Linear Diophantine Fuzzy Sets: Image Edge Detection Techniques based on Similarity Measures

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Abstract: - In the digital imaging process, fuzzy logic provides many advantages, including uncertainty management, adaptability to variations, noise tolerance, and adaptive classification. One of the techniques of digital image processing is the edge detection. The edge detection process is an essential tool to segment the foreground objects from the image background. So, it facilitates subsequent analysis and comprehension of the image's underlying structural properties. This process can be moved on with the notion of fuzzy sets and their generalizations. The concept of Linear Diophantine fuzzy sets is a generalization of fuzzy sets where reference parameters correspond to membership and non-membership grades. This study aims to apply linear Diophantine fuzzy sets (LDFSs) to edge detection of images. The novelty of this paper is twofold. The first one is that we conduct a comprehensive evaluation to ascertain the similarity values using the linear Diophantine fuzzy similarity measure by leveraging the gray normalized membership values associated with fundamental edge detection techniques. The other is to modify the image pixels into the LDFSs and then filter the images by using the presented similarity measure operators given in the LDFS environment.

Key-Words: - Image processing, Edge detection, Filtering, Distance, Similarity, Linear Diophantine fuzzy sets.

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1 Introduction

In 1965, fuzzy set (FS) theory was given in [1], to overcome ambiguous and uncertain information. Then, FS theory has been applied in different areas from economics, information sciences, computer sciences, and medical sciences to the social sciences. While the authors applied this theory effectively, some of them have stated that there are some situations where this theory is inadequate. For this reason, different generalizations of FS theory have been presented such as intuitionistic fuzzy set (IFS), Pythagorean fuzzy set (PyFS), picture fuzzy set (PFS), spherical fuzzy set (SFS), and LDFSs. These generalized fuzzy sets have been the subject of numerous studies in the literature, and their applications have been extensively explored in various domains. The authors in [2], [3], [4], [5], [6], [7], [8], have successfully applied these theories to the decision-making problems. Also, distance, similarity, and entropy measures have been defined in the mentioned environment and applied to real-life problems, [9], [10], [11], [12].

The notion of IFSs was introduced in [13], according to this theory, every element is defined by a degree of membership and a degree of non-membership, with the condition that the total of these levels does not surpass one. There are a lot of applications of IFSs such as decision-making, image processing, expert systems, pattern recognition, and multi-criteria decision analysis, [14], [15], [16], [17]. Then a more generalized version of IFSs, named LDFSs, was given in [18], by incorporating reference parameters to offer a more flexible and efficient approach. The theory of LDFSs expands the range of membership and non-membership degrees through the use of reference parameters. There are many application areas where LDFSs find practical use that have been described in the current literature [19], [20], [21], [22]. These studies cover the extent and various domains and include a wide variety of problem-solving. Moreover, some works on image processing can be found in [23], [24]. In this work, we aim to give an application of LDFSs to image processing that efficiently provides numerous benefits such as effectively handling uncertainty, adapting to variations, tolerating noise and enable adaptive classification. These benefits significantly increase the precision and adaptability of image processing, enabling more accurate and flexible handling of visual data. Image processing consists of multiple stages and edge detection is a crucial first step. Accurate edge detection is crucial for the correct execution of the subsequent stages. Therefore, we first establish some new similarity measures and show that an image can be represented as LDFSs. Then, using the similarity measures, we show that the edge detection process can be effectively managed and executed using LDFS principles. In conclusion, with this work we can show that image processing is a suitable application area for theoretical mathematics topics.

2 Preliminaries

2.1 Linear Diophantine Fuzzy Sets

In this subsection, we give the concepts of FSs, IFSs, and LDFSs. Then, we recall some distance and similarity measures for LDFSs. Furthermore, we give the definition of the Minkowski distance measure which is a generalization of the mentioned distance measures.

Definition 1, [1], [13], Let U be a non-empty set.
(i) A fuzzy set (FS) F on U is given by

$$F = \{(u, \mu_F(u)) : u \in U\}$$

where $\mu_F: U \to [0,1]$ is the function that represent the membership of u to the F.
(ii) An intuitionistic fuzzy set (IFS) A on U is given by

$$A = \{(u, \mu_A(u), \nu_A(u)): u \in U\}$$

where $\mu_A, \nu_A: U \to [0,1]$ are the functions that represent the membership and non-membership function of u to the A, respectively, satisfying $0 \le \mu_A(u) + \nu_A(u) \le 1$ for all $u \in U$.

Remark 2 Each FS can be taken as an IFS by considering the non-membership function $\nu_A(u) = 1 - \mu_A(u)$. So, the collection of FSs is the subset of the collection of IFSs.

As a more general form, the notion of LDFSs is described in [18], in the following manner:

Definition 3, [18], Let U be a non-empty set. A LDFS L on U is given by

$$L = \{(u, \langle \mu_L(u), \nu_L(u)\rangle, \langle \alpha_L, \beta_L\rangle): u \in U\}$$

where $\mu_L, \nu_L: U \to [0,1]$ are the functions that represent the membership and non-membership function of u to the L, respectively, and α_L, $\beta_L \in [0,1]$ denotes the reference parameters value satisfying $0 \le \alpha_L\mu_L(u) + \beta_L\nu_L(u) \le 1$, for all $u \in U$, with $0 \le \alpha_L + \beta_L \le 1$. This reference parameters help us to define or classify a particular system.

The hesitation value π_L is calculated by $\pi_L(u) = 1 - (\alpha_L\mu_L(u) + \beta_L\nu_L(u))$ for all $u \in U$. We use the pair $(\langle\mu, \nu\rangle, \langle\alpha, \beta\rangle)$ to denote the linear Diophantine fuzzy number (LDFN) if the conditions $0 \le \alpha\mu + \beta\nu \le 1$ and $0 \le \alpha + \beta \le 1$ are satisfied. The collection of all LDFSs on U will be represented by L(U).

