

Performance of some modified test statistics for testing the population signal to noise ratio: Simulation and Applications

SAMANTHA MENENDEZ, FLORENCE GEORGE, AND B M GOLAM KIBRIA
Department of Mathematics and Statistics
Florida International University
11200 SW 8th St, Miami, FL 33199
UNITED STATES OF AMERICA

Abstract: - This paper considers fifteen existing and proposed test statistics for testing the population SNR. A theoretical comparison among the test statistics is not possible, a Monte Carlo simulation study has been conducted. The performance of the test statistics is based on the empirical size and power of the tests considering a significance level 0.05. The simulation study resulted in some existing and proposed methods performing well in some conditions. For testing SNR=1, 2 and 5, Method 10 performed the best in all simulation conditions. However, for testing SNR=0.5 the proposed Method 12 and for testing SNR=10, the proposed Method 15 performed the best. Two real life data sets are analyzed to illustrate the performance of the test statistics.

Key-words: Normal Distribution; Gamma Distribution; Power of the test; Signal to Noise Ratio; Simulation Study; Size of the test

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1 Introduction

Mean is known as an expected value that represents the central tendency of a probability distribution which is known as the average of a data set. When we are interested in the measure of how dispersed the distribution of data is in relation to the mean, we refer to the standard deviation. Standard deviation can vary between high and low values, when the standard deviation is high the values fall far from the mean and when standard deviation is low the values are closer to the mean.

Standard deviation and noise relate vice versa as variability within a gathered data sample. Improper filtering where data was measured, and random errors can be introduced can result in unexpected noise. Noise has two main sources; errors introduced by measurement tools and random errors introduced by processing. Noise can be any data that has been received and changed in a manner that it cannot be read which adversely affects results of any data analysis.

While the mean describes what is being measured, standard deviation represents noise. Signal to noise ratio (SNR) is a measure of the strength of the desired data relative to the undesigned data or signal known as noise. Population SNR is equal to the population mean divided by the population standard deviation ($\frac{\mu}{\sigma}$). In practice, commonly in digital communications and image processing, a high SNR means that the signal strength is stronger in relation to the noise level. Having higher SNR means there is more useful information than unwanted noise data in the output. The application on SNR can be found in John (2007) and Tania (2008) among others.

SNR is the reciprocal of the commonly used coefficient of variation (CV) which is unitless and is defined as the standard deviation divided by the mean. Since SNR is the reciprocal, when there is better data there is a higher value for the SNR and a lower value for the CV. This can also be seen when, putting SNR into practice, the larger the standard deviation (noise) the smaller the SNR. Therefore, it is possible to construct both confidence interval and hypopaper testing from CV for SNR. The information on testing the population SNR is limited. Nevertheless, there are various methods available for estimating the confidence interval (CI) for a population CV, such as parametric, nonparametric, modified, and bootstrapping (Banik and Kibria, 2010). For more information on the CI for the CV, we refer Koopmans et al. (1964), Miller (1991), Sharma and Krishna (1994), McKay (1932), Vangel (1996), and Curto and Pinto (2009), George and Kibria (2012), Banik et al. (2012), Albatineh et al. (2014), and recently Abu-Shaweish et al. (2019) among others. First Kibria and George (2014) consider various test statistics based on the confidence interval of CV for testing the population SNR.

The objective of this paper is to review and propose some test statistics for testing the population SNR based on parametric, and modified methods for confidence intervals for SNR. The organization of the paper is as follows: We will review and propose some test statistics for testing the SNR in Section 2. A simulation will be conducted in Section 3. Two real life data are analyzed in Section. Finally, some concluding remarks are given in Section 5.

2 Statistical Methodology

Let $X_1, X_2, \ldots, X_n$ be an independently and identically distributed (iid) random sample of size n from a population from a population with finite mean, μ, and finite variance, σ^2. Let $\bar{x}$ be the sample mean and s be the sample standard deviation. Then $\widehat{SNR} = \frac{\bar{x}}{s}$ would be the estimated values of the population SNR $(\frac{\mu}{\sigma})$ and $\widehat{CV} = \frac{s}{\bar{x}}$ would be the estimated values of the population CV $(\frac{\sigma}{\mu})$. The objective of this paper is to test the population SNR. The null and alternative hypothesis are defined as follows:

$$H_0: SNR = SNR_0$$
$$H_a: SNR \neq SNR_0$$

Following Kibria and George (2014), we have considered the following eleven (11) promising test statistics, For details about these tests, we refer to Kibria and George (2014).

2.1 Miller (1991) Method

Method 1. Miller demonstrated that the estimator, $\frac{s}{\bar{x}}$ has an approximate normal distribution with mean $\frac{\sigma}{\mu}$ and variance of $(1/(n-1))(\frac{\sigma}{\mu})^2[0.5+(\frac{\sigma}{\mu})^2]$. Then the approximate upper and lower confidence limits the population SNR can be constructed. From the confidence intervals, we will reject the null hypothesis when SNR_0 is less than the lower limit or greater than the upper limit. The upper and lower limits for SNR_0 are given as follows:

$$SNR_0 < \left(\frac{s}{\bar{x}} + Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}\left(\frac{s}{\bar{x}}\right)^2\left[0.5 + \left(\frac{s}{\bar{x}}\right)^2\right]}\right)^{-1}$$

$$or$$

$$SNR_0 > \left(\frac{s}{\bar{x}} - Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}\left(\frac{s}{\bar{x}}\right)^2\left[0.5 + \left(\frac{s}{\bar{x}}\right)^2\right]}\right)^{-1} \quad (1)$$

We will reject the null hypothesis when SNR is more than the above upper limit or less than the lower limit.

2.2 Sharma and Krishna's (1994) Method

Method 2. Sharma and Krishna (1994) developed the asymptotic sampling distribution of the inverse of the coefficient of variation for making statistical inferences about population coefficient of variation (CV). Following Sharma and Krishna (1994), based on the CI for inverted SNR, we reject the null hypothesis when SNR_0 is less than the lower limit or greater than the upper limit, which are given below:

$$SNR_0 < \frac{\bar{x}}{s} - \frac{Z_{\alpha/2}}{\sqrt{n}} \quad or \quad SNR_0 > \frac{\bar{x}}{s} + \frac{Z_{\alpha/2}}{\sqrt{n}} \quad (2)$$

2.3 Curto and Pinto's (2009) Method

Method 3. Using the approximate confidence interval (CI) for the population inverted SNR, we will reject the null hypothesis, whe

$$SNR_0 < \left(\frac{s}{\bar{x}} + Z_{\alpha/2}\sqrt{\frac{1}{n}\left(\left(\frac{s}{\bar{x}}\right)^4 + 0.5\left(\frac{s}{\bar{x}}\right)^2\right)}\right)^{-1}$$

$$or$$

$$SNR_0 > \left(\frac{s}{\bar{x}} - Z_{\alpha/2}\sqrt{\frac{1}{n}\left(\left(\frac{s}{\bar{x}}\right)^4 + 0.5\left(\frac{s}{\bar{x}}\right)^2\right)}\right)^{-1} \quad (3)$$

2.4 McKay's (1932) Method

Method 4. Using the approximate confidence interval (CI) for the population inverted SNR, we will reject the null hypothesis, when

$$SNR_0 < \left[\frac{s}{\bar{x}}\sqrt{\left[\left(\frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n} - 1\right)\left(\frac{s}{\bar{x}}\right)^2 + \frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n-1}\right]}\right]^{-1}$$

$$or$$

$$SNR_0 > \left[\frac{s}{\bar{x}}\sqrt{\left[\left(\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n} - 1\right)\left(\frac{s}{\bar{x}}\right)^2 + \frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n-1}\right]}\right]^{-1} \quad (4)$$

where $\chi^2_{n-1,\frac{\alpha}{2}}$ and $\chi^2_{n-1,1-\frac{\alpha}{2}}$ are $(\alpha/2)^{th}$ and $(1-\alpha/2)^{th}$ percentile points for a chi-square distribution with (n-1) degrees of freedom

2.5 Modified McKay Confidence Interval (MMcK)

Method 5. Modified McKay confidence interval is Vangel (1996) modifying McKay's original 1932 interval. Using the approximate confidence interval (CI) for the population inverted SNR, we will reject the null hypothesis, when

$$SNR_0 < [\frac{s}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,\frac{\alpha}{2}}+2}{n}-1\right)\left(\frac{s}{\bar{x}}\right)^2+\frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n-1}}]]^{-1}$$
$$or$$
$$SNR_0 > [\frac{s}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}+2}{n}-1\right)\left(\frac{s}{\bar{x}}\right)^2+\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n-1}}]]^{-1} \quad (5)$$

2.6 Panichkitkosolkul's (2009) Method

Method 6. Panichkitkosolkul's (2009) modified the Modified Mckay (in section 2.5) method by replacing the sample inverted SNR with k the maximum likelihood estimator for a normal distribution, where $k = \frac{\sqrt{\sum(x-\bar{x})^2}}{\sqrt{n}\bar{x}}$. Using the approximate confidence interval (CI) for the population inverted SNR, we will reject the null hypothesis, when

$$SNR_0 < [k\sqrt{[\left(\frac{\chi^2_{n-1,\frac{\alpha}{2}}+2}{n}-1\right)k^2+\frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n-1}}]]^{-1}$$
$$or$$
$$SNR_0 > [k\sqrt{[\left(\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}+2}{n}-1\right)k^2+\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n-1}}]]^{-1} \quad (6)$$

2.7 Median Modified Miller

For skewed data the median describes the center of the distribution better than the mean (Kibria (2006), and Shi and Kibria (2007)) expressing that it makes more sense to measure sample variability in terms of median. The following median modifications are proposed by Kibria and George (2014) to improve performance for skewed distributions and represent both parametric and nonparametric methods.

