The Valuation of Futures Options for Emissions Allowances under the Term Structure of Stochastic Multi-factors

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Abstract: - We develop the general model of the futures options valuation under the term structure of stochastic multi factors. Our model analysis suggest the futures options function carry information about the volatility and adjustment speed of arbitrary multi factors, the correlation among multi-factors, and the time to maturity of futures and options contract. We provide numerical examples compare their difference between the actual and theoretical futures options valuation. Our empirical results show the term structure of arbitrary multi factors has significant effect on the futures options valuation for CO_2 emissions allowances, we can estimate the theoretical futures options valuation by using historical market information. We can determine the reasonable options price for CO_2 emissions allowances and then make right options trading policy in the emissions allowances options markets.

Keywords: -emissions allowances; term structure; stochastic multi-factor; futures pricing; kalman filter; volatility; futures options pricing;

1 Introduction

Compared with the other physical commodities markets, emission allowances markets are becoming the most promising and liquid within the EU emissions trading scheme (EU ETS). According to research report on state and trend of carbon market in 2011 by the World Bank, after five consecutive years of robust growth, the total value of the global emissions allowances markets stalled at $142 billion in 2011[1]. Emissions allowances rights are given specific property, the producers owned excess emissions allowances quotas can obtain additional economical benefits by selling emissions allowances quotas. Accordingly emissions allowances rights are similar with other physical commodities, they are also significantly valuable assets for the producers, investors, hedgers and other market practitioners.

The stochastic behaviours of commodity price always play a significant role in the model of futures pricing and options valuation for physical commodities. Assumed by the constant convenience yields and interest rate, early studies developed one-factor model of futures pricing and options pricing to the valuation and hedging of the physical commodities. The assumption can fit approximate valuation of the physical commodity since it implies the volatility of the futures price is the same as the volatility of spot price. Gibson and Schwartz develop two-factor model of the commodity price, where the spot prices and convenience yields of the commodity are state variable, and the convenience yields is assumed to follow mean-reverting O-U process [2]. Schwartz [3], Miltersen and Schwartz [4] propose three-factor model of the commodity price, where the third instantaneous interest rate is also assumed to follow mean-reversion process. Manoliu and Tompaidis [5], Cortarzar and Naranjo [6] and Wang et al. [7] present stochastic N-factor affine model of term structure for futures price, accordingly the commodities' futures prices of are composed of multi random non-observable state variables.

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Chang et al. propose a new N-factor affine model for CO_2 futures price and estimate parameters in the new affine model by using the Kalman filter technique, and their empirical results show that CO_2 futures price follow significant mean-reversion process [8]. Nikolaos present that effective decisions on financial management and corporate management can be supported by hybrid systems, recurrent neural networks and genetic algorithms optimize the results of significant precision [9]. Neri discuss learning predictive models for financial time series by using agent based simulations, his empirical evidence show the effectiveness of our learning simulation system and then adjust its parameters by learning system [10].

The implication of options concepts to value physical commodities has been significant topics in the theory and practice of finance. Futures pricing and futures options has been especially successful implications in the valuation and hedging of the commodities. An option of futures contract provides its holders with the right to buy or sell the underlying assets at the exercise price of the options [11]. Schwartz [12], Hilliard and Reis [13] propose the futures option prices carry information about the spot price volatility, the convenience yields volatility, the correlation and the adjustment factor between spot price and convenience yields. Wang compare the term structure relationship with implied volatilities for the underlying exchange rates, short maturity options overreact to the dynamics currency assets, and long maturity options overreact to short maturity options [14]. Kuo and Lin present multi-factor term structure models for pricing and hedging Eurodollar futures options, and compare between one, two, three-factor model in the HJM class and Black's implied volatility function. Accordingly the arbitrary spot prices, convenience yields and interest rate have a significant impact on the options value of futures contracts [15].

