Closed form Delay/Doppler/Propagation Factor Acquisition for GPS Signals

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Abstract: In this article, a delay/Doppler/propagation factor acquisition method for global positioning system (GPS) signal is proposed. Utilising the Fourier transform (FFT) of the autocorrelation function (ACF) for the received signal, together with some mathematical simplifications, a closed form solution is derived to estimate the delay/Doppler/propagation factor for the received signal. Unlike previous related estimation methods for this case, the proposed method has the advantage of not requiring any searching over the different parameters to perform the estimation. Simulation results for the proposed method are presented in this article to assess its performance.

Key–Words: GPS; Acquisition; autocorrelation function; Signal Processing.

1 Introduction

Positioning in global positioning system (GPS) requires estimating distances between different satellites and the GPS device. These distances are deduced from estimating the signal's flight time from the GPS satellite to the GPS device and consequently finding the time delays from different satellites. Due to the movement of the satellites in their orbits and the possibility of the movement of the GPS device on the earth surface, the received signal will suffer from Doppler shift, which will introduce errors in the delays estimation. Thus, it is imperative to account for the Doppler shift to produce acceptable performance.

Many methods were proposed in the literature to estimate the delay and Doppler shift [1, 2, 3, 4, 5]. The method presented in [1] uses fractional fast Fourier transform (FFT) to estimate the delay and Doppler shift. The method in [2] presents thresholds on the counter for the searching process to estimate the parameters. In [3], the authors use block searches with large-scale FFT to perform the estimation. The method presented in [4] is FFT-based acquisition method which utilises circular/linear convolution methods for the parameters estimation. In [5], the authors investigate using partial differential post-correlation processing techniques to perform parameter estimation. All the previously mentioned methods require searching process to perform the estimation. In this article however, a closed form estimator for delay and Doppler shift is presented for GPS signals in addition to the propagation factor.

The rest of this article is organised as follows: Section II presents the system model. Section III explains the proposed estimator. Simulations are shown in Section IV to assess the proposed method performance. Finally, conclusions are presented in Section V.

2 System Model

For an asynchronous code division multiple access (CDMA) system that employs BPSK modulation, let the number of involved satellites be L. The data bit duration is T and the chip duration is $T_c = T/N$, where N is the number of chips per bit. The PN spreading waveforms are assumed to be of period NT_c and each chip has the waveform denoted by $\check{P}_{T_c}(t)$. The transmitted baseband signal of the kth user over the mth bit interval, $\check{s}_l(t)$, is formed by modulating the mth data bit, $d_{l,m}$, with the spreading code waveform, $\check{c}_l(t)$, as

$$\check{s}_l(t) = d_{l,m}\check{c}_l(t - (m-1)T) \tag{1}$$

for $(m-1)T \le t < mT$ where $d_{l,m} \in$ takes one of the values {-1,+1} with equal probabilities and $\check{c}_l(t)$ is the transmitted spreading code sequence, which takes the form of

$$\check{c}_l(t) = \sum_{n=0}^{N-1} a_{l,n}\check{P}_{T_c}(t - nT_c) \tag{2}$$

where $a_{l,n} \in \{-1, 1\}$ is the