New online RKPCA-RN Kernel method Applied to Tennessee Eastman Process

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Abstract: -This paper proposes a new method for online identification of a nonlinear system using RKHS models. The RKHS model is a linear combination of kernel functions applied to the used training set observations. For large datasets, this kernel based to severs computational problems and makes identification techniques unsuitable to the online case. For instance, in the KPCA scheme the Gram matrix order grows with the number of training observations and its eigen decomposition. The proposed method is based on Reduced Kernel Principal Component Analysis technique (RKPCA), to extract the principal component will be time consuming.

Key-Words: - *RKHS, RN, SLT, Kernel method, RKPCA, Online RKPCA-RN, Tennessee.*

1 Introduction

The last few years have registered the birth of a new modelling method of nonlinear systems using Reproducing Kernel Hilbert Space (RKHS) [1], [7], [15], [16], [17], [21], [22] called kernel methods. These methods have been applied to a large class of problems, such as face recognition [12], [24], time series prediction [4], identification of nonlinear system [11], [14], [23], …. The proposed method proceeds in two steps, first in an off line phase we select a set of kernels functions using the RKPCA technique, then in the online phase a RKHS model is constructed and successively updated.

An eigen decomposition of Gram matrix can simply become too time-consuming to extract the principal components and therefore the system parameter identification becomes a tough task. To overcome this burden recently a theoretical foundation for online learning algorithm with kernel method in reproducing kernel Hilbert spaces was proposed [8], [10]. When the system to be identified is time varying, the online kernel algorithm is more useful, because these algorithms can automatically track changes of system Behaviour with time-varying and time lagging characteristic.

In this paper we propose a new method for online identification of a non linear system parameters modeled on Reproducing Kernel Hilbert Space (RKHS). This method uses the Reduced Kernel Principal Component Analysis (RKPCA) that selects the observations data to approach the Principal Components Analysis [11]. The selected observations are used to build an RKHS model with a reduced parameter number. The numbers of reduced observations are fixed and we actualise the parameters on minimizing a criterion. The proposed technique may be very helpful to design an adaptive control strategy of non linear systems.

The paper is organized as follows. In section 2, we remind the Reproducing Kernel Hilbert Space (RKHS). In section 3, we remind the Reduced Kernel Principal Component Analysis RKPCA method. In section 4, we propose the new online RKPCA-RN method. The proposed algorithm has been tested to identify the nonlinear system and a Tennessee Eastman process.

2 Reproducing Kernel Hilbert Space

Let $X \subset \mathbb{R}^d$ an input space and $L^2(X)$ the Hilbert space of square integrable functions defined on X. Let $k: X^2 \to \mathbb{R}$ be a continuous positive definite kernel. It is proved [18] that it exists a sequence of an orthonormal eigen functions $(\psi_1, \psi_2, ..., \psi_l)$ in $L^2(X)$ and a sequence of corresponding real positive eigenvalues $(\sigma_1, \sigma_2, ..., \sigma_l)$ (where l can be infinite) so that:

$$k(x, t) = \sum_{j=1}^{l} \sigma_j \psi_j(x) \psi_j(t) \quad ; \quad x, t \in X \tag{1}$$

Let $H \subset L^2(X)$ be a Hilbert space associated to the kernel k and defined by:

$$H = \left\{ f \in L^2(X) / f = \sum_{i=1}^{l} w_i \varphi_i \ \text{ and } \ \sum_{j=1}^{l} \frac{w_j^2}{\sigma_j} < +\infty \right\} \tag{2}$$

Where $\varphi_i = \sqrt{\sigma_i}\, \psi_i \ \ i = 1, ..., l$. The scalar product in the space H is given by:

$$< f, g >_H = \ < \sum_{i=1}^{l} w_i \varphi_i, \sum_{j=1}^{l} z_j \varphi_j >_H \ = \sum_{i=1}^{l} w_i z_i \tag{3}$$

K is a Reproduisant Kernel for the space H if

* $\forall x \in X$, the function k_x such as

$$\begin{aligned} k_x &: X \to \mathbb{R} \\ t &\mapsto k_x(t) = k(x, t) \end{aligned} \tag{4}$$

is a function of the space H.

* $\forall x \in X \ ; \ \forall f \in H \ \ \langle f, k_x \rangle_H = f(x)$ (5)

H is a Reproducing Kernel Hilbert Space (RKHS) for the kernel k.

