# The term structure model of corporate bond yields

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*Abstract:* - We build the term structure of corporate bond yields with N-factor affine model, and we estimate the parameters by using Kalman filtering. We choose weekly average corporate bond yields data in Shanghai Stock Exchange and Shenzhen Stock Exchange. We find the one-factor model and two-factor model could do one-step forward forecasting well, but the three-factor model could fit the observable data well.

*Key-Words:* - corporate bond; yields; term structure; Kalman filtering

## 1 Introduction

Many scholars research on term structure affine models of bonds. The literatures are as below. Some scholars find the three factor model fits observable data well. Dai, Singleton(2000) [1] analyzes the structural differences and goodness-of-fits of affine term structure models. Some models are good at modeling the conditional correlation, some are good at modeling volatilities of the risk factors. He extends N-factor affine model into N+1-factor affine model. Vasicek (1977) Cox, Ingersoll, and Ross (1985) [2,3]assume instantaneous short rate r(t) is the equation of N-factor state variable Y(t), and r(t)=$\alpha_0 + \alpha_y^{'}$Y(t), and Y(t) follows Gaussian and square root diffusions. Some scholars extend Markov one factor short rate model, and add in a stochastic long-run mean $\theta(t)$ and a volatility v(t) of r(t), dr(t)=($\theta - r(t)$)dt+$\sqrt{v}dB(t)$. These models come from bond pricing and interest-rate derivatives. Duffee(2002)[4] considers affine model can't forecast treasury yields. He thinks assuming yields follow stochastic random walk and forecasting results are good. He considers the models failure for the reason that variation of risk compensation is related with interest-rate volatility. He raises essential affine model, and the model keeps the advantage of standard model, but it makes interest-rate variation independent from interest-rate volatility, and this is important for forecasting future yield. Jong(2000) [22] analyzes term structure affine model combining with time series and cross-section information, and he uses discretization continuous time to do Kalman filtering. He finds the three factors model could fit cross-section and dynamic term structure model. Duffie, Kan(1996) [5] finds yields with fixed maturity follow stochastic volatility multi-parameters Markov diffusion process by using continuous no arbitrage multi-factor model of interest-rate term structure. He uses jump-diffusion to solve interest-rate term structure model. Longstaff and Schwartz(1995) [6] evaluate corporate bonds value which have default risk and interest-rate risk by using simple methods. He finds the relation between default risk and interest-rate has important effect on credit spread. Also, he finds credit spread correlates with interest-rate negatively, and the risky bond duration depends on interest rate. He uses V to represent corporate total asset value, and it follows dynamic variation below: dv=$\mu V dt + \sigma v dz_1$, and $\sigma$ is constant, and $z_1$ is

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standard Wiener process. He uses r to represent risk-free interest rate, and dr=($\varepsilon - \beta r$)dt+$\propto dz_2$, and $\varepsilon, \beta$ and $\propto$ are constant, and $z_2$ is standard Wiener process, and the correlation of $dz_1$ and $dz_2$ is $\rho dt$. Cox,Ingersoll and Ross(1985)[2]study intertemporal interest-rate term structure by using ordinary equilibrium asset pricing model. In the model, anticipation, risk aversion, investment choice and consumption preferences have impact on bond price, and he provides bond pricing formula and it fits the data well. Vasicek(1977) [3] assumes: (1) instantaneous interest rate follows diffusion process; (2) discount bond price depends on instantaneous term; (3) market is efficient. He finds bond expected yields are proportional with standard deviation. Asileiou(2006) [7]evaluates bond value by using non-default bond until maturity, and he finds semi-Markov property holds, and he provides algorithm for forward transition probability. Lamoureux, Witte(2002) [8]uses Bayes model to do research. He finds the three factors model is better.

Some scholars add default factor into bond term structure model. Duffee(1999) [9]analyzes default risk in corporate bond price by using term structure model. He builds square root diffusion transition process model of corporate instantaneous default probability, but the model correlates with default free interest rate. He analyzes time series and cross-section term structure of corporate bond price by using extended Kalman filtering. The model fits corporate bond yields well, and also parameters are the main factors of yield spread term structure. Duan and Simonato(1999) [10] build exponent term structure model for estimating parameters of state space model. He uses Kalman filtering with the conditional mean and conditional variance. Duarte （2004） [11]tries to solve the contradiction in affine term structure model for fitting mean interest and interest rate volatility. Dai and Singleton （2002） [12]find yield curve slope is the linear function of returns, and this is conflict with traditional expectation theory. Cheridito, Filipovic and Kimmel(2007) [13] extend measuring criteria of market price of affine yield model. His research could be used into other asset pricing model. Lando(1998) [14] builds the model of defaultable security and credit derivative, and it includes market risk factors and credit risk. He tests how to use term structure model and price affine model in bonds with different credit ratings. He analyzes one factor term structure affine model by using closed method. Jarrow and Turnbull (1995) [15] provide a new method for credit risk derivative pricing. There are two kinds of credit risks, and one is the default risk in derivatives of basic assets, the other is default risk of the writer of derivative bonds. Duffee(1998) [16] considers bond spreads depend on callability of corporate bond. He tests the assumptions of investment grade corporate bond. Carr and Linetsky(2010)[17] take defaultable stock price as time varying Markov diffusion process with volatility and default intensity. Dai and Singleton(2003) [18]observe dynamic term structure model, and it fits on treasury and swap yield curve, and default factor follows diffusion, jump diffusion. Duffie and Lando (2001) [19] study on corporate bond credit spread term structure with imperfect information. He assumes bond investors can’t observe the assets of bond issuers, and they only get the imperfect accounting reports. He considers corporate assets follow Geometric Brownian Motion, and the credit spread has accounting information character.

