Digital camera calibration, relative orientation and essential matrix parameters

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Abstract: The fundamental matrix, based on the co-planarity condition, even though it is very interesting for theoretical issues, it does not allow finding the camera calibration parameters, and the base and rotation parameters altogether. In this work we present an easy calibration method for calculating the internal parameters: pixel dimensions and image center pixel coordinates. We show that the method is slightly easier if the camera rotation angles, in relation with the general referential system, are small. The accuracy of the four calibration parameters are evaluated by simulations. In addition, a method to improve the accuracy is explained. When the calibration parameters are known, the fundamental matrix can be reduced to the essential matrix. In order to find the relative orientation parameters in stereo vision, there is also presented a new method to extract the base and the camera rotation by means of the essential matrix. The proposed method is simple to implement. We also include a simpler method for the relative orientation when the relative rotation angles between the two cameras are small.

Keywords: Fundamental matrix, essential matrix, camera calibration

1 Introduction

If a point in a 3D referential system is imaged as $\boldsymbol{P_L}$ in the left view, and as $\boldsymbol{P_R}$ in the right view, then the image point direction vectors in the same referential system satisfy the equation $\boldsymbol{P_L} \cdot \boldsymbol{b} \times \boldsymbol{P_R} = 0$, where $\boldsymbol{b}$ is the vector joining the two camera points of view (Figure 1). That is, the triple product is zero, since these 3 vectors are in the same plane (epipolar geometry). Using the skew-symmetric matrix B obtained from the vector $\boldsymbol{b}$, the triple product condition can be stated as $\boldsymbol{P_L} B \boldsymbol{P_R} = 0$.

The pixel coordinates $\boldsymbol{W_L}$ and $\boldsymbol{W_R}$ of both points in digital images, are related to points in length units as the ones used in the 3D referential system, by a calibration matrix C. If the left camera determines the

3D referential system, the coordinates in the right camera have to be multiplied by the rotation matrix R that accounts for the right camera orientation. Then, $\boldsymbol{W_L} {C^{-1}}^T BRC^{-1}\boldsymbol{W_R} = 0$. The fundamental matrix [1,2] is $F = {C^{-1}}^T BRC^{-1}$. It is a 3 × 3 matrix of rank 2, and, even if it is very interesting for theoretical issues related to the epipolar geometry, it is not possible to obtain the 9 parameters involved: 4 parameters for the camera calibration, 3 for the rotation and 2 for the base (known up to scale).

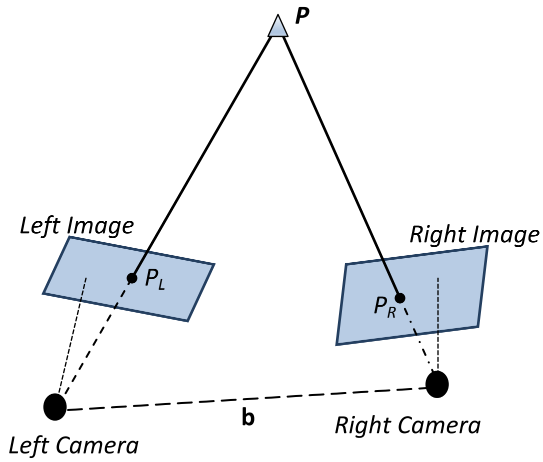

Figure 1. Epipolar geometry

When the calibration is known, only the parameters of B and R are unknown. In this case, it is easy to obtain the essential matrix $E = BR$ [3] from the fundamental matrix, since $E = C^T FC$. There are several approaches to obtain the 5 involved parameters [4,5,6,7]. The usual method involves the singular value decomposition of the essential matrix [8].

There are some methods to obtain the calibration of the camera. What we propose here is a very simple calibration scheme, which is also easy to implement. We assume that it is quite easy to measure the X and Y coordinates of the camera point of view in relation to a 3D referential system when the camera optical axis has a small rotation angle with the Z axis. We then accept that the error in the Z coordinate of the point of view is less than +/- 1 cm. Under these assumptions, we evaluate the error in the calibration parameters obtained by simulation. In a later step, assuming that two cameras have a small angle in relation to the Z axis, the calibration parameters error can be evaluated and corrected, knowing the distance between two benchmarks in relation to the one calculated by stereoscopy.