The relation (5) describes the property of reproducibility of the function for the space H [1]. In other words the scalar product between each function $f \in H$ and the function k_x enables to determine $f(x)$.

Let's define the application Φ :

$$\begin{aligned} \Phi &: X \to IR^l \\ x &\mapsto \Phi(x) = \begin{pmatrix} \varphi_1(x) \\ \vdots \\ \varphi_l(x) \end{pmatrix} \end{aligned} \tag{6}$$

Where φ_i are given in (2).

The kernel trick [1] is so that:

$$k(x, x') = \langle \Phi(x), \Phi(x') \rangle \quad x, x' \in X \tag{7}$$

Vapnik [18] proposes to adopt the (Structural Risk Minimisation: SRM)

$$D(f) = \frac{1}{N} \sum_{i=1}^{N} V(y_i, f(x_i)) + \lambda \| f \|_H^2 \tag{8}$$

Based on the representer theorem [19] the optimal function f_{opt} which minimizes $D(f)$ can be written as:

$$f_{opt}(x) = \sum_{i=1}^{N} a_i \ k(x_i, x) \tag{9}$$

Where $a_i, \ i = 1, ..., N$ are the model parameters.

3 RKPCA method

Let a nonlinear system with an input $u \in \mathbb{R}$ and an output $y \in \mathbb{R}$ from which we extract a set of observations $\{u_i, y_i\}_{i=1, ..., N}$. Let H an RKHS space with kernel k. To build the input vector x_i of the RKHS model we use the NARX (Nonlinear auto regressive with eXogeneous input) structure as:

$$x_i = \left\{ u_i, ..., u_{i-m_u}, y_{i-1}, ..., y_{i-m_y} \right\}^T \ ; \ m_u, m_y \in \mathbb{N} \tag{10}$$

The set of observations becomes $D = \{x_i, y_i\}_{i=1, ..., N}$ where $x_i \in \mathbb{R}^{m_u + m_y + 1}$ and $y_i \in \mathbb{R}$.

and the RKHS model of this system based on (9) can be written as:

$$\tilde{y}_j = \sum_{i=1}^{N} a_i \, k(x_i, x_j) \tag{11}$$

Let the application Φ :

$$\Phi : X \to \mathbb{R}^l$$

$$x \mapsto \Phi(x) = \begin{pmatrix} \varphi_1(x) \\ . \\ . \\ . \\ \varphi_l(x) \end{pmatrix} \quad (12)$$

Where φ_i are given in (13).

The Gram matrix K associated to the kernel k is an N - dimensional square matrix, so that:

$$k_{i,j} = k(x_i, x_j) \text{ for } i, j = 1, ..., N \quad (13)$$

The kernel trick [1] is so that:

$$< \Phi(x), \Phi(x') > = k(x, x') \; x, x' \in X \quad (14)$$

We assume that the transformed data $\{\Phi(x_i)\}_{i=1, ..., N} \in \mathbb{R}^l$ are centered [11]. The empirical covariance matrix of the transformed data is symmetrical and l - dimensional. It is written as following:

$$C_\phi = \frac{1}{M} \sum_{i=1}^{M} \Phi(x_i) \, \Phi(x_i)^T, \; C_\phi \in \mathbb{R}^{l \times l} \quad (15)$$

Let l' the number of the eigenvectors $\{V_j\}_{j=1, ..., l'}$ of the C_ϕ matrix that corresponding to the non zeros positive eigenvalues $\{\lambda_j\}_{j=1, ..., l'}$. It is proved in [11] that the number l' is less or equal to N.

Due to the large size l of C_ϕ, the calculus of $\{V_j\}_{j=1, ..., l'}$ can be difficult. The KPCA method shows that these $\{V_j\}_{j=1, ..., l'}$ are related to the eigenvectors $\{\beta_j\}_{j=1, ..., l'}$ of the gram matrix K according to [11]:

$$V_j = \sum_{i=1}^{N} \beta_{j,i} \; \Phi(x_i) \;\; , \;\; j = 1, ..., l' \quad (16)$$

Where $(\beta_{j,i})_{j=1, ..., P}$ are the components of $\{\beta_j\}_{j=1,...,l'}$ associated to their nonzero eigenvalues $\mu_1 > ... > \mu_{l'}$.