Benz and Truck propose the price of emission allowances is directly determined by the expected market scarcity induced by the current demand and supply, empirical results show the prices of spot and futures are strong similarity of time-varying trend[16]. Benz and Truck analyze the short-term spot price behaviour of carbon dioxide (CO_2) emission allowances in the new EU-wide CO_2 emissions trading system (EU ETS) [17]. Botterud et al. show the relationship between spot and futures prices is clearly linked to the physical state of the system, such as hydro inflow, reservoir levels, and demand [18]. Chevallier find the risk premium in CO_2 spot and futures prices exhibit time-varying trend and positive relationship between risk premium and time-to-maturity [19]. Cetin and Merschuere discuss the pricing and hedging of EUA contracts traded within the EU ETS scheme [20]. Daskalakis et al. develop an empirically and theoretically valid framework for the pricing and hedging of intra-phase and inter-phase futures and futures options [21]. Victor et al. propose a multi-objective approach to address the scheduling of thermal power systems with emissions constraints, and the trade-off curve between fuel cost and emissions pollution in a way to aid decisions [22]. Zhou and Mi calculate energy consumption and CO_2 emissions in the year 2010-2030 by taking Chinese industrial structure and energy consumption in each industry into account, and their empirical results show CO_2 emissions can be reduced 1.95 billion tons in 2030 if clear energy account for 20% of total energy consumption [23]. Hajek and Olej present air quality modeling by using various structures of Kohonen's self-organizing feature maps and the classification by Learning Vector Quantization neural networks, and its modeling generates well-separated clusters and has good generalization ability as well [24].

The spot and futures prices for emissions allowances are randomly changed in the EU ETS, in reality, the convenience yields and interest rate experiences significant change. Thereby those signs are important for the market participants to accurately capture changes on the term structure of futures price and the options value of futures contracts.

Our paper has the two major goals: the first goal is to explicitly analyze the term structure of futures price and the volatility feature of futures contracts with varying maturities for CO_2 emissions allowances. The second goal is to compare the theoretical and realistic options valuation change for CO_2 futures contracts with the varying delivery dates. The remainder of the paper is organized as follows: Section II describes the date sample source. Section III proposes N-factor affine term structure model of futures price and gives the parameter coefficients in the one, two and three-factor model. Section IV presents the futures price volatility and European call futures options valuation for emissions allowances. Section V shows the empirical analysis results of European call futures options valuation for emissions allowances. Section VI provides some brief conclusions.

2 Date description

To facilitate the reduction of greenhouse gases caused by mankind, the Kyoto Protocol provides three flexible mechanisms, which are the clean development mechanism (CDM), joint implementation (JI), and emissions trading scheme (ETS). For CO_2 emission reductions resulted from JI and CDM projects, Emission Reduction Units (ERU) and Certified Emission Reductions (CER) is given the right to emit one tone of CO_2 into the atmosphere. In order to attain emissions target in a cost-effective way, the EU member states introduced the emissions trading scheme since 2005. In the EU ETS, the EU ETS has the existing two phrases: the Pilot phase (2005-2007) and the Kyoto phase (2008-2012). Emissions allowances rights are new tradable credit assets, one European Union allowance (EUA) has the right to emit one tone of CO_2 into the atmosphere. Now the EU ETS has become the largest CO_2 emissions trading system in the world.

European Climate Exchange (ECX), merged by ICE now, is the most promising and liquid platform for emissions allowances futures in European. The minimum trading volumes for each normal futures contract are 1,000 tons of CO_2 equivalent amounts. The trading of futures contracts with vintages December 2013 and 2014 were introduced on April 8, 2008. Considered the continuity of study samples, we choose the date samples cover the period from April 8, 2008 to June 30, 2011 in the Kyoto phrase. The total empirical period has 840 samples.

We select the samples are time-varying daily settlement prices on EUA futures contracts with the varying maturities going from December 2011 to December 2014. The settlement dates for the ECX futures contracts of maturity December 2011 through 2014 are summarized in the Table 1. Here F_1 denotes the closest to maturity for EUA futures contract, F_2 denotes the second closest to maturity for EUA futures contract, and so on. In the following numerical illustration, we assume constant free-risk risk is 0.02.