Regarding the essential matrix [9], we propose an original solution focused on extracting the base B and the rotation R, involving the solution of linear systems.

The rest of the paper is organized as follows. In section 2, a method to calculate the camera calibration parameters is exposed. Section 3 describes the calculation of position parameters (base and rotation) from the essential matrix. Then, in section 4, the solution for the relative orientation with small relative rotations is presented. In section 5, the numerical simulations to evaluate accuracy of the calibration parameters are shown. Section 6 exposes the procedure to reduce the parameter errors using stereoscopy. Finally, section 7 presents the conclusions.

2 Camera Calibration.

Let $\boldsymbol{W} = (u, v, 1)^T$ be the pixel coordinates of a point in a digital image, and $\boldsymbol{V} = (x, y, 1)^T$ the image coordinates in length units. The relationship between these coordinates is

$$\begin{aligned} u &= \alpha x + u_0 \\ v &= \beta y + v_0 \end{aligned} \qquad (1)$$

where (u_0,v_0) is the image principal point, that is the point in the image that belongs to the optical axis, and $\alpha = f.d_x$ and $\beta = f.d_y$ are focal lengths in pixels, product of focal distance by scale factors in horizontal and vertical directions.

This relationship can be stated as

$$\boldsymbol{W} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{bmatrix} \alpha & 0 & uo \\ 0 & \beta & vo \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = C.\boldsymbol{V} \qquad (2)$$

The matrix

$$C = \begin{bmatrix} \alpha & 0 & uo \\ 0 & \beta & vo \\ 0 & 0 & 1 \end{bmatrix} \qquad (3)$$

is the intrinsic parameter matrix.

Then,

$$\boldsymbol{V} = C^{-1}.\boldsymbol{W} = \begin{bmatrix} 1/\alpha & 0 & -uo/\alpha \\ 0 & 1/\beta & -vo/\beta \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \quad (4)$$

Let $\boldsymbol{P} = (X, Y, Z)^T$ be the coordinates of a point in the space, with respect to a fixed 3D referential system. Let us assume a camera in the point of coordinates $\boldsymbol{b} = (X_c, Y_c, Z_c)^T$, and rotated respect to the referential; and this rotation is given by a rotation matrix R. Thus,

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} - \begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = \lambda R\boldsymbol{V} = \lambda RC^{-1} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \qquad (5)$$

Without loss of generality, let us assume $\boldsymbol{b} = (0,0,0)^T$.

Now, let us name $RC^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$. Then,

$$\begin{aligned} X &= (a_{11}u + a_{12}v + a_{13})\lambda \\ Y &= (a_{21}u + a_{22}v + a_{23})\lambda \\ Z &= (a_{31}u + a_{32}v + a_{33})\lambda \end{aligned} \qquad (6)$$

From here we take,

$$\begin{aligned} \frac{X}{Y} &= \frac{a_{11}u + a_{12}v + a_{13}}{a_{21}u + a_{22}v + a_{23}} \\ \frac{X}{Z} &= \frac{a_{11}u + a_{12}v + a_{13}}{a_{31}u + a_{32}v + a_{33}} \\ \frac{Y}{Z} &= \frac{a_{21}u + a_{22}v + a_{23}}{a_{31}u + a_{32}v + a_{33}} \end{aligned} \qquad (7)$$

Reordering,

$$\begin{aligned} &a_{21}uX + a_{22}vX + a_{23}X - a_{11}uY - a_{12}vY - a_{13}Y = 0 \\ &a_{31}uX + a_{32}vX + a_{33}X - a_{11}uZ - a_{12}vZ - a_{13}Z = 0 \\ &a_{21}uZ + a_{22}vZ + a_{23}Z - a_{31}uY - a_{32}vY - a_{33}Y = 0 \end{aligned} \qquad (8)$$

Having a set of scene points whose coordinates are known, $\boldsymbol{P_k} = (X_k, Y_k, Z_k)$, projected to $(u_k, v_k, 1)$ in the image, the equations (8) constitute a linear system of equations for the unknowns a_{ij}.