Remark 4 Each IFS can be taken as an LDFS. That is, if $A = \{(u, \mu_A(u), \nu_A(u)): u \in U\}$ is an IFS and the parameter values $\alpha_A, \beta_A \in [0,1]$ satisfies $0 \le \alpha_A + \beta_A \le 1$, then we have that $0 \le \alpha_A\mu_A(u) + \beta_A\nu_A(u) \le \mu_A(u) + \nu_A(u) \le 1$. Hence the set $A = \{(u, \langle\mu_A(u), \nu_A(u)\rangle, \langle\alpha_A, \beta_A\rangle): u \in U\}$ satisfying $\alpha_A, \beta_A \in [0,1]$ and $0 \le \alpha_A + \beta_A \le 1$, for all $u \in U$, is an LDFS.

We obtain the following results from Remark 2 and Remark 4:

Corollary 5 If $F = \{(u, \mu_F(u)): u \in U\}$ is a FS, then we have that the set $A = \{(u, \langle\mu_A(u), 1 - \mu_A(u)\rangle, \langle\alpha_A, \beta_A\rangle): u \in U\}$ satisfying $0 \le \alpha_A + \beta_A \le 1$ is an LDFS.

Definition 6, [18], A LDFS on L(U) of the form $L_U = \{(u, \langle 1,0\rangle, \langle 1,0\rangle): u \in U\}$ is called absolute LDFS and $L_0 = \{(u, \langle 0,1\rangle, \langle 0,1\rangle): u \in U\}$ is called null (empty) LDFS.

Definition 7, [18], Let $L \in L(U)$. Then the complement of L, represented by L^c, is given by

$L^c = \{(u, \langle \nu_L(u), \mu_L(u)\rangle, \langle \beta_L(u), \alpha_L(u)\rangle): u \in U\}$.

Algebraic operations between LDFSs were defined in [18], as follows:

Definition 8, [18], Let $L, L_1, L_2 \in L(U)$ and $k > 0$. Then,

- $L_1 \oplus L_2 = \{(u, \langle \mu_{L_1}(u) + \mu_{L_2}(u) - \mu_{L_1}(u)\mu_{L_2}(u), \nu_{L_1}(u)\nu_{L_2}(u)\rangle, \langle \alpha_{L_1}(u) + \alpha_{L_2}(u) - \alpha_{L_1}(u)\alpha_{L_2}(u), \beta_{L_1}(u)\beta_{L_2}(u)\rangle): u \in U\}$
- $L_1 \otimes L_2 = \{ (u, \langle \mu_{L_1}(u)\mu_{L_2}(u), \nu_{L_1}(u) + \nu_{L_2}(u) - \nu_{L_1}(u)\nu_{L_2}(u)\rangle, \langle \alpha_{L_1}(u)\alpha_{L_2}(u), \beta_{L_1}(u) + \beta_{L_2}(u) - \beta_{L_1}(u)\beta_{L_{2(u)}}\rangle): u \in U\}$
- $k.L = \{\left(u, \langle 1 - \left(1 - \mu_L(u)\right)^k, \nu_L(u)^k\rangle, \langle 1 - \left(1 - \alpha_L(u)\right)^k, \beta_L(u)^k\rangle\right): u \in U\}$,
- $L^k = \{\left(u, \langle \mu_L(u)^k, 1 - \left(1 - \nu_L(u)\right)^k\rangle, \langle \alpha_L(u)^k, 1 - \left(1 - \beta_L(u)\right)^k\rangle\right): u \in U\}$.

Proposition 9, [18], For LDFSs $L, L_1, L_2 \in L(U)$ and $k > 0$, $L_1 \oplus L_2$, $L_1 \otimes L_2$, $k.L$ and L^k are also LDFSs.

The concept of distance measurement is a very important tool because it shows how different or far away two objects are from each other. The definition of distance measure in LDFS environment was given as follows:

Definition 10, [20], Let U be a non-empty set. Then a mapping $D: L(U) \times L(U) \to [0,1]$ is said to be a distance measure on LDFSs if the following conditions hold for all $L_1, L_2 \in L(U)$:

- $0 \le D(L_1, L_2) \le 1$,
- $D(L_1, L_2) = D(L_2, L_1)$,
- $D(L_1, L_2) = 0$ if $L_1 = L_2$,
- If $L_1 \sqsubseteq L_2 \sqsubseteq L_3$ then $D(L_1, L_3) \ge D(L_1, L_2)$ and $D(L_1, L_3) \ge D(L_2, L_3)$.

Theorem 11, [20], Let $U = \{u_1, u_2, .., u_n\}$, $L_1, L_2 \in L(U)$ and the mappings $D_H, D_E: L(U) \times L(U) \to [0,1]$ defined as follows:

$$D_H(L_1, L_2) = \frac{1}{4n}\sum_{i=1}^{n}[\,\left|\mu_{L_1}(u_i) - \mu_{L_2}(u_i)\right| + \left|\nu_{L_1}(u_i) - \nu_{L_2}(u_i)\right| + \left|\alpha_{L_1}(u_i) - \alpha_{L_2}(u_i)\right| + \left|\beta_{L_1}(u_i) - \beta_{L_2}(u_i)\right|]$$

$$D_E(L_1, L_2) = (\tfrac{1}{4n}\textstyle\sum_{i=1}^{n}[\left(\mu_{L_1}(u_i) - \mu_{L_2}(u_i)\right)^2 + \left(\nu_{L_1}(u_i) - \nu_{L_2}(u_i)\right)^2 + \left(\alpha_{L_1}(u_i) - \alpha_{L_2}(u_i)\right)^2 + \left(\beta_{L_1}(u_i) - \beta_{L_2}(u_i)\right)^2])^{\frac{1}{2}}$$

Then the mappings D_H and D_E are distance measures on LDFSs. These distance measures are called the normalized Hamming distance and the normalized Euclidean distance, respectively.