The principle of the KPCA method consists in organizing the eigenvectors $\{\beta_j\}_{j=1, ..., l'}$ in the decreasing order of their corresponding eigenvalues $\{\mu_j\}_{j=1, ..., l'}$. The principal components are the P first vectors $\{V_j\}_{j=1, ..., P}$ associated to the highest eigenvalues and are often sufficient to describe the structure of the data [11]. The number P satisfies the Inertia Percentage criterion IPC given by:

$$P^* = \arg(IPC \geq 99) \quad (17)$$

Where

$$IPC = \frac{\sum_{i=1}^{P} \mu_i}{\sum_{i=1}^{M} \mu_i} * 100 \quad (18)$$

The RKHS model provided by the KPCA method is [1].

$$\tilde{y}_{new} = \sum_{q=1}^{P} w_q \; \sum_{i=1}^{N} \beta_{q,i} \; k(x_i, x_{new}) \quad (19)$$

Since the principal components are a linear combination of the transformed input data $\{\Phi(x_i)\}_{i=1, ..., N}$ [12], the Reduced KPCA approaches each vector $\{V_j\}_{j=1,...,P}$ by a transformed input data $\Phi(x_j^b) \in \{\Phi(x_i)\}_{i=1, ..., N}$ having a high projection value in the direction of V_j [5].

The projection of the $\Phi(x_i)$ on the V_j called $\tilde{\Phi}(x_i)_j \in \mathbb{R}$ and can be written as:

$$\tilde{\Phi}(x_i)_j = \langle V_j, \Phi(x_i) \rangle; \quad j = 1, ..., P \quad (20)$$

According to (18) and (16), the relation (22) is written:

$$\tilde{\Phi}(x_i)_j = \sum_{m=1}^{N} \beta_{j,m} \; k(x_m, x_i); \quad j = 1, ..., P \quad (21)$$

To select the vectors $\{\Phi(x_i^b)\}$, we project all the $\{\Phi(x_i)\}_{i=1,\ldots,N}$ vectors on each principal component $\{V_j\}_{j=1,\ldots,P}$ and we retained $x_j^b \in \{x_i\}_{i=1,\ldots,N}$ that satisfies

$$\begin{cases} \Phi(x_j^b)_j = \underset{i=1,\ldots,N}{Max}\ \tilde{\Phi}(x_i)_j \\ and \\ \Phi(x_j^b)_{i\neq j} < \zeta \end{cases} \quad (22)$$

Where ζ is a given threshold.

Once the $\{x_j^b\}_{j=1,\ldots,P}$ corresponding to the P principal component $\{V_j\}_{j=1,\ldots,P}$ are determined, we transform the vector $\Phi(x)\in\mathbb{R}^l$ to the $\hat{\Phi}(x)\in\mathbb{R}^P$ vector that belongs to the space generated by $\{\Phi(x_j^b)\}_{j=1,\ldots,P}$ and the proposed reduced model is:

$$\tilde{y}_{new} = \sum_{j=1}^{P} \hat{a}_j\ \hat{\Phi}(x_{new})_j \quad (23)$$

Where :

$$\hat{\Phi}(x_{new})_j = \langle \Phi(x_j^b), \Phi(x_{new}) \rangle; \qquad j = 1, \ldots, P \quad (24)$$

And according to the kernel trick (7), the model (23) is:

$$\tilde{y}_{new} = \sum_{j=1}^{P} \hat{a}_j\ k_j(x_{new}) \quad (25)$$

Where:

$$k_j(x) = k(x_j^b, x) \quad for\ j = 1, \ldots, P \quad (26)$$

The model (23) is less complicate than that provided by the KPCA. The identification problem can be formulated as a minimization of the regularized least square written as:

$$J_r(\hat{a}) = \frac{1}{2}\sum_{i=1}^{N}\left(y_i - \sum_{j=1}^{P} \hat{a}_j\ k_j(x_i)\right)^2 + \frac{\rho}{2}\|\hat{a}\|^2 \quad (27)$$