The system is homogeneous and the matrix has one-dimension kernel. The solution can be calculated up to scale factor:

$$\begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{31} \\ a_{32} \\ a_{33} \end{pmatrix} = \rho \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \\ s_9 \end{pmatrix} \qquad (9)$$

where the s_i are obtained by solving the system, and ρ is a scale factor to be determined later. Therefore,

$$RC^{-1} = \begin{bmatrix} r_{11}/\alpha & r_{12}/\beta & r_{13} - \frac{r_{11}u_o}{\alpha} - \frac{r_{12}v_0}{\beta} \\ r_{21}/\alpha & r_{22}/\beta & r_{23} - \frac{r_{21}u_0}{\alpha} - \frac{r_{22}v_0}{\beta} \\ r_{31}/\alpha & r_{32}/\beta & r_{33} - \frac{r_{31}u_0}{\alpha} - \frac{r_{32}v_0}{\beta} \end{bmatrix} = \rho \begin{bmatrix} s_1 & s_2 & s_3 \\ s_4 & s_5 & s_6 \\ s_7 & s_8 & s_9 \end{bmatrix} \qquad (10)$$

Now, it is evident that

$$\begin{aligned} r_{21} &= r_{11}\frac{s_4}{s_1} \quad r_{31} = r_{11}\frac{s_7}{s_1} \\ r_{22} &= r_{12}\frac{s_5}{s_2} \quad r_{32} = r_{12}\frac{s_8}{s_2} \end{aligned} \qquad (11)$$

Since R is an orthogonal matrix, its columns are orthogonal vectors. Then, we can assert

$$r_{11}^2 + r_{21}^2 + r_{31}^2 = r_{11}^2\left(1 + \left(\frac{s_4}{s_1}\right)^2 + \left(\frac{s_7}{s_1}\right)^2\right) = 1$$

$$r_{11}^2 = \frac{s_1^2}{s_1^2 + s_4^2 + s_7^2} \qquad (12)$$

Similarly for r_{12}

$$r_{12}^2 = \frac{s_2^2}{s_2^2 + s_5^2 + s_8^2} \qquad (13)$$

At this point, two columns of R, R_{*1} and R_{*2} are known. The third column can be calculated since it must be orthogonal to R_{*1} and R_{*2}. Then, $R_{*3} = R_{*1} \times R_{*2}$.

Knowing R, the calibration matrix arises as

$$C^{-1} = \begin{bmatrix} \frac{1}{\alpha} & 0 & -\frac{u_o}{\alpha} \\ 0 & \frac{1}{\beta} & -\frac{v_0}{\beta} \\ 0 & 0 & 1 \end{bmatrix} = \rho R^{-1} \begin{bmatrix} s_1 & s_2 & s_3 \\ s_4 & s_5 & s_6 \\ s_7 & s_8 & s_9 \end{bmatrix} \qquad (14)$$

Clearly, the value of ρ is the one that makes the (3,3) element of C^{-1} equal to 1.

Admitting that point measurements has always errors, it is convenient to use more points to over-determine system (8), and solve it by means of least square minimization.

2.1. Approximation for small rotation

Consider that the camera has a small rotation angle in relation with the 3D referential. In this case, the rotation matrix can be approximated with

$$R \approx \begin{bmatrix} 1 & -w_z & w_y \\ w_z & 1 & -w_x \\ -w_y & w_x & 1 \end{bmatrix} \qquad (15)$$

Thus,

$$RC^{-1} \cong \begin{bmatrix} \frac{1}{\alpha} & -\frac{w_z}{\beta} & -\frac{u_0}{\alpha} + \frac{w_z v_0}{\beta} + w_y \\ \frac{w_z}{\alpha} & \frac{1}{\beta} & \frac{w_z u_0}{\alpha} - \frac{v_0}{\beta} - w_x \\ -\frac{w_y}{\alpha} & \frac{w_x}{\beta} & \frac{w_y u_0}{\alpha} - \frac{w_x v_0}{\beta} + 1 \end{bmatrix} \qquad (16)$$

From where,

$$w_z = \frac{s_4}{s_1}, w_y = -\frac{s_7}{s_1}, w_x = \frac{s_8}{s_5} \qquad (17)$$

And consequently R is known. Then, the calibration coefficients came from (14).

2.2. Calibration Platform

The 3D general reference system consists of a vertical grid whose axis X is horizontal. The optical axis of the camera is placed pointing to the grid. The distance between the camera and the grid may be around 1m or 1.5 m.