Method 7. Null hypothesis will be rejected, when,

$$SNR_0 < (\frac{\tilde{s}}{\bar{x}}+Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}(\frac{\tilde{s}}{\bar{x}})^2[0.5+(\frac{\tilde{s}}{\bar{x}})^2]})^{-1}$$
$$or$$
$$SNR_0 > (\frac{\tilde{s}}{\bar{x}}-Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}(\frac{\tilde{s}}{\bar{x}})^2[0.5+(\frac{\tilde{s}}{\bar{x}})^2]})^{-1} \quad (7)$$

where $\tilde{s} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x-med(x))^2}$ is the modified sample variance.

2.8 Median Modification of McKay

Method 8. Reject the null hypothesis when,

$$SNR_0 < [\frac{\tilde{s}}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n}-1\right)\left(\frac{\tilde{s}}{\bar{x}}\right)^2+\frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n-1}}]]^{-1}$$
$$or$$
$$SNR_0 > [\frac{\tilde{s}}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n}-1\right)\left(\frac{\tilde{s}}{\bar{x}}\right)^2+\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n-1}}]]^{-1} \quad (8)$$

2.9 Median Modification of Modified McKay

Method 9. Reject the null hypothesis when,

$$SNR_0 < [\frac{\tilde{s}}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,\frac{\alpha}{2}}+2}{n}-1\right)\left(\frac{\tilde{s}}{\bar{x}}\right)^2+\frac{\chi^2_{n-1,\frac{\alpha}{2}}}{n-1}}]]^{-1}$$
$$or$$
$$SNR_0 > [\frac{\tilde{s}}{\bar{x}}\sqrt{[\left(\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}+2}{n}-1\right)\left(\frac{\tilde{s}}{\bar{x}}\right)^2+\frac{\chi^2_{n-1,1-\frac{\alpha}{2}}}{n-1}}]]^{-1} \quad (9)$$

2.10 Median Modification of Curto and Pinto (2009)

Method 10. Reject the null hypothesis when,

$$SNR_0 < (\frac{\tilde{s}}{\bar{x}}+Z_{\alpha/2}\sqrt{\frac{1}{n}((\frac{\tilde{s}}{\bar{x}})^4+0.5(\frac{\tilde{s}}{\bar{x}})^2})^{-1}$$
$$or$$
$$SNR_0 > (\frac{\tilde{s}}{\bar{x}}-Z_{\alpha/2}\sqrt{\frac{1}{n}((\frac{\tilde{s}}{\bar{x}})^4+0.5(\frac{\tilde{s}}{\bar{x}})^2})^{-1} \quad (10)$$

2.11 Kibria and George (2014)

Kibria and George (2014) developed Method 11 based on the normality assumption. First they developed the confidence interval for inverse of the population variance and then after some algebraic simplification they obtained the confidence interval for population SNR. From the confidence interval they found the upper and lower limits for the SNR_0, which are given below.

Method 11. Reject the null hypothesis when,

$$SNR_0 < \sqrt{\frac{\chi^2_{v,\frac{\alpha}{2}}}{(n-1)}}\widehat{SNR} \ \ or \ \ SNR_0 > \sqrt{\frac{\chi^2_{v,1-\alpha/2}}{(n-1)}}\widehat{SNR} \quad (11)$$

Now, we want to propose some new test statistics. Kibria and George (2014) modified Sharma and

Krishna's method using bootstrap technique, which is computationally expensive and time consuming and did not improve that much. In this paper, we will modify Sharma and Krishna's method using the modified standard deviation and MAD. We also modified Miller's methods and provided them in the following subsections.

2.12 Modified Kibria and George (2014)

Method 12. Reject the null hypothesis when,

$$SNR_0 < \sqrt{\frac{\chi^2_{v,\frac{\alpha}{2}}}{(n-1)}}SNR^*$$

or

$$SNR_0 > \sqrt{\frac{\chi^2_{v,1-\alpha/2}}{(n-1)}}SNR^* \quad (12)$$

where $SNR^* = \frac{\bar{x}}{\tilde{s}}$ and

$$\tilde{s} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x - med(x))^2}.$$

2.13 Modification of Miller (1991)

Method 13. Reject the null hypothesis when

$$SNR_0 < \left(\frac{s_{MAD}}{\bar{x}} + Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}}\left(\frac{s_{MAD}}{\bar{x}}\right)^2\left[0.5 + \left(\frac{s_{MAD}}{\bar{x}}\right)^2\right]\right)^{-1}$$

$$or$$

$$SNR_0 > \left(\frac{s_{MAD}}{\bar{x}} - Z_{\alpha/2}\sqrt{\frac{1}{(n-1)}}\left(\frac{s_{MAD}}{\bar{x}}\right)^2\left[0.5 + \left(\frac{s_{MAD}}{\bar{x}}\right)^2\right]\right)^{-1} \quad (13)$$

where,

$$s_{MAD} = 1.4826 \times Median(|x_i - Mean(x_j)|)$$

This formula is aa modification of Rousseeuw and Croux (1993).

2.14 Modification of Sharma and Krishna's (1994) Method

Method 14. We will reject the null hypothesis when SNR_0 is less than the lower limit or greater than the upper limit.

$$SNR_0 < \frac{\bar{x}}{s_{MAD}} - \frac{Z_{\alpha/2}}{\sqrt{n}} \quad or \quad SNR_0 > \frac{\bar{x}}{s_{MAD}} + \frac{Z_{\alpha/2}}{\sqrt{n}}$$

2.15 Median Modification of Sharma and Krishna's (1994) Method

Method 15. We reject the null hypothesis when SNR_0 is less than the lower limit or greater than the upper limit.

$$SNR_0 < \frac{\bar{x}}{\tilde{s}} - \frac{Z_{\alpha/2}}{\sqrt{n}} \quad or \quad SNR_0 > \frac{\bar{x}}{\tilde{s}} + \frac{Z_{\alpha/2}}{\sqrt{n}}$$

$$\tilde{s} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x - med(x))^2}.$$

Since a theoretical comparison among the test statistics is not possible, a Monte Carlo simulation study using R 4.2.1 to find the empirical size and power of the tests is conducted in the following Section.

3. Simulation Study

3.1 Simulation Techniques

In this section, we will compare the performance of the test statistics that are given in section 2. Simulation study is done under the symmetric (normal) and skewed (Gamma) distributional conditions.

The performance of the test statistics is examined for both small and large sample sizes (n = 30, 50, 100). Simulation will be run for 5,000. The simulation results are presented only for α=0.05, a widely used significance level. The simulation results of empirical type I error rate and the power of the tests are presented in Tables 3.1 to 3.30 for various parametric conditions. In all Tables (3.1 to 3.30), column number 6 represents the empirical size and the rest of the columns are empirical power of the tests. For more on simulation studies, we refer the readers to Shi and Kibria (2007), Banik and Kibria (2010), Kibria and George (2014), Andrew et al. (2015) and very recently Panichkitosolkul (2022) among others.

3.2 Simulation Results Discussion

3.2.1 When Data is Generated from a Normal Distribution.

We will consider a test is good test when the test attains the empirical nominal level at most 0.06 [(05-1.96*sqrt((.95*.05)/5000) =0.04 and

05+1.96*sqrt((.95*.05)/5000)=0.06)] for 5% level of significance test. When we review Tables 3.1 to 3.3, we can see that for testing SNR=0.5, and sample sizes 30 and 50, none of the tests but Methods 2 and 15 achieves the empirical nominal level 0.05. Methods 2 and 15 produce high powers on both ends. However, when the sample size is 100, Methods 1, 2, 3, 7, 10 and 15 achieved the nominal level 0.05. When we review Tables 3.4 to 3.6, we can see that for testing SNR=1, and sample size 30, none of the tests achieves the empirical nominal level 0.05 and cannot be used for testing for SNR. However, when the sample sizes are 50 and 100, all methods except methods 2, 11 to 15 achieved the nominal level 0.05. If we review Tables 3.5 and 3.6, and consider lower and upper tail powers, it appears that methods 1, 3, 7 and 10 are performing better than Methods 4, 5, 6, 8 and 9. However, method 10 proposed by Kibria and George (2014) has performed the best in the sense of attaining the nominal size and high empirical power. When we review Tables 3.7 to 3.9, we can see that for testing SNR=2, and for all small sample sizes, all methods except methods 2, 11 to 15 achieved the nominal level 0.05 and gave high empirical power. Again, method 10 has performed the best in the sense of attaining the nominal size and high empirical power. When we review Tables 3.10 to 3.12, we can see that for testing SNR=5, all methods except 2, 13 to 15 achieved the nominal level 0.05 for sample sizes 50 and 100. Proposed method 12 performed better than Method 11 for testing the high values of SNR, while method 11 for low values. When we review Tables 3.13 to 3.15, we can see that for testing SNR=10, all methods except 2, 13 to 15 achieved the nominal level 0.05 for all sample sizes. However, the proposed method 12 performed better than Method 11 for testing the high values of SNR, while method 11 performed better than Method 12 for low values of SNR.

3.2.2 When Data is Generated from a Gamma Distribution.

The simulated empirical sizes and powers are tabulated in Tables 3.16 to 3.30 when data is generated from a Gamma distribution. When we review Table 3.16, we can see that for testing SNR=0.5, and small sample size 30, Methods 2, 7, 10, and proposed methods 13- 15 attained the nominal level. Proposed method 15 performed the best among all methods. Tables 3.17 and 3.18 indicate that the proposed Method 15 performed the best. When we review Tables 3.19 to 3.21, we can see that for testing SNR=1, all methods except Methods 11-14 attained the nominal level. Proposed method 15 produces the highest power at the high value of SNR and methods 1 & 2 for the low values. When we review Tables 3.22-324, we can see that for testing SNR=2, and n=30, 50 and 100, methods 1, 3, 4 through 10 attained the nominal level and produced high power for large sample sizes.

Method 10 performed the best in the sense of highest power at both ends. When we review Tables 3.25-3.27, we can see that for testing SNR=5, Methods 1, 3, 4, 5, 7 through 12 attained the nominal level for all sample sizes. Proposed method 15 produces the highest power at the high value of SNR and method 3 for the low values. All empirical powers are lower than 55%. When we review Tables 3.28-3.30, we can see that for testing SNR=10, Methods 1, 3, 4, 5, 7 through 12 attained the nominal level for all sample sizes. Proposed method 15 produces the highest power at the high value of SNR, while Method 3 for the low values.