Some scholars study term structure model of commodity future. Schwartz and Smith(2000) [20] use a two-factor model of commodity prices, and it allows mean-reversion in short-term prices and uncertainty in equilibrium level to which prices revert. They estimate the parameters of the model using prices for oil futures contracts and then apply the model to some hypothetical oil-linked assets to demonstrate its use and some of its advantages over the Gibson-Schwartz model. Casassus and Collin-Dufresne(2005)[21] three factor model with commodity spot prices, convenience yield and interest rate, and convenience yield relies on spot price and interest rate, and there is time varying risk premium. Chen(2009) [23] predicts Taiwan 10-year government bond yield. Neri(2012) [24]shows how L-FABS can be applied in a partial knowledge learning scenario or a full knowledge learning scenario to approximate financial time series. Neri (2011) [29] Learns and Predicts Financial Time Series by Combining Evolutionary Computation and Agent Simulation. Neri(2012)[30] makes Quantitative estimation of market sentiment: A discussion of two alternatives. Wang(2013)[31] finds Idiosyncratic volatility has an impact on corporate bond spreads: Empirical evidence from Chinese bond markets.

In China, Fan longzhen and Zhang guoqing （2005） [25] analyze time continuous two-factor generalized Gaussian affine model by using Kalman filtering. The model could reflect cross-section characteristic of interest rate term structure, but it can’t reflect the time series character. Wang xiaofang, Liu fenggen and Hanlong（2005）[26] build interest rate term structure curve by using cubic spline function. Fan longzhe （2005）[27]estimates bond interest rate by using term structure of yields with three-factor Gaussian essential affine model. Fan longzhen（2003）[28] estimates treasury time continuous two factor Vasicek model by using Kalman filtering. There are many literatures on interest rate term structure model, the abroad research focuses on commodity futures, corporate bond pricing, and some of corporate bond spread and bond yield. In China, they are mainly about treasury term structure and few of corporate bond term structure. We research on corporate bond yield term structure in Shanghai and Shenzhen Exchange by using Kalman filtering, and few scholars has ever researched on it by using the method, and also we plan to research on the complex factors on corporate bond spread in Shanghai and Shenzhen Exchange.

## 2. Data description

We choose corporate bond yields in Shanghai Exchange and Shenzhen Exchange. We choose bonds with more than 1 year to maturity, because bonds with less than 1 year to maturity are very sensitive to interest rate. We choose corporate bonds weekly average returns with 3 years, 5 years, 7 years and 10 years maturity from January 1st 2012 to December 31st 2012. The data descriptive statistics are in table1. We can see the long term bonds have lower weekly average yields than short term bonds. According to JB values, only 7 years bonds don’t follow normal distribution, and others follow normal distribution.

Table1 descriptive statistics

| | Y1 | Y2 | Y3 | Y4 |
|---|---|---|---|---|
| **Mean** | 5.6770 | 5.6300 | 5.8959 | 4.2039 |
| **Median** | 5.4073 | 5.3979 | 5.7922 | 4.6498 |
| **Max** | 6.8343 | 6.8026 | 6.7393 | 5.4585 |
| **Min** | 4.8176 | 4.6666 | 5.1545 | 1.4359 |
| **St.d** | 0.6288 | 0.6465 | 0.4826 | 1.0274 |
| **skewness** | 0.6356 | 0.5716 | 0.3868 | -1.481 |
| **kurtosis** | 2.0264 | 1.8824 | 1.9326 | 4.1198 |
| **JB** | 5.4480 | 5.4311 | 3.6929 | 21.301 |
| **P** | 0.0656 | 0.0662 | 0.1578 | 0.0000 |

## 3. Term structure affine model

Vasicek (1977) and Cox, Ingersoll and Ross(1985) assume instantaneous short term interest rate r(t) is the affine equation of N-factor state vector Y(t). We assume the equation of r(t) and Y(t) as below:

$$r_t = a_0 + \delta_1 f_{1t} + \delta_2 f_{2t} + \delta_3 f_{3t} + \cdots + \delta_n f_{nt} \quad (1)$$

$r_t$ is short term interest rate, $a_0$ is constant and $f_{1t}, f_{2t}, f_{3t}, \ldots f_{nt}$ are the N-state variables which decide interest rate value. According to short term interest rate model of Longstaff and Schwartz (1995), state variables follow mean reversion in the condition of risk neutral probability.

$$\mathrm{d}F_t = KF_t dt + \sigma dW_t \quad (2)$$

The equation is as below:

$$d\begin{bmatrix} f_{1t} \\ f_{2t} \\ f_{3t} \\ \vdots \\ f_{nt} \end{bmatrix} = \begin{bmatrix} -k_1 & 0 & 0 & 0 & 0 \\ 0 & -k_2 & 0 & 0 & 0 \\ 0 & 0 & -k_3 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & -k_n \end{bmatrix} \begin{bmatrix} f_{1t} \\ f_{2t} \\ f_{3t} \\ \vdots \\ f_{nt} \end{bmatrix} dt +$$

$$\begin{bmatrix} \sigma_1 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \sigma_n \end{bmatrix} \begin{bmatrix} dw_{1t} \\ dw_{2t} \\ dw_{3t} \\ \vdots \\ dw_{nt} \end{bmatrix} \quad (3)$$