Where: ρ is a regularization parameter and $\hat{a} = (\hat{a}_1, \ldots, \hat{a}_P)^T$ is the parameter estimate vector
The solution of the problem (27) is:

$$\hat{a}^* = (F + \rho I_P)^{-1} G \quad (28)$$

With :

$$G = \begin{pmatrix} \sum_{i=1}^{N} k_1(x_i)\, y_i \\ \vdots \\ \sum_{i=1}^{N} k_b(x_i)\, y_i \end{pmatrix} \in \mathbb{R}^P$$

and

$$F = \begin{pmatrix} \sum_{i=1}^{N} k_1(x_i)k_1(x_i) & \cdots & \sum_{i=1}^{N} k_1(x_i)k_b(x_i) \\ \vdots & & \\ \sum_{i=1}^{N} k_b(x_i)k_1(x_i) & \cdots & \sum_{i=1}^{N} k_b(x_i)k_b(x_i) \end{pmatrix} \in \mathbb{R}^{P\times P}$$

And $I_P \in \mathbb{R}^{P\times P}$ is the P identity matrix

The RKPCA algorithm is summarised by the five following steps:

1. Determine the nonzero eigenvalues $\{\mu_j\}_{j=1,\ldots,l'}$ and the eigenvectors $\{\beta_j\}_{j=1,\ldots,l'}$ of Gram matrix K .
2. Order the $\{\beta_j\}_{j=1,\ldots,l'}$ on the decreasing way with respect to the corresponding eigenvalues.
3. For the P retained principal components, choose the $\{(x_j^b)\}_{j=1,\ldots,P}$ that satisfy (22).
4. Solving (29) to determine $\hat{a}^* \in \mathbb{R}^P$
5. The reduced RKHS model is given by (2)

4 Online RKPCA-RN method

Prior to the online identification, we start with offline identification on a set of observations $D=\{(x_1, y_1), \ldots, (x_n, y_n)\}$.

In this offline phase we apply the RKPCA technique to reduce the number of the parameters of the RKHS model. The model provided by the RKPCA is given by:

$$\tilde{y}(x)=\sum_{j=1}^{P} a_j\ k\left(x_j^b, x\right) \tag{29}$$

The on line identification phase begins from the instant $n+1$. At this instant a new couple of observations is available $\{x_{n+1}, y_{n+1}\}$.

From the equation (29), we calculate the output RKHS model given by:

$$\tilde{y}_{n+1}=\sum_{j=1}^{P} a_j\ k\left(x_j^b, x_{n+1}\right) \tag{30}$$

The error between the estimated output and the measured on actual one is:

$$e_{n+1}=\left(\tilde{y}_{n+1}-y_{n+1}\right) \tag{31}$$

If

$$\left|e_{n+1}\right|<\varepsilon_1 \tag{32}$$

where ε_1 is a given threshold, we can say the model approaches sufficiently the system behavior. If not, we update the parameters $\{a_j\}$ by minimizing the criterion $J_{r,n+1}$

$$\begin{aligned} J_{r,n+1}(A_{n+1})=&\frac{1}{2}\sum_{i=1}^{P}\left(y_i^b-\sum_{j=1}^{P} a_j\ k\left(x_j^b, x_n\right)\right)^2+\\ &\frac{1}{2}\left(y_{n+1}-\sum_{j=1}^{P} a_j\ k\left(x_j^b, x_n\right)\right)^2+\frac{\rho}{2}\left\|A_{n+1}\right\|^2 \end{aligned} \tag{33}$$

Where

$$A_{n+1}=\begin{pmatrix} a_1 \\ \vdots \\ a_P \end{pmatrix}\in IR^P$$

We can write

$$J_{r,n+1}\left(A_{n+1}\right)=\frac{1}{2}\left(\left\|K_{(n+1)P}\ A_{n+1}-Y_{n+1}\right\|^2+\rho\left\|A_{n+1}\right\|^2\right)$$

Where

$$K_{(n+1)P}=\begin{pmatrix} k\left(x_1^b, x_1\right) & \ldots & k\left(x_P^b, x_1\right) \\ \vdots & \ldots & \vdots \\ k\left(x_1^b, x_P\right) & \ldots & k\left(x_P^b, x_P\right) \\ k\left(x_1^b, x_{n+1}\right) & \ldots & k\left(x_P^b, x_{n+1}\right) \end{pmatrix}\in IR^{(P+1)\times P} \quad \text{and}$$

$$Y_{n+1}=\begin{pmatrix} y_1^b \\ \vdots \\ y_P^b \\ y_{n+1} \end{pmatrix}\in IR^{P+1} \tag{34}$$