Table 3.1: Empirical type I error rate and power of tests for Normal (2.5, 25), SNR = 0.5, n = 30

SNR0	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900
Method1	1.000	0.973	0.714	0.491	0.354	0.287	0.263	0.256	0.253
Method2	0.593	0.395	0.223	0.122	0.072	0.096	0.200	0.379	0.577
Method3	1.000	0.964	0.707	0.485	0.345	0.277	0.252	0.244	0.241
Method4	1.000	1.000	1.000	1.000	1.000	1.000	0.397	0.122	0.023
Method5	1.000	1.000	1.000	1.000	1.000	0.530	0.177	0.041	0.011
Method6	1.000	1.000	1.000	1.000	1.000	0.569	0.195	0.064	0.013
Method7	1.000	0.972	0.711	0.490	0.354	0.291	0.268	0.260	0.259
Method8	1.000	1.000	1.000	1.000	1.000	1.000	0.354	0.087	0.024
Method9	1.000	1.000	1.000	1.000	1.000	0.531	0.173	0.035	0.012
Method10	1.000	0.964	0.703	0.483	0.346	0.280	0.256	0.249	0.247
Method11	0.989	0.927	0.775	0.593	0.494	0.524	0.626	0.749	0.846
Method12	0.989	0.925	0.769	0.585	0.493	0.528	0.634	0.756	0.852
Method13	1.000	0.972	0.727	0.525	0.397	0.325	0.288	0.273	0.266
Method14	0.589	0.403	0.255	0.153	0.103	0.124	0.228	0.394	0.577
Method15	0.585	0.387	0.216	0.115	0.070	0.096	0.202	0.384	0.587

Table 3.2: Empirical type I error rate and power of tests for Normal (2.5, 25), SNR = 0.5, n = 50

SNR0	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900
Method1	1.000	0.901	0.609	0.319	0.152	0.090	0.073	0.069	0.068
Method2	0.798	0.561	0.309	0.129	0.067	0.122	0.301	0.561	0.791
Method3	1.000	0.900	0.609	0.319	0.149	0.087	0.070	0.065	0.065
Method4	1.000	1.000	1.000	1.000	1.000	0.328	0.082	0.011	0.004
Method5	1.000	1.000	1.000	1.000	0.688	0.189	0.044	0.007	0.002
Method6	1.000	1.000	1.000	1.000	0.707	0.192	0.050	0.007	0.002
Method7	1.000	0.898	0.605	0.313	0.149	0.089	0.074	0.070	0.070
Method8	1.000	1.000	1.000	1.000	1.000	0.332	0.079	0.012	0.004
Method9	1.000	1.000	1.000	1.000	0.693	0.185	0.046	0.007	0.002
Method10	1.000	0.898	0.605	0.313	0.146	0.087	0.072	0.067	0.067
Method11	0.999	0.968	0.845	0.626	0.495	0.562	0.723	0.865	0.944
Method12	0.999	0.968	0.841	0.623	0.496	0.567	0.729	0.870	0.947
Method13	1.000	0.891	0.604	0.338	0.189	0.109	0.086	0.078	0.074
Method14	0.789	0.549	0.314	0.162	0.094	0.150	0.318	0.567	0.774
Method15	0.793	0.552	0.300	0.126	0.064	0.124	0.305	0.570	0.795

Table 3.3: Empirical type I error rate and power of tests for Normal (2.5, 25), SNR = 0.5, n = 100

SNR0	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900
Method1	1.000	0.959	0.724	0.317	0.066	0.007	0.002	0.002	0.677
Method2	0.980	0.841	0.523	0.187	0.062	0.181	0.507	0.837	0.968
Method3	1.000	0.959	0.725	0.320	0.066	0.007	0.002	0.001	0.691
Method4	1.000	1.000	1.000	1.000	0.230	0.021	0.001	0.000	0.000
Method5	1.000	1.000	1.000	0.829	0.153	0.012	0.001	0.000	0.000
Method6	1.000	1.000	1.000	0.837	0.153	0.014	0.001	0.000	0.000
Method7	1.000	0.958	0.720	0.312	0.063	0.007	0.002	0.002	0.683
Method8	1.000	1.000	1.000	1.000	0.231	0.020	0.001	0.000	0.000
Method9	1.000	1.000	1.000	0.830	0.153	0.013	0.001	0.000	0.000
Method10	1.000	0.959	0.721	0.314	0.064	0.007	0.002	0.002	0.695
Method11	1.000	0.998	0.943	0.709	0.504	0.629	0.861	0.968	0.996
Method12	1.000	0.998	0.942	0.705	0.503	0.631	0.866	0.969	0.996
Method13	1.000	0.955	0.711	0.333	0.095	0.018	0.003	0.002	0.657
Method14	0.975	0.836	0.522	0.219	0.091	0.199	0.510	0.801	0.947
Method15	0.979	0.837	0.518	0.183	0.060	0.183	0.513	0.840	0.969

Table 3.4: Empirical type I error rate and power of tests for Normal (5, 25), SNR = 1, n = 30

SNR0	0.400	0.600	0.700	0.800	1.000	1.200	1.300	1.400	1.500
Method1	0.962	0.705	0.491	0.298	0.082	0.016	0.007	0.003	0.001
Method2	0.882	0.584	0.410	0.258	0.122	0.246	0.398	0.566	0.723
Method3	0.964	0.710	0.498	0.305	0.084	0.017	0.007	0.003	0.001
Method4	1.000	1.000	0.741	0.456	0.128	0.025	0.011	0.004	0.001
Method5	1.000	0.877	0.610	0.366	0.102	0.020	0.008	0.003	0.001
Method6	1.000	0.888	0.629	0.396	0.114	0.023	0.010	0.004	0.001
Method7	0.960	0.694	0.477	0.285	0.076	0.015	0.006	0.002	0.001
Method8	1.000	1.000	0.736	0.445	0.123	0.023	0.010	0.003	0.001
Method9	1.000	0.868	0.596	0.354	0.097	0.019	0.008	0.002	0.001
Method10	0.961	0.700	0.484	0.290	0.079	0.015	0.007	0.002	0.001
Method11	0.992	0.831	0.627	0.415	0.252	0.425	0.557	0.680	0.787
Method12	0.991	0.820	0.613	0.403	0.255	0.441	0.571	0.696	0.799
Method13	0.942	0.678	0.500	0.340	0.134	0.051	0.031	0.019	0.012
Method14	0.850	0.581	0.432	0.311	0.195	0.320	0.439	0.573	0.693
Method15	0.877	0.570	0.396	0.247	0.120	0.251	0.411	0.580	0.736

Table 3.5: Empirical type I error rate and power of tests for Normal (5, 25), SNR = 1, n = 50

SNR0	0.400	0.600	0.700	0.800	1.000	1.200	1.300	1.400	1.500
Method1	0.995	0.850	0.632	0.373	0.064	0.005	0.061	0.217	0.434
Method2	0.981	0.774	0.568	0.341	0.115	0.323	0.543	0.745	0.882
Method3	0.995	0.855	0.635	0.378	0.065	0.008	0.071	0.233	0.453
Method4	1.000	0.882	0.654	0.380	0.068	0.006	0.002	0.001	0.000
Method5	1.000	0.844	0.614	0.352	0.059	0.005	0.002	0.001	0.000
Method6	1.000	0.854	0.636	0.376	0.069	0.006	0.002	0.001	0.000
Method7	0.994	0.842	0.622	0.358	0.059	0.005	0.064	0.226	0.445
Method8	1.000	0.877	0.644	0.366	0.063	0.006	0.002	0.001	0.000
Method9	1.000	0.836	0.601	0.340	0.055	0.005	0.002	0.001	0.000
Method10	0.995	0.846	0.627	0.364	0.060	0.008	0.074	0.241	0.465
Method11	0.999	0.945	0.785	0.535	0.248	0.501	0.678	0.824	0.909
Method12	0.999	0.943	0.777	0.523	0.251	0.514	0.689	0.831	0.913
Method13	0.988	0.808	0.612	0.397	0.112	0.023	0.098	0.256	0.445
Method14	0.964	0.741	0.558	0.378	0.188	0.375	0.541	0.705	0.830
Method15	0.980	0.765	0.557	0.330	0.111	0.331	0.554	0.757	0.889

Table 3.6: Empirical type I error rate and power of tests for Normal (5, 25), SNR = 1, n = 100

SNR0	0.400	0.600	0.700	0.800	1.000	1.200	1.300	1.400	1.500
Method1	1.000	0.979	0.842	0.533	0.055	0.160	0.428	0.728	0.908
Method2	1.000	0.963	0.817	0.528	0.112	0.499	0.791	0.936	0.985
Method3	1.000	0.979	0.845	0.537	0.056	0.164	0.433	0.735	0.908
Method4	1.000	0.964	0.812	0.499	0.050	0.000	0.000	0.132	0.695
Method5	1.000	0.960	0.799	0.481	0.047	0.000	0.000	0.371	0.778
Method6	1.000	0.963	0.810	0.498	0.051	0.000	0.000	0.357	0.767
Method7	1.000	0.978	0.838	0.526	0.053	0.164	0.436	0.738	0.910
Method8	1.000	0.963	0.807	0.489	0.048	0.000	0.000	0.134	0.704
Method9	1.000	0.959	0.795	0.472	0.045	0.000	0.000	0.383	0.785
Method10	1.000	0.978	0.841	0.529	0.054	0.169	0.441	0.743	0.914
Method11	1.000	0.998	0.951	0.743	0.250	0.660	0.864	0.957	0.987
Method12	1.000	0.997	0.949	0.736	0.251	0.666	0.869	0.959	0.988
Method13	1.000	0.957	0.799	0.523	0.116	0.216	0.453	0.677	0.844
Method14	1.000	0.935	0.775	0.519	0.200	0.520	0.740	0.883	0.954
Method15	1.000	0.961	0.812	0.519	0.111	0.508	0.797	0.940	0.985