Parameters $k_1$, $k_2$, $k_3$,… $k_n$ indicate state variables, and $f_{1t}$, $f_{2t}$, $f_{3t}$, … $f_{nt}$ indicate mean reversion rate, and $\sigma_1$, $\sigma_2$, $\sigma_3$, $\cdots \sigma_n$ indicate state variables volatility, and $w_{1t}$, $w_{2t}$, $w_{3t}$, …$w_{nt}$ indicate N independent Standard Brown Motions. In risk neutral probability, the unconditional mean of state variable is 0. $a_0$ denotes long term mean of short term interest rate in risk neutral probability. In real probability P, the state variables change as below:

$$d\begin{bmatrix} f_{1t} \\ f_{2t} \\ f_{3t} \\ \vdots \\ f_{nt} \end{bmatrix} = \begin{bmatrix} -k_1 & 0 & 0 & 0 & 0 \\ 0 & -k_2 & 0 & 0 & 0 \\ 0 & 0 & -k_3 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & -k_n \end{bmatrix} \begin{bmatrix} f_{1t} \\ f_{2t} \\ f_{3t} \\ \vdots \\ f_{nt} \end{bmatrix} dt +$$

$$\begin{bmatrix} \sigma_1 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \sigma_n \end{bmatrix} \begin{bmatrix} dw_{1t} \\ dw_{2t} \\ dw_{3t} \\ \vdots \\ dw_{nt} \end{bmatrix} + \begin{bmatrix} \beta_{10} \\ \beta_{20} \\ \beta_{30} \\ \vdots \\ \beta_{n0} \end{bmatrix} dt +$$

$$\begin{bmatrix} \beta_{11} & 0 & 0 & 0 & 0 \\ 0 & \beta_{22} & 0 & 0 & 0 \\ 0 & 0 & \beta_{33} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \beta_{nn} \end{bmatrix} \begin{bmatrix} f_{1t} \\ f_{2t} \\ f_{3t} \\ \vdots \\ f_{nt} \end{bmatrix} dt$$

=

$$\begin{bmatrix} k_1-\beta_{11} & 0 & 0 & 0 & 0 \\ 0 & k_2-\beta_{22} & 0 & 0 & 0 \\ 0 & 0 & k_3-\beta_{33} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & k_n-\beta_{nn} \end{bmatrix} \times \left[ \begin{bmatrix} k_1-\beta_{11} & 0 & 0 & 0 & 0 \\ 0 & k_2-\beta_{22} & 0 & 0 & 0 \\ 0 & 0 & k_3-\beta_{33} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & k_n-\beta_{nn} \end{bmatrix}^{-1} \times \begin{bmatrix} \beta_{10} \\ \beta_{20} \\ \beta_{30} \\ \vdots \\ \beta_{n0} \end{bmatrix} - \begin{bmatrix} f_{1t} \\ f_{2t} \\ f_{3t} \\ \vdots \\ f_{nt} \end{bmatrix} \right] dt + \begin{bmatrix} \sigma_1 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \sigma_n \end{bmatrix} \begin{bmatrix} dw_{1t} \\ dw_{2t} \\ dw_{3t} \\ \vdots \\ dw_{nt} \end{bmatrix} \quad (4)$$

$\beta_{10}, \beta_{20}, \beta_{30}, \dots \beta_{n0}$ denote the fixed interest rate risk premium. $\beta_{11}, \beta_{22}, \beta_{33}, \dots \beta_{nn}$ denote the time varying interest rate risk premium.

$$K = \begin{bmatrix} k_1-\beta_{11} & 0 & 0 & 0 & 0 \\ 0 & k_2-\beta_{22} & 0 & 0 & 0 \\ 0 & 0 & k_3-\beta_{33} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & k_n-\beta_{nn} \end{bmatrix},$$

$$\theta = \begin{bmatrix} \frac{\beta_{10}}{k_1-\beta_{11}} & 0 & 0 & 0 & 0 \\ 0 & \frac{\beta_{20}}{k_2-\beta_{22}} & 0 & 0 & 0 \\ 0 & 0 & \frac{\beta_{30}}{k_3-\beta_{33}} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \frac{\beta_{n0}}{k_n-\beta_{nn}} \end{bmatrix},$$

$$\partial = \begin{bmatrix} \sigma_1 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \sigma_n \end{bmatrix}, \quad F_t = \begin{bmatrix} f_{1t} \\ f_{2t} \\ f_{3t} \\ \vdots \\ f_{nt} \end{bmatrix}$$

In the real probability P, state variables mean reversion follow the equation below.

$$dF_t = K(\theta - F_t)dt + \sigma dW_t \quad (5)$$

$$\emptyset = \begin{bmatrix} \exp(\beta_{11}-k_1) & 0 & 0 & 0 & 0 \\ 0 & \exp(\beta_{22}-k_2) & 0 & 0 & 0 \\ 0 & 0 & \exp(\beta_{33}-k_3) & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \exp(\beta_{nn}-k_6) \end{bmatrix},$$

$$Q = \begin{bmatrix} \sigma_1^2\frac{1-e^{-2(k_1-\beta_{11})}}{2(k_1-\beta_{11})} & 0 & 0 & 0 & 0 \\ 0 & \sigma_2^2\frac{1-e^{-2(k_2-\beta_{22})}}{2(k_2-\beta_{22})} & 0 & 0 & 0 \\ 0 & 0 & \sigma_3^2\frac{1-e^{-2(k_3-\beta_{33})}}{2(k_3-\beta_{33})} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \sigma_n^2\frac{1-e^{-2(k_n-\beta_{nn})}}{2(k_n-\beta_{nn})} \end{bmatrix}$$