With a length greater than the distance from the camera to the grid, we join the center of the camera with the grid in two symmetrical points, in vertical way and again in horizontal way. The midpoint of these two pairs of points determines respectively the X and Y coordinates of the optical center of the camera.

We do not know with certainty the Z coordinate of the optical center within the camera, we just know *approximately* the Z coordinate of the optical point of view. This uncertainty will cause an error in the calibration parameters.

In section 4 we estimate by simulation these errors in the calibration parameters. Later, we present how to correct these calibration parameters. We will compare the distance between two benchmarks against the one obtained by stereoscopy using the calibration values.

3 Essential matrix parameters.

The essential matrix allows expressing the connection between corresponding points in a pair of stereo images from calibrated cameras.

Fixing a 3D referential system in left camera, with Z-axis parallel to its optical axis, let us denote $\boldsymbol{b} = (X_c, Y_c, Z_c)^T$ the coordinates of the right camera with respect to this reference, and R the rotation matrix that gives the orientation of the right camera.

As it was stated before, the vectors $\boldsymbol{V_L}$, $R\boldsymbol{V_R}$ and $\boldsymbol{b}$ are coplanar, where $\boldsymbol{V_L}$ and $\boldsymbol{V_R}$ are the coordinates of corresponding image points. Therefore,

$$\boldsymbol{V_L} \cdot (\boldsymbol{b} \times R\boldsymbol{V_R}) = \boldsymbol{V_L^T} BR\boldsymbol{V_R} = 0 \qquad (18)$$

Here, B is the skew-symmetric matrix corresponding to $\boldsymbol{b}$, that is,

$$B = \begin{pmatrix} 0 & -Z_c & Y_c \\ Z_c & 0 & -X_c \\ -Y_c & X_c & 0 \end{pmatrix} \qquad (19)$$

$E = BR$ is the essential matrix. E can be determined up to scale, knowing 8 pairs of homologous points.

Thus, it can be calculated $\widetilde{\mathrm{E}} = \lambda E = \lambda BR$, where λ is an unknown scale factor. The columns of E are orthogonal to $\boldsymbol{b}$. Then, the cross product of two columns of E is a vector parallel to $\boldsymbol{b}$. Let $\boldsymbol{b}'$ be a unit vector parallel to $\boldsymbol{b}$;

$$\frac{\tilde{E}_{*1} \times \tilde{E}_{*2}}{\left|\tilde{E}_{*1} \times \tilde{E}_{*2}\right|} = \boldsymbol{b}' = \gamma \boldsymbol{b} \tag{20}$$

then $\gamma = \frac{1}{|\boldsymbol{b}|}$. Let us denote B' the skew-symmetric matrix related to $\boldsymbol{b}'$.

The matrix

$$\widetilde{\mathrm{E}}^{\mathrm{T}}\widetilde{\mathrm{E}} = \lambda^2 R^T B^T BR = R^{\mathrm{T}}(\lambda^2 B^{\mathrm{T}} B)R \tag{21}$$

is similar to $\lambda^2\mathrm{B}^{\mathrm{T}}\mathrm{B}$, and therefore they have the same trace,

$$\mathrm{tr}\left(\widetilde{\mathrm{E}}^{\mathrm{T}}\widetilde{\mathrm{E}}\right) = \lambda^2 \mathrm{tr}\left(\mathrm{B}^{\mathrm{T}}\mathrm{B}\right) = \lambda^2 2|\mathbf{b}|^2 \tag{22}$$

From this,

$$\lambda = \pm\frac{\sqrt{\mathrm{tr}\left(\widetilde{\mathrm{E}}^{\mathrm{T}}\widetilde{\mathrm{E}}\right)}}{\sqrt{2}|\boldsymbol{b}|} \tag{23}$$

Then,

$$\widetilde{\mathrm{E}} = \lambda\frac{\mathrm{B}'}{\gamma}\mathrm{R} = \pm\frac{\sqrt{\mathrm{tr}(\widetilde{\mathrm{E}}^{\mathrm{T}}\widetilde{\mathrm{E}})}}{\sqrt{2}}\mathrm{B}'\,\mathrm{R} \tag{24}$$

Thereafter,

$$\pm\widetilde{\mathrm{E}}\frac{\sqrt{2}}{\sqrt{\mathrm{tr}(\widetilde{\mathrm{E}}^{\mathrm{T}}\widetilde{\mathrm{E}})}} = \mathrm{E}' = \mathrm{B}'\,\mathrm{R} \tag{25}$$

Now, E' and B' are known. Since B' is singular, it cannot be inverted to solve for R. However, being R an orthogonal matrix, it can be calculated as follows, according to literature.