Table 3.7: Empirical type I error rate and power of tests for Normal (10, 25), SNR = 2, n = 30

SNR0	1.200	1.400	1.600	1.800	2.000	2.200	2.400	2.600	2.800
Method1	0.926	0.712	0.413	0.186	0.069	0.028	0.061	0.169	0.349
Method2	0.951	0.813	0.584	0.365	0.269	0.355	0.555	0.745	0.877
Method3	0.929	0.719	0.421	0.191	0.073	0.032	0.069	0.187	0.369
Method4	0.931	0.724	0.426	0.195	0.073	0.022	0.008	0.056	0.218
Method5	0.925	0.715	0.415	0.190	0.072	0.021	0.022	0.109	0.285
Method6	0.939	0.748	0.457	0.220	0.084	0.027	0.020	0.094	0.252
Method7	0.916	0.692	0.394	0.174	0.063	0.028	0.066	0.183	0.367
Method8	0.922	0.708	0.409	0.181	0.068	0.020	0.007	0.062	0.232
Method9	0.915	0.694	0.398	0.176	0.065	0.019	0.025	0.119	0.306
Method10	0.920	0.703	0.403	0.178	0.067	0.033	0.076	0.201	0.386
Method11	0.923	0.678	0.358	0.152	0.107	0.193	0.360	0.557	0.724
Method12	0.914	0.658	0.337	0.145	0.111	0.208	0.379	0.576	0.744
Method13	0.854	0.655	0.442	0.276	0.161	0.117	0.161	0.267	0.402
Method14	0.890	0.740	0.568	0.451	0.431	0.503	0.619	0.721	0.816
Method15	0.945	0.799	0.561	0.346	0.265	0.365	0.570	0.762	0.885

Table 3.8: Empirical type I error rate and power of tests for Normal (10, 25), SNR = 2, n = 50

SNR0	1.200	1.400	1.600	1.800	2.000	2.200	2.400	2.600	2.800
Method1	0.990	0.866	0.532	0.206	0.058	0.050	0.178	0.429	0.688
Method2	0.994	0.934	0.718	0.409	0.256	0.404	0.682	0.882	0.964
Method3	0.990	0.871	0.539	0.210	0.061	0.053	0.184	0.441	0.700
Method4	0.990	0.873	0.542	0.212	0.058	0.025	0.112	0.344	0.626
Method5	0.989	0.862	0.531	0.206	0.057	0.034	0.142	0.382	0.657
Method6	0.991	0.883	0.566	0.231	0.065	0.031	0.121	0.350	0.623
Method7	0.988	0.856	0.513	0.192	0.056	0.054	0.188	0.447	0.703
Method8	0.988	0.863	0.523	0.201	0.056	0.026	0.119	0.362	0.646
Method9	0.988	0.853	0.512	0.194	0.054	0.036	0.152	0.405	0.677
Method10	0.988	0.861	0.520	0.198	0.058	0.056	0.198	0.459	0.713
Method11	0.993	0.879	0.523	0.193	0.108	0.236	0.495	0.737	0.888
Method12	0.992	0.870	0.504	0.182	0.111	0.251	0.514	0.749	0.896
Method13	0.954	0.780	0.530	0.293	0.163	0.164	0.279	0.452	0.634
Method14	0.969	0.858	0.657	0.477	0.430	0.530	0.689	0.817	0.900
Method15	0.994	0.928	0.703	0.395	0.255	0.415	0.699	0.889	0.967

Table 3.9: Empirical type I error rate and power of tests for Normal (10,25), SNR = 2, n = 100

SNR0	1.200	1.400	1.600	1.800	2.000	2.200	2.400	2.600	2.800
Method1	1.000	0.989	0.777	0.301	0.052	0.122	0.466	0.808	0.961
Method2	1.000	0.997	0.903	0.526	0.262	0.523	0.856	0.980	0.998
Method3	1.000	0.989	0.779	0.304	0.053	0.124	0.471	0.810	0.962
Method4	1.000	0.989	0.778	0.304	0.049	0.087	0.419	0.779	0.956
Method5	1.000	0.988	0.771	0.298	0.049	0.097	0.436	0.793	0.960
Method6	1.000	0.990	0.790	0.314	0.053	0.089	0.415	0.773	0.954
Method7	1.000	0.987	0.767	0.292	0.050	0.129	0.479	0.814	0.963
Method8	1.000	0.987	0.768	0.295	0.047	0.092	0.431	0.790	0.960
Method9	1.000	0.986	0.763	0.289	0.046	0.104	0.447	0.802	0.961
Method10	1.000	0.988	0.769	0.295	0.051	0.132	0.483	0.818	0.964
Method11	1.000	0.994	0.811	0.319	0.105	0.336	0.708	0.929	0.989
Method12	1.000	0.993	0.802	0.312	0.105	0.348	0.718	0.933	0.990
Method13	0.999	0.939	0.708	0.365	0.169	0.237	0.478	0.725	0.889
Method14	0.999	0.967	0.817	0.556	0.439	0.576	0.789	0.926	0.978
Method15	1.000	0.996	0.897	0.512	0.262	0.534	0.863	0.981	0.998

Table 3.10: Empirical type I error rate and power of tests for Normal (25, 25), SNR = 5, n = 30

SNR0	4.000	4.400	4.600	4.800	5.000	5.200	5.400	5.600	6.400
Method1	0.489	0.246	0.165	0.105	0.069	0.048	0.047	0.066	0.297
Method2	0.869	0.707	0.643	0.610	0.611	0.631	0.680	0.729	0.916
Method3	0.498	0.254	0.172	0.109	0.072	0.050	0.052	0.072	0.313
Method4	0.495	0.252	0.170	0.109	0.070	0.048	0.044	0.063	0.290
Method5	0.493	0.250	0.169	0.108	0.070	0.049	0.048	0.067	0.299
Method6	0.544	0.292	0.197	0.127	0.084	0.056	0.045	0.058	0.257
Method7	0.467	0.227	0.153	0.098	0.064	0.046	0.052	0.072	0.319
Method8	0.472	0.231	0.157	0.100	0.065	0.047	0.049	0.067	0.313
Method9	0.470	0.230	0.156	0.099	0.066	0.047	0.051	0.072	0.321
Method10	0.473	0.235	0.158	0.102	0.067	0.051	0.056	0.079	0.336
Method11	0.329	0.139	0.090	0.062	0.062	0.083	0.122	0.184	0.528
Method12	0.312	0.131	0.083	0.060	0.065	0.087	0.136	0.205	0.548
Method13	0.503	0.337	0.281	0.239	0.214	0.198	0.200	0.212	0.377
Method14	0.818	0.750	0.730	0.726	0.737	0.745	0.769	0.798	0.888
Method15	0.854	0.691	0.631	0.612	0.617	0.637	0.691	0.741	0.922

Table 3.11: Empirical type I error rate and power of tests for Normal (25,25), SNR = 5, n = 50

SNR0	4.000	4.400	4.600	4.800	5.000	5.200	5.400	5.600	6.400
Method1	0.659	0.312	0.181	0.100	0.057	0.044	0.067	0.109	0.541
Method2	0.944	0.785	0.689	0.614	0.598	0.623	0.696	0.774	0.970
Method3	0.667	0.320	0.186	0.104	0.059	0.047	0.070	0.114	0.548
Method4	0.667	0.318	0.186	0.103	0.058	0.044	0.064	0.107	0.536
Method5	0.665	0.315	0.185	0.102	0.058	0.045	0.066	0.109	0.541
Method6	0.701	0.351	0.207	0.118	0.064	0.045	0.058	0.095	0.501
Method7	0.640	0.297	0.170	0.094	0.053	0.046	0.073	0.119	0.560
Method8	0.645	0.302	0.174	0.096	0.053	0.045	0.071	0.117	0.555
Method9	0.644	0.300	0.173	0.095	0.054	0.046	0.072	0.119	0.560
Method10	0.646	0.302	0.175	0.097	0.056	0.047	0.076	0.124	0.568
Method11	0.536	0.206	0.115	0.064	0.057	0.085	0.146	0.240	0.714
Method12	0.516	0.193	0.108	0.062	0.061	0.092	0.158	0.257	0.729
Method13	0.605	0.389	0.300	0.236	0.204	0.197	0.216	0.256	0.530
Method14	0.880	0.774	0.737	0.731	0.735	0.756	0.775	0.804	0.928
Method15	0.937	0.771	0.672	0.605	0.596	0.634	0.707	0.782	0.975

Table 3.12: Empirical type I error rate and power of tests for Normal (25, 25), SNR = 5, n = 100

SNR0	4.000	4.400	4.600	4.800	5.000	5.200	5.400	5.600	6.400
Method1	0.892	0.480	0.263	0.130	0.064	0.059	0.126	0.263	0.867
Method2	0.993	0.892	0.761	0.634	0.598	0.667	0.777	0.862	0.997
Method3	0.894	0.483	0.265	0.132	0.065	0.061	0.129	0.268	0.868
Method4	0.895	0.483	0.265	0.131	0.064	0.058	0.124	0.259	0.866
Method5	0.894	0.482	0.263	0.130	0.064	0.059	0.126	0.262	0.867
Method6	0.906	0.506	0.284	0.145	0.071	0.053	0.115	0.241	0.852
Method7	0.885	0.467	0.255	0.126	0.061	0.061	0.134	0.276	0.872
Method8	0.887	0.470	0.256	0.127	0.061	0.059	0.131	0.273	0.871
Method9	0.886	0.469	0.255	0.127	0.061	0.061	0.133	0.275	0.871
Method10	0.886	0.470	0.256	0.128	0.062	0.064	0.136	0.281	0.873
Method11	0.852	0.403	0.212	0.104	0.061	0.106	0.226	0.397	0.917
Method12	0.843	0.390	0.200	0.100	0.060	0.113	0.238	0.411	0.923
Method13	0.786	0.496	0.365	0.276	0.225	0.225	0.289	0.366	0.757
Method14	0.951	0.833	0.775	0.751	0.737	0.765	0.810	0.849	0.966
Method15	0.992	0.885	0.751	0.628	0.598	0.672	0.785	0.866	0.998