In real probability P, the conditional expectation and variance of state variables are below:

$$E(F_{t+1}|F_t) = \theta + \emptyset[F_t - \theta] \quad (6)$$

$$var(F_{t+1}|F_t) = Q \quad (7)$$

When short term interest rate and state variable are certain, bond price and long term interest rate will be determined by short term interest rate in risk neutral probability. According to literatures, the bond with maturity at time T and par value 1$, its pricing model is as below.

$$P_t^{(T-t)} = E_t^Q\left[\exp\left(-\int_t^T r_s ds\right)\right] =$$

$$E_t^Q\left[\exp\left(-\int_t^T (a_0 + \delta_1 f_{1t} + \delta_2 f_{2t} + \delta_3 f_{3t} + \dots + \delta_n f_{nt})ds\right)\right] \quad (8)$$

After derivation, the bond with term $\tau$, at time t, the spot interest rate is below:

$$y_t^\tau = a_0 - \frac{\sigma_1^2}{k_1^2} - \frac{\sigma_2^2}{k_2^2} - \frac{\sigma_3^2}{k_3^2} - \cdots - \frac{\sigma_n^2}{k_n^2} + \frac{2\sigma_1^2(1-e^{-k_1\tau})}{k_1^3\tau} - \frac{\sigma_1^2(1-e^{-2k_1\tau})}{2k_1^3\tau} + \frac{2\sigma_2^2(1-e^{-k_2\tau})}{k_2^3\tau} - \frac{\sigma_2^2(1-e^{-2k_2\tau})}{2k_2^3\tau} + \frac{2\sigma_3^2(1-e^{-k_3\tau})}{k_3^3\tau} - \frac{\sigma_3^2(1-e^{-2k_3\tau})}{2k_3^3\tau} + \cdots + \frac{2\sigma_n^2(1-e^{-k_n\tau})}{k_n^3\tau} - \frac{\sigma_n^2(1-e^{-2k_n\tau})}{2k_n^3\tau} + \frac{1-e^{-k_1\tau}}{k_1\tau}f_{1t} + \frac{1-e^{-k_2\tau}}{k_2\tau}f_{2t} + \frac{1-e^{-k_3\tau}}{k_3\tau}f_{3t} + \cdots + \frac{1-e^{-k_n\tau}}{k_n\tau}f_{nt} \quad (9)$$

## 4. Kalman filtering

Kalman filtering is made up of recursive mathematical formulas, and the signal equation indicates the relation between bond yields which could be observed and state variables which can't be observed. The state equation indicates the changing process of state variables. We give initial value for state variable, and we can estimate the parameters combining with maximum likelihood estimation model. According to equation (9), we mark

$$C_0^\tau = a_0 - \frac{\sigma_1^2}{k_1^2} - \frac{\sigma_2^2}{k_2^2} - \frac{\sigma_3^2}{k_3^2} - \cdots - \frac{\sigma_n^2}{k_n^2} + \frac{2\sigma_1^2(1-e^{-k_1\tau})}{k_1^3\tau} - \frac{\sigma_1^2(1-e^{-2k_1\tau})}{2k_1^3\tau} + \frac{2\sigma_2^2(1-e^{-k_2\tau})}{k_2^3\tau} - \frac{\sigma_2^2(1-e^{-2k_2\tau})}{2k_2^3\tau} + \frac{2\sigma_3^2(1-e^{-k_3\tau})}{k_3^3\tau} - \frac{\sigma_3^2(1-e^{-2k_3\tau})}{2k_3^3\tau} + \cdots + \frac{2\sigma_n^2(1-e^{-k_n\tau})}{k_n^3\tau} - \frac{\sigma_n^2(1-e^{-2k_n\tau})}{2k_n^3\tau}$$

$$C_1^\tau = \frac{1-e^{-k_1\tau}}{k_1\tau},\quad C_2^\tau = \frac{1-e^{-k_2\tau}}{k_2\tau},\quad C_3^\tau = \frac{1-e^{-k_3\tau}}{k_3\tau},$$

$$\ldots C_n^\tau = \frac{1-e^{-k_n\tau}}{k_n\tau}。$$

Equation (9) could be written as below:

$$y_t^\tau = C_0^\tau + C_1^\tau f_{1t} + C_2^\tau f_{2t} + C_3^\tau f_{3t} + \cdots + C_n^\tau f_{nt} \quad (10)$$

We choose corporate bond yields data from Shanghai Exchange and Shenzhen Exchange and the bonds with maturity of 3 years, 5 years, 7 years and 10 years.

$$Y_t = \begin{bmatrix} y_t^3 \\ y_t^5 \\ y_t^7 \\ y_t^{10} \end{bmatrix},\quad C_0 = \begin{bmatrix} C_t^3 \\ C_t^5 \\ C_t^7 \\ C_t^{10} \end{bmatrix},\quad e_t = \begin{bmatrix} e_t^3 \\ e_t^5 \\ e_t^7 \\ e_t^{10} \end{bmatrix},$$

$$C_1 = \begin{bmatrix} C_1^3 & C_2^3 & C_3^3 & C_4^3 & C_5^3 & C_6^3 & \cdots & C_n^3 \\ C_1^5 & C_2^5 & C_3^5 & C_4^5 & C_5^5 & C_6^5 & \cdots & C_n^5 \\ C_1^7 & C_2^7 & C_3^7 & C_4^7 & C_5^7 & C_6^7 & \cdots & C_n^7 \\ C_1^{10} & C_2^{10} & C_3^{10} & C_4^{10} & C_5^{10} & C_6^{10} & \cdots & C_6^3 \end{bmatrix}$$

