A Data Envelopment Analysis Evaluation and Financial Resources Reallocation for Brazilian Olympic Sports

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Abstract: - This paper proposes the use of a Data Envelopment Analysis model to evaluate the Brazilian Olympic sports efficiency and also to reallocate the financial resources received by them. The sports selected were those that received financial resources from the Agnelo/Piva Law in 2011. As proposed in previous works, we use as inputs the funds received and medals offered, as a proxy for difficulty measure in winning a medal; and the results obtained (gold, silver and bronze medal) as outputs. We proposed to use the results from the Pan-American Games, specifically from the 2011 Pan-American Games, since we believe that the Olympic Games results were scarce to assess the sports efficiency as there are many null results for lots of sports. Therefore, the medals related to the 2011 Pan American Games are used in this paper. A DEA non-radial model with weights restrictions is formulated to perform the Olympic sports efficiency evaluation. With these results a financial resources reallocation is proposed using a ZSG-DEA non-radial approach. Results show that using data with minimal null medals leads to a good financial resources reallocation, based on the sports efficiency, without the need of including anymore variables or imposing additional weight restrictions.

Key-Words: - Data Envelopment Analysis, Olympic sports, sports efficiency, financial resources reallocation

1 Introduction

Brazil will be soon the host country of two major world sporting events: the 2014 FIFA World Cup and the 2016 Olympic Games. It represents a unique opportunity for the country to take advantage of the large investment that will be made and to leave a great impression all over the world, whether the events are well organized and achieved. Moreover, in this position of great international visibility, the country performance in both events is a growing concern. In order for the country to achieve a good performance during the sporting events, it is necessary a high investment in sports. The main source of financial funds for Brazilian Olympic Sports is the Agnelo/Piva Law. This Law was sanctioned in 2001 and determines that 2% of the gross revenues from the Brazilian federal lotteries must be destined to the Brazilian Olympic Committee. Since the creation of this Law, we haven't been noticing great improvements on the Brazilian sporting performance and, consequently, the Brazilian Olympic Committee have been suffering harsh criticism regarding the application of the funds and how it has been distributed [1, 2].

With the aim of contributing on the improvement of its sporting performance, this paper proposes the use of Data Envelopment Analysis (DEA) to firstly evaluate some Brazilian Olympic sports efficiency, based on the country results in the 2011 Guadalajara Pan-American Games. Posteriorly a financial resources reallocation is made considering the funds transferred to each sport committee as defined by the Agnelo/Piva Law in 2011. Therefore, those who have good performance (high efficiency) will receive more funds; and those who haven't, will receive smaller funds as they are not using them properly, i.e., transforming them into results.

This paper is divided into five sections. In section 1 an introduction and the motivation of the work are presented. Posteriorly, in section 2, the theoretical explanation of the methodology used and its main features are exposed. Still in this section, some studies using DEA concerning financial resources destined to sports are presented. In section 3, it is shown how Data Envelopment Analysis was used to reach the results presented in section 4, where it is found a discussion about these results. Finally, in section 5, final comments are made.

2 Data Envelopment Analysis

Data Envelopment Analysis (DEA) [3] is a mathematical technique used to evaluate the efficiency of a group of units, called Decision Making Units (DMUs). The DEA method involves the use of Linear Programming (LP) to determine the relative efficiency of each DMU. A group of DMUs represents productive units in a broader sense, not only those involved in production processes, but units with the same targets and with the use of the same kind of resources (inputs), generating the same kind of products (outputs). Traditionally, DEA models can have two orientations: input orientation, when the objective is to decrease the inputs while maintaining outputs level constant, and output orientation, when the objective is to increase the outputs keeping the inputs level constant.

The DEA efficiency is obtained by the ratio of the outputs weighted sum to the inputs weighted sum. There is no imposition of fixed values for the weights used on the inputs and outputs weighing, which allows each DMU to find the most favourable weights set for itself. This flexibility represents an advantage of DEA, since the DMUs considered inefficient cannot claim that such inefficiency is due to an unfair weights distribution.