Table 1: Expiration dates for ECX futures contracts

Futures contract	Last trade	First delivery	Last delivery
Dec-2011(F_1)	12/19/2011	12/20/2011	12/22/2011
Dec-2012(F_2)	12/17/2012	12/18/2012	12/20/2012
Dec-2013(F_3)	12/30/2013	12/31/2013	1/3/2014
Dec-2014(F_4)	12/29/2014	12/30/2014	1/2/2015

Source: Bloomberg

3 N-factor model of futures price for emissions allowances

3.1 N-factor term structure model for futures price

In this section we propose N-factor affine model of futures pricing for emissions allowances and provide parameter coefficients in the affine model. Since spot and futures prices for emission allowances exhibit time-varying trend, the volatility of futures return for CO_2 emissions allowances exhibits also time-varying trend.

Now we capture the term structure of futures contracts with different maturities, these signs are significant to understand the volatility features and futures options valuation for emissions allowances.

Futures price is composed of arbitrary number of non-observable state variables, and each state variable follows mean-reverting process [2-8]. Although the spot price for emissions allowances is observable state variable, there we consider the spot price is unobservable state variable. Let S_t denotes the spot price for emissions allowances at time t, we assume Log spot price for emissions allowances can be expressed as a sum of N non-observable state variables [5][7].

$$\ln S_t = \sum_{i=1}^{n} x_i \tag{1}$$

By assuming constant market price risk λ, the vector of state variables x_i with the risk-adjusted process follows zero mean-reversion process by the stochastic differential equation [6].

$$dx = -(Kx_t + \lambda)dt + \Sigma dz_t \tag{2}$$

Where $K=\begin{bmatrix} k_1 & 0 & \cdots & 0 \\ 0 & k_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & k_n \end{bmatrix}$, $\Sigma=\begin{bmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_n \end{bmatrix}$

is a $n \times n$ diagonal matrices of mean-reversion speed rate and the volatility of state variables, $\lambda = \begin{bmatrix} \lambda_1 & \lambda_2 & \dots & \lambda_n \end{bmatrix}^T$ is a $n \times 1$ vector of constant market price risk of state variables. dz_t is a $n \times 1$ vector of correlated Brownian motion increments such that $(dz_t) \times (dz_t)^T = \Omega dt$, where the (i, j) element of Ω is the instantaneous correlation between state variables *i* and *j,* and $\rho_{ij} \in [-1,1]$.

Let $F(t,T)$ denotes the futures price for emissions allowances at time *t* with the delivery date *T,* accordingly the futures price for emissions allowances can be expressed as the expected value of spot price at the maturity date *T* under the risk-neutral measure Q [25].

$$F(x_t,t,T) = E_t^Q(S_T) \tag{3}$$

Based on referring futures pricing model by Manoliu and Tompaidid [5], Cortarzar and Naranjo [6] and Wang et al. [7], we provide the futures price for emissions allowances at time *t* and maturing at *T* in equation (3) can be defined as:

$$F(x_t,t,T) = \exp\Big(\sum_{i=2}^{N} e^{-k_i(T-t)} x_i + \mathrm{A}(T-t)\Big)$$
$$A(T-t) = -\sum_{i=1}^{N}(1-e^{-k_i(T-t)})\frac{\lambda_i}{k_i} + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_i\sigma_j\rho_{ij}\frac{1-e^{-(k_i+k_j)(T-t)}}{k_i+k_j} \tag{4}$$

Equation (4) is shown in the Appendix A. Since each state variable follows mean-reversion Brownian motion process, the futures prices for emissions allowances follow mean-reverting process. The volatility of futures price returns can be obtained from the equation (2) and (4).

$$\sigma_F^2(T) = \sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\sigma_i\sigma_j e^{-(k_i+k_j)(T-t)} \tag{5}$$

As is shown in the equation (5), the volatility of futures price for emissions allowances is independent of the value of the state variables. Equation (5) carries information that the volatility of futures price depends on the volatility of these stochastic state variables, the correlation among them, the adjustment speed rate of state variables, and the time to maturity.