Table 3.13: Empirical type I error rate and power of tests for Normal (250, 25), SNR = 10, n = 30

SNR0	6.000	7.000	8.000	9.000	10.000	11.000	12.000	13.000	14.000
Method1	0.991	0.869	0.503	0.202	0.067	0.059	0.146	0.359	0.572
Method2	1.000	0.995	0.946	0.825	0.794	0.843	0.921	0.970	0.989
Method3	0.992	0.873	0.513	0.208	0.070	0.065	0.159	0.376	0.588
Method4	0.991	0.871	0.508	0.205	0.068	0.060	0.147	0.362	0.574
Method5	0.991	0.871	0.508	0.205	0.068	0.062	0.149	0.364	0.575
Method6	0.995	0.897	0.555	0.240	0.081	0.054	0.123	0.315	0.530
Method7	0.989	0.850	0.482	0.185	0.061	0.064	0.162	0.385	0.596
Method8	0.989	0.854	0.488	0.189	0.063	0.065	0.163	0.389	0.598
Method9	0.989	0.853	0.487	0.189	0.063	0.065	0.165	0.389	0.601
Method10	0.990	0.856	0.491	0.192	0.065	0.068	0.176	0.401	0.611
Method11	0.963	0.707	0.330	0.104	0.053	0.128	0.343	0.570	0.765
Method12	0.957	0.682	0.311	0.096	0.057	0.142	0.369	0.593	0.779
Method13	0.938	0.752	0.515	0.329	0.219	0.209	0.301	0.423	0.548
Method14	0.996	0.965	0.905	0.867	0.868	0.897	0.914	0.947	0.969
Method15	1.000	0.994	0.942	0.813	0.796	0.849	0.928	0.974	0.990

Table 3.14: Empirical type I error rate and power of tests for Normal (250, 25), SNR = 10, n = 50

SNR0	6.000	7.000	8.000	9.000	10.000	11.000	12.000	13.000	14.000
Method1	0.999	0.969	0.693	0.262	0.065	0.088	0.308	0.616	0.838
Method2	1.000	0.999	0.983	0.875	0.776	0.867	0.953	0.989	0.998
Method3	0.999	0.971	0.697	0.268	0.068	0.093	0.318	0.623	0.844
Method4	0.999	0.970	0.696	0.265	0.067	0.088	0.308	0.616	0.838
Method5	0.999	0.970	0.695	0.265	0.066	0.089	0.310	0.617	0.839
Method6	0.999	0.977	0.729	0.296	0.075	0.080	0.276	0.583	0.811
Method7	0.999	0.965	0.671	0.245	0.062	0.096	0.329	0.632	0.847
Method8	0.999	0.965	0.676	0.248	0.063	0.097	0.330	0.633	0.848
Method9	0.999	0.965	0.674	0.248	0.063	0.098	0.331	0.634	0.849
Method10	0.999	0.965	0.678	0.252	0.065	0.102	0.338	0.640	0.852
Method11	0.999	0.931	0.546	0.161	0.056	0.174	0.476	0.754	0.914
Method12	0.999	0.923	0.525	0.153	0.058	0.187	0.498	0.772	0.921
Method13	0.989	0.883	0.632	0.360	0.220	0.251	0.398	0.576	0.730
Method14	0.999	0.991	0.940	0.885	0.861	0.892	0.939	0.967	0.983
Method15	1.000	0.999	0.981	0.868	0.781	0.874	0.957	0.990	0.998

Table 3.15: Empirical type I error rate and power of tests for Normal (250, 25), SNR = 10, n = 100

SNR0	6.000	7.000	8.000	9.000	10.000	11.000	12.000	13.000	14.000
Method1	1.000	0.999	0.917	0.369	0.055	0.194	0.654	0.926	0.993
Method2	1.000	1.000	0.999	0.927	0.776	0.914	0.989	1.000	1.000
Method3	1.000	0.999	0.920	0.374	0.057	0.197	0.659	0.926	0.993
Method4	1.000	0.999	0.919	0.372	0.056	0.194	0.653	0.926	0.993
Method5	1.000	0.999	0.919	0.372	0.056	0.195	0.655	0.926	0.993
Method6	1.000	0.999	0.930	0.398	0.059	0.175	0.629	0.919	0.991
Method7	1.000	0.999	0.910	0.358	0.055	0.205	0.667	0.929	0.993
Method8	1.000	0.999	0.911	0.359	0.056	0.205	0.667	0.929	0.993
Method9	1.000	0.999	0.911	0.359	0.056	0.206	0.668	0.929	0.993
Method10	1.000	0.999	0.911	0.361	0.057	0.209	0.672	0.930	0.993
Method11	1.000	0.999	0.862	0.279	0.056	0.295	0.756	0.957	0.996
Method12	1.000	0.999	0.854	0.265	0.056	0.312	0.765	0.959	0.996
Method13	1.000	0.983	0.808	0.438	0.219	0.330	0.592	0.815	0.931
Method14	1.000	0.999	0.980	0.905	0.867	0.902	0.966	0.988	0.997
Method15	1.000	1.000	0.999	0.922	0.780	0.920	0.989	1.000	1.000

Table 3.16: Empirical type I error rate and power of tests for Gamma (0.25,2), SNR = 0.5, n = 30

SNR0	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900
Method1	1.000	0.988	0.689	0.268	0.073	0.039	0.036	0.036	0.036
Method2	0.842	0.527	0.203	0.044	0.006	0.001	0.018	0.122	0.421
Method3	1.000	0.986	0.695	0.273	0.072	0.036	0.032	0.032	0.032
Method4	1.000	1.000	1.000	1.000	1.000	1.000	0.125	0.000	0.000
Method5	1.000	1.000	1.000	1.000	1.000	0.278	0.028	0.000	0.000
Method6	1.000	1.000	1.000	1.000	1.000	0.340	0.021	0.000	0.000
Method7	1.000	0.975	0.512	0.139	0.066	0.058	0.057	0.057	0.057
Method8	1.000	1.000	1.000	1.000	1.000	1.000	0.000	0.000	0.000
Method9	1.000	1.000	1.000	1.000	1.000	0.500	0.000	0.000	0.000
Method10	1.000	0.973	0.516	0.138	0.060	0.051	0.050	0.050	0.050
Method11	1.000	0.999	0.939	0.605	0.220	0.223	0.472	0.737	0.915
Method12	1.000	0.999	0.898	0.401	0.148	0.353	0.690	0.901	0.975
Method13	1.000	1.000	1.000	1.000	0.086	0.008	0.001	0.001	0.000
Method14	1.000	1.000	1.000	0.116	0.015	0.003	0.001	0.000	0.000
Method15	0.740	0.309	0.070	0.012	0.002	0.001	0.026	0.209	0.631

Table 3.17: Empirical type I error rate and power of tests for Gamma (0.25,2), SNR = 0.5, n = 50

SNR0	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900
Method1	1.000	0.985	0.746	0.260	0.026	0.002	0.001	0.001	0.001
Method2	0.974	0.768	0.350	0.067	0.004	0.005	0.084	0.409	0.813
Method3	1.000	0.985	0.752	0.265	0.027	0.001	0.001	0.001	0.001
Method4	1.000	1.000	1.000	1.000	1.000	0.048	0.000	0.000	0.000
Method5	1.000	1.000	1.000	1.000	0.393	0.015	0.000	0.000	0.000
Method6	1.000	1.000	1.000	1.000	0.447	0.012	0.000	0.000	0.000
Method7	1.000	0.971	0.568	0.080	0.005	0.001	0.001	0.001	0.001
Method8	1.000	1.000	1.000	1.000	1.000	0.000	0.000	0.000	0.000
Method9	1.000	1.000	1.000	1.000	0.409	0.000	0.000	0.000	0.000
Method10	1.000	0.973	0.574	0.083	0.005	0.001	0.001	0.001	0.001
Method11	1.000	1.000	0.978	0.703	0.260	0.327	0.684	0.921	0.990
Method12	1.000	1.000	0.955	0.494	0.192	0.535	0.884	0.985	0.999
Method13	1.000	1.000	1.000	1.000	0.266	0.005	0.000	0.000	0.000
Method14	1.000	1.000	1.000	0.992	0.029	0.002	0.000	0.000	0.000
Method15	0.951	0.603	0.137	0.012	0.001	0.010	0.157	0.649	0.949

Table 3.18: Empirical type I error rate and power of tests for Gamma (0.25,2), SNR = 0.5, n = 100

SNR0	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900
Method1	1.000	0.999	0.891	0.326	0.018	0.000	0.000	0.000	0.706
Method2	0.999	0.969	0.663	0.134	0.007	0.038	0.385	0.890	0.996
Method3	1.000	0.999	0.892	0.329	0.019	0.000	0.000	0.000	0.724
Method4	1.000	1.000	1.000	1.000	0.060	0.000	0.000	0.000	0.000
Method5	1.000	1.000	1.000	0.765	0.031	0.000	0.000	0.000	0.000
Method6	1.000	1.000	1.000	0.763	0.030	0.000	0.000	0.000	0.000
Method7	1.000	0.997	0.775	0.095	0.001	0.000	0.000	0.000	0.921
Method8	1.000	1.000	1.000	1.000	0.011	0.000	0.000	0.000	0.000
Method9	1.000	1.000	1.000	0.608	0.004	0.000	0.000	0.000	0.000
Method10	1.000	0.997	0.778	0.097	0.001	0.000	0.000	0.000	0.927
Method11	1.000	1.000	0.996	0.817	0.292	0.528	0.916	0.996	1.000
Method12	1.000	1.000	0.989	0.629	0.259	0.805	0.989	1.000	1.000
Method13	1.000	1.000	1.000	1.000	1.000	0.004	0.000	0.000	0.000
Method14	1.000	1.000	1.000	1.000	0.642	0.001	0.000	0.000	0.575
Method15	0.999	0.933	0.385	0.021	0.001	0.086	0.670	0.984	1.000