The criterion (33) can be written as the following :

$$\min_{A_{n+1}\in IR^P} J_{r,n+1}(A_{n+1})=$$

$$\begin{aligned} &\min_{A_{n+1}\in IR^P}\frac{1}{2}\left(\left\|K_{(n+1)P}\ A_{n+1}-Y_{n+1}\right\|^2+\rho\left\|A_{n+1}\right\|^2\right)\\ &=\min_{A_{n+1}\in IR^P}\left(\frac{1}{2}\ \left(K_{(n+1)P}\ A_{n+1}-Y_{n+1}\right)^T\left(K_{(n+1)P}\ A_{n+1}-Y_{n+1}\right)+\frac{\rho}{2}{A_{n+1}}^T A_{n+1}\right) \end{aligned} \tag{35}$$

The minimum of $J_{r,n+1}$ is reached for :

$$\frac{\partial J_{r,n+1}}{\partial A_{n+1}}=0$$

$$\frac{\partial J_{r,n+1}}{\partial A_{n+1}}=$$

$$\begin{aligned} &\frac{\partial}{\partial A}\left[\frac{1}{2}\left({A_{n+1}}^T K_{(n+1)P}^T-{Y_{n+1}}^T\right)\left(K_{(n+1)P}.A_{n+1}-Y_{n+1}\right)+\frac{\rho}{2}{A_{n+1}}^T A_{n+1}\right]\\ &=\frac{1}{2}\frac{\partial}{\partial A_{n+1}}\left[{A_{n+1}}^T K_{(n+1)P}^T.K_{(n+1)P}.A_{n+1}-{A_{n+1}}^T K_{(n+1)P}^T.Y_{n+1}\right.\\ &\left.-{Y_{n+1}}^T.K_{(n+1)P}.A_{n+1}-{Y_{n+1}}^T Y_{n+1}+\rho{A_{n+1}}^T A_{n+1}\right] \end{aligned} \tag{36}$$

As ${A_{n+1}}^T K_{(n+1)P}.Y_{n+1}$ et ${Y_{n+1}}^T.K_{(n+1)P}.A_{n+1}$ are scalar and transposed, then

$$\frac{\partial J_{r,n+1}}{\partial A_{n+1}} = \frac{\partial}{\partial A_{n+1}}\Big[{A_{n+1}}^T K_{(n+1)P}^T K_{(n+1)P}.A_{n+1} - 2.{Y_{n+1}}^T.K_{(n+1)P}.A_{n+1} - {Y_{n+1}}^T Y_{n+1} + \rho {A_{n+1}}^T A_{n+1}\Big] \quad (37)$$

Therefore

$$\frac{\partial J_{r,n+1}}{\partial A_{n+1}} = K_{(n+1)P}^T K_{(n+1)P} A_{n+1} - K_{(n+1)P}^T Y_{n+1} + \rho A_{n+1}$$

$$= \left(K_{(n+1)P}^T K_{(n+1)P} + \rho I_P\right) A_{n+1} - K_{(n+1)P}^T Y_{n+1}$$

Then

$$A_{n+1} = \left(K_{(n+1)P}^T K_{(n+1)P} + \rho I_P\right)^{-1} K_{(n+1)P}^T Y_{n+1} \quad (38)$$

To every new observation, we apdate the parameters of the model (30) using the relation (38).

Online RKPCA- NR algorithm

Offline phase:

According to (17) and (18) we determine the P retained principal components resulting from the processing of an N measurement set.

1- Then we determine the $I = \{x_j^b\}_{j=1,\ldots,P}$ set according to (22), used during the online phase.

2- Write RKHS model obtained by RKPCA

$$\tilde{y}_n = \sum_{j=1}^{P} a_j \; k\left(x_j^b, x_n\right)$$

Online phase:

For a new couple of observation $\{x_{n+1}, y_{n+1}\}$

1 - Calculate the output of the model RKHS

$$\tilde{y}_{n+1} = \sum_{j=1}^{P} a_j \; k\left(x_j^b, x_{n+1}\right)$$

2- Calculate the value of e_{n+1}.

3- If the condition (32) is satisfied $\Rightarrow$ we comes back to 1 for a new observation $\{x_{n+2}, y_{n+2}\}$

Otherwise:

- Estimate the parameters $\{a_j\}$ $j = 1, \ldots, P$ using the relation (38).

5 Simulations

The proposed method has been tested for modelling a nonlinear system and a Tennessee Eastman process.