3.2 Parameter estimation

The Kalman filter is an estimation methodology which may be applied to estimate the parameter coefficients of non-observable state variables by using previous time series information. Early scholars are used to estimating and implementing arbitrary multi-factor models of commodities price by the Kalman filter technique [3][5-8]. Estimated coefficients in the one, two, three-factor affine model is shown in the following table with standard errors in parentheses.

Table 2: Estimated parameter coefficients of affine model from EUA futures dates

Parameter	*One-factor*	*Two-factor*	*Three-factor*
k_1	0.0265*** (0.001)	0.142*** (0.000)	0.105*** (0.002)
k_2		0.150*** (0.000)	0.124*** (0.003)
k_3			0.021***(0.001)
σ_1	0.099*** (0.038)	0.220*** (0.001)	0.204*** (0.004)
σ_2		0.182*** (0.003)	0.160*** (0.005)
σ_3			0.124*** (0.060)
λ_1	- 0.1345*** (0.0018)	- 3.135*** (0.077)	0.002*** (0.000)
λ_2		2.772*** (0.084)	-0.599*** (0.021)
λ_3			- 0.010***(0.003)
ρ_{12}		- 0.254*** (0.001)	-0.207*** (0.000)
ρ_{13}			-0.027*** (0.001)
ρ_{23}			0.025*** (0.001)
Log likelihood	8605.8	9988.5	11548.5

Note: ***,**,* denotes significant 1%, 5%, 10% level, measured standard error is shown in parentheses.

All mean-reversion parameters k_i are significantly unequal to zero at the significant level 1%, and their corresponding standard deviations of the measure errors approximately go to zero when the number of unobservable state variables is one, two and three, these signs indicate the state variables x_{1t}, x_{2t}, x_{3t} follow strong mean-reversion process. The volatility and correlation parameters are also highly significant, and market risk premium parameters are significant at the confident level 1%. All the measure errors of standard deviation are lower and they show very significant.

4 Futures option valuation for emission allowances

Futures option for emission allowances has been a steady growth in the number of financial assets. Futures options are option contracts on the futures contracts of the physical commodities or financial instruments, which give the holders the right to purchase (or sell) futures contracts at a prespecified futures price, called the exercise price *K,* and the futures option contracts may expire at the date T_1 prior the maturity *T* of the underlying futures contracts, that is $T_2<T_1$ [26-27].

In this study, our hypothesis is that futures price for emission allowances follows a geometric Brownian motion process and the volatility of futures options price exhibits time-varying trend which is composed of term structure of multi random factors.

$$dF(x_t,t,T)=\mu F(x_t,t,T)dt+\sigma(t,T)F(x_t,t,T)dZ_t \tag{6}$$

Where μ,σ denote the expected instantaneous price change and the instantaneous standard deviation relative of the futures contracts, dZ_t is Brownian motion increment. In the following section, we assume *t* is current time, *T,* T_1 denote the delivery date of a futures contract and expire date of a European call options contract, and $T_1<T$. $\tau=T-t,\tau_1=T_1-t$ denotes the time to maturity of the futures contract and the time to expire date of the call options contract. $K,V(t,T_1,T)$ denotes the strike price of the call options and the value at time *t* of a futures call options that expires at time T_1 for the futures contract with the maturity *T*. Since Black [28] developed the standard model of European call futures options, we assume *r* is currently constant free-risk interest rate, it is straight to show European call options valuation on futures contract for emissions allowances under the risk-neutral world, this implies

$$V(t,T_1,T)=e^{-r\tau_1}(F(x_t,t,T)N(d_1)-KN(d_2))$$

$$d_1=\frac{\ln(F(x_t,T_1,T)/K+0.5w^2}{w},d_2=d_1-w \tag{7}$$

In general, futures volatility for emissions allowances exhibits time-varying stochastic trend and different from spot price volatility. Accordingly accurate prediction of futures volatility for emissions allowances is significant for the market participants to assess the valuation of futures options and attain the hedging credit assets. In the expired date of futures options contracts, we induce the volatility of futures options for emissions allowances is the integral of futures return volatility by equation (5). In the following section, we imply one, two, three, and multi-factor volatility of futures options contracts for emissions allowances.