There are two DEA classical models. The first one is the most basic DEA model and it is called CCR, that stands for Charnes, Cooper e Rhodes, the model's creators [3]. This model works with constant returns to scale, being used for situations where inputs variations cause proportional variations on the outputs. The second DEA classical model is the BCC, that stands for Banker, Charnes e Cooper [4]. The BCC model works with variable returns to scale, being suitable for a set of DMUs in different scales, avoiding possible problems caused by imperfect competition situations. The input oriented version of the BCC model is presented in (1).

$$
\begin{aligned}
& Min\, h_0 \\
& Subject\, to \\
& h_0 x_{i0} \geq \sum_{k=1}^{n} \lambda_k x_{ik} \;, \forall i \\
& y_{j0} \leq \sum_{k=1}^{n} \lambda_k y_{jk} \;, \forall j \\
& \sum_{k=1}^{n} \lambda_k = 1 \\
& \lambda_k \geq 0 \;, \forall k
\end{aligned}
\tag{1}
$$

In this programme, h_0 is the efficiency of the DMU o, if h_0 is equal to 1, then DMU o is efficient, if h_0<1 then DMU o is inefficient. Also, x_{ik} and y_{jk} represent, respectively, the value of inputs i and outputs j of a DMU k; λ_k represents the contribution of each DMU k in the composition of the target of the DMU o. This model is called the envelopment model; its dual is called the multipliers model. Both models provide the efficiency of the DMU under evaluation, h_o, but they deliver different information: benchmarks and targets (envelopment model) and the variables weights (multipliers model) [5].

Since the very first DEA model, many others have been proposed accounting for several different characteristics of the variables, case studies, etc. In this paper, we use a non-radial model, which basically means that not all inputs (outputs) reduce (increase) proportionally. In our case, we will use a non-radial model proposed by [6], which takes into account the existence of non-controllable variables, variables that cannot be modified by the decision maker (for more information about non-controllable variables see, for instance, [7]). The input oriented model that takes into account variable returns to scale and non-controllable inputs is presented in (2).

$$
\begin{aligned}
& Min\, h_0 \\
& Subject\, to \\
& h_0 x_{i0}^{C} \geq \sum_{k=1}^{n} \lambda_k x_{ik}^{C} \ , \forall i \in C \\
& x_{i0}^{NC} \geq \sum_{k=1}^{n} \lambda_k x_{ik}^{NC} \ , \forall i \in NC \\
& y_{j0} \leq \sum_{k=1}^{n} \lambda_k y_{jk} \ , \forall j \\
& \sum_{k=1}^{n} \lambda_k = 1 \\
& \lambda_k \geq 0 \ , \forall k
\end{aligned}
\tag{2}
$$

As mentioned previously, programmes (1) and (2) are very similar being the only difference that in programme (2) the inputs are divided into two groups, the controllable inputs denoted by C, and the non- controllable inputs denoted by NC. We can observe that the first set of restrictions concerns only the controllable inputs, which are multiplied by the term h_0 in the left part of the equation. However, the second set of restrictions, very similar to the first one, concerns only the non-controllable inputs, that are not multiplied by the term h_0 in the left part of the equation.

Moreover, in this paper we have additional information to include in the model. When a priori information or value judgements about the variables must be taken into account we use weight restrictions [8]. There are different types of weights restrictions, depending on the kind of information available. In our case, we will use the Assurance Region Method, as termed by Thompson et al. [9]. This type of weight restrictions makes a direct comparison between the variables. When we compare either inputs, or outputs, we are using Assurance Region I, or ARI, as used by [10] and Kornbluth [11]. The Assurance Region II, also proposed by [10], is used for comparisons between inputs and outputs. Since we just need to compare two variables, these are the most used weight restrictions, and at the same time they preserve the DEA spirit of providing some freedom to determine variables weights. In (3), $A^T\gamma$ represents the coefficients matrix of the outputs weights restrictions, $Au \leq 0$, as presented in [12] and also used in [13]. The composition and form of this matrix will be explained in the next Section.

$$
\begin{aligned}
& Min\, h_0 \\
& Subject\, to \\
& h_0 x_{i0}^{C} \geq \sum_{k=1}^{n} \lambda_k x_{ik}^{C} \ , \forall i \\
& x_{i0}^{NC} \geq \sum_{k=1}^{n} \lambda_k x_{ik}^{NC} \ , \forall i \\
& y_{j0} \leq \sum_{k=1}^{n} \lambda_k y_{jk} - A^T \gamma_i , \forall j \\
& \sum_{k=1}^{n} \lambda_k = 1 \\
& \lambda_k \geq 0 \ , \forall k
\end{aligned}
\tag{3}
$$