5. 1 System nonlinear

We consider the nonlinear system

$$y(i) = \log\left(\left|u(i-1)^2 - u(i-2) + 0.6\; y(i-2)\right| + 0.4\; y(i-1) + 1\right) + e(i) \quad (39)$$

Where $e(i)$ is a gaussian noise. The input vector of RKHS model has the structure

$$x(i) = [\, u(i-1), u(i-2),\; y(i-1)]^T$$

$u(i)$ is the process input chosen as gaussian signal. To build the RKHS model we use the ERBF Kernel (Exponential Radial Basis Function)

$$k(x, x') = \exp\left(-\sqrt{\frac{\|x - x'\|}{\mu}}\right) \quad (40)$$

With $\mu = 5$ and $\|\ \|$ is the euclidean norm. The term of regularisation $\lambda = 10^{-6}$

The chosen threshold is: $\varepsilon_1 = 0.04$

We performed the online identification using the online RKPCA-RN algorithm developed in section 4.

The number of observations in identification offline phase is 150 and the number of principal component analysis obtained by RKPCA method is equal to 5.

The number of observations in online identification phase is 300.

The minimal normalized mean square error between real output and estimated one (NMSE) is defined.

$$NMSE = \frac{\sum_{i=1}^{N} (\tilde{y}_i - y_i)^2}{\sum_{i=1}^{N} (y_i)^2} \quad (41)$$

Where $\tilde{y}_i$ and y_i design the model RKHS output and the system output respectively.

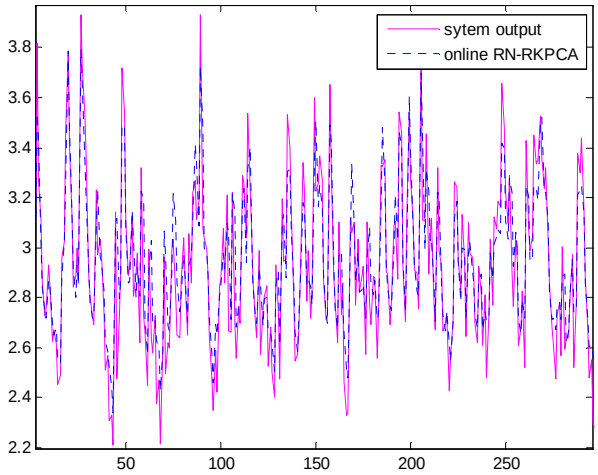

Figure 1: System and model outputs during the online Identification

In Figure 1, we represent the online RKPCA-RN output as well as the system output. We remark that the model output is in concordance with the system output, indeed the Normalized mean Square Error is equal to $0,002$. This shows the good performances of the proposed online identification method.

To evaluate the performance of the proposed method we plot in Figure 2, the evolution of the NMSE.

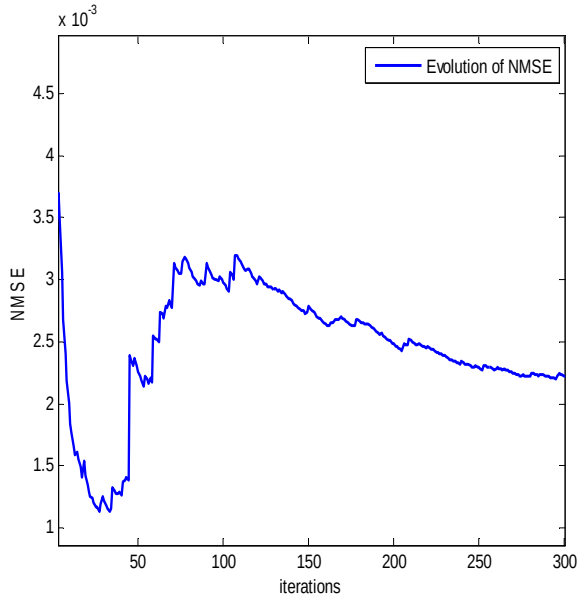

Figure 2: Evolution of NMSE

5. 2 Tennessee Process

To illustrate the efficiency of the proposed model we proceed to its validation on Tennessee Eastman process.