The general of the multi-factor volatility of futures options contracts for emissions allowances

$$w^2(t,T_1,T)=\int_t^{T_1}\sum_{i=1}^{n}\sum_{j=1}^{n}e^{-(k_i+k_j)(T-v)}\rho_{ij}\sigma_i\sigma_j dv \tag{8}$$

The one-factor volatility of futures options contracts for emissions allowances

$$w^2(t,T_1,T)=\int_t^{T_1}\sigma_1^2e^{-2k_1(T-v)}dv$$
$$=\frac{\sigma_1^2}{2k_1}(e^{-2k_1(T-T_1)}-e^{-2k_1(T-t)}) \tag{9}$$

The two-factor volatility of futures options contracts for emissions allowances

$$w^2(t,T_1,T)=\int_t^{T_1}\sigma_1^2e^{-2k_1(T-v)}dv+\int_t^{T_1}\rho_{12}\sigma_1\sigma_2e^{-(k_1+k_2)(T-v)}dv+\int_t^{T_1}\sigma_2^2e^{-2k_2(T-v)}dv$$
$$=\frac{\sigma_1^2}{2k_1}(e^{-2k_1(T-T_1)}-e^{-2k_1(T-t)})+$$
$$\frac{\rho_{12}\sigma_1\sigma_2}{k_1+k_2}(e^{-(k_1+k_2)(T-T_1)}-e^{-(k_1+k_2)(T-t)}) \tag{10}$$

The three-factor volatility of futures options contracts for emissions allowances

$$w^2(t,T_1,T)=\int_t^{T_1}\sigma_1^2e^{-2k_1(T-v)}dv+\int_t^{T_1}\rho_{12}\sigma_1\sigma_2e^{-(k_1+k_2)(T-v)}dv$$
$$+\int_t^{T_1}\sigma_2^2e^{-2k_2(T-v)}dv+\int_t^{T_1}\sigma_3^2e^{-2k_3(T-v)}dv+\int_t^{T_1}\rho_{13}\sigma_1\sigma_3e^{-(k_1+k_3)}$$
$$+\int_t^{T_1}\rho_{23}\sigma_2\sigma_3e^{-(k_2+k_3)(T-v)}dv$$
$$=\frac{\sigma_1^2}{2k_1}(e^{-2k_1(T-T_1)}-e^{-2k_1(T-t)})$$
$$+\frac{\rho_{12}\sigma_1\sigma_2}{k_1+k_2}(e^{-(k_1+k_2)(T-T_1)}-e^{-(k_1+k_2)(T-t)})$$
$$+\frac{\sigma_2^2}{2k_2}(e^{-2k_1(T-T_1)}-e^{-2k_1(T-t)})$$
$$+\frac{\sigma_3^2}{2k_3}(e^{-2k_3(T-T_1)}-e^{-2k_3(T-t)})$$

$$+\frac{\rho_{13}\sigma_1\sigma_3}{k_1+k_3}(e^{-(k_1+k_3)(T-T_1)}-e^{-(k_1+k_3)(T-t)})$$
$$+\frac{\rho_{22}\sigma_2\sigma_3}{k_2+k_3}(e^{-(k_2+k_3)(T-T_1)}-e^{-(k_2+k_3)(T-t)})$$

(11)

As is shown from equation (8) to equation (11), w^2 denotes the integral of the futures return volatility during the maturity of futures options contract. They demonstrate the volatility of futures option contracts for emissions allowances depends on the volatility and adjustment speed of arbitrary multi factors, the correlation among them, the time to expire date of a options contract, and the time to maturity of a futures contract. The European call futures options valuation for emissions allowances does not depend explicitly on the level of arbitrary multi factors, the long-run mean of arbitrary multi factors, and the marker price risk premium of multi factors in the equation (7). Thereby the European call futures options valuation for emissions allowances depends directly on these variables through the level of the exogenous multi-factor futures price.