In the following, we define the Minkowski distance between LDFSs:

Definition 12 Let $U = \{u_1, u_2, .., u_n\}$, $L_1, L_2 \in L(U)$ and the mapping $D_M: L(U) \times L(U) \to [0,1]$ defined as follows:

$$D_M(L_1, L_2) = (\frac{1}{4n}\sum_{i=1}^{n}[\,\left|\mu_{L_1}(u_i) - \mu_{L_2}(u_i)\right|^a + \left|\nu_{L_1}(u_i) - \nu_{L_2}(u_i)\right|^a + \left|\alpha_{L_1}(u_i) - \alpha_{L_2}(u_i)\right|^a + \left|\beta_{L_1}(u_i) - \beta_{L_2}(u_i)\right|^a])^{\frac{1}{a}}.$$

Then the mapping D_M is a distance measure on LDFSs. This distance measure is called the normalized Minkowski distance.

Note that, in the definition of the normalized Minkowski distance, if $a = 1$ and $a = 2$, then we have the definitions of the normalized Hamming distance and the normalized Euclidean distance, respectively.

Definition 13, [20], Let U be a non-empty set. Then a mapping $S: L(U) \times L(U) \to [0,1]$ is said to be a similarity measure on LDFSs if the following conditions hold for all $L_1, L_2 \in L(U)$:

- $0 \le S(L_1, L_2) \le 1$,
- $S(L_1, L_2) = S(L_2, L_1)$,
- $S(L_1, L_2) = 1$ if $L_1 = L_2$,
- If $L_1 \sqsubseteq L_2 \sqsubseteq L_3$ then $S(L_1, L_3) \le S(L_1, L_2)$ and $S(L_1, L_3) \le S(L_2, L_3)$.

Next, we show that a similarity measure can be induced via distance measure:

Theorem 14 Let $f: [0,1] \to [0,1]$ be a decreasing function and D be a distance measure on LDFSs. Suppose that the mapping $S_D^f: L(U) \times L(U) \to [0,1]$ is defined by $S_D^f(L_1, L_2) = \frac{f(D(L_1, L_2)) - f(1)}{f(0) - f(1)}$. Then the mapping S_D^f is a similarity measure on LDFSs. This similarity measure is said to be an f-similarity measure based on the distance measure on LDFSs. Now, we give some specific examples to demonstrate the Theorem 14.

Example 15 (1) Let $f_1:[0,1]\to[0,1]$ be defined by $f_1(u)=1-u$ and D be a distance measure on LDFSs. Then, we obtain

$$S_D^{f_1}(L_1,L_2) = \frac{f_1(D(L_1,L_2))-f_1(1)}{f_1(0)-f_1(1)}=\frac{1-D(L_1,L_2)-0}{1-0} = 1-D(L_1,L_2).$$

(2) Let $f_2:[0,1]\to[0,1]$ be defined by $f_2(u)=e^{-u}$ and D be a distance measure on LDFSs. Then, we obtain

$$S_D^{f_2}(L_1,L_2) = \frac{f_2(D(L_1,L_2))-f_2(1)}{f_2(0)-f_2(1)}=\frac{e^{-D(L_1,L_2)}-e^{-1}}{e^0-e^{-1}} = \frac{e^{-D(L_1,L_2)}-e^{-1}}{1-e^{-1}}.$$

(3) Let $f_3:[0,1]\to[0,1]$ be defined by $f_3(u)=\frac{1}{1+u}$ and D be a distance measure on LDFSs. Then, we obtain

$$S_D^{f_3}(L_1,L_2) = \frac{f_3(D(L_1,L_2))-f_3(1)}{f_3(0)-f_3(1)}=\frac{\frac{1}{1+D(L_1,L_2)}-\frac{1}{2}}{1-\frac{1}{2}} = \frac{1-D(L_1,L_2)}{1+D(L_1,L_2)}.$$

Using the distance measures introduced and described above, we can obtain similarity measures to use in the process of edge detection as follows;

Corollary 16 Let $U=\{u_1,u_2,..,u_n\}$ and $L_1,L_2\in L(U)$. Then the similarity measures $S_{D_H}^{f_1}, S_{D_E}^{f_1}, S_{D_M}^{f_1}, S_{D_H}^{f_2}, S_{D_E}^{f_2}, S_{D_M}^{f_2}, S_{D_H}^{f_3}, S_{D_E}^{f_3}$ and $S_{D_M}^{f_3}$ are obtained as follows:

$$S_{D_H}^{f_1}(L_1,L_2)=1-D_H(L_1,L_2) = 1-(\frac{1}{4n}\sum_{i=1}^{n}[\,|\mu_{L_1}(u_i)-\mu_{L_2}(u_i)| + |\nu_{L_1}(u_i)-\nu_{L_2}(u_i)| + |\alpha_{L_1}(u_i)-\alpha_{L_2}(u_i)|+|\beta_{L_1}(u_i)-\beta_{L_2}(u_i)|])$$

$$S_{D_E}^{f_1}(L_1,L_2)=1-D_E(L_1,L_2) = 1-(\frac{1}{4n}\sum_{i=1}^{n}[\,|\mu_{L_1}(u_i)-\mu_{L_2}(u_i)|^2 + |\nu_{L_1}(u_i)-\nu_{L_2}(u_i)|^2 + |\alpha_{L_1}(u_i)-\alpha_{L_2}(u_i)|^2 + |\beta_{L_1}(u_i)-\beta_{L_2}(u_i)|^2])^{1/2}$$

$$S_{D_M}^{f_1}(L_1,L_2)=1-D_M(L_1,L_2) = 1-(\frac{1}{4n}\sum_{i=1}^{n}[\,|\mu_{L_1}(u_i)-\mu_{L_2}(u_i)|^a + |\nu_{L_1}(u_i)-\nu_{L_2}(u_i)|^a + |\alpha_{L_1}(u_i)-\alpha_{L_2}(u_i)|^a + |\beta_{L_1}(u_i)-\beta_{L_2}(u_i)|^a])^{1/a}$$