The signal equation is as below:

$$Y_t = C_0 + C_1 F_t + e_t \quad (11)$$

According to financial theory, interest rate is determined by state variables. The mean value of $e_t$ is 0, and it follows the equation below:

$$\mathrm{var}(e_t) = \mathrm{M}$$

According to (5), we get the state equation below:

$$F_{t+1} = \theta + \emptyset(F_t - \theta) + \varepsilon_{t+1} \quad (12)$$

$\varepsilon_{t+1}$ is the stochastic error of state variable, and its mean value is 0, and its variance is Q. $F_t$ has initial value and initial variance as below:

$$\widehat{F_0} = \theta,\quad \mathrm{vec}(\hat{P}_0) = [I - (\emptyset \times \emptyset)]^{-1} vec(Q)$$

The predicting equation of $\widehat{F}_t$ is below:

$$F_{(t|t-1)} = \theta + \emptyset(F_{t-1} - \theta) \quad (13)$$

The conditional variance of predicting value is below:

$$P_{(t|t-1)} = \emptyset P_{t-h}\acute{\emptyset} + Q \quad (14)$$

$$U_t = Y_t - C_0 - C_1 F_{(t|t-1)}, \quad V_t = C_1 P_{(t|t-1)} \acute{C_1} + M$$

and $U_t$ follows normal distribution, so the likelihood equation is below:

$$-2\ln L_t = ln|V_t| + \acute{U}_t V_t^{-1} U_t \quad (15)$$

The parameters meet the condition below:

$$\min_{(parameters)} \sum_t 2lnL_t$$

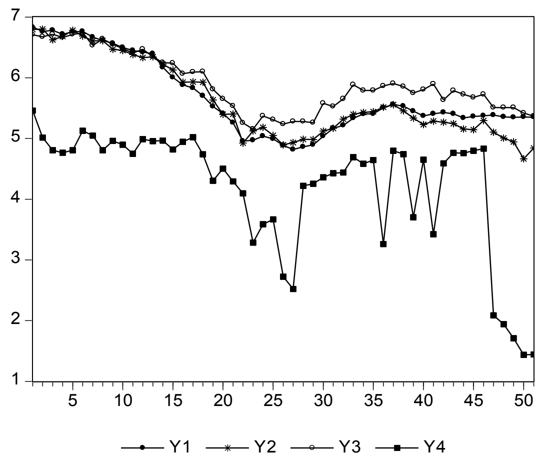
In Kalman filtering analysis,

$$K_t = P_{(t|t-1)} \acute{C_1} V_t^{-1}, \quad L_t = I - K_t C_1$$

Recursive Algorithm is below:

$$\widehat{F}_t = F_{(t|t-1)} + K_t U_t, \quad \widehat{P}_t = L_t P_{(t|t-1)}$$

## 5. Empirical results analysis




Graph1 observable bonds yields

Graph1 indicates corporate bond weekly average yields in Shanghai Exchange and Shenzhen Exchange, and Y1 shows corporate bond yields with 3 years maturity, and Y2 shows corporate bond yields with 5 years maturity, and Y3 shows corporate bond yields with 7 years maturity, and Y4 means corporate bonds yields with 10 years maturity. We can see bonds with short term have higher weekly average yields.

### 5.1 one-factor empirical analysis

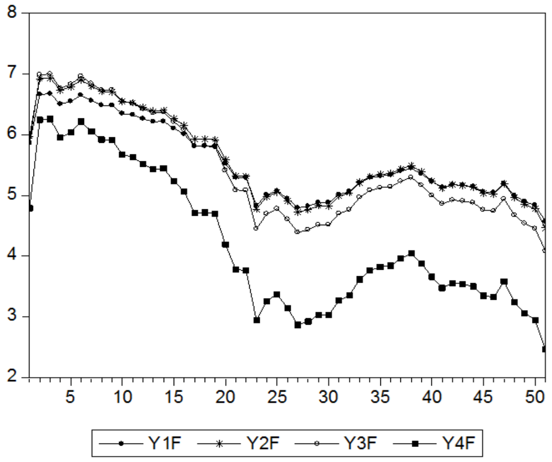
With given initial values of parameters, we get parameters in table2. From table2 we know $a_0$ is significant at 5% level. $\sigma_1$ is significant at 1% confidence level, and it means corporate bond yields fluctuate. $k_1$ is significant at 1% confidence level, and it means bond yields have mean reversion, but they reverse slowly. $\beta_1$ isn't significant at 1%.

Table2 one-factor affine model results

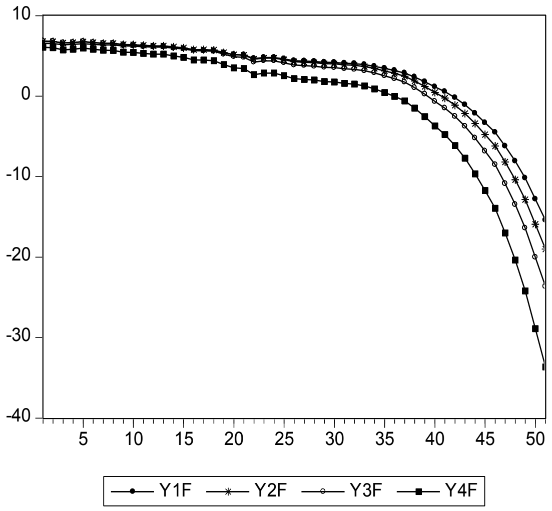
| | **parameters** | **St.d** | **Z** | **Prob.** |
|---|---|---|---|---|
| $\mathbf{a_0}$ | 3.691** | 1.581 | 2.33 | 0.0196 |
| $\sigma_1$ | 0.161*** | 0.036 | 4.50 | 0.0000 |
| $k_1$ | -0.146*** | 0.018 | --8.24 | 0.0000 |
| $\beta_1$ | -0.265 | 0.178 | -1.48 | 0.1377 |

*** denotes statistical variables are significant at 1% confidence level and ** denotes statistical variables are significant at 5% confidence level.