So far, model (3) is used for the performance evaluation. Once the evaluation is performed, we also propose a financial resources reallocation using the input targets from the non-radial model. This reallocation is made based on a ZSG non-radial approach, as in Santos et al [14]. The DEA Zero Sum Gains model (DEA-ZSG) was proposed to solve problems where the total sum of some inputs or outputs values must be constant [12, 15]. As the total sum of the financial resources received by all sports in this study must be constant, and the targets determined by model (3) will not ensure that, the reallocation is made based on this approach. Equation (4) shows how the reallocation of an input is calculated using the ZSG non-radial approach. The term $x_{io}^{reallocated}$ is the new value of the input i for the DMU o; x_{io}^{target} is the target of the input i for the DMU o obtained with the non-radial model (3); $x_{ik}^{original}$ is the original input i for a DMU k; x_{ik}^{target} is the target of the input i for a DMU k obtained with the non-radial model; n is the total number of DMUs.

$$
x_{io}^{reallocated} = x_{io}^{target} \times \left(\sum_{k=1}^{n} x_{ik}^{original} \div \sum_{k=1}^{n} x_{ik}^{target} \right)
\tag{4}
$$

At the end of the reallocation, all DMUs must be efficient and the model is run again to verify that all have reach maximum efficiency. Also, those who were efficient previously would receive more input and the inefficient ones will lose some input. Sometimes, when using a non-radial model with weight restrictions, it is necessary to perform many iterations, i.e., reallocate input and verify efficiency of DMUs, for all DMUs to be efficient [13].

2.1 DEA in sports

Data Envelopment Analysis has been used in sports, especially in determining rankings in Olympic sports or other international events. A brief survey of DEA in sport can be found in [16]. Among, all this papers, we highlight the works of [12] that present and use the ZSG-DEA model to show a redistribution of medals among countries for all to be efficient and also to rank winning medal countries. An application of DEA in determining a final ranking for the Olympic Games can be found in [17]. Also [18] determine a ranking and proposed a way to benchmark inefficient countries and [19] proposed a model to rank countries taking into account integer values. Moreover, regarding target setting and redistribution we have [20] and [14].

3 Brazilian Olympic sports – Evaluation and Financial Resources Reallocation

Sports are in the spotlight in Brazil due to the two upcoming worldwide events to be host in the country: the 2014 FIFA World Cup and the 2016 Olympic Games. Therefore there is been more interest in the results obtained by Brazilian representatives in international events. Moreover, the below average performance of some popular sports has raised issues about the financial funds received by Olympic sports in general.

The main source of financial funds for Brazilian Olympic Sports is the Agnelo/Piva Law, sanctioned in 2001. This law determines that 2% of the gross revenues from the Brazilian federal lotteries must be destined to the Brazilian Olympic Committee (COB – *Comitê Olímpico Brasileiro* in Portuguese), which receives 85% of the amount, and to the Brazilian Paralympic Committee, which receives the 15% remaining. Both these Committees must invest 75% in the Brazilian Olympic Confederations. COB uses the funds in expenses related to sports, hiring international coaches, the maintenance of managerial and technical staff, equipment and material acquisition, maintenance of the training centres, training abroad, Brazilian technical staff qualification, the participation of delegation on national and international events, etc [21].

However, as mentioned earlier, results obtained in these sports were below expectation, which carried harsh criticism and questioning about the distribution of funds among the sports being made based on political agenda and popularity instead of technical basis.

In this paper, we proposed a DEA model to assess the sports efficiency regarding their results, taking into account the funds received and the opportunities given to win a medal, that is, the efficient sports will be the ones which succeeded better at obtaining medals compared to the others, by meritocracy. Moreover, the reallocation of financial resources will be based on their efficiency, how well did they performed in comparison to the others. This reallocation will be made based on technical reasons, in an objective way, and this is possible using DEA.

We observed that work of Santos et al [14] wasn't reallocating the funds properly for the efficient countries, they even suggested including additional variables. However, we can also observe that Brazilian results in the Olympics were very scarce, with many null values in the data set, as no medals were won. Therefore, we propose to use the results, medals won, in the 2011 Guadalajara Pan American Games, as Brazil has always a better performance at the Pan American Games [22].