$$S_{D_H}^{f_2}(L_1,L_2)=\frac{e^{-D_H(L_1,L_2)}-e^{-1}}{1-e^{-1}} = (exp(1-1/4n\sum_{i=1}^{n}[\,|\mu_{L_1}(u_i)-\mu_{L_2}(u_i)| + |\nu_{L_1}(u_i)-\nu_{L_2}(u_i)| + |\alpha_{L_1}(u_i)-\alpha_{L_2}(u_i)| + |\beta_{L_1}(u_i)-\beta_{L_2}(u_i)|])) - 1/(e-1)$$

$$S_{D_E}^{f_2}(L_1,L_2)=\frac{e^{-D_E(L_1,L_2)}-e^{-1}}{1-e^{-1}} = (exp(1-\frac{1}{4n}\sum_{i=1}^{n}[\,|\mu_{L_1}(u_i)-\mu_{L_2}(u_i)|^2 + |\nu_{L_1}(u_i)-\nu_{L_2}(u_i)|^2 + |\alpha_{L_1}(u_i)-\alpha_{L_2}(u_i)|^2 + |\beta_{L_1}(u_i)-\beta_{L_2}(u_i)|^2])^{1/2}) - 1/(e-1)$$

$$S_{D_M}^{f_2}(L_1,L_2)=\frac{e^{-D_M(L_1,L_2)}-e^{-1}}{1-e^{-1}} = (exp(1-\frac{1}{4n}\sum_{i=1}^{n}[\,|\mu_{L_1}(u_i)-\mu_{L_2}(u_i)|^a + |\nu_{L_1}(u_i)-\nu_{L_2}(u_i)|^a + |\alpha_{L_1}(u_i)-\alpha_{L_2}(u_i)|^a + |\beta_{L_1}(u_i)-\beta_{L_2}(u_i)|^a])^{1/a}) - 1/(e-1)$$

$$S_{D_H}^{f_3}(L_1,L_2)=\frac{1-D_H(L_1,L_2)}{1+D_H(L_1,L_2)} = (1-\frac{1}{4n}\sum_{i=1}^{n}[\,|\mu_{L_1}(u_i)-\mu_{L_2}(u_i)| + |\nu_{L_1}(u_i)-\nu_{L_2}(u_i)| + |\alpha_{L_1}(u_i)-\alpha_{L_2}(u_i)| + |\beta_{L_1}(u_i)-\beta_{L_2}(u_i)|]) /(1+\frac{1}{4n}\sum_{i=1}^{n}[\,|\mu_{L_1}(u_i)-\mu_{L_2}(u_i)| + |\nu_{L_1}(u_i)-\nu_{L_2}(u_i)| + |\alpha_{L_1}(u_i)-\alpha_{L_2}(u_i)| + |\beta_{L_1}(u_i)-\beta_{L_2}(u_i)|])$$

$$S_{D_E}^{f_3}(L_1,L_2)=\frac{1-D_E(L_1,L_2)}{1+D_E(L_1,L_2)} =(1-\frac{1}{4n}\sum_{i=1}^{n}[\,|\mu_{L_1}(u_i)-\mu_{L_2}(u_i)|^2 + |\nu_{L_1}(u_i)-\nu_{L_2}(u_i)|^2$$

$$+\left|\alpha_{L_1}(u_i)-\alpha_{L_2}(u_i)\right|^2+\left|\beta_{L_1}(u_i)-\beta_{L_2}(u_i)\right|^2]^{\frac{1}{2}})$$
$$/(1+\frac{1}{4n}\sum_{i=1}^{n}[\,\left|\mu_{L_1}(u_i)-\mu_{L_2}(u_i)\right|^2$$
$$+\left|\nu_{L_1}(u_i)-\nu_{L_2}(u_i)\right|^2$$
$$+\left|\alpha_{L_1}(u_i)-\alpha_{L_2}(u_i)\right|^2+\left|\beta_{L_1}(u_i)-\beta_{L_2}(u_i)\right|^2]^{1/2})$$

$$S_{D_M}^{f_3}(L_1,L_2)=\frac{1-D_M(L_1,L_2)}{1+D_M(L_1,L_2)}$$
$$=(1-\frac{1}{4n}\sum_{i=1}^{n}[\,\left|\mu_{L_1}(u_i)-\mu_{L_2}(u_i)\right|^a$$
$$+\left|\nu_{L_1}(u_i)-\nu_{L_2}(u_i)\right|^a+\left|\alpha_{L_1}(u_i)-\alpha_{L_2}(u_i)\right|^a$$
$$+\left|\beta_{L_1}(u_i)-\beta_{L_2}(u_i)\right|^a]^{\frac{1}{a}})$$
$$/(1+\frac{1}{4n}\sum_{i=1}^{n}[\,\left|\mu_{L_1}(u_i)-\mu_{L_2}(u_i)\right|^a$$
$$+\left|\nu_{L_1}(u_i)-\nu_{L_2}(u_i)\right|^a+\left|\alpha_{L_1}(u_i)-\alpha_{L_2}(u_i)\right|^a$$
$$+\left|\beta_{L_1}(u_i)-\beta_{L_2}(u_i)\right|^a]^{1/a})$$

2.2 Basic Edge Detection Techniques

Edge detection plays a crucial role in various image-related tasks such as image processing, image analysis, image pattern recognition, and computer vision techniques. The outcome of the edge detection process on an image provides a collection of connected curves that represent object boundaries, surface markings, and variations in surface orientation. By applying an edge detection algorithm to an image, it becomes possible to significantly reduce the data volume for processing, effectively filtering out less relevant information while preserving the essential structural characteristics of the image. Successful execution of the edge detection step allows for a simplified interpretation of the information contained in the original image.

Brief information about the commonly used edge detection techniques Sobel, Prewitt, LoG (Laplacian of Gaussian), Canny and Roberts are given below and these techniques are applied for a grey level image on MATLAB and given in Figure 1, Figure 2, Figure 3, Figure 4 and Figure 5.

In the Sobel technique, edges are identified by employing a 3×3 image filter in the local neighborhood. This technique gives some smoothing effect against the random noise of the image, making the edges appear thicker and brighter.