5. 2. 1 Process description

The Tennessee Eastman (TE) process [9] is a highly non linear, non-minimum phase, and open-loop unstable chemical process consisting of a reactor/separator/recycle arrangement. This process produces two products G and H from four reactants A, C, D and E. Also a byproduct F is present in the process. The simultaneous, irreversible and exothermic gas-liquid reactions are:

$$A(g)+C(g)+D(g)\rightarrow G(\text{lig}),\text{Product 1}$$
$$A(g)+C(g)+E(g)\rightarrow G(\text{lig}),\text{Product 2}$$
$$A(g)+E(g)\rightarrow F(\text{lig}),\text{byroduct}$$
$$3D(g)\rightarrow 2F(\text{lig}),\text{byroduct}$$

The process has 12 valves available for manipulation and 41 measurements available for monitoring or control. The detailed description of these variables, process disturbances and base case operating conditions, is given in [3]. The process flowsheet is presented in Figure 3

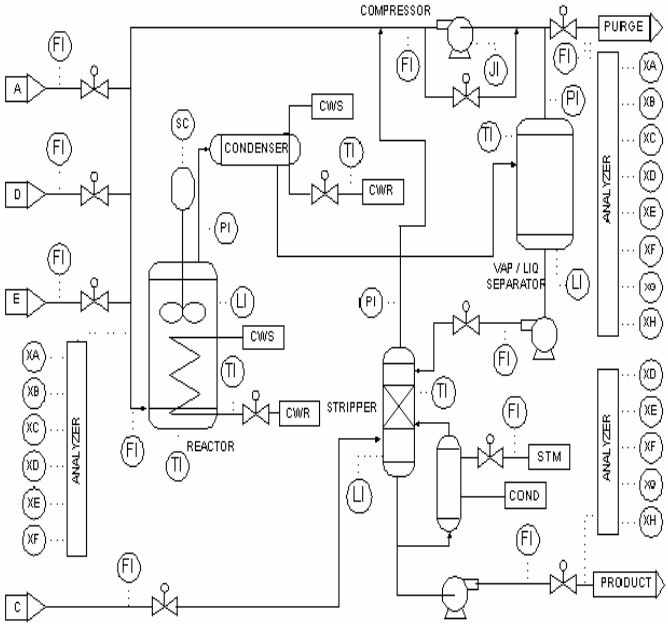

Figure 3: Tennessee Process

The modelling and the identification of the Tennessee Eastman process represent a challenge for the control community. It has been the subject of several studies [2] but most of them have tackled the process control without giving importance to modeling step. [13] have used input/ output process data to identify an autoregressive (AR) model parameters.

In our paper we intend to identify the parameters of the corresponding RKPCA- RN method of this process using the same technique of [20] for generating the data.

5. 2. 2 Data extraction

The input/output data used to built the model were generated from the model of Tennessee implemented for the program Matlab © in the toolbox Simulink [9]. The process has 12 inputs and 41 outputs. According to the work of the process was divided into two fields. The first with a PID controller to maintain the process stability. The second field is devoted to the identification where only four inputs (reactor pressure, reactor level, D feed flow and E feed flow) are tuned and the others are maintained as suggested by mode 3 of Simulink model. Assuming the reactor outputs, we select the separator temperature product.

5. 2. 3 Knowledge model of Tennessee Process

In this section, we consider for the knowledge model of the Tennessee process that suggested by [9] as shown by Figure 4.

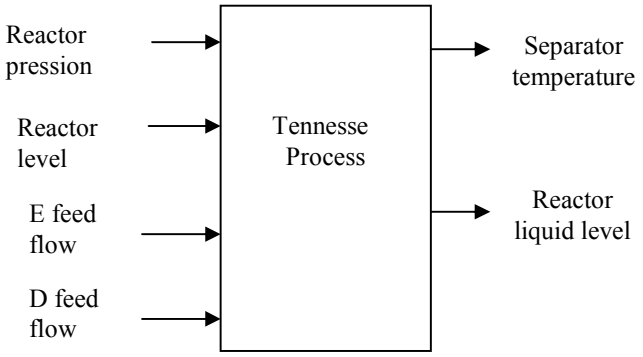

Figure 4: Sheme input / output of Tennessee process

* Output equations:

The first output, separator temperature is given by:

$$T_S = \left(T_{CW,S,out} - T_{CW,S,in}\right) \quad (42)$$

Where:

$T_{CW,S,out}$ Cooling water outlet temperature in the separator

$T_{CW,S,\text{in}}$ Cooling water inlet temperature in the separator

The temperature T_S is linked to the energy Q_S removed for the separator by the differential equation:

$$\dot{Q}_S = m_{CW,S} c_{p,CW} \left(T_{CW,S,out} - T_{CW,S,in}\right) \quad (43)$$

Where:

$C_{p,CW}$ Specific heat capacity cooling water, kJ kg^{-1} K^{-1}

$m_{CW,S}$ Cooling water flow rate separator, kg h^{-1}

For the second output, the reactor liquid level is given by:

$$V_{Lr} = \sum \frac{N_{i,r}}{\rho_i} \; ; i = D, E, F, G, H \quad (44)$$

Where: ρ_i Molar density of component i, mol m^{-3} , $N_{i,r}$ is the total molar holdup of the component i in the reactor.