As we want to evaluate how well the financial resources were used by the Olympic Committees to obtain results, the Olympic sports are the DMUs. These were the ones who participated in the 2011 Guadalajara Pan American Games and that also received funds from the Agnelo/Piva Law in 2011. It is important to point out that some sports, as Soccer, Bowling, Karate and Squash, despite having taken part of these Games, are not considered as DMUs in this paper since they didn't receive any funds coming from this Law. On the other hand, Hockey on Grass is considered as a DMU since it received funds from aforementioned Law, even not having participated in the games. It is also important to highlight that it was necessary to group some sports according to the Confederations to which they belong. This is the case of Water Sports, Gymnastics and Volleyball. It was necessary because Agnelo/Piva Law funds are distributed among the Confederations and not for each sport individually. In total there are 26 Sports Confederations considered as DMUs in this study.

The model was formulated using two inputs and three outputs, which values of each DMU considered on this paper are presented in Table 1.

Olympic Confederations	Inputs		Outputs		
	Financial Resources (R$)	Gold Medals Offer	Gold Medals	Silver Medals	Bonze Medals
Athletics	3.000.000,00	47	10	6	7
Badminton	1.300.000,00	5	0	0	1
Basketball	2.100.000,00	2	0	0	1
Boxing	1.700.000,00	13	0	2	5
Canoeing	2.300.000,00	12	0	2	2
Cycling	2.300.000,00	18	0	0	0
Water Sports	3.000.000,00	46	10	9	11
Fencing	1.100.000,00	12	0	0	3
Gymnastics	2.800.000,00	24	6	3	5
Handball	3.000.000,00	2	1	1	0
Horse Riding	2.900.000,00	6	0	1	2
Hockey on Grass	1.300.000,00	2	0	0	0
Judo	3.000.000,00	14	6	3	4
Weightlifting	1.100.000,00	15	1	0	0
Wrestling	1.500.000,00	18	0	1	1
Modern Pentathlon	1.300.000,00	2	0	1	0
Oar	1.900.000,00	14	0	2	0
Rugby	500.000,00	1	0	0	0
Taekwondo	1.200.000,00	8	0	0	1
Tennis	1.800.000,00	5	0	1	1
Table Tennis	2.300.000,00	4	1	0	0
Archery	1.300.000,00	4	0	0	0
Sports Shooting	2.000.000,00	15	1	0	5
Triathlon	2.000.000,00	2	1	0	1
Sailing	3.000.000,00	9	5	1	1
Volleyball	3.000.000,00	4	4	0	0
TOTAL	**52.700.000,00**				

Table 1 – Inputs and Outputs Values

The first input is represented by the funds coming from the Agnelo/Piva Law that were transferred to each Olympic Confederation by the Brazilian Olympic Committee in 2011. This input measures the amount of funds that each one receives for investments in maintenance and development of the athletes. It is important to say that in this paper the financial resources considered are only the ones coming from the Agnelo/Piva Law, which means that funds coming from other sources, as private sponsorship, are not being taken into account.

The second input is the number of gold medals offered for each sport at the 2011 Guadalajara Pan American Games. The use of this variable in the model allows us to consider the disparity in chances of winning a medal for each sport, indicating a proxy for the difficulty that each one has to win a medal. It is necessary to consider this input since each sport has a different number of competitions and, the more competitions they have, the easier it is to win a medal. Sports like swimming or athletics, for example, have a much larger number of competitions than basketball or handball, which have only two possible medals each one, one of their men's team and the other one of their women's team. That is the way for the model to take into account the difficulty in winning a medal, as a gold medal in basketball may be considered more valuable than a gold medal in swimming in term of the effort involved. This second output represents the non-controllable variable of the problem, since this input values for each Sport is defined by the Committee responsible for the Pan American Games organization and it cannot be changed. As various sports are aggregated into confederations, the gold medals offered were added.

The outputs are the number of gold, silver and bronze medals won by each confederation during the 2011 Guadalajara Pan American Games. These three outputs represent the results obtained by each sport and it is linked to the investments made using the financial resources of the Agnelo/Piva Law, since everything that is involved in the athletes training needs funds to happen. For the Confederation of Hockey on Grass, which received funds from the Agnelo/Piva Law in 2011 but did not have Brazilian representatives at the 2011 Pan American Games, null values were assigned to its three outputs.