E admits a singular value decomposition $E = USV^T$, where U and V are orthogonal matrices and S is a diagonal matrix with the singular values of E in its diagonal. Then, B and R are obtained as,

$$B = UASU^T \tag{26}$$

$$R = UAV^T \tag{27}$$

with $A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

There is also another solution $R = UA^TV^T$.

Since there is a sign ambiguity in E from (25), there are other two possible solutions for the factorization B-R, using the opposite sign.

Another original way of extracting B and R from E, without singular value decomposition, is as follows.

Each column of R satisfies the equation $E'_{*i} = B'R_{*i}$. Since B' has rank 2, the solution space is one-dimensional. Solving it, $R_{*i} = R^0_{*i} + \gamma_i N$, where N is a unit vector in the null space of B' (that is, a vector such that $B'N = 0$) and R^0_{*i} is a particular solution of the system.

Then,

$$R = [R_{*1} \quad R_{*2} \quad R_{*3}] = [R^0_{*1} \quad R^0_{*2} \quad R^0_{*3}] + N[\gamma_1 \quad \gamma_2 \quad \gamma_3] \tag{28}$$

for some scalars γ_i. The objective now is to calculate these scalars.

The orthogonality of R implies $R^TR = I$, that is $R^T_{*i}\,R_{*j} = 0$ if $i \neq j$; and $R^T_{*i}\,R_{*i} = 1$.

From this point,

$$1 = R^T_{*i}R_{*i} = {R^0_{*i}}^T R^0_{*i} + 2\gamma_i {R^0_{*i}}^T N + \gamma_i^2 N^T N = |R^0_{*i}|^2 + 2\gamma_i {R^0_{*i}}^T N + \gamma_i^2 |N|^2 \tag{29}$$

Eq. (29) gives a quadratic equation for γ_i. N is a unit vector, then the possible values of γ_i are

$$\gamma_i = -{R^0_{*i}}^T N + \sqrt{({R^0_{*i}}^T N)^2 + 1 - |R^0_{*i}|^2} \tag{30}$$

$$\gamma_i = -{R^0_{*i}}^T N - \sqrt{({R^0_{*i}}^T N)^2 + 1 - |R^0_{*i}|^2} \tag{31}$$

Orthogonality of R also implies

$$0 = R^T_{*i}R_{*j} = {R^0_{*i}}^T R^0_{*j} + \gamma_i N^T R^0_{*j} + \gamma_j {R^0_{*i}}^T N + \gamma_j\gamma_i \tag{32}$$

for i,j=1,2,3, $i > j$.

With the values of γ_i found previously in (30) and (31), Eq. (32) takes the form:

$$\pm\sqrt{({R^0_{*i}}^T N)^2 + 1 - |R^0_{*i}|^2}\sqrt{({R^0_{*j}}^T N)^2 + 1 - |R^0_{*j}|^2} = ({R^0_{*i}}^T N)({R^0_{*j}}^T N) - {R^0_{*i}}^T R^0_{*j} \qquad (33)$$

Here, the + sign corresponds to take Eq. (30) for both *i* and *j*, or Eq. (31) for both *i* and *j*; and the – sign corresponds to take Eq. (30) for *i* and Eq. (31) for *j*, or vice versa.

The strategy is then, to calculate the right hand side of Eq. (33), $({R^0_{*i}}^T N)({R^0_{*j}}^T N) - {R^0_{*i}}^T R^0_{*j}$. If it is positive, γ_i and γ_j are calculated both with Eq. (30) or both with Eq. (31). And if it is negative, γ_i is calculated with Eq. (30) and γ_j is calculated with Eq. (31), or vice versa.