5 The empirical results of futures option for emissions allowances

5.1 The volatility of futures options for emissions allowances

Manoliu and Tompaidis [5], Cortarzar and Naranjo [6] and Wang et al. [7] propose the futures prices are affected by arbitrary number state variables. In general, the volatility of each state variable exhibits time-varying trend. Thereby the volatility of each state variable is significant for the market participants and hedgers to estimate accurately the futures options volatility and the theoretical futures options valuation. We select the daily settlement price of EUA futures contracts with the varying maturity going from December 2011 to December 2014 as the empirical samples.

Table 3: The one-factor volatility of futures options for emissions allowances

τ_1	$\sigma(F_1,\tau_1)$	$\sigma(F_2,\tau_1)$	$\sigma(F_3,\tau_1)$	$\sigma(F_4,\tau_1)$
0.252 year	0.0485	0.0473	0.0460	0.0448
0.504 year	0.0691	0.0673	0.0654	0.0637
0.732 year	0.0832	0.0827	0.0805	0.0784

Table 4: The two-factor volatility of futures options for emissions allowances

τ_1	$\sigma(F_1,\tau_1)$	$\sigma(F_2,\tau_1)$	$\sigma(F_3,\tau_1)$	$\sigma(F_4,\tau_1)$
0.252 year	0.1226	0.1061	0.0913	0.0790
0.504 year	0.1767	0.1529	0.1315	0.1138
0.732 year	0.2165	0.1873	0.1611	0.1394

Table 5: The three-factor volatility of futures options for emissions allowances

τ_1	$\sigma(F_1,\tau_1)$	$\sigma(F_2,\tau_1)$	$\sigma(F_3,\tau_1)$	$\sigma(F_4,\tau_1)$
0.252 year	0.1473	0.1346	0.1230	0.1132
0.504 year	0.1988	0.1816	0.1659	0.1526
0.732 year	0.2373	0.2168	0.1979	0.1820

In this section, we take an example for analyzing the volatility of futures options contracts for emissions allowances, we choose on March 28, 2011 as the study initial date. As is shown in the above table 2, 4, 5, $\sigma^2(F_1,\tau_1)$ denote the volatility of futures options contract with the closest time to maturity, $\sigma^2(F_2,\tau_1)$ is the volatility of futures options contract with the second closest time to maturity, and so on. Based on the estimated parameters coefficients of term structure for CO_2 futures price, the one, two, three-factor volatility of futures options contracts for emissions allowances is shown from table 3 to table 5. From the table 3, the one-factor volatility of EUA futures options contracts with the different maturities is the lower extremely, it does not indicate accurately actual volatility of EUA futures options contracts. In the table 4 and 5, compared with one-factor model, the two and three-factor volatility of EUA futures options contracts with the different delivery dates are the higher, they show objectively actual volatility of EUA futures options contracts. From the table 4, 5, with the time to expire date of options contracts increased, the two-factor and three-factor volatility of EUA futures options contracts with the different maturities enhance. In the same time to expired date of futures options contracts, the three-factor volatility of EUA futures options contracts a little higher than two–factor volatility of emissions futures options contracts with the different maturities. The volatility of futures options contracts with the different delivery dates shows recursively decreasing trend in the same time to maturity of options contracts, which fits mean-reversion process in the affine term structure model of futures price for emissions allowances.

5.2 Futures options valuation for emissions allowances

In this section, we assume the volatility of EUA futures options contracts exhibits time-varying motion trend, we provide the numerical illustration of theoretical futures options valuation for emissions allowances by using a standard Black-Scholes approach. We choose the empirical period going from May 2, 2011 to June 30, 2011 as numerical analysis samples. Seen from the ECX market, traded options contracts are the closer time to maturity of futures contracts for emissions allowances. Accordingly we take futures contract with the maturity of December 2011 as the empirical sample. Used the estimated coefficients in the table 2, we estimate the futures options valuation in one, two and three-factor model by the equation (7).