As we know, the different medals, gold, silver and bronze, don’t have the same importance. The final classification of the Pan American Games, as well as other international events, is based on the number of gold medals won by each country. The number of silver medals and, posteriorly, the number of bronze medals are only used if there is a draw between two or more countries. This is a multicriteria method called the Lexicographic Method, which main disadvantage is the overvaluation of the gold medal [12]. In order to take into account these differences, in other words, to take into account that each medal must have a different weight without imposing an actual weight value, this additional information was included in the form of weight restrictions, of the Assurance Region I, mentioned in the previous section. So, in our non-radial model we will use the same restrictions used by Lins et al [12]. The three weight restrictions included can be seen in (5), (6) and (7). (For other methods to include value judgements or other information in a DEA model see [23])

$$u_{gold} \geq u_{silver} \tag{5}$$

$$u_{silver} \geq u_{bronze} \tag{6}$$

$$u_{gold} - u_{silver} \geq u_{silver} - u_{bronze} \tag{7}$$

The first weight restriction (5) indicates that the gold medal associated weight must be equal to or greater than the silver one, which means that the gold medal is more important, or at least equal to the silver medal. The second one (6) indicates that the silver medal associated weight must be equal to or greater than the bronze one, which means that the silver medal is more important or at least equal to the bronze. Finally, the third weight restriction (7) indicates that the difference between the gold medal and the silver medal associated weights must be equal to or greater than the difference between the silver medal and the bronze medal associated weights. Those statements were also used in [24] and [25].

The weight restrictions in general are introduced in the multiplier version of the DEA models. In (2) we presented the envelopment model version, so it is necessary to formulate the dual form of these restrictions (5-7). Since one is dual of the other, additional restrictions in the multipliers model (primal) generate new variables in the envelopment form (dual). Taking the variables on the restrictions left hand side, we may express the coefficient of these restrictions as a matrix A shown in (8).

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & -2 & 1 \end{bmatrix} \tag{8}$$

Then, the transpose of the matrix A was multiplied by the vector of the dual variables γ, which has three components, γ_1, γ_2 and γ_3, each one for an Assurance Region added to the model. Finally, each line of the matrix $-A^T\gamma$ could be included in the associated output restriction of the Envelopment Model, which results in (3).

We use the data shown in Table 1 and the input oriented non radial model, shown in (3) to evaluate the performance of the 26 Confederations. This evaluation aims to identify the ones that better use the funds received, obtaining good results, and also the ones that couldn't obtain good results, which will be considered inefficient. The model is input oriented since the objective is to reallocate the financial resources, which represent an input of the modeling. Once the efficiency evaluation if performed, the reallocation of the financial resources is done using a ZSG non-radial approach, based on the results obtained by the performance evaluation. This reallocation aims to allow all DMUs to be efficient. Therefore those who have been efficient will receive more funds and those who were not will receive fewer funds.

4 Results and discussions

This section is divided into two parts: first of all we analyse the results concerning the efficiency of each sport regarding the financial resources received and the results obtained by each sport at the 2011 Pan American Games. Also, based on the efficiency index we perform the financial resources reallocation. These results are depicted in Table 2.

By analysing the second column of this table we can note that there are eight DMUs with maximum efficiency: Athletics, Water Sports, Handball, Judo, Rugby, Triathlon, Sailing and Volleyball. Among these eight DMUs, all of them won at least one gold medal, except the Rugby Confederation. This Confederation was among the DMUs with maximum efficiency, in spite of not having won any medal, since for the models with variable returns to scale, the DMU with the smallest values of inputs has maximum efficiency, even if it has null outputs. Besides, the Confederation of Athletics was not considered Pareto Efficient, even having reached the maximum efficiency and, consequently, being in the efficiency frontier.

While the sports mentioned above are considered the most efficient ones, there is a group of sports needing urgent performance improvement: Badminton, Basketball, Canoeing, Cycling, Horse Riding, Hockey on Grass, Wrestling, Oar, Taekwondo, Tennis, Table Tennis and Archery. Together, these 12 Confederations received approximately 42% of the total amount of funds distributed by the Agnelo/Piva Law in 2011 and they had together 98 gold medals being offered during the 2011 Guadalajara Pan American Games, that is 98 possibilities of winning a medal. However, they won only one gold medal, which shows their inefficiency.