From graph2 we can see, it's one-step forward forecasting of corporate bond weekly average yields in Shanghai Exchange and Shenzhen Exchange. The yields curves in graph2 are similar with the yields curves in graph1, so the model fits one-step forward forecasting well. Graph3 indicates the modeling of real yields in graph1, we can see it can't fit the real curve well. So one-factor Kalman filtering model can't fit real value well.




Graph2 one-step forward forecasting of yields




Graph3 modeling real curve of yields

## 5.2 Two-factor empirical analysis

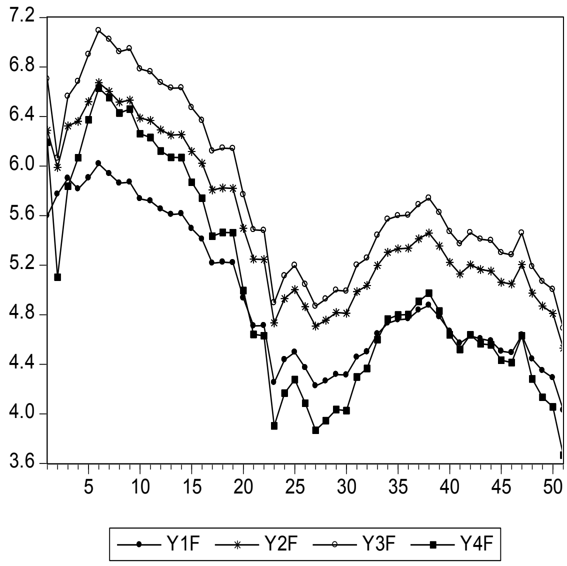
From table3 we can see $a_0$ isn't significant. $\sigma_1$ is significant at 1% confidence level, and $\sigma_2$ is not significant, and we infer may be they represent default risk and liquidity risk. $k_1$ is significant at 1% confidence level, and $k_2$ is not significant. $\beta_1$ is significant at 1% confidence level, also $\beta_2$ is significant at 1% confidence level, so there are risk premium in both state variable 1 and state variable 2.

Table3 two-factor affine model results

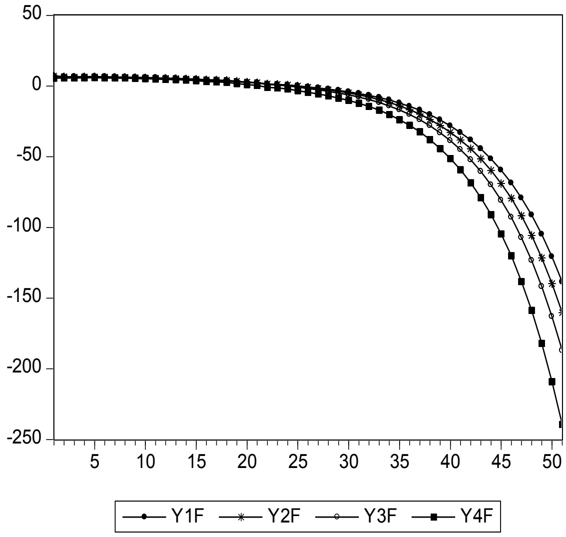
| | parameters | St.d | Z | Prob. |
|---|---|---|---|---|
| **$a_0$** | 1.087 | 8.948 | 0.122 | 0.9033 |
| $\sigma_1$ | 0.182*** | 0.0238 | 7.664 | 0.0000 |
| $k_1$ | -0.215*** | 0.006 | -38.93 | 0.0000 |
| $\beta_1$ | -0.215*** | 0.011 | -19.86 | 0.0000 |
| $\sigma_2$ | 2.330 | 6.890 | 0.338 | 0.7353 |
| $k_2$ | 1.330 | 2.495 | 0.533 | 0.5941 |
| $\beta_2$ | -4.484*** | 1.679 | -2.671 | 0.0076 |

*** denotes statistical variables are significant on the 1% confidence level.

From graph4 we can see it's the one step-forward forecasting of corporate bond weekly average yields in Shanghai Exchange and Shenzhen Exchange, graph4 is similar with graph1, and it means Kalman filtering two-factor model could forecast yields well. Graph5 is modeling the real yields, and we can see graph5 and graph1 is very different, so the two-factor Kalman filtering model can't fit real curve well.




