

# An Algorithm for Indoor Photo Diode-based Visible Light Positioning

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*Abstract:* - In recent years, indoor visible light positioning using white LEDs is getting more attention. An algorithm for indoor photo diode-based visible light positioning is proposed, where the time difference of arrival of the received signal, together with the scale and azimuth angle of the receiver is used to determine the location of the receiver. Hyperbola or straight line equations are established and the crossing point of these equations determines the location of the receiver. In a moderate model room, the performance of the proposed algorithm is demonstrated through simulation, with the positioning accuracy usually in the order of millimeters, suitable for various indoor location-based services.

*Key-Words:* - indoor positioning system; visible light; photo diode; white LEDs; time difference of arrival; scale; azimuth angle

## 1 Introduction

Energy efficient white light emitting diodes (LEDs) are not only used for indoor lighting but also for optical wireless communication [1]. Recently, indoor visible light positioning using white LEDs has attracted increasing attention. A number of visible light positioning systems using white LEDs have been proposed. According to the photosensitive elements utilized in the receiver, indoor visible light positioning can be divided into the imaging positioning [2-3] and the non-imaging positioning [4-14]. Image sensors are used as photosensitive elements in the image positioning systems, and often integrated into a high-speed camera. At least one pixel is processed to derive the location of the target via image and digital processing technique. Most research on the imaging positioning is now in laboratory stage.

In the non-imaging positioning systems, photo diode (PD) are used as photosensitive element, and the PD-based visible light positioning is the most widely used indoor visible light positioning systems. Depending on whether fingerprints are collected or not, the PD-based visible light positioning can further be putted into two categories: fingerprinting

and trilateration. In fingerprinting, the fingerprints are first collected associated with one particular scene and the target's location is then discriminated by matching real-time measurements to these fingerprints. The most commonly used fingerprints are received signal strength (RSS) fingerprints [4-6]. However, the RSS profile is usually affected by factors, such as the room size, number of targets or LEDs and layout of the room etc. Hence, it is highly environment dependent and has poor robustness.

To apply trilateration method [7-14] to the PD-based visible light positioning, it is necessary to estimate parameters such as RSS, time of arrival (TOA), time difference of arrival (TDOA), phase difference of arrival (PDOA), angel of arrival or the combination of them. Unfortunately, all the trilateration methods mentioned above have viewed the receiver as a point without taking into account the scale of the receiver, which is not always true in practice where smart phones, mobile robots or packets in warehouse etc. are used as receivers.

In this paper, an algorithm for indoor PD-based visible light positioning has been proposed, where the scale of the receiver is taken into consideration. Furthermore, the azimuth angle of the receiver can

be obtained with gyro-sensors since most smart phones or tablets are now provided with built-in sensors such as PDs and gyro-sensors.

This paper is organized as follows. The system model and relevant notations are established in Section 2. The algorithm for indoor PD-based visible light positioning is proposed in Section 3. Simulation and results are given in Section 4. The conclusion is drawn in Section 5.

## 2 System Model

The indoor PD-based visible light positioning system model is shown in Fig.1. The white LEDs  $\mathbf{T}_1$  and  $\mathbf{T}_2$  are with known location of  $\mathbf{T}_1: [X_1, Y_1, Z_1]$  and  $\mathbf{T}_2: [X_2, Y_2, Z_2]$ , where  $Z_1 = Z_2 = H$ , and  $H$  is the vertical distance between the transmitter's plane and the receiver's plane. The length of the receiver  $L$  is negligible. Two PDs  $\mathbf{R}_1$  and  $\mathbf{R}_2$  are respectively at either end of the receiver. If the mid-point of the receiver is  $[x, y, 0]$ , then the location of  $\mathbf{R}_1$  can be expressed as  $[x-L\cos\theta/2, y-L\sin\theta/2, 0]$  and the location of  $\mathbf{R}_2$  can be expressed as  $[x+L\cos\theta/2, y+L\sin\theta/2, 0]$ , where the azimuth angle of the receiver is  $\theta$ , which is the angle of the receiver deviating from the X-axis in the world coordinate system. The azimuth angle of the receiver is measured in counter-clock and lie in  $(-\pi, \pi)$ . For example, the  $\theta$  in Fig.1 is positive.