Regarding the Fencing and Weightlifting efficiencies index, it is possible to notice the action of the weight restrictions presented in (5), (6) and (7). If the weight of the gold, silver and bronze medals were the same, probably Fencing would achieve an efficiency index higher than the Weightlifting one, since both received the same amount of money, R$ 1,100,000.00, Fencing was offered fewer gold medals than Weightlifting, and the total number of medals won by Fencing is greater than that achieved by Weightlifting, as we can see in Table 1. However, taking into account the weight restrictions that added to the model a greater importance to the gold medals, and knowing that the three medals won by Fencing are bronze medals and the only medal won by Weightlifting is a gold one, both DMUs reached the same efficiency index in the modelling proposed here, which was 0.681818.

Olympic Confederations	Efficiency Score	Original Resources (R$)	Reallocated Resources (R$)
Athletics	1.0000	3,000,000.00	4,534,431.80
Badminton	0.4487	1,300,000.00	881,695.17
Basketball	0.3297	2,100,000.00	1,046,406.29
Boxing	0.6373	1,700,000.00	1,637,433.96
Canoeing	0.3623	2,300,000.00	1,259,564.94
Cycling	0.2174	2,300,000.00	755,737.58
Water Sports	1.0000	3,000,000.00	4,534,431.80
Fencing	0.6818	1,100,000.00	1,133,607.65
Gymnastics	0.8163	2,800,000.00	3,454,807.17
Handball	1.0000	3,000,000.00	4,534,431.80
Horse Riding	0.2586	2,900,000.00	1,133,609.31
Hockey on Grass	0.3846	1,300,000.00	755,737.88
Judo	1.0000	3,000,000.00	4,534,431.80
Weightlifting	0.6818	1,100,000.00	1,133,607.65
Wrestling	0.4444	1,500,000.00	1,007,650.50
Modern Pentathlon	0.5325	1,300,000.00	1,046,406.60
Oar	0.3539	1,900,000.00	1,016,338.17
Rugby	1.0000	500,000.00	755,738.63
Taekwondo	0.4861	1,200,000.00	881,694.87
Tennis	0.3704	1,800,000.00	1,007,650.50
Table Tennis	0.3727	2,300,000.00	1,295,552.61
Archery	0.3846	1,300,000.00	755,737.88
Sports Shooting	0.5000	2,000,000.00	1,511,477.27
Triathlon	1.0000	2,000,000.00	3,022,954.54
Sailing	1.0000	3,000,000.00	4,534,431.80
Volleyball	1.0000	3,000,000.00	4,534,431.80
TOTAL		**52,700,000.00**	**52,700,000.00**

Table 2 – Efficiency, Original Resource and Reallocated Resource for each DMU

But it is also important to notice that the isolated fact of winning one or more gold medals won't determine that a DMU is more efficient than another one which won no gold medals. There are several other factors influencing the model. A good example is the Table Tennis, which despite having won a gold medal, achieved a lower efficiency index than other Olympic sports that did not win gold medals, as Fencing and Taekwondo, and even than other sports that won no medal, as Archery. This probably occurred because Table Tennis received a large amount of financial resources, higher than the amounts received by Fencing, Taekwondo, Archery and many other DMUs.

The fourth column of the table represents the data obtained by the financial resources reallocation made using a DEA-GSZ non radial approach. As mentioned previously, sometimes it is necessary to expect more than one iteration when reallocating funds. However, rounding the first iteration average efficiency up to six decimal places, the value found is 1.000000. In addition, new iterations would produce minimal changes in the values of the input, which in practice does not represent significant changes. Therefore, the reallocation of resources from the first iteration, shown in the fourth column of the table, is considered as the one that allows all DMUs to reach the efficiency frontier. Comparing these data with the ones in the second column, the original distribution of resources, we can reach several important conclusions.

Firstly, the DMUs that had originally received a great amount of resources and reached the maximum efficiency had more than R$ 3,000,000.00 of resources after the reallocation. It happened for all of DMUs with maximum efficiency, except for the Rugby. This sport didn't have a big amount of money in the original distribution and it was efficient only because it has the lowest values of inputs. Therefore, it seems that the model recognizes that this sport doesn't need much more money to keep its efficiency. That is why the Rugby Confederation didn't receive much more funds in the reallocation, compared with how much it had already received originally. Still among the DMUs with maximum efficiency, those that had received exactly R$ 3,000,000.00 were transferred the same value after the resources reallocation: R$ 4,534,431.80. It happened because the calculation of the reallocated input for each DMU is proportional to the efficiency of this DMU and to its original input.