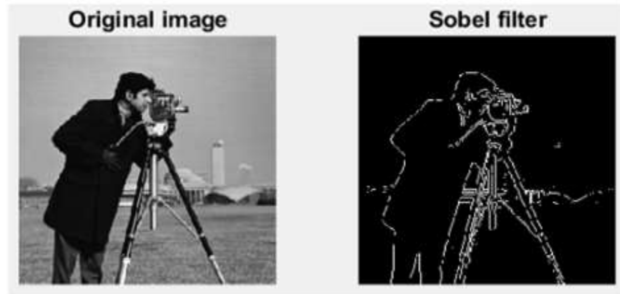

Fig. 1: The results of Sobel technique

The Prewitt technique which is very similar to the Sobel technique estimates edge detection by utilizing a simplified 3×3 image filter in the local neighborhood.

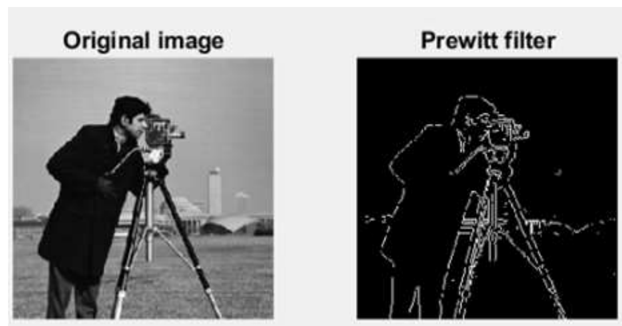

Fig. 2: The results of Prewitt technique

The Laplacian technique detects edges by searching for points where the second derivative crosses zero. The second derivative is more effective in capturing fine details compared to the first derivative. However, it has the disadvantage of being highly sensitive to noise. Therefore, to mitigate this, a Gaussian filter is applied beforehand to remove noise.

Fig. 3: The results of LoG technique

The Canny technique is designed to reduce noise, emphasize actual edges, and increase the sensitivity of edge detection. It involves several steps to identify edges. Firstly, it uses a Gaussian filter to reduce the details and noise in the image, finds the edge candidates by determining the direction and magnitude of the gradient for each pixel, and then selects the most suitable edges by removing the weak pixels with the thresholding method.

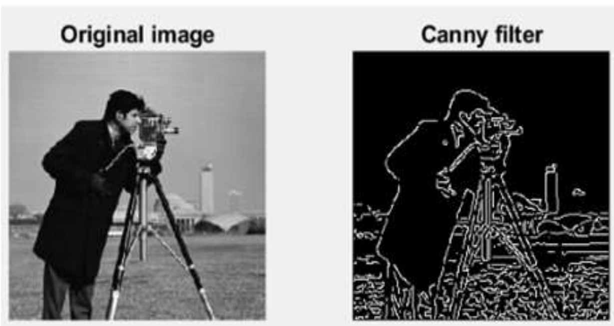

Fig. 4: The results of Canny technique

The Roberts method employs two separate 2x2 convolutional masks to compute the gradient magnitude within the image. This involves applying these masks to specific pixel neighborhoods through a process known as convolution, enabling the derivation of gradients across the image.

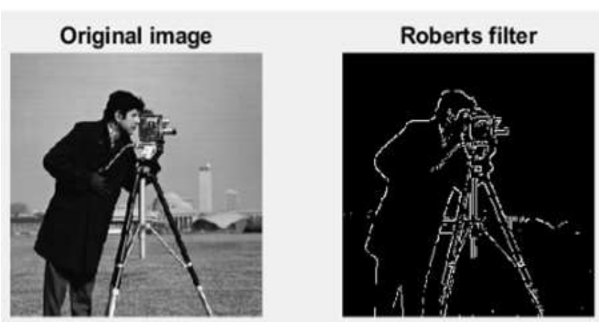

Fig. 5: The results of Roberts technique

To facilitate a comparative analysis among the aforementioned edge detection techniques, each method is individually employed on the grayscale version of a different image. Subsequently, a collective presentation of the outcomes is provided in the combined Figure 6 for easy comparison.

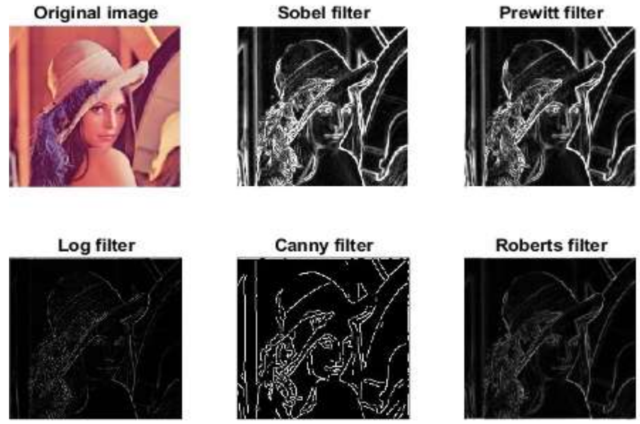

Fig. 6: Comparison of edge detection techniques

Fuzzification of an image is particularly useful when working with algorithms or techniques that require a specific range of inputs, such as image processing algorithms based on normalized values. Generally, fuzzification of an image involves rescaling pixel values to the range [0, 1], this can be made in various ways using different membership functions. This process allows for easier analysis and processing of image data while ensuring consistency and compatibility with image processing techniques and algorithms.

The above-mentioned techniques are employed on the image after fuzzification, and the resulting outputs are displayed in Figure 7.