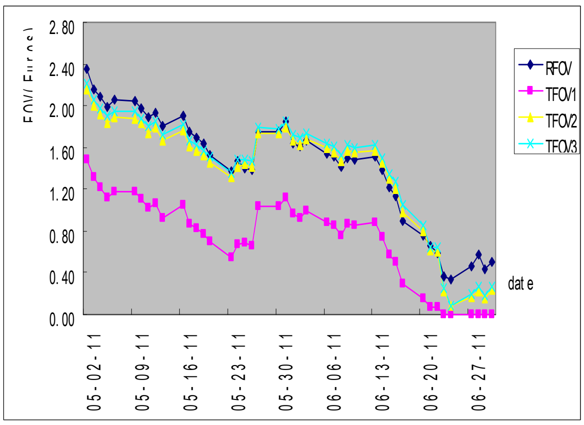

Fig 1. The comparison of the theoretical and realistic options valuation for CO_2 futures contracts when exercise price is 16 Euros from May 2, 2011 to June 30, 2011.

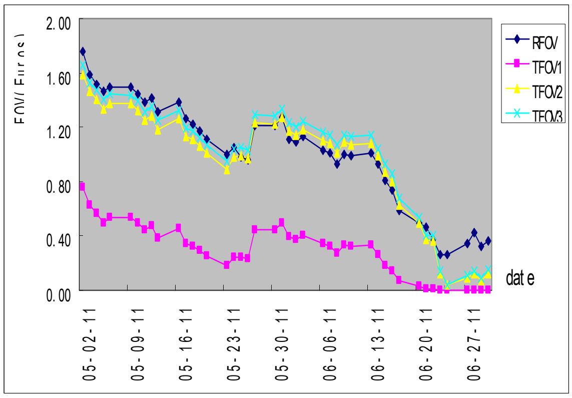

Fig 2. The comparison of the theoretical and realistic options valuation for CO_2 futures contracts when exercise price is 17 Euros from May 2, 2011 to June 30, 2011.

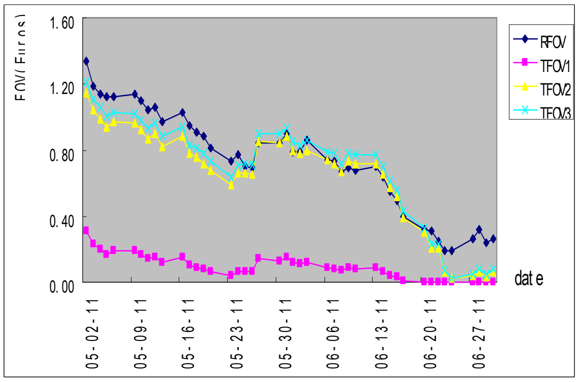

Fig3. The comparison of the theoretical and realistic options valuation for CO_2 futures contracts when exercise price is 18 Euros from May 2, 2011 to June 30, 2011.

As is shown in the above figure 1, 2 and 3, when the exercise price K=16, 17, 18, we provide the comparison of the realistic settlement price for CO_2 options contracts and the theoretical CO_2 options valuation by the Black-Scholes approach. In the figure 1, 2, 3, *RFOV* denote the actual options valuation of EUA futures contracts in the ECX market, *TFOV1, TFOV2, TFOV3* denote the theoretical options valuation of EUA futures contracts in the one, two and three-factor model. We know the two, and three-factor volatility of options contracts is much higher than the one-factor volatility of options contracts for emissions allowances from the above empirical results. The theoretical options valuation of futures contracts under the term structure of two and three factors are close approximately to the actual options settlement price for EUA futures contracts, however the theoretical options valuation under the term structure of one factor is removed seriously from the realistic options valuation.

6 Conclusions

In this paper, we propose the general model of theoretical options valuation of futures contracts for emissions allowances under the term structure of stochastic multi factors. Under Gaussian assumptions, we can estimate the parameter coefficients of the term structure of futures price for emissions allowances under arbitrary multi factors by using the Kalman filter technique. The analysis from equation (7) to equation (11) demonstrate that the futures options function carry information not only the volatility of random multi factors, but also the correlation among them, the adjustment speed, the time to maturity of futures and options contracts. We provide the numerical example arbitrary multi factors have a significant effect on futures options valuation.