Furthermore, all DMUs with efficiency equal to or greater than 0.6818 received a larger amount of funds after the reallocation. However, all DMUs with efficiency equal to or less than 0.6373 lost part of the original amount of funds. Therefore, according to the DEA model used, the greater is the DMU efficiency reached using the non-radial model with weight restrictions, the bigger is the amount of resources it should receive by the reallocation with a DEA-GSZ non radial approach, in order for all DMUs to reach the maximum efficiency.

5 Final Comments

This paper, using Data Envelopment Analysis, could evaluate sports performance, based on their results at the 2011 Pan American Games and on the funds coming from the Agnelo/Piva Law in 2011, and could also propose a reallocation of those funds, allowing all DMUS to be efficient. The results obtained pointed out many efficient DMUs but also many others that need urgent improvements, the ones with the lowest values of efficiency. However, those sports needing urgent improvements received fewer funds with the financial resources reallocation. It didn't happen because the DEA-ZSG approach used intends to be punitive, but because it intends to reward those DMUs with best performance and to serve as a warning signal for those with worst performance. The DMUs with worst performance should face the results as an indicative that they have to do something to improve their performance.

The approach used in this paper is very similar to the one used in [14]. The major difference is that in this study the data used as outputs, representing the number of gold, silver and bronze medals conquered by each DMU, came from the 2011 Pan-American Games, while in the mentioned paper, they came from the 2008 Olympic Games. This choice provided more robust results for this paper compared with the other one, since data coming Olympic Games have many null results for Brazilian sports. Furthermore, [14] concluded that the inclusion of the number of gold medals offered for each sport as an input in the model used in that study didn't add any value for the results. However, it could have been caused, one more time, by the countless null results in the data used for that study, inasmuch as in this paper the use of this input was fundamental for the results obtained. Also, we saw that the inclusion of this input allow us to really take into account the effort for winning a medal in a given sport.

From a technical standpoint, the models proved to be well suited to what it was intended to do. In the evaluation as well as in the resources reallocation, we could reach satisfactory results, considering the specificities of the study. We can say that this study validated the use of the DEA-GSZ non radial approach for financial resources reallocation on sports.

We also would like to mention another proposal of modelling this problem, which may probably bring interesting conclusions. This proposal is quite simple and it would use the same variables considered on the modelling presented in this paper.

The only difference between them is that, while the modelling already presented uses the number of gold medals offered for each sport as an input, the new modelling would use this variable as the denominator of the three outputs considered. Thus, the outputs would be the ratio of the number of gold, silver and bronze medals won to the number of gold medals offered for each sport. The only input considered would be the financial resources allocated to each DMU. The intention behind using this variable is verify if the proportion of won medals is a better way to include the "effort in winning a medal" and also we would have one less variable. Research is being done regarding this modelling.

It is very important to notice that this study only considered one Brazilian Olympic Confederations funds source: the Agnelo/Piva Law. However, there are many others public and private sources sponsoring some Confederations studied, which were not considered. Therefore, it is suggested the consideration of such sources for future work, in order to obtain more complete results. Even take into account other financial assets for important decisions [26].

Moreover, we identified a limitation in this present study, which is the non-consideration of the maintenance costs for each sport. We believe that it is necessary to take into account this cost to propose a fairer reallocation, since it represents a variable that, combined with the funds available for each Confederation, affects the results of them. Also, we know that some sports are more "expensive" than others. Considering this variable, we expect that the efficiencies obtained with the model for each DMU will be more robust. This addition may also solve the problem of sports with high maintenance costs receiving a less amount of funds with the financial resources reallocation.

Finally, it is interesting to highlight the importance of studies like this for Brazilian sport, due to the current situation of the country: a country in full development, host of the next FIFA World Cup and the next Olympic Games, with enormous international visibility, but also with serious problems and at the same time, basic, like a poor level of education and high rates of violence, which may find its solution with the aid of the sport. There are very few scientific studies using DEA applied to investments in sports. This is another one and it serves as an incentive for future works.

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