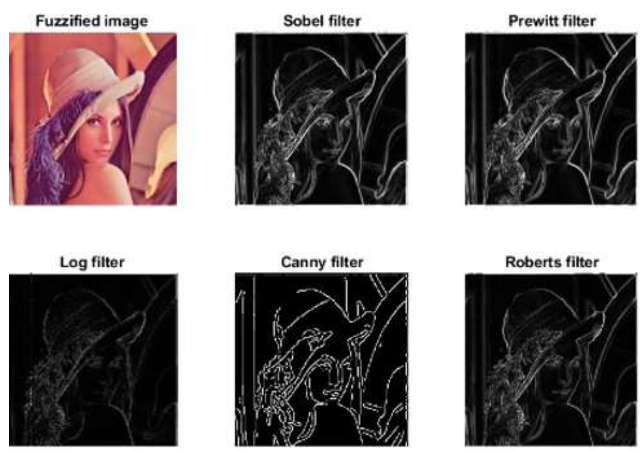

Fig. 7: Edge detection for fuzzified image

3 Modification of an Image to LDFS

The process of modification of an image into LDFS entails the incorporation of the specific characteristic of being LDFN into the data, wherein said data possesses pixel values falling within the predefined range of [0, 1]. For example, if image A satısfies the condition $a(i,j)*\alpha+\left(1-a(i,j)\right)*\beta))<1$ where $a(i,j)$ is its (i,j)th pixel and α,β are randomly chosen, then we can define each pixel of A as $a(i,j)=<\left(a(i,j),1-a(i,j)\right),(\alpha,\beta)>$. Therefore, it can be argued that every image has the potential to be expressed and characterized within the LFDS framework.

Implementing such a modification facilitates the seamless utilization of operators specifically defined for LDFS, enabling straightforward integration into various operations and processes applied to the image.

The modified version of the Lena image and traditional edge detection filters applied can be seen in Figure 8 and Figure 9 for different $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

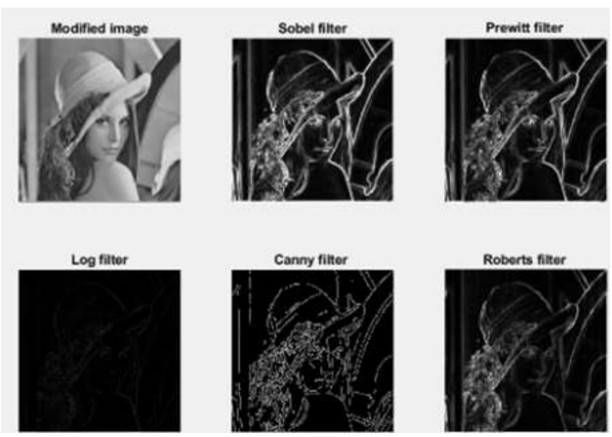

Fig. 8: Edge detection for modified image with $\alpha=0.0971$ and $\beta=0.8235$

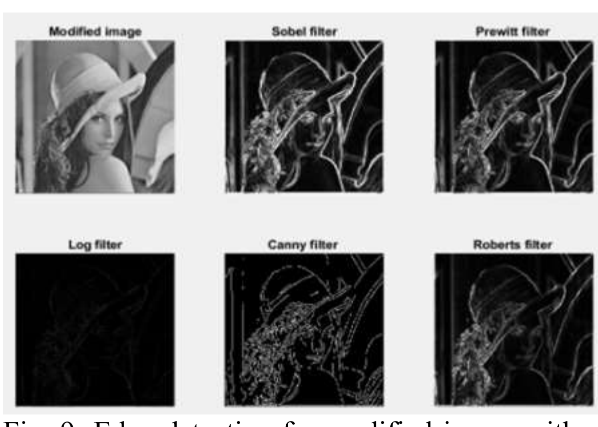

Fig. 9: Edge detection for modified image with $\alpha=0.1869$ and $\beta=0.4898$

The modification has some disadvantages as well as advantages. As seen in Figure 8 and Figure 9, this process does not work in harmony with the LoG filter for the selected alpha and beta values and does not give a clear output. However, compatible alpha and beta values can be found. The results obtained from the LoG filter for some values of α and β are given in Figure 10 below.

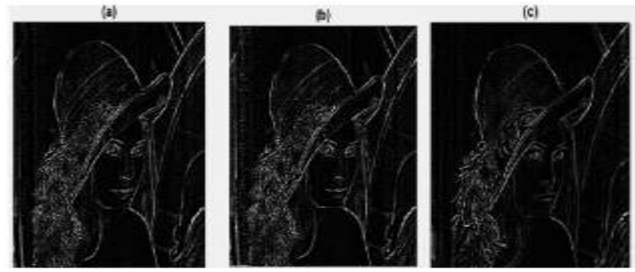

Fig. 10: Results of LoG filter: (a) $\alpha=0.4018$ and $\beta=0.0760$, (b) $\alpha=0.2399$ and $\beta=0.1233$, (c) $\alpha=0.3685$ and $\beta=0.6256$

On the other hand, it is imperative to acknowledge its performance, mainly when operating with filters rooted in matrix-based methodologies. This noteworthy observation underscores the remarkable compatibility between this approach and the application of matrix-oriented filtering techniques.

4 An Application of the LDFSs Similarity for Edge Detection

The similarities between each edge detection filter were calculated based on the similarity measures obtained from the above-mentioned distance measures and by using random $\alpha\ and\ \beta$ parameters. Different outcomes are obtained when employing various $\alpha\ and\ \beta$ parameters, which are randomly chosen. To illustrate, specified $\alpha\ and\ \beta$ values for each filter given in Table 1 are used and the corresponding outcomes are presented in Table 2, Table 3 and Table 4.