Graph4 two-factor one-step forward forecasting




Graph5 modeling real curve of yields

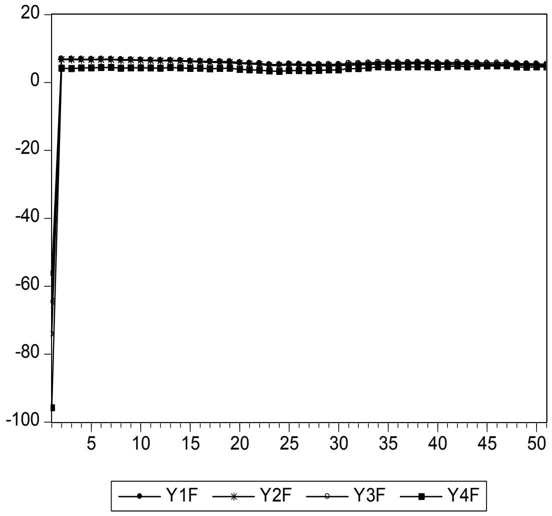
### 5.3 Three-factor empirical analysis

From table4 we can see $a_0$ is significant. $\sigma_1$ is significant at 1% confidence level, and it means the state variable 1 fluctuates with time, and $\sigma_2$ isn't significant, also $\sigma_3$ isn't significant. $k_1$ is significant at 1% confidence level, and it means state variable 1 follows mean reversion, and $k_2$ is significant at 5% confidence level, and it means state variable 2 follows mean reversion, and $k_2 > k_1$, means state variable 2 reverses more quickly than state variable 1, and $k_3$ is significant at 1% confidence level, and means state variable 3 follows mean reversion, but it reverses more slowly than variable2. $\beta_{11}$ is significant at 10% confidence level, and means state variable 1 has time varying risk premium, and $\beta_{22}$ is significant at 5% confidence level, and means state variable 2 has time varying risk premium, also $\beta_{33}$ is significant at 1% confidence level, and it means state variable 3 has time varying risk premium, and state variable 2 has the largest time varying risk premium. $\beta_{10}$ is significant at 10% confidence level, and it means state variable 1 has fixed risk premium, but both $\beta_{20}$ and $\beta_{30}$ aren't significant.

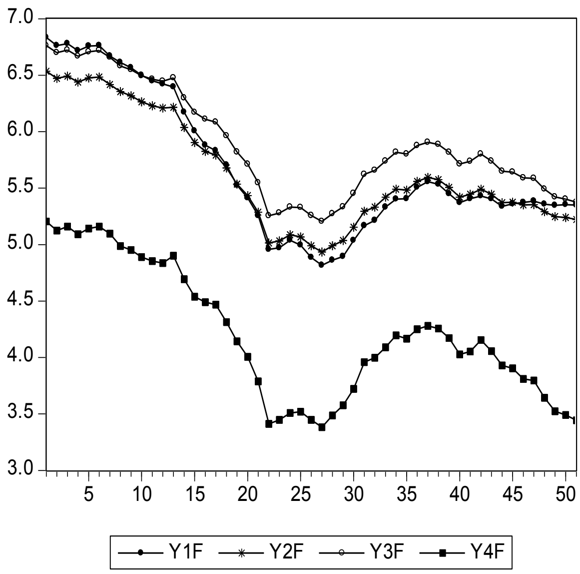
Table4 three-factor affine model results

| | parameters | St.d | Z | Prob. |
|---|---|---|---|---|
| $a_0$ | 3.681 | 24.374 | 0.151 | 0.8800 |
| $\sigma_1$ | 0.152*** | 0.058 | 2.631 | 0.0085 |
| $k_1$ | -0.229*** | 0.072 | -3.182 | 0.0015 |
| $\beta_{11}$ | -0.635* | 0.383 | -1.658 | 0.0974 |
| $\sigma_2$ | -0.764 | 16.977 | -0.045 | 0.9641 |
| $k_2$ | 0.969** | 0.388 | 2.497 | 0.0125 |
| $\beta_{22}$ | 0.865** | 0.434 | 1.992 | 0.0464 |
| $\sigma_3$ | 0.103 | 9.854 | 0.010 | 0.9917 |
| $k_3$ | 0.203*** | 0.054 | 3.735 | 0.0002 |
| $\beta_{33}$ | 0.187*** | 0.056 | 3.367 | 0.0008 |
| $\beta_{10}$ | 0.976* | 0.547 | 1.787 | 0.074 |
| $\beta_{20}$ | 1.162 | 4.954 | 0.234 | 0.815 |
| $\beta_{30}$ | -0.033 | 2.001 | -0.017 | 0.987 |

*** denotes statistical variables are significant at the 1% confidence level. ** denotes statistical variables are significant at the 5% confidence level. * denotes statistical variables are significant at 10% confidence level.




Graph 6 Three-factor one-step forward forecasting




Graph 7 modeling real curve of yields

From graph 6 we can see it's one-step forward forecasting of average weekly corporate bond yields in Shanghai Exchange and Shenzhen Exchange, and it's very different with graph 1, so the forecasting isn't good. Graph 7 is the modeling of real curve, and it's similar with graph 1, so the three-factor model could fit real data well.

## 6. Conclusion

We analyze corporate bond yields term structure in Shanghai Exchange and Shenzhen Exchange by using Kalman filtering model. We build N-factor affine term structure model, and then we use Kalman filtering to estimate the parameters of one-factor model, two-factor model and three-factor model. The results indicate one-factor model and two-factor model could do one-step forward forecasting well, and they have fixed risk premium, but they can't fit the real data well. Three-factor model can't forecast well, but it could fit real data well, and we add the time varying risk premium factor into three-factor model, and find they are all significant, so the three state variables have time varying risk premium. But only state variable 1 has the significant fixed risk premium. And the results are similar with other scholars. I would do further research on corporate bond spread by using Kalman filtering.

*Reference*


[1] Dai Q, Singleton K J, Specification Analysis of Affine Term Structure Models, *The Journal of finance*,Vol. 5, No.4, 2000, pp. 1943-1978.

[2] Vasicek, Oldrich A, An equilibrium characterization of the term structure, *Journal of Financial Economics*, Vol.5, 1977, pp. 177–188.