The goal of indoor PD-based visible light positioning is to determine the mid-point of the straight line joining  $\mathbf{R}_1$  and  $\mathbf{R}_2$ .

The input of the visible light channel from two LEDs are separately expressed as

$$x_i(t) = P_0 + P_0 \cos(2\pi f_i t), \quad i = 1, 2 \quad (1)$$

where the signal  $x_i(t)$  represents instantaneous optical power, which is nonnegative. The maximum transmitted intensity is  $P_0$ . That the frequency  $f_1$  is different from  $f_2$  is designed in order to obtain the arrival time difference of certain frequency signal in the receiver.

In this paper, it is assumed that only the line of sight (LOS) path exists and the diffused reflection path is negligible [15]. The direct current (DC) gain  $h$  of the intensity modulation and direct detection (IM/DD) channel is

$$h = \frac{m+1}{2\pi d^2} \cos^m(\psi) \cdot \cos(\varphi) \text{rect}\left(\frac{\varphi}{\text{FOV}}\right) \cdot A_R R_p \quad (2)$$

where  $\psi$  is the angle of irradiance with respect to the

normal of LED transmitter plane and  $\varphi$  is the angle of incidence with respect to the normal of receiver plane. The transmission distance between transmitter and receiver is  $d$ , and  $m$  is the order of Lambertian radiation with  $m = -\ln 2 / \ln(\cos \psi_{1/2})$  where  $\psi_{1/2}$  is given by the semi-angle at half illuminance of an LED. For example, if  $\psi_{1/2} = 60^\circ$ ,  $m = 1$ . Function  $\text{rect}(\cdot)$  is a rectangular function and FOV is the field of view of the receiver. The parameters  $A_R$  and  $R_p$  are respectively the effective detecting area and responsivity of the PDs.

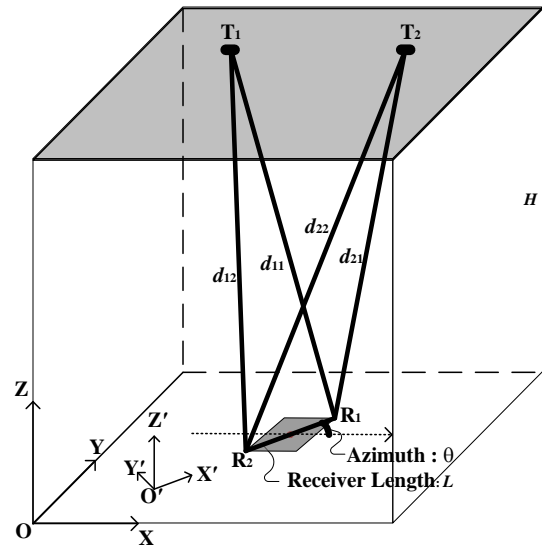


Fig. 1. Indoor PD-based visible light positioning system model.

After the propagating of such an IM/DD channel and the photoelectric converting of the two PDs, the obtained electrical signals are expressed as

$$\begin{cases} r_1(t) = h_{11}s_1(t - \tau_{11}) + h_{21}s_2(t - \tau_{21}) \\ r_2(t) = h_{12}s_1(t - \tau_{12}) + h_{22}s_2(t - \tau_{22}) \end{cases} \quad (3)$$

where different propagation channel has different gain  $h$  and time delay  $\tau$ . For example, the LOS channel gain between LED  $\mathbf{T}_1$  and PD  $\mathbf{R}_1$  is  $h_{11}$  and the time delay of such a particular LOS channel is  $\tau_{11}$ , and so on.

## 3 Algorithm for Indoor PD-based Visible Light Positioning

The proposed algorithm for indoor PD-based visible light positioning is performed via three steps, as shown in Fig.2.