Thus, two possible sets of values $(\gamma_1, \gamma_2, \gamma_3)$, are obtained, for each choice of sign in Eq. (25). Two orthogonal matrices *R* are obtained; one of them is a direct orthogonal matrix R_1 (it has determinant 1; pure rotation) and the other is an inverse orthogonal matrix R_2 (it has determinant -1; rotation and symmetry). Then, $-R_2$ is a direct orthogonal matrix; and it is one of the matrices that would be obtained assuming opposite sign in (25).
Summarizing, four orthogonal matrices, R_1, $-R_1$, R_2 and $-R_2$, satisfy $\pm\widetilde{\mathrm{E}}\frac{\sqrt{2}}{\sqrt{\mathrm{tr}\,(\widetilde{\mathrm{E}}^{\mathrm{T}}\widetilde{\mathrm{E}})}} = \mathrm{B'R}$, but only two of them are rotation matrices. Both are valid solutions of the system. But one of them predicts points behind the camera, so it should be discarded.

4 Relative orientation with small relative rotations

Let us assume that the relative rotations between the two cameras are very small.
Consider a referential whose origin is located in left camera position and the *x*-axis is parallel to the vector joining left and right camera. So, left camera is located at $\boldsymbol{b_L}=(0,0,0)^T$ and right camera is located at point $\boldsymbol{b_R} = (b,0,0)^T$. Then, the relative rotation around the *x*-axis can be considered only in one of the two cameras ($w_x = 0$). Therefore the simplified rotation matrixes for the left and the right camera are:

$$R_L = \begin{bmatrix} 1 & -w_z & w_y \\ w_z & 1 & 0 \\ -w_y & 0 & 1 \end{bmatrix} \qquad (34)$$

$$R_R = \begin{bmatrix} 1 & -\mu_z & \mu_y \\ \mu_z & 1 & -\mu_x \\ -\mu_y & \mu_x & 1 \end{bmatrix} \qquad (35)$$

The homologous vectors related to image point in each camera referential are:

$$\boldsymbol{V_L} = \begin{pmatrix} x_L \\ y_L \\ 1 \end{pmatrix} \quad \boldsymbol{V_R} = \begin{pmatrix} x_R \\ y_R \\ 1 \end{pmatrix} \qquad (36)$$

Multiplying by the corresponding rotation matrices, the vectors are

$$R_L\boldsymbol{V_L} = \begin{pmatrix} x_L - w_z y_L + w_y \\ x_L w_z + y_L \\ 1 - x_L w_y \end{pmatrix} \qquad (37)$$

$$R_R\boldsymbol{V_R} = \begin{pmatrix} x_R - \mu_z y_R + \mu_y \\ x_R\mu_z + y_R - \mu_x \\ 1 - x_R\mu_y + y_R\mu_x \end{pmatrix} \qquad (38)$$

Normalizing these vectors such that the third component is equal to 1, we have vectors in the same plane with third coordinate equal to 1, then the transversal parallaxes is equal to 0 that means the second coordinates are equal. Therefore,

$$\frac{x_L w_z + y_L}{1 - x_L w_y} = \frac{x_R\mu_z + y_R - \mu_x}{1 - x_R\mu_y + y_R\mu_x} \qquad (39)$$

So we obtain for each pair of vectors an equation with 9 unknowns:

$$\begin{aligned} x_L w_z - x_L x_R w_z \mu_y + x_L y_R w_z \mu_x + y_L \\ - x_R y_L \mu_y + y_L y_R \mu_x \\ = x_R\mu_z + y_R - \mu_x - x_R x_L w_y \mu_z \\ + x_L y_R w_y - x_L w_y \mu_x \end{aligned} \qquad (40)$$

With very small rotations, we can neglect the second order terms and then we obtain:

$$\begin{aligned} x_L w_z + y_L - x_R y_L \mu_y + y_L y_R \mu_x \\ = x_R\mu_z + y_R - \mu_x + x_L y_R w_y \end{aligned} \qquad (41)$$

Or, reordering terms

$$\begin{aligned} x_L w_z - x_R y_L \mu_y + (1 + y_L y_R)\mu_x - x_R\mu_z \\ - x_L y_R w_y = y_R - y_L \end{aligned} \qquad (42)$$

For each pair of points, a linear equation with the 5 unknowns is obtained. Then, 5 points are required to obtain the rotation parameters. However, due to measurement errors, it is advisable to use more than 5 points and solve the system using least square solution.

5 Simulation: Evaluating accuracy of the calibration parameters

Numerical simulations were performed, in order to evaluate the accuracy of the proposed calibration method, and the sensitivity to the Z_c coordinate (which in practical situations is hard to determine with precision, as it was explained before).