Table 3.19: Empirical type I error rate and power of tests for Gamma (1,2), SNR = 1, n = 30

SNR0	0.400	0.600	0.700	0.800	1.000	1.200	1.300	1.400	1.500
Method1	0.998	0.847	0.589	0.314	0.041	0.002	0.001	0.000	0.000
Method2	0.972	0.718	0.476	0.260	0.048	0.096	0.248	0.491	0.711
Method3	0.998	0.852	0.598	0.323	0.044	0.003	0.001	0.000	0.000
Method4	1.000	1.000	0.725	0.393	0.055	0.004	0.001	0.000	0.000
Method5	1.000	0.910	0.613	0.325	0.044	0.002	0.001	0.000	0.000
Method6	1.000	0.916	0.647	0.358	0.053	0.004	0.001	0.000	0.000
Method7	0.995	0.751	0.471	0.235	0.032	0.002	0.001	0.000	0.000
Method8	1.000	1.000	0.676	0.344	0.051	0.003	0.001	0.000	0.000
Method9	1.000	0.857	0.527	0.265	0.037	0.002	0.001	0.000	0.000
Method10	0.995	0.761	0.480	0.242	0.035	0.002	0.001	0.000	0.000
Method11	1.000	0.944	0.756	0.435	0.102	0.280	0.474	0.657	0.803
Method12	1.000	0.895	0.631	0.333	0.133	0.411	0.590	0.744	0.858
Method13	1.000	0.914	0.656	0.380	0.085	0.018	0.008	0.004	0.002
Method14	1.000	0.793	0.537	0.332	0.092	0.057	0.190	0.433	0.645
Method15	0.943	0.586	0.365	0.193	0.041	0.164	0.373	0.608	0.783

Table 3.20: Empirical type I error rate and power of tests for Gamma (1,2), SNR = 1, n = 50

SNR0	0.400	0.600	0.700	0.800	1.000	1.200	1.300	1.400	1.500
Method1	0.999	0.941	0.747	0.397	0.032	0.000	0.012	0.098	0.337
Method2	0.998	0.894	0.664	0.360	0.047	0.192	0.478	0.749	0.916
Method3	0.999	0.943	0.753	0.404	0.033	0.001	0.017	0.112	0.363
Method4	1.000	0.941	0.733	0.388	0.032	0.001	0.000	0.000	0.000
Method5	1.000	0.926	0.700	0.361	0.029	0.000	0.000	0.000	0.000
Method6	1.000	0.931	0.728	0.388	0.034	0.001	0.000	0.000	0.000
Method7	0.999	0.887	0.593	0.285	0.022	0.000	0.027	0.193	0.491
Method8	1.000	0.887	0.584	0.280	0.022	0.000	0.000	0.000	0.000
Method9	1.000	0.853	0.549	0.256	0.020	0.000	0.000	0.000	0.000
Method10	0.999	0.890	0.599	0.289	0.023	0.001	0.037	0.213	0.516
Method11	1.000	0.989	0.901	0.605	0.122	0.415	0.662	0.844	0.946
Method12	1.000	0.975	0.807	0.460	0.154	0.562	0.763	0.897	0.964
Method13	1.000	0.993	0.852	0.510	0.084	0.009	0.003	0.027	0.222
Method14	1.000	0.970	0.779	0.469	0.100	0.108	0.367	0.658	0.842
Method15	0.994	0.795	0.511	0.255	0.042	0.323	0.618	0.827	0.945

Table 3.21: Empirical type I error rate and power of tests for Gamma (1,2), SNR = 1, n = 100

SNR0	0.400	0.600	0.700	0.800	1.000	1.200	1.300	1.400	1.500
Method1	1.000	0.995	0.921	0.605	0.022	0.080	0.352	0.736	0.946
Method2	1.000	0.989	0.900	0.595	0.049	0.437	0.810	0.971	0.998
Method3	1.000	0.995	0.922	0.608	0.022	0.084	0.360	0.741	0.948
Method4	1.000	0.989	0.894	0.559	0.019	0.000	0.000	0.082	0.701
Method5	1.000	0.988	0.885	0.541	0.017	0.000	0.000	0.289	0.798
Method6	1.000	0.989	0.893	0.558	0.020	0.000	0.000	0.275	0.780
Method7	1.000	0.983	0.822	0.413	0.009	0.180	0.540	0.850	0.973
Method8	1.000	0.976	0.778	0.363	0.007	0.000	0.000	0.168	0.825
Method9	1.000	0.972	0.761	0.343	0.006	0.000	0.000	0.481	0.896
Method10	1.000	0.983	0.824	0.416	0.010	0.187	0.548	0.856	0.975
Method11	1.000	1.000	0.984	0.827	0.138	0.646	0.899	0.985	0.999
Method12	1.000	0.999	0.958	0.677	0.193	0.786	0.951	0.995	0.999
Method13	1.000	1.000	0.992	0.803	0.076	0.010	0.161	0.551	0.852
Method14	1.000	1.000	0.987	0.795	0.101	0.234	0.652	0.911	0.985
Method15	1.000	0.974	0.789	0.405	0.047	0.631	0.905	0.989	0.999

Table 3.22: Empirical type I error rate and power of tests for Gamma (4,2), SNR = 2, n = 30

SNR0	1.200	1.400	1.600	1.800	2.000	2.200	2.400	2.600	2.800
Method1	0.955	0.749	0.421	0.167	0.051	0.018	0.032	0.128	0.314
Method2	0.974	0.856	0.614	0.350	0.213	0.296	0.525	0.755	0.898
Method3	0.957	0.759	0.433	0.174	0.054	0.020	0.040	0.142	0.337
Method4	0.959	0.763	0.438	0.178	0.054	0.015	0.004	0.029	0.173
Method5	0.955	0.750	0.426	0.172	0.052	0.015	0.011	0.071	0.249
Method6	0.964	0.792	0.472	0.206	0.066	0.019	0.009	0.058	0.208
Method7	0.930	0.697	0.380	0.144	0.043	0.020	0.053	0.169	0.365
Method8	0.936	0.712	0.397	0.154	0.047	0.014	0.003	0.050	0.223
Method9	0.929	0.699	0.385	0.148	0.045	0.014	0.017	0.105	0.300
Method10	0.934	0.707	0.391	0.151	0.047	0.024	0.061	0.187	0.388
Method11	0.954	0.712	0.352	0.123	0.069	0.146	0.328	0.543	0.737
Method12	0.929	0.662	0.316	0.116	0.087	0.190	0.376	0.586	0.768
Method13	0.917	0.715	0.475	0.267	0.142	0.083	0.097	0.193	0.340
Method14	0.942	0.805	0.610	0.436	0.360	0.426	0.558	0.707	0.824
Method15	0.954	0.811	0.562	0.320	0.225	0.340	0.570	0.782	0.910

Table 3.23: Empirical type I error rate and power of tests for Gamma (4,2), SNR = 2, n = 50

SNR0	1.200	1.400	1.600	1.800	2.000	2.200	2.400	2.600	2.800
Method1	0.995	0.913	0.567	0.198	0.043	0.026	0.128	0.389	0.678
Method2	0.997	0.959	0.761	0.423	0.208	0.348	0.665	0.892	0.978
Method3	0.995	0.916	0.574	0.201	0.044	0.029	0.138	0.404	0.690
Method4	0.995	0.917	0.578	0.204	0.044	0.013	0.074	0.307	0.611
Method5	0.994	0.911	0.567	0.199	0.042	0.017	0.091	0.344	0.645
Method6	0.995	0.923	0.605	0.225	0.053	0.015	0.079	0.312	0.608
Method7	0.990	0.873	0.512	0.172	0.038	0.041	0.175	0.450	0.715
Method8	0.990	0.878	0.522	0.178	0.037	0.018	0.106	0.364	0.659
Method9	0.989	0.869	0.511	0.173	0.036	0.027	0.134	0.406	0.687
Method10	0.990	0.877	0.518	0.175	0.040	0.045	0.186	0.461	0.728
Method11	0.996	0.922	0.559	0.181	0.066	0.183	0.461	0.738	0.900
Method12	0.992	0.883	0.503	0.162	0.086	0.238	0.517	0.771	0.913
Method13	0.986	0.852	0.571	0.300	0.134	0.103	0.207	0.405	0.606
Method14	0.994	0.917	0.717	0.475	0.373	0.454	0.645	0.808	0.912
Method15	0.993	0.934	0.705	0.378	0.220	0.401	0.705	0.906	0.980

Table 3.24: Empirical type I error rate and power of tests for Gamma (4,2), SNR = 2, n = 100

SNR0	1.200	1.400	1.600	1.800	2.000	2.200	2.400	2.600	2.800
Method1	1.000	0.993	0.804	0.289	0.035	0.095	0.438	0.826	0.976
Method2	1.000	0.998	0.925	0.550	0.221	0.488	0.873	0.988	0.999
Method3	1.000	0.993	0.807	0.293	0.035	0.098	0.443	0.829	0.976
Method4	1.000	0.993	0.806	0.294	0.033	0.066	0.381	0.797	0.971
Method5	1.000	0.992	0.799	0.286	0.032	0.075	0.400	0.808	0.972
Method6	1.000	0.994	0.816	0.309	0.038	0.067	0.378	0.792	0.969
Method7	1.000	0.984	0.741	0.242	0.035	0.143	0.513	0.856	0.981
Method8	1.000	0.983	0.741	0.246	0.029	0.108	0.457	0.832	0.977
Method9	1.000	0.982	0.737	0.239	0.030	0.119	0.477	0.842	0.979
Method10	1.000	0.984	0.743	0.245	0.036	0.145	0.518	0.859	0.982
Method11	1.000	0.996	0.840	0.314	0.076	0.304	0.725	0.946	0.993
Method12	1.000	0.990	0.781	0.266	0.100	0.374	0.768	0.957	0.994
Method13	1.000	0.980	0.769	0.387	0.129	0.162	0.426	0.716	0.896
Method14	1.000	0.995	0.882	0.577	0.373	0.525	0.785	0.931	0.979
Method15	1.000	0.996	0.879	0.477	0.244	0.560	0.899	0.989	0.999