First, the photoelectric conversion outputs from the two PDs, i.e.  $r_1(t)$  and  $r_2(t)$ , are fed into the band-pass filters with different central frequency in Band Pass Filter Unit. The filtered signals are separately expressed with the speed of light  $c$  by

$$\begin{cases} r_{11}(t) = h_{11}P_0 \cos[2\pi f_1 t - 2\pi f_1 \frac{d_{11}}{c}] \\ r_{21}(t) = h_{21}P_0 \cos[2\pi f_2 t - 2\pi f_2 \frac{d_{21}}{c}] \\ r_{12}(t) = h_{12}P_0 \cos[2\pi f_1 t - 2\pi f_1 \frac{d_{12}}{c}] \\ r_{22}(t) = h_{22}P_0 \cos[2\pi f_2 t - 2\pi f_2 \frac{d_{22}}{c}] \end{cases} \quad (4)$$

Then, the phase difference  $\phi^i$  ( $i = 1, 2$ ) of the same frequency signal is detected by the Phase Detector Unit, which can be written as

$$\begin{cases} \phi^1 = \phi_{12} - \phi_{11} = 2\pi f_1 \frac{d_{11} - d_{12}}{c} \\ \phi^2 = \phi_{22} - \phi_{21} = 2\pi f_2 \frac{d_{21} - d_{22}}{c} \end{cases} \quad (5)$$

where the phase difference  $\phi^i$  ( $i = 1, 2$ ) can be realized by cross-correlation method [16]. Correspondingly, the transmission distance difference  $D^i$  ( $i=1, 2$ ) can be expressed by

$$\begin{cases} D^1 = d_{11} - d_{12} = \frac{\phi^1}{2\pi f_1} \cdot c \\ D^2 = d_{21} - d_{22} = \frac{\phi^2}{2\pi f_2} \cdot c \end{cases} \quad (6)$$

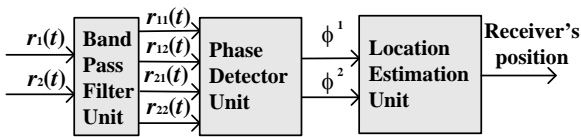


Fig. 2. The algorithm for indoor PD-based visible light positioning.

Finally, in the Location Estimation Unit, we use the **Euclidean** distance between each pair of LED-PD

$$\begin{aligned} d_{11} &= \sqrt{(X_1 - x + L \cos \theta / 2)^2 + (Y_1 - y + L \sin \theta / 2)^2 + H^2} \\ d_{12} &= \sqrt{(X_1 - x - L \cos \theta / 2)^2 + (Y_1 - y - L \sin \theta / 2)^2 + H^2} \\ d_{21} &= \sqrt{(X_2 - x + L \cos \theta / 2)^2 + (Y_2 - y + L \sin \theta / 2)^2 + H^2} \\ d_{22} &= \sqrt{(X_2 - x - L \cos \theta / 2)^2 + (Y_2 - y - L \sin \theta / 2)^2 + H^2} \end{aligned} \quad (7)$$

Together with the obtained the transmission distance difference  $D^1$  and  $D^2$ , we can deduce the following for any  $i$  ( $i=1, 2$ )

$$2L(X_i - x) \cos \theta + 2L(Y_i - y) \sin \theta + (D^i)^2 = 2D^i \sqrt{(X_i - x + L \cos \theta / 2)^2 + (Y_i - y + L \sin \theta / 2)^2 + H^2} \quad (8)$$

Let

$$\begin{cases} X_i - x = x_i \\ Y_i - y = y_i \end{cases} \quad (9)$$

Then (8) can be rearranged for any  $i$  ( $i=1, 2$ ) as

$$\begin{aligned} x_i^2 (4L^2 \cos^2 \theta - 4(D^i)^2) + y_i^2 (4L^2 \sin^2 \theta - 4(D^i)^2) \\ + 8L^2 x_i y_i \cdot \sin \theta \cos \theta = (D^i)^2 (L^2 - (D^i)^2 + 4H^2) \end{aligned} \quad (10)$$

It should be noted from (10) that it is a quadratic equation of  $(x_i, y_i)$  for any  $i$  ( $i=1, 2$ ). According to the relationship between three sides of a triangle, the sum of two sides is always greater than the third side, and the difference of two sides is always less than the third side. As shown in Fig.1,  $|D^i| < L$  is true, i.e.  $L^2 - (D^i)^2 > 0$  is true. Since the vertical height  $H > 0$ , part of the constant term of (10), i.e.  $L^2 - (D^i)^2 + 4H^2 > 0$  is always true.

In the following, it is necessary to take into account two cases of (10):  $D^i \neq 0$  and  $D^i = 0$ .