5. 2. 4 Modelling in RKHS

To generate the data from simulink, the simulation step size was 0.0005 s and the data were collected every 0.02 s.

To build the RKHS model we use the Kernel RBF (Radial Basis Function)

$$K(x, x') = \exp\left(-\frac{\|x - x'\|}{\mu}\right) \quad (45)$$

Where $\mu = 120$, and $\| \; \|$ is the euclidean norm. The term of regularisation $\lambda = 10^{-6}$

The chosen threshold are:
$\varepsilon_1 = 0.01$

The number of observations in phase of identification offline is 300 and the number of principal component analysis obtained by RKPCA method is equal to 10.

The number of observations in online identification phase is 5000.

In Figure 5, we represent the online RKPCA-RN output as well as the Tennessee Eastman output. We remark that the model output is in concordance with the system output, indeed the Normalized mean Square Error is equal to $5,6310^{-6}$. This shows the good performances of the proposed online identification method.

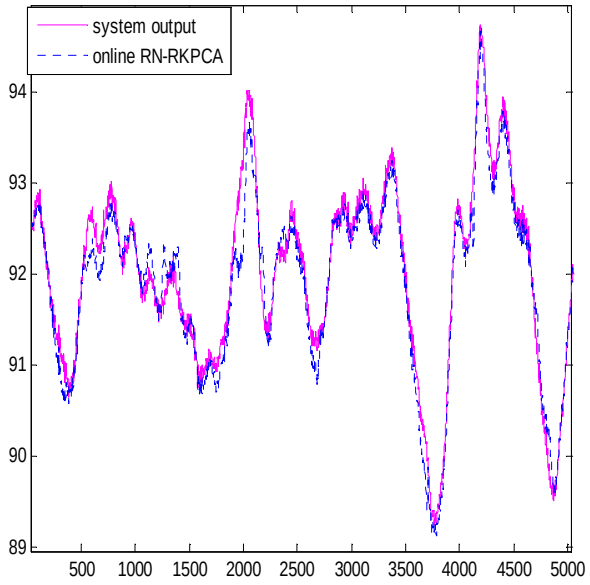

Figure 5: Validation phase

To evaluate the performance of the proposed method we plot in Figure 6, the evolution of the NMSE.

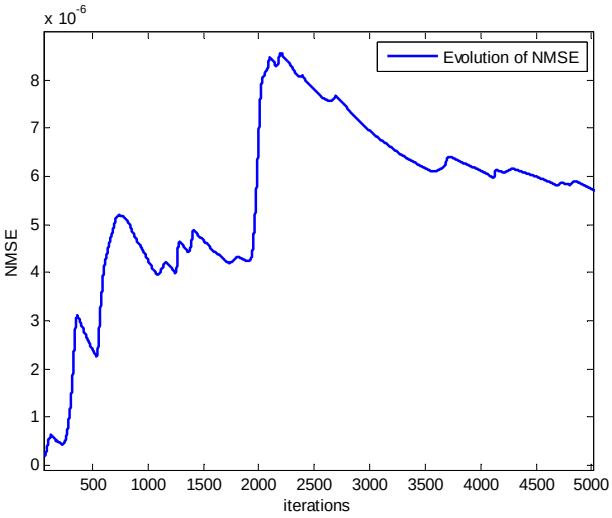

Figure 6: Evolution of NMSE

6 Conclusion

In this paper, we have proposed an online reduced kernel principal component analysis method for nonlinear system parameter identification. Through several experiments, we showed the accuracy and good scaling properties of the proposed method. This algorithm has been tested for identifying a nonlinear system and a Tennessee Eastman process and the results were satisfactying. The proposed technique may be very helpful to design an adaptive control strategy of non linear systems.

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