Samantha Menendez,
Florence George, B. M. Golam Kibria

Table 3.25: Empirical type I error rate and power of tests for Gamma (25,2), SNR = 5, n = 30

SNR0	4.000	4.400	4.600	4.800	5.000	5.200	5.400	5.600	6.400
Method1	0.492	0.249	0.158	0.095	0.062	0.043	0.042	0.058	0.275
Method2	0.882	0.729	0.647	0.609	0.600	0.617	0.671	0.712	0.921
Method3	0.502	0.258	0.164	0.101	0.065	0.046	0.046	0.063	0.289
Method4	0.500	0.256	0.162	0.098	0.064	0.042	0.039	0.056	0.269
Method5	0.497	0.254	0.161	0.098	0.064	0.043	0.042	0.059	0.276
Method6	0.552	0.297	0.194	0.120	0.075	0.049	0.040	0.049	0.236
Method7	0.467	0.231	0.147	0.088	0.057	0.045	0.048	0.066	0.302
Method8	0.474	0.236	0.149	0.090	0.058	0.043	0.046	0.063	0.296
Method9	0.472	0.235	0.149	0.090	0.058	0.045	0.048	0.067	0.304
Method10	0.475	0.239	0.150	0.092	0.061	0.047	0.054	0.073	0.317
Method11	0.336	0.133	0.081	0.055	0.054	0.073	0.111	0.170	0.521
Method12	0.315	0.122	0.076	0.056	0.062	0.084	0.129	0.193	0.546
Method13	0.501	0.336	0.279	0.235	0.199	0.186	0.182	0.195	0.364
Method14	0.823	0.748	0.719	0.713	0.726	0.738	0.771	0.800	0.882
Method15	0.861	0.708	0.636	0.610	0.604	0.630	0.680	0.727	0.927

Table 3.26: Empirical type I error rate and power of tests for Gamma (25,2), SNR = 5, n = 50

SNR0	4.000	4.400	4.600	4.800	5.000	5.200	5.400	5.600	6.400
Method1	0.668	0.307	0.183	0.097	0.053	0.042	0.057	0.101	0.540
Method2	0.954	0.792	0.684	0.607	0.577	0.621	0.700	0.783	0.972
Method3	0.672	0.312	0.186	0.099	0.055	0.044	0.060	0.106	0.552
Method4	0.672	0.311	0.186	0.098	0.054	0.042	0.055	0.097	0.534
Method5	0.670	0.309	0.184	0.097	0.054	0.042	0.056	0.100	0.540
Method6	0.707	0.340	0.211	0.116	0.064	0.042	0.048	0.088	0.498
Method7	0.638	0.286	0.167	0.088	0.051	0.044	0.066	0.119	0.568
Method8	0.645	0.290	0.170	0.090	0.051	0.045	0.064	0.115	0.564
Method9	0.642	0.288	0.170	0.089	0.051	0.045	0.066	0.118	0.568
Method10	0.646	0.291	0.171	0.091	0.054	0.048	0.069	0.126	0.580
Method11	0.544	0.210	0.111	0.062	0.052	0.075	0.138	0.231	0.721
Method12	0.514	0.193	0.104	0.060	0.057	0.088	0.157	0.255	0.740
Method13	0.614	0.387	0.299	0.233	0.200	0.189	0.208	0.247	0.523
Method14	0.881	0.777	0.738	0.725	0.728	0.748	0.773	0.808	0.924
Method15	0.944	0.773	0.668	0.596	0.583	0.635	0.715	0.793	0.975

Table 3.27: Empirical type I error rate and power of tests for Gamma (25,2), SNR = 5, n = 100

SNR0	4.000	4.400	4.600	4.800	5.000	5.200	5.400	5.600	6.400
Method1	0.904	0.480	0.264	0.115	0.053	0.052	0.119	0.257	0.880
Method2	0.995	0.902	0.764	0.636	0.597	0.649	0.759	0.869	0.997
Method3	0.905	0.484	0.266	0.118	0.054	0.053	0.122	0.262	0.883
Method4	0.905	0.485	0.266	0.118	0.054	0.052	0.117	0.253	0.880
Method5	0.905	0.483	0.266	0.117	0.053	0.052	0.118	0.256	0.880
Method6	0.917	0.511	0.286	0.128	0.055	0.048	0.103	0.233	0.867
Method7	0.888	0.458	0.246	0.108	0.052	0.060	0.134	0.285	0.889
Method8	0.891	0.463	0.248	0.111	0.053	0.059	0.132	0.281	0.887
Method9	0.889	0.461	0.247	0.110	0.052	0.060	0.134	0.284	0.889
Method10	0.890	0.462	0.248	0.111	0.053	0.062	0.137	0.289	0.890
Method11	0.865	0.403	0.202	0.083	0.054	0.095	0.217	0.392	0.934
Method12	0.848	0.383	0.187	0.080	0.060	0.109	0.242	0.415	0.940
Method13	0.798	0.494	0.353	0.257	0.211	0.212	0.267	0.354	0.769
Method14	0.951	0.845	0.780	0.736	0.729	0.759	0.801	0.850	0.972
Method15	0.993	0.887	0.744	0.627	0.604	0.660	0.774	0.878	0.997

Table 3.28: Empirical type I error rate and power of tests for Gamma (100,2), SNR = 10, n = 30

SNR0	6.000	7.000	8.000	9.000	10.000	11.000	12.000	13.000	14.000
Method1	0.991	0.861	0.531	0.207	0.069	0.058	0.152	0.337	0.557
Method2	1.000	0.995	0.949	0.847	0.788	0.840	0.924	0.967	0.988
Method3	0.992	0.867	0.541	0.215	0.071	0.061	0.161	0.353	0.575
Method4	0.991	0.864	0.537	0.210	0.069	0.058	0.153	0.339	0.561
Method5	0.991	0.863	0.536	0.210	0.070	0.058	0.154	0.341	0.562
Method6	0.994	0.890	0.586	0.250	0.086	0.052	0.128	0.292	0.509
Method7	0.987	0.844	0.502	0.192	0.064	0.063	0.168	0.360	0.582
Method8	0.987	0.848	0.507	0.195	0.065	0.064	0.169	0.362	0.585
Method9	0.987	0.848	0.507	0.195	0.065	0.065	0.171	0.364	0.586
Method10	0.988	0.851	0.513	0.200	0.067	0.069	0.178	0.377	0.600
Method11	0.966	0.725	0.334	0.107	0.052	0.133	0.320	0.554	0.756
Method12	0.958	0.702	0.313	0.098	0.059	0.146	0.342	0.579	0.775
Method13	0.944	0.764	0.520	0.326	0.216	0.213	0.292	0.413	0.546
Method14	0.995	0.969	0.908	0.860	0.859	0.895	0.917	0.948	0.967
Method15	1.000	0.993	0.943	0.839	0.786	0.844	0.927	0.971	0.989

Table 3.29: Empirical type I error rate and power of tests for Gamma (100,2), SNR = 10, n = 50

SNR0	6.000	7.000	8.000	9.000	10.000	11.000	12.000	13.000	14.000
Method1	1.000	0.968	0.683	0.250	0.060	0.090	0.317	0.625	0.845
Method2	1.000	1.000	0.981	0.866	0.783	0.868	0.963	0.987	0.998
Method3	1.000	0.970	0.689	0.255	0.063	0.094	0.326	0.634	0.851
Method4	1.000	0.970	0.686	0.253	0.061	0.090	0.318	0.627	0.845
Method5	1.000	0.970	0.686	0.252	0.061	0.091	0.320	0.627	0.846
Method6	1.000	0.975	0.718	0.285	0.067	0.082	0.288	0.588	0.823
Method7	1.000	0.962	0.664	0.233	0.059	0.100	0.337	0.646	0.855
Method8	1.000	0.964	0.667	0.236	0.060	0.101	0.337	0.646	0.855
Method9	1.000	0.964	0.667	0.235	0.060	0.102	0.338	0.648	0.856
Method10	1.000	0.965	0.668	0.237	0.061	0.104	0.344	0.656	0.860
Method11	0.999	0.929	0.547	0.155	0.054	0.178	0.475	0.767	0.923
Method12	0.999	0.920	0.525	0.145	0.057	0.198	0.501	0.782	0.930
Method13	0.990	0.889	0.629	0.351	0.222	0.247	0.396	0.586	0.730
Method14	1.000	0.991	0.942	0.871	0.859	0.906	0.937	0.971	0.982
Method15	1.000	1.000	0.975	0.858	0.784	0.877	0.965	0.989	0.998

Table 3.30: Empirical type I error rate and power of tests for Gamma (100,2), SNR = 10, n = 100

SNR0	6.000	7.000	8.000	9.000	10.000	11.000	12.000	13.000	14.000
Method1	1.000	0.999	0.919	0.377	0.052	0.192	0.643	0.930	0.990
Method2	1.000	1.000	0.999	0.926	0.777	0.914	0.991	0.999	1.000
Method3	1.000	0.999	0.921	0.380	0.052	0.197	0.646	0.931	0.990
Method4	1.000	0.999	0.920	0.379	0.052	0.191	0.643	0.930	0.990
Method5	1.000	0.999	0.920	0.378	0.052	0.192	0.643	0.930	0.990
Method6	1.000	1.000	0.928	0.404	0.055	0.174	0.622	0.918	0.989
Method7	1.000	0.999	0.911	0.364	0.050	0.207	0.658	0.934	0.990
Method8	1.000	0.999	0.913	0.366	0.051	0.206	0.657	0.934	0.990
Method9	1.000	0.999	0.913	0.366	0.051	0.207	0.658	0.934	0.990
Method10	1.000	0.999	0.914	0.366	0.051	0.210	0.662	0.935	0.990
Method11	1.000	0.999	0.868	0.285	0.048	0.291	0.752	0.959	0.994
Method12	1.000	0.999	0.857	0.272	0.049	0.305	0.762	0.962	0.996
Method13	1.000	0.984	0.806	0.425	0.222	0.337	0.601	0.817	0.931
Method14	1.000	1.000	0.979	0.906	0.870	0.916	0.966	0.989	0.998
Method15	1.000	1.000	0.998	0.921	0.775	0.921	0.992	1.000	1.000

4. Applications

In this section we will consider and analyze two (2) real life data sets to illustrate the performance of the test statistics.