Table 1. Parameters for filters

Filters	α	β
Sobel	0.7447	0.1890
Prewitt	0.6868	0.1835
LoG	0.3685	0.6256
Canny	0.7802	0.0811
Roberts	0.4868	0.4359

Table 2. Similarity results under $S_{D_H}^{f_1}$, $S_{D_E}^{f_1}$ and $S_{D_M}^{f_1}$

		$S_{D_H}^{f_1}$	$S_{D_E}^{f_1}$	$S_{D_M}^{f_1}$ $(a=3)$
Sobel	Prewitt	0.9998	0.9966	0.9913
Sobel	LoG	0.9991	0.9804	0.9460
Sobel	Canny	0.9975	0.9518	0.8679
Sobel	Roberts	0.9987	0.9814	0.9544
Prewitt	LoG	0.9990	0.9813	0.9484
Prewitt	Canny	0.9976	0.9545	0.8754
Prewitt	Roberts	0.9989	0.9843	0.9613
LoG	Canny	0.9966	0.9467	0.8630
LoG	Roberts	0.9990	0.9840	0.9593
Canny	Roberts	0.9976	0.9626	0.9036

Table 3. Similarity results under $S_{D_H}^{f_2}$, $S_{D_E}^{f_2}$ and $S_{D_M}^{f_2}$

		$S_{D_H}^{f_2}$	$S_{D_E}^{f_2}$	$S_{D_M}^{f_2}$ $(a=3)$
Sobel	Prewitt	0.9997	0.9947	0.9863
Sobel	LoG	0.9985	0.9694	0.9169
Sobel	Canny	0.9961	0.9256	0.8042
Sobel	Roberts	0.9980	0.9709	0.9294
Prewitt	LoG	0.9984	0.9707	0.9205
Prewitt	Canny	0.9962	0.9296	0.8147
Prewitt	Roberts	0.9983	0.9753	0.9400
LoG	Canny	0.9946	0.9179	0.7975
LoG	Roberts	0.9984	0.9749	0.9368
Canny	Roberts	0.9962	0.9420	0.8547

Table 4. Similarity results under $S_{D_H}^{f_3}$, $S_{D_E}^{f_3}$ and $S_{D_M}^{f_3}$

		$S_{D_H}^{f_3}$	$S_{D_E}^{f_3}$	$S_{D_M}^{f_3}$ $(a=3)$
Sobel	Prewitt	1	0.9933	0.9828
Sobel	LoG	0.9981	0.9616	0.8976
Sobel	Canny	0.9951	0.9081	0.7666
Sobel	Roberts	0.9975	0.9636	0.9127
Prewitt	LoG	0.9980	0.9633	0.9019
Prewitt	Canny	0.9952	0.9130	0.7785
Prewitt	Roberts	0.9979	0.9690	0.9255
LoG	Canny	0.9932	0.8988	0.7590
LoG	Roberts	0.9979	0.9685	0.9217
Canny	Roberts	0.9953	0.9280	0.8242

The similarities between the fuzzified images obtained from each edge detection technique were calculated with the defined similarity measures.

According to the results obtained, it is seen that the Sobel and Prewitt filters are largely similar to each other. This similarity gives close results even if the alpha and beta parameters change. Both filters are popular image processing methods used for edge detection. Both perform a gradient calculation to detect the horizontal and vertical edges in the image and use the differences between pixel values to determine the boundaries. However, there are also some differences. For example, the Sobel filter does more computation than the Prewitt filter and may therefore have a higher computational cost. Also, the Sobel filter may have a better edge redirection ability than the Prewitt filter. In general, however, Sobel and Prewitt filters give similar results and are interchangeable.

We can apply the above calculations on the modified image and filters, similarly. The results obtained by using the alpha and beta values in Table 1 are shown in Table 5, Table 6 and Table 7.

Table 5. Similarity results of modified image filters under $S_{D_H}^{f_1}$, $S_{D_E}^{f_1}$ and $S_{D_M}^{f_1}$

		$S_{D_H}^{f_1}$	$S_{D_E}^{f_1}$	$S_{D_M}^{f_1}$ $(a=3)$
Sobel	Prewitt	0.9821	0.9704	0.9633
Sobel	LoG	0.7580	0.6962	0.6647
Sobel	Canny	0.8874	0.8024	0.7110
Sobel	Roberts	0.8443	0.8018	0.7736
Prewitt	LoG	0.7716	0.7116	0.6789
Prewitt	Canny	0.8743	0.7980	0.7097
Prewitt	Roberts	0.8573	0.8174	0.7874
LoG	Canny	0.6860	0.5964	0.5418
LoG	Roberts	0.8801	0.8416	0.7957
Canny	Roberts	0.7591	0.6974	0.6470

Table 6. Similarity results of modified image filters under $S_{D_H}^{f_2}$, $S_{D_E}^{f_2}$ and $S_{D_M}^{f_2}$

		$S_{D_H}^{f_2}$	$S_{D_E}^{f_2}$	$S_{D_M}^{f_2}$ $(a=3)$
Sobel	Prewitt	0.9720	0.9538	0.9429
Sobel	LoG	0.6600	0.5856	0.5493
Sobel	Canny	0.8316	0.7163	0.6030
Sobel	Roberts	0.7719	0.7156	0.6795
Prewitt	LoG	0.6770	0.6037	0.5656
Prewitt	Canny	0.8132	0.7107	0.6014
Prewitt	Roberts	0.7896	0.7359	0.6970
LoG	Canny	0.5737	0.4746	0.4185
LoG	Roberts	0.8213	0.7683	0.7077
Canny	Roberts	0.6613	0.5870	0.5295

Table 7. Similarity results of modified image filters under $S_{D_H}^{f_3}$, $S_{D_E}^{f_3}$ and $S_{D_M}^{f_3}$

		$S_{D_H}^{f_3}$	$S_{D_E}^{f_3}$	$S_{D_M}^{f_3}$ $(a=3)$
Sobel	Prewitt	0.9649	0.9999	0.9291
Sobel	LoG	0.6104	0.9986	0.4977
Sobel	Canny	0.7977	0.9991	0.5516
Sobel	Roberts	0.7305	0.9991	0.6308
Prewitt	LoG	0.6282	0.9987	0.5139
Prewitt	Canny	0.7767	0.9991	0.5501
Prewitt	Roberts	0.7502	0.9992	0.6494
LoG	Canny	0.5220	0.9981	0.3715
LoG	Roberts	0.7859	0.9993	0.6607
Canny	Roberts	0.6117	0.9986	0.4782

5 Conclusion

In this paper, we combine the structure of LDFSs with edge detection techniques using the presented similarity measures defined on LDFSs to generalize the fuzzy edge detection domain. We give numerical results of this technique on some images, analyze the validity of the techniques, and show these results in tables. Different generalizations of FS theory can be applied to the edge detection process for future work. Some different operators can be created to be used in this process. Moreover, LDFSs can be applied to problems such as image denoising and recognition.

