2. Detailed results for sport X in the fictitious Olympic Games

Country	Sport X				
	Number of Medals			Lexicographic Ranking	Borda Points
	Gold	Silver	Bronze		
A	2	0	0	1st	1
B	1	1	0	2nd	2
H	0	1	1	4th	4
C	0	1	0	5th	5
D	0	0	0	6th	7
E	1	0	0	3rd	3
F	0	0	0	6th	7
G	0	0	0	6th	7

Comparing table 1 with table 5 we can observe that country H was in the last position in the quasi-

official method because it had not won any gold medals. In the proposed methodology, it is in third position in front of countries that have won gold medals.

Table 3. Detailed results for sport Y in the fictitious Olympic Games

Country	Sport Y				
	Number of Medals			Lexicographic Rankings	Borda Points
	Gold	Silver	Bronze		
A	1	0	0	1st	1,5
B	0	0	0	6th	7
H	0	1	0	3rd	3
C	1	0	0	1st	1,5
D	0	0	1	4th	4,5
E	0	0	1	4th	4,5
F	0	0	0	6th	7
G	0	0	0	6th	7

Table 4. Detailed results for sport Z in the fictitious Olympic Games

Country	Sport Z				
	Number of Medals			Lexicographic Ranking	Borda Points
	Gold	Silver	Bronze		
A	0	0	0	6th	7
B	1	1	1	1st	1
H	0	2	0	5th	5
C	0	0	0	6th	7
D	1	0	0	2nd	3
E	0	0	0	6th	7
F	1	0	0	2nd	3
G	1	0	0	2nd	3

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Table 7 - Rank for Athletics

Country	Gold	Silver	Bronze	Lexicographic Ranking	Borda points
United States	9	13	7	1	1
Russia	8	5	5	2	2
Jamaica	4	4	4	3	3
UK	4	1	1	4	4
Ethiopia	3	1	3	5	5
Kenya	2	4	5	6	6
Germany	1	4	3	7	7
Australia	1	2	0	8	8
DominicanRepublic	1	1	0	9	10,5
France	1	1	0	9	10,5
Poland	1	1	0	9	10,5
Turkey	1	1	0	9	10,5
China	1	0	5	13	13
Trinidad and Tobago	1	0	3	14	14
CzechRepublic	1	0	1	15	15
Grenada	1	0	0	16	19,5
Croatia	1	0	0	16	19,5
Bahamas	1	0	0	16	19,5
Algeria	1	0	0	16	19,5
New Zealand	1	0	0	16	19,5
Kazakhstan	1	0	0	16	19,5
Hungary	1	0	0	16	19,5
Uganda	1	0	0	16	19,5
Ukraine	0	1	2	24	24
Cuba	0	1	1	25	25
South Africa	0	1	0	26	29
Iran	0	1	0	26	29
Tunisia	0	1	0	26	29
Slovenia	0	1	0	26	29
Botswana	0	1	0	26	29
Guatemala	0	1	0	26	29
Colombia	0	1	0	26	29
Bahrain	0	0	1	33	37
Canada	0	0	1	33	37
Puerto Rico	0	0	1	33	37
Qatar	0	0	1	33	37
Italy	0	0	1	33	37
Estonia	0	0	1	33	37
Finland	0	0	1	33	37
Japan	0	0	1	33	37
Morocco	0	0	1	33	37
Allother countries	0	0	0		64

Table 5. Final results for the fictitious Olympic Games

Country	Borda Points			Total Borda	Rank
	X	Y	Z		
A	1	1,5	7	9,5	1st
B	2	7	1	10	2nd
H	4	3	5	12	3rd
C	5	1,5	7	13,5	4th
D	7	4,5	3	14,5	5th
E	3	4,5	7	14,5	5th
F	7	7	3	17	7th
G	7	7	3	17	7th

4. Results for the 2012 SummerOlympic Games

In this section we will apply the proposed methodology to the 2012 London Olympic Games. In both the Lexicographic and Bordamethods, the alternatives are all the 85 countries that won at least one medal in the 2012 London Olympic Games. For the first step, the Lexicographic Method, the decision criteria are the numbers of gold, silver and bronze medals for each sport. In the second step, the Borda Method, the decision criteria are the sports of the Summer Olympic Games. As the Borda Method needs an ordinal scale, we use the rankings obtained in each with the Lexicographic Method.Table6exemplifies the rank for Archeryand table 7 exemplifies the rank for Athletics.

Table 6 - Rank for Archery

Country	Gold	Silver	Bronze	Lexicographic Ranking	Borda points
South Korea	3	0	1	1	1
Italy	1	0	0	2	2
Mexico	0	1	1	3	4
China	0	1	1	3	4
Japan	0	1	1	3	4
United States	0	1	0	6	6
All other countries	0	0	0		46

In the second step, we use the Borda Method to aggregate all the rankings obtained. The final ranking according to our proposed methodology is presented in Table 8, as well as the ranking obtained using only the Lexicographic Method and the discrepancy between the positions in the two rankings.

Table 8 - Ranking of Borda and Lexicographic Methods and the discrepancy

Country	Total Borda Points	Final Ranking	Lexicographic rank	Discrepancy
United States	793	1	1	0
China	844	2	2	0
Russia	854	3	4	1
UK	888	4	3	-1
Germany	1057	5	6	1
France	1100	6	7	1
Italy	1137	7	8	1
Australia	1140	8	10	2
Japan	1195	9	11	2
South Korea	1255	10	5	-5
Spain	1290	11	21	10
Canada	1328	12	36	24
Netherlands	1331	13	13	0
Ukraine	1362	14	14	0
Brazil	1377	15	22	7
....	...	...	...	...
Hong Kong	1714	78	79	1
Portugal	1714	78	69	-9
Moldova	1714	78	75	-3
Tajikistan	1717	81	79	-2
Botswana	1718	82	69	-13
Guatemala	1718	82	69	-13
Kuwait	1719	84	79	-5
Morocco	1726	85	79	-6

Countries that in our methodology are better ranked than they are in the standard Lexicographic Method have more medals which were won in collective sports. Among them we can cite Brazil. This result is in line with the conclusions obtained in [16] when a different methodology (Data Envelopment Analysis) was used, and the data was from the Beijing Olympic Games. This vindicates a tendency for Brazil to invest mainly in collective sports.

Also, among the countries that are ranked worse we can cite Jamaica. This country has a concentration of medals in individual sports mainly athletics. We should remember that athletics is one of the sports with a larger number of medals distributed.

We may point out that in the two first positions there are no differences between the two rankings.

5. Conclusions and Future Works

The use of Lexicographic Methods for each sport together with the Borda Method to aggregate the various sports rankings has an important consequence: the gold medal is not as overvalued as it is in the pure Lexicographic Method. This is one advantage of our methodology. Another advantage is that it is possible to take into account the difference between medals won in different sports in a much simpler way than the method used in [2]. In fact, simplicity is the main advantage of our method, since it does not need advanced mathematical tools. Olympic rankings are based on ordinal data and except for the quasi-official Lexicographic Method, they have been treated with cardinal methods. In this paper, we have succeeded in dealing with the problems of the over-valuing of gold medals and different values for medals in different sports using only ordinal information and ordinal methods. The rankings we have obtained can be used as inputs for time series forecasting in order to estimate the rank for a country in future games, using the methods shown in [37, 38].

We also should note that there are other important problems related to the Olympic Games. One of the most important is the problem of modern urban facilities for the Games [39], mainly the need for high speed mobile communications [40].

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