Let us first consider the case when  $D^i \neq 0$ . The right hand of (10) is always positive for this case. After the coordinate axis transformation, (10) can be rearranged as

$$\frac{x_i'^2}{a^2} - \frac{y_i'^2}{b^2} = 1, \quad i=1,2 \quad (11)$$

The graph expressed by (11) is a standard hyperbola in the new coordinate axes of  $x_i \cos \theta + y_i \sin \theta = 0$  and  $x_i \sin \theta - y_i \cos \theta = 0$  with

$$a^2 = \frac{(D^i)^2 (L^2 - (D^i)^2 + 4H^2)}{4(L^2 - (D^i)^2)}; b^2 = \frac{(L^2 - (D^i)^2 + 4H^2)}{4} \quad (12)$$

For the other case when  $D^i = 0$ , the right hand of (10) is always equal to zero. And (10) can be simplified as a straight line, which is expressed as

$$x_i \cos \theta + y_i \sin \theta = 0, \quad i=1,2 \quad (13)$$

In a word, in our algorithm, hyperbola or straight line equations are first established, which is based on the TDOA of the received signal combined with the scale and azimuth angle of the receiver, and then the crossing point of hyperbola or straight line equations are used to determine the location of the receiver.

### 4 Simulation and Results

To investigate the performance of the proposed algorithm, simulation has been performed. It is supposed to be a 5 by 5 by 3 meter model room. Two LEDs are separately placed on the ceiling at (2m, 4m, 3m) and (4m, 2m, 3m). They are respectively modulated by  $f_1 = 1\text{MHz}$  and  $f_2 = 2\text{MHz}$ . The length of the receiver is supposed to be 12.7cm, which is appropriate to most commonly used smartphones or tablets.

Fig.3 shows the obtained hyperbolas at point (2.5m, 2m) when the receiver is placed on a desk (0.85m above the floor), facing up with azimuth angle  $\theta = 30^\circ$  and  $L = 12.7\text{cm}$ . It can be seen from Fig.3 that the crossing of the two hyperbolas perfectly indicates the location of the receiver.

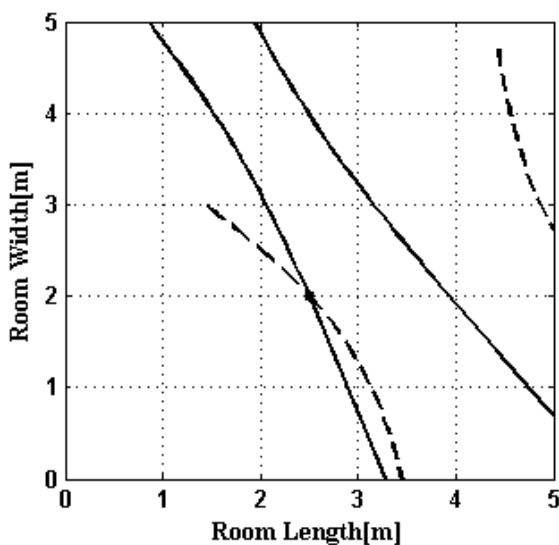


Fig. 3. The obtained hyperbolas at point (2.5m, 2m) when the receiver is placed on a desk (0.85m above the floor), facing up, with azimuth angle  $\theta = 30^\circ$  and  $L = 12.7\text{cm}$ .

The effect of the measurement error during phase detection on the positioning accuracy is investigated. we assume that the measurement noise is a Gaussian process with zero mean and standard deviation of  $10^{-3}$ . As shown in Fig. 4, the estimated location is (2.4597m, 2.0767m), with the positioning error of

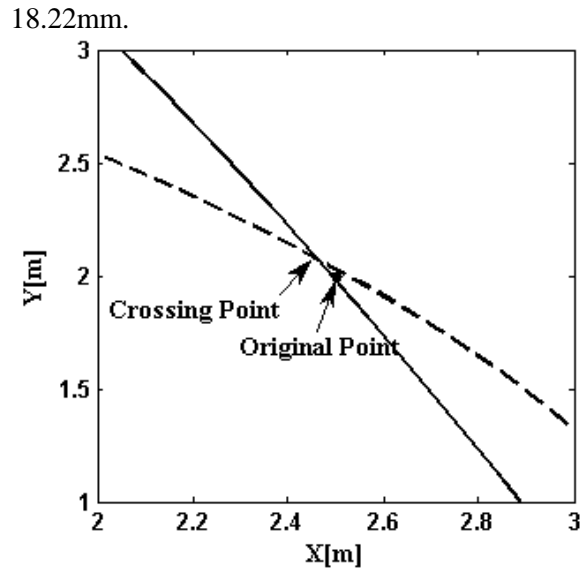


Fig.4. The influence of measurement error on positioning accuracy.