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References:

[1] Yang, H., Song, H., Li, W., Qin, K., Shi, H., Jiao, Q.: Social image annotation based on image captioning. *WSEAS Transactions on Signal Processing*, 18, 109-115 (2022), https://doi.org/10.37394/232014.2022.18.15.

[2] Aydoğdu, E., Güner, E., Aldemir, B., Aygün, H.: Complex spherical fuzzy TOPSIS based on entropy. *Expert Systems with Applications*, 215, 119331 (2023), doi: 10.1016/j.eswa.2022.119331.

[3] Aydoğdu, E., Aldemir, B., Güner, E., Aygün, H.: A novel entropy measure with its application to the COPRAS method in complex spherical fuzzy environment. *Informatica*, 1-33 (2023), doi: 10.15388/23-INFOR539.

[4] Dursun, M., Goker, N.: Fuzzy cognitive map methodology for evaluating material selection criteria. *Engineering World*, 2, 15-19 (2020).

[5] Jurio, A., Paternain, D., Bustince, H., Guerra, C., Beliakov, G.: A construction method of Atanassov's intuitionistic fuzzy sets for image processing. *In 2010 5th IEEE International Conference Intelligent Systems* pp. 337-342, (2010), doi: 10.1109/IS.2010.5548390.

[6] Kilic, M., Dursun, M., Goker, N.: Evaluation of R&D projects using fuzzy MCDM method. *Engineering World*, 2, 150-154 (2020).

[7] Mahmood, T., Rehman, U.U.: A novel approach towards bipolar complex fuzzy sets and their applications in generalized similarity measures. *International Journal of Intelligent Systems*, 37.1: 535-567 (2022). doi: 10.1002/int.22639.

[8] Srebrenkoska, S., Apostolova, A., Dzidrov, M., Krstev, D.: Application of Analytical Hierarchy Process (AHP) in the selection of a flexible production system. *Engineering World*, 5, 138-143 (2023), https://doi.org/10.37394/232025.2023.5.15.

[9] Cuong, B. C., Kreinovich, V.: Picture fuzzy sets-A new concept for computational intelligence problems. *In 2013 third world congress on information and communication technologies (WICT 2013)*, pp. 1-6, (2013), doi: 10.1109/WICT.2013.7113099.

[10] Ejegwa P.A.: Distance and similarity measures for Pythagorean fuzzy sets. *Granular Computing*, 5(2): 225-238 (2020), doi: 10.1007/s41066-018-00149-z.

[11] Xuecheng, L.: Entropy, distance measure and similarity measure of fuzzy sets and their relations. *Fuzzy Sets and Systems*, 52(3), 305-318 (1992), doi: 10.1016/0165-0114(92)90239 Z.

[12] Zadeh, L.A.: Fuzzy sets. *Information and Control*, 8(3), 338-353 (1965). doi:10.1016/S0019-9958(65)90241-X

[13] Atanassov, K.: Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1), 87-96 (1986), doi: 10.1016/s0165-0114(86)80034-3.

[14] Bustince, H., Barrenechea, E., Pagola, M., Orduna, R.: Image thresholding computation using Atanassov's intuitionistic fuzzy sets. *Journal of Advanced Computational Intelligence and Intelligent Informatics*, 11(2), 187-194 (2007), doi: 10.20965/jaciii.2007.p0187.

[15] De, S.K., Biswas, R., Roy, A.R.: An application of intuitionistic fuzzy sets in medical diagnosis. *Fuzzy Sets and Systems*, 117(2), 209-213 (2001), doi: 10.1016/S0165-0114(98)00235-8.

[16] Liu, S., Zhang, J., Niu, B., Liu, L., He, X.: A novel hybrid multi-criteria group decision-making approach with intuitionistic fuzzy sets to design reverse supply chains for COVID-19 medical waste recycling channels. *Computers and Industrial Engineering*, 169, 108228 (2022), doi: 10.1016/j.cie.2022.108228.

[17] Xu, Z.S., Chen, J.: An overview of distance and similarity measures of intuitionistic fuzzy sets. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 16(04), 529-555 (2008), doi:

10.1142/S0218488508005406.
[18] Riaz M., Hashmi M.R.: Linear Diophantine fuzzy set and its applications towards multi-attribute decision-making problems, *Journal of Intelligent and Fuzzy Systems*, 37(4): 5417-5439 (2019), doi: 10.3233/JIFS-190550.
[19] Aldring, J., Santhoshkumar, S., Ajay, D.: A decision-making approach using linear diophantine fuzzy sets with Dombi operations. *In International Conference on Intelligent and Fuzzy Systems. Cham: Springer International Publishing*, 684-692 (2022). doi:10.1007/978-3-031-09176-6-76.
[20] Gül S., Aydoğdu A.: Novel distance and entropy definitions for linear Diophantine fuzzy sets and an extension of TOPSIS (LDF-TOPSIS). *Expert Systems*, 40(1): e13104 (2023), doi:10.1111/exsy.13104.
[21] Kamacı, H.: Linear Diophantine fuzzy algebraic structures. *Journal of Ambient Intelligence and Humanized Computing*, 12(11), 10353-10373, (2021).
[22] Mohammad, M.M.S., Abdullah, S., Al-Shomrani, M.M.: Some linear Diophantine fuzzy similarity measures and their application in decision-making problem. *IEEE Access*, 10, 29859-29877, (2022), doi: 10.1109/ACCESS.2022.3151684.
[23] Greer, K.: Recognising image shapes from image parts, not neural parts. *WSEAS Transactions on Signal Processing*, 19, 77-82 (2023), https://doi.org/10.37394/232014.2023.19.9.
[24] Shanthi S.A., Valarmathi R.: Edge detection on fuzzy near sets, *Materials Today*: Proceedings, 51: 2504-2511 (2022), doi: 10.1016/j.matpr.2021.12.120.

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