A simulated camera was defined, using parameter values α=400, β=380, u_0=600 and v_0=500. It was assumed that the camera orientation is rotated an angle of π/50 about the direction $(1,1,1)^T$. A set of points in a vertical plane were considered for calibration scheme.

Table 1 shows the results of the numerical simulation. Columns 2 to 5 show the calculated values of the intrinsic parameters, considering certain error in the value of $Z_c = 100 + \Delta Z_c$.

Table 1: Numerical results from simulation

Parameter	ΔZ_c			
	-0.1	0.1	-1	1
α	399.6	400.4	396.01	403.98
β	379.6	380.4	376.2	383.78
u_0	599.97	600.03	599.71	600.28
v_0	500.03	499.97	500.23	499.72

Assuming the simplified rotation matrix, valid for small rotation, the estimated parameter values are very close to the one obtained with full rotation matrix. Table 2 shows the results.

Table 2: Numerical simulation using simplified rotation matrix

Parameter	ΔZ_c			
	-0.1	0.1	-1	1
α	399.5	400.4	396.0	403.9
β	379.6	380.4	376.18	383.7
u_0	599.4	599.4	599.14	599.7
v_0	499.5	499.5	499.77	499.2

In practice, points in the space are measured with certain errors. In the next experimental analysis, noise was added to the measured points. Column 2 and column 3 show the parameters obtained when points have errors of 0.1% and 1% respectively.

Table 3: Parameters obtained with noisy data

Parameter	Error 0.1%	Error 1%
α	399.69	397.35
β	379.38	379.76
u_0	600.33	609.3
v_0	500.15	495.3

As it can be seen, quite accurate estimation was obtained, even admitting imprecise data within reasonable limits.

6 Parameter error correction

Since the Z coordinate of the optical point of view is not known with precision, the error in this value carries errors in the parameter calibration.

In order to correct these errors a simple procedure can be applied. The idea is to take images of two benchmarks in a scene with two cameras, to reconstruct the 3D position of these benchmarks with stereoscopy, using the estimated calibration. Then, the distance between the two target points is calculated with the estimated position. Since the real distance is known, the error in the distance estimation can be evaluated.

Then, varying ΔZ and using dichotomic search, the error in the distance is reduced, conveying precision to calibration parameters.

In detail, let $\boldsymbol{P_1}$ and $\boldsymbol{P_2}$ two points in the scene, and let $\boldsymbol{d}$ the distance between them. The distance is invariant to changes in the reference system. Let $\boldsymbol{W}_R^1$, $\boldsymbol{W}_R^2, \boldsymbol{W}_I^1$, y $\boldsymbol{W}_I^2$ the images coordinates of $\boldsymbol{P_1}$ and $\boldsymbol{P_2}$ obtained with two cameras. The cameras are separated by a distance b.

Since the intrinsic parameter matrix is approximately known, the essential matrix can be estimated from the fundamental matrix, $E = C^T FC$. Then, from the essential matrix, the rotation matrix of right camera can be obtained as it was explained in section 3.

Now, the 3D coordinates of $\boldsymbol{P_1}$ and $\boldsymbol{P_2}$ in the reference system determined by the left camera are estimated and, particularly, the distance **d** is estimated, obtaining $\boldsymbol{d}_{est}$.

Following, the Z coordinate used in the intrinsic calibration is changed to $Z + \varDelta Z$, with $\varDelta Z = 1$, and the procedure is repeated, obtaining a new estimated $\boldsymbol{d}_{est}$.

In the next iterations, $\varDelta Z$ is changed by dichotomy in order to get better approximations of the real value **d,** leading to improved estimations of the calibration parameters.

7 Conclusions

In this work, we presented a very simple method for obtaining calibration parameters of a camera. It is based on taking an image of a set of points in a vertical grid and solving a system of linear equations. In a first step, the rotation matrix is obtained, which allows obtaining the intrinsic calibration parameters. The precision of the method was evaluated by simulation, obtaining very acceptable errors, even with added noise. Moreover, if necessary, we describe a procedure for correcting the calibration parameters, based on stereoscopy and the invariance of length by changes of reference system.

Finally, the proposed method to obtain the 5 camera position parameters in terms of relative orientation is original and simple. We present the method to resolve the relative orientation when the relative rotations are very small, because it is very easy to implement, and it is not very known.

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