4.1 Chemotherapy Treatment

In this section, we will consider the chemotherapy data, which was given on a group of individuals who had chemotherapy treatment exclusively for 45 years (Bekker *et al*. 2000). The observations are as follows: 0.047, 0.115, 0.121, 0.132, 0.164, 0.197, 0.203, 0.260, 0.282, 0.296, 0.334, 0.395, 0.458, 0.466, 0.501, 0.507, 0.529, 0.534, 0.540, 0.641, 0.644, 0.696, 0.841, 0.863, 1.099, 1.219, 1.271, 1.326, 1.447, 1.485, 1.553, 1.581, 1.589, 2.178, 2.343, 2.416, 2.444, 2.825, 2.830, 3.578, 3.658, 3.743, 3.978, 4.003, 4.033.

The histogram of the failure time data is presented in Figure 4.1, which looks like a gamma distribution. The skewness and kurtosis are obtained as 0.97 and 2.66 respectively.

Using R 4.2.2, we obtained the estimated parameters of the gamma distribution as 1.1 and 0.82. Now using the KS test, we found the P-value as 0.6029 and concluded that the chemotherapy data follow a gamma distribution with parameters 1.1 and 0.82. The histogram, density plot, Q-Q plot, empirical and theoretical cdf plot and P-P plot are provided in Figure 4.2, which also supported the gamma distribution. That means the population signal to ratio will be $\sqrt{1.1} = 1.05$. Then it is logical to test the true signal to ratio as, H_0: SNR=1.05. The lower and upper critical values for all tests are presented in Table 4.1.

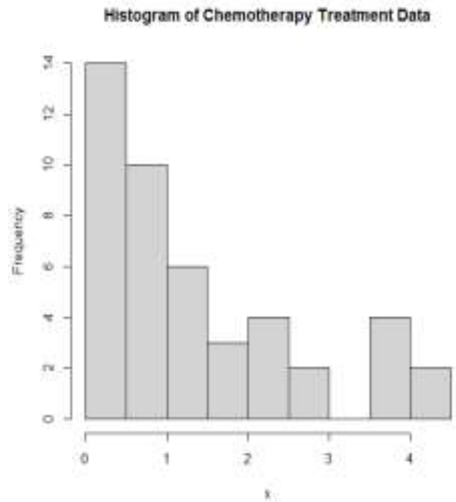

Figure 4.1: Histogram of chemotherapy data

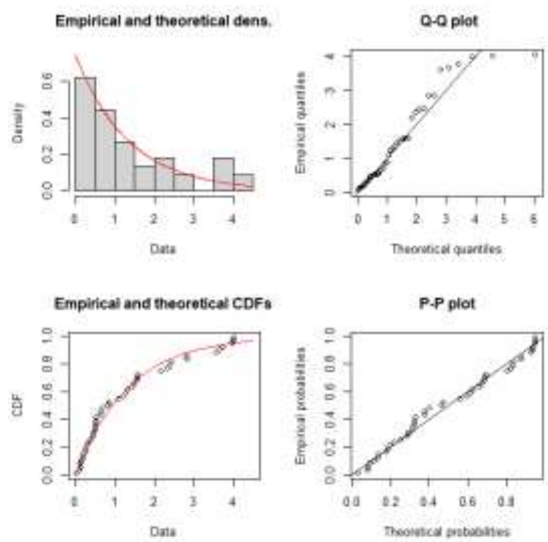

Figure 4.2: Empirical and theoretical density, QQ plot, CDF and P-P plots of chemotherap data

Table 4.1: Lower and upper critical values of the chemotherapy treatment data

Method	Lower Limit	Upper Limit
1	0.800	1.643
2	0784	1.368
3	0.802	1.633
4	0.796	1.990
5	0.788	1.871
6	0.799	1.874
7	0.804	1.313
8	0.726	2.047
9	0.717	1.878
10	0.806	1.306
11	0.852	1.300
12	0.789	1.204
13	0.696	1.525
14	0.664	1.248
15	0.705	1.289

From Table 4.1, we observed that all methods including our proposed methods 12 to 15 have accepted the null hypothesis H_0: SNR=1.05. These results are consistent with the simulation results. It is noted that the proposed method 15 performed very well in the simulation results and it is consistent with the real data.

4.2 Mathematics Grades

The below datasets contain the 2013 mathematics grades for 48 students enrolled in the slow-paced program. From Linhart and Zucchini (1986), the following observations were obtained: 29, 25, 50, 15, 13, 27, 15, 18, 7, 7, 8, 19, 12, 18, 5, 21, 15, 86, 21, 15, 14, 39, 15, 14, 70, 44, 6, 2,58, 19, 50, 23, 11, 6, 34, 18, 28, 34, 12, 37, 4, 60, 20, 23, 40, 65, 19, 31.

The histogram of the mathematics grade data is presented in Figure-4.5, which looks like a gamma distribution. The skewness and kurtosis are obtained as 1.29 and 4.22 respectively.

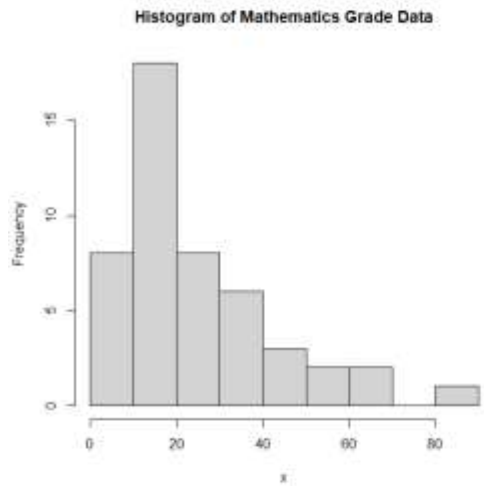

Figure 4.5: Histogram of mathematics grade data

The estimated parameters of the gamma distribution are obtained as 1.97 and 0.08. Now using the KS test, we found the P-value as 0.958 and concluded that the mathematics grade data follow a gamma distribution with parameters 1.97 and 0.82. The histogram, density plot, Q-Q plot, empirical and theoretical cdf plot and P-P plot are provided in Figure 4.6, which also supported the gamma distribution. That means the population signal to ratio will be $\sqrt{1.97} = 1.40$. Then it is logical to test the true signal to ratio as, H_0: SNR=1.40. The lower and upper critical values for all tests are presented in Table 4.3.

From Table 4.3, we observed that all methods including our proposed methods 12 to 15 have accepted the null hypothesis H_0: SNR=1.40. These results are consistent with the simulation results.

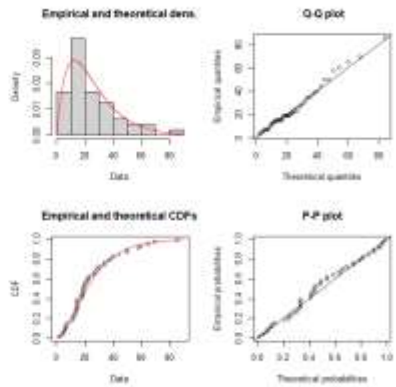

Figure 4.6: Empirical and theoretical density, QQ plot, CDF and P-P plots of mathematics grade data

Table 4.3: Lower and upper critical values of the mathematics grades data

Method	Lower Limit	Upper Limit
1	1.041	1.904
2	1.063	1.629
3	1.043	1.896
4	1.041	2.053
5	1.034	1.999
6	1.046	2.012
7	1.001	1.744
8	0.976	1.999
9	0.969	1.938
10	0.888	2.243
11	1.074	1.616
12	1.016	1.528
13	1.173	2.075
14	1.216	1.781
15	0.989	1.555

5 Conclusion

In this paper we consider fifteen different test statistics (Methods 1 to 15) for testing the population SNR. We consider some existing test statistics, which were developed from the confidence interval of SNR and from the work of Miller (1991), Sharma and Krishna (1994), Curto and Pinto (2009), McKay's (1932), Panichkitkosolkul (2009) and Kibria and George (2014) among others. We also propose a few new test statistics based on the robust estimator of population standard deviation σ. Since a theoretical comparison among the test statistics is not possible, a Monte Carlo simulation study has been conducted under both symmetric and skewed distributions to compare the performance of the test statistics. The performance of the test statistics is determined based on the empirical size and power of the tests. We have considered the most popular and widely used significance level 0.05 for finding the size and power of the test. To see the impact of sample size on the test statistics, we considered n=30, 50 & 100. From simulation study it appears that Methods 1, 3, 4, 5, 6, 7, 8, 9,10 and proposed Methods 12 and 15 are promising and performed well in some conditions. However, Method 10 proposed by Kibria and George (2014) performed the best when testing for SNR=1, 2 and 5 in all simulation conditions both at the lower and upper end of the alternative hypotheses. However, for testing SNR=0.5 the proposed Method 12 and for testing SNR=10, the proposed Method 15 performed the best. Two real life data on chemotherapy treatment and mathematics grades are analyzed to illustrate the performance of the test statistics. It appears that the simulation results and applications are consistent to some extent. The conclusions of this paper are restricted to the given simulation conditions of this paper. For a definite statement one might need more simulation conditions and more sample sizes and do a simulation under various distributional conditions. Hope the findings of the paper will be an asset for the practitioners.

This study was done for two-tailed tests. However, it would be interesting to compare these methods for one tail test. Additionally, it would be interesting to use t statistic instead of z-statistic to compare the performance of the test statistics under small sample sizes.

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Conflict of Interest

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