The empirical results suggest we can estimate the theoretical futures options valuation for emissions allowances by using historical market information. The numerical illustration shows the two and three-factor volatility of emissions futures options contracts with the different maturities when τ_1 =0.252, 0.504,and 0.723. The volatility of emissions futures options contracts with the different delivery dates decrease recursively in the same time to maturity τ_1 . The empirical results indicate the theoretical options valuation for emissions futures contracts under two and three-factor model are close approximately to the realistic options settlement price for emissions futures contracts, however the one-factor model is clearly inappropriate with the abnormal emissions market conditions. Therefore the term structure of arbitrary multi factors is a significant prerequisite to obtain more accurate futures options valuation, they have significant effect on futures options valuation for emissions allowances. The theoretical options valuation of futures contracts will be applied to determine the reasonable options price and then market participants make right options trading policy in the emissions options market.

Our empirical results propose that market players in the options market of emissions allowances can estimate futures options value and then optimize assets portfolio of futures contracts with varying maturity by the three-factor theoretical options valuation extended historical information set of futures settlement price. Our official decision-makers should provide full symmetric market information and make effective decisions to keep stable options price for emissions allowances. The direction of future work is to study the implications of theoretical options pricing for futures contracts and make effective decisions to build full options market.

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Appendix A

Based on referring the multi-factor affine model of futures price by Manoliu and Tompaidis [5], Cortarzar and Naranjo [6], and Wang et al. [7], this Appendix deduces equation (4) by using equation (3). Because the conditional normal distribution for the spot price S_T is the lognormal, it follows that

$$E^Q(S_T) = \exp(E_t^Q(\ln S_T) + \frac{1}{2}VAR_t^Q(\ln S_T)) \tag{A1}$$

Where

$$E_t^Q(\ln S_T) = l^T E_t^Q(x_T)$$

$$VAR_t^Q(\ln S_T) = l^T \operatorname{cov}_t^Q(x_T) l$$

Where $l = [1,1,\cdots\cdots,1]^T$ is a $n \times 1$ vector, $E_t^Q(x_T)$ is the expected value of the state variable $x_T = [x_{1t}, x_{2t}, \cdots\cdots, x_{nt}]^T$, $\operatorname{cov}_t^Q(x_T)$ is a covariance matrix of the state variable x_T

From equation (2) the conditional moments of $\boldsymbol{x}_t$ are

$$E_t^Q(x_T) = e^{-K(T-t)}x_t - \lambda\int_0^{T-t} e^{-K\tau}d\tau \tag{A2}$$

$$\operatorname{cov}_t^Q(x_T) = \int_0^{T-t} e^{-K\tau}\Sigma Q \Sigma^T (e^{-K\tau})^T d\tau \tag{A3}$$

Where $(dZ_t)(dZ_t)^T = \Omega dt$, thus

$$E_t^Q(x_T) = e^{-k_i(T-t)}x_{it} - (1-e^{-k_i(T-t)})\frac{\lambda_i}{k_i}, i = 1,2,\cdots\cdots,n \quad (A4)$$

$$\mathrm{cov}_t^Q(x_T) = \rho_{ij}\sigma_i\sigma_j \frac{1-e^{-(k_i+k_j)(T-t)}}{k_i+k_j}, i, j = 1,2,\cdots\cdots,n \quad (A5)$$

The futures price for emissions allowances *F(t, T)* can be defined as the expected value of spot price at the delivery date *T* under the risk-neutral measure *Q*, $F(x_t,t,T) = E_t^Q(S_T)$. The valuation formula (4) is obtained by inserting equations (A1), (A4) and (A5) into equation (4).

$$F(x_t,t,T) = \exp(\sum_{i=2}^{N} e^{-k_i(T-t)}x_i + \mathrm{A}(T-t))$$
$$A(T-t) = -\sum_{i=1}^{N}(1-e^{-k_i(T-t)})\frac{\lambda_i}{k_i} + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_i\sigma_j\rho_{ij}\frac{1-e^{-(k_i+k_j)(T-t)}}{k_i+k_j} \quad (A6)$$