[3] Cox, John C., Jonathan E. Ingersoll, Stephen A. Ross, A theory of the term structure of interest rates, *Econometrica*, Vol.53, 1985, pp. 385–408.

[4] Duffee R, Term Premia and Interest Rate Forecasts in Affine Models, *The Journal of finance*, Vol.1, No.6, 2002, pp. 405-443.

[5] Duffie, Kan, A yield-factor model of interest rates, *Mathematical Finance*, Vol.4, No.6, 1996, pp. 379-406.

[6] Longstaff A, Schwartz S, A Simple Approach to Valuing Risky Fixed and Floating Rate Debt, *The Journal of finance*, 1995, Vol.3, No.3, pp. 789-819.

[7] Vasileiou A, Vasileiou G, An inhomogeneous semi-markov model for the term structure of credit risk spreads, *Advances in Applied Probability*, Vol.1, No.38, 2006, pp. 171-198.

[8] Lamoureux G, Witte H, Empirical Analysis of the Yield Curve: The Information in the Data Viewed through the Window of Cox, Ingersoll, and Ross, *The Journal of finance*, Vol.3, No.6, 2002, pp. 1479-1520.

[9] Duffee R, Estimating the Price of Default Risk, *The Review of Financial Studies*, Vol.1, No.12, 1999, pp. 197-226.
[10] Duan, Simonato, Estimating and Testing Exponential-Affine Term Structure Models by Kalman Filter, *Review of Quantitative Finance and Accounting*, Vol.13, 1999, pp. 111-135.
[11] Duarte, Evaluating an Alternative Risk Preference in Affine Term Structure Models, *The Review of Financial Studies*, Vol.2, No.17, 2004, pp. 379-404.
[12] Dai Q, Singleton K J, Expectation puzzles, time-varying risk premia, and affine models of the term structure, *Journal of Financial Economics*, Vol.63, 2002, pp. 415-441.
[13] Cheridito, Filipovic, Kimmel L R, Market price of risk specifications for affine models: Theory and evidence, *Journal of Financial Economics*, Vol.83, 2007,pp. 123-170.
[14] Lando, On Cox Processes and Credit Risky Securities, *Review of Derivatives Research*, Vol.2, 1998, pp. 99-120.
[15] Jarrow A R, Turnbull M S, Pricing Derivatives on Financial Securities Subject to Credit Risk, *The Journal of finance*, Vol.1, No.5, 1995, pp. 53-85.
[16] Duffee R G. The Relation Between Treasury Yields and Corporate Bond Yield Spreads [J]. *The Journal of finance*, 1998, 6 (53): 2225-2241.
[17] Carr, Linetsky, Time-changed markov processes in unified credit-equity modeling, *Mathematical Finance*, Vol.20, No.4, 2010, pp. 527–569.
[18] Dai, Singleton, Term Structure Dynamics in Theory and Reality, *The Review of Financial Studies*, Vol.16, No.3, 2003, pp. 631-678.
[19] Duffie, Lando, Term structure of credit spreads with incomplete accounting information, *Econometrica*, Vol.69, No.3, 2001, pp. 633-664.
[20] Schwartz, Smith J E, Short-Term Variations and Long-Term Dynamics in Commodity Prices, *Management Science*, Vol.7, No. 46, 2000, pp.893–911.
[21] Casassus, Collin-Dufresne, Stochastic Convenience Yield Implied from Commodity Futures and Interest Rates, *The Journal of finance*, Vol.5, 2005,pp. 2283-2331.
[22] Jong D F, Time Series and Cross-section information in Affine Term-Structure Models, *Journal of Business & Economic Statistics*, Vol.18, No.3, 2000,pp. 300-314.
[23] K. Chen; H. Lin; T. Huang, The Prediction of Taiwan 10-Year Government Bond Yield, *WSEAS Transactions on Systems*, Vol.8, No. 9, 2009, pp. 1051-60.
[24] F. Neri, Agent Based Modeling Under Partial and Full Knowledge Learning Settings to Simulate Financial Markets, *AI Communications*, Vol.25, No.4, 2012, pp. 295-305.
[25] Fan longzhen, Zhang guoqing, Modeling yield curves in the SSE with two-factor affine and Gaussian essential affine models, *Journal of Industrial*, Vol.19, No.3, 2005, pp. 97-101.
[26] Wang xiaofang, Liu fenggen, Han long, Formatting the term structure curve of interest rates of China's treasury bonds based on cubic spline functions, *Journal of system engineering*, Vol.23, No.6, 2005, pp. 85-89.
[27] Fan longzhen, Modeling the term structure of yields in the SSE with three-factor Gaussian essential affine model, *Journal of Industrial*, Vol. 1, No.19, 2005, pp. 81-86.
[28] Fan longzhen, Modeling the term structure of yields in the SSE with two-factor Vasicek mode, *Journal of FUDAN University*, Vol. 42, No.5, 2003, pp. 773-778.
[29] F. Neri, Learning and Predicting Financial Time Series by Combining Evolutionary Computation and Agent Simulationm, *Applications of Evolutionary Computation*, LNCS 6625, 2011, pp. 111-119.
[30] F. Neri, Quantitative estimation of market sentiment: A discussion of two alternatives, *WSEAS Transactions on Systems*, Vol.11, No.12, 2012, pp. 691-702.
[31] S S Wang, J M huang, Idiosyncratic volatility has an impact on corporate bond spreads: Empirical evidence from Chinese bond markets, *WSEAS Transactions on Systems*, Vol.12, No.5, 2013, pp. 280-289.