The effect of the dimension of the room on the positioning accuracy is also investigated. We simulate by changing the dimension from  $1 \times 1 \times 1 \text{ m}^3$  to  $5 \times 5 \times 5 \text{ m}^3$ . As shown in Fig. 5, the positioning accuracy is gradually decreasing with increasing the dimension of the room. This is because the signals transmitted from LEDs are much affected by the visible light channel while the dimension of the room is increased.

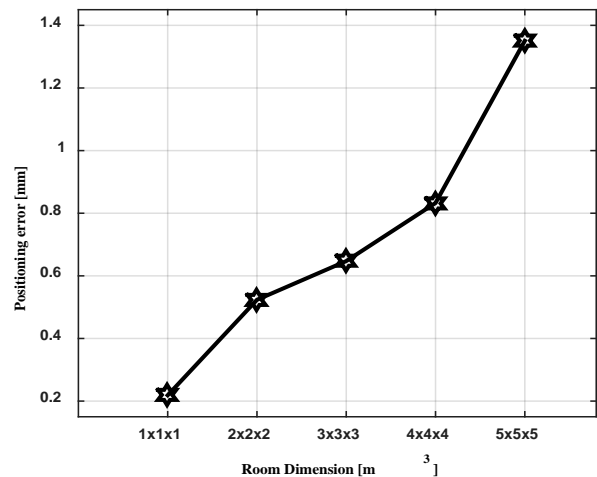


Fig.5. The influence of the dimension of the room on positioning accuracy.

In the end, the positioning errors at various discrete positions in the model room are also shown in Fig.6. It is supposed that the floor is divided into grids with equal spacing of 0.2 m, and the receiver is located in the centre of each grid on the floor. As

shown in Fig. 6, the most serious errors occur at the corner of the room. Among all the discrete points on the floor, the maximum positioning error is 5.568 mm. However, the mean location error is only 0.691 mm, much more suitable for indoor location-based services.

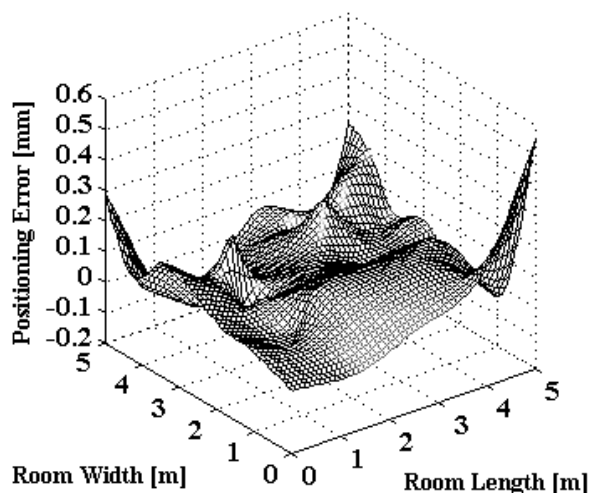


Fig. 6. Positioning error profile.

## 5 Conclusion

Proposed in this paper is a novel trilateration algorithm for indoor photo diode-based visible light positioning using white LEDs. Using the time difference of the received signal from the same LED, together with the scale and azimuth angle of the receiver, we determine the location of the receiver. A hyperbola or straight line equations are established and the crossing point of these equations is just the location of the receiver. For typical practical scenes, such as smart phones, mobile robots or packets in warehouse etc. the scale of the receiver should not be negligible in the proposed algorithm, which is different from conventional TDOA-based method. Simulation results show that the positioning accuracy of the proposed algorithm is usually in the order of millimetres in a moderate model room, suitable for various indoor location-based services.

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## References:

- [1] Karunatilaka D., Zafar F., Kalavally V., Parthiban R. , LED Based Indoor Visible Light Communications: State of the Art, *IEEE Communications Surveys & Tutorials*, vol. PP, no. 99, 2015, pp.1–1.
- [2] R. Mohammad, S. M. Haque, and K. D. Kim, High precision indoor positioning using lighting LED and image sensor, *Proc. Int. Conf. Computer and Information Technology*, Dhaka, Bangladesh, Dec. 2011, pp.309-314.
- [3] T. Tanaka and S. Haruyama, New position detection method using image sensor and visible light LEDs, *2<sup>nd</sup> Int. Conf. on Machine Vision*, Dubai, United Arab Emirates, Dec. 2009, pp. 150–153.
- [4] J. Vongkulbhisal, B. Chantaramolee, Y. Zhao, and W. S. Mohammed, A fingerprinting-based indoor localization system using intensity modulation of light emitting diodes, *Microwave and Optical Technol. Lett.*, vol.54, no. 5, May.2012, pp. 1218–1227.
- [5] S. Yang, D. Kim, H. Kim, Y. Son, and S. Han, Visible light based high accuracy indoor localization using the extinction ratio distributions of light signals, *Microwave and Optical Technol. Lett.*, vol. 55, no. 6, June 2013, pp. 1385–1389.
- [6] S. Y. Jung, S. Hann, S. Park, and C. S. Park, Optical wireless indoor positioning system using light emitting diode ceiling lights, *Microwave and Optical Technol. Lett.*, vol. 54, no. 7, July.2012, pp.1622–1626.
- [7] S. Y. Jung , S. R. Lee and C. S. Park, Indoor location awareness based on received signal strength ratio and time division multiplexing using light-emitting diode light, *Opt. Eng.*, Vol.53, no. 1, Jan. 2014, pp. 016106-1–016106-5.
- [8] S.-H. Yang, H. S. Kim, Y. H. Son, and S. K. Han, Three-Dimensional Visible Light Indoor Localization Using AOA and RSS With Multiple Optical Receivers, *J. Lightwave Technol.*, vol. 32, no. 14, July 2014, pp. 2480–2485.
- [9] T. Q. Wang, Y. A. Sekercioglu, A. Neild, and Jean Armstrong, Position Accuracy of Time-of-Arrival Based Ranging Using Visible Light With Application in Indoor Localization

- Systems, *J. Lightwave Technol.*, vol. 31, no. 20, Oct.2013, pp. 3302–3308.
- [10] U. Nadeem, N.U. Hassan, M.A. Pasha and C. Yuen, Highly accurate 3D wireless indoor positioning system using white LED lights, *Electronics Letters*, vol. 50, no. 11, May. 2014, pp. 828–830.
- [11] S.-Y. Jung, S. Hann, and C. S. Park, TDOA-based optical wireless indoor localization using LED ceiling lamps, *IEEE Trans. Consumer Electron.*, vol. 57, no.4, Nov. 2011, pp. 1592–1596.
- [12] K. Panta and J. Armstrong, Indoor localization using white LEDs, *Electronics Letters*, vol. 48, no. 4, Feb. 2012, pp. 228–230.
- [13] G. B. Prince, T. D. C. Little, A Two Phase Hybrid RSS/AoA Algorithm for Indoor Device Localization Using Visible Light, *IEEE GLOBECOM*, California, USA, Dec. 2012, pp. 3347-3352.
- [14] M. Yasir, S. W. Ho, and B. N. Vellambi, Indoor Positioning System Using Visible Light and Accelerometer, *J. Lightwave Tech.*, vol. 32, no. 19, Oct. 2014, pp. 3306–3316.
- [15] L. Zeng, D. C. O’Brien, H. L. Minh, G. E. Faulkner, K. Lee, D. Jung, Y. Oh, and E. T. Won, High Data Rate Multiple Input Multiple Output (MIMO) Optical Wireless Communications Using White LED Lighting, *IEEE Journal on Selected Areas in Communications*, vol. 27, no. 9, Dec. 2009, pp. 1654-1662.
- [16] S. L. Marple, Jr., Estimating group delay and phase delay via discrete-time “analytic” cross-correlation, *IEEE Trans. Signal Processing*, vol. 47, no. 9 , Sep.1999, pp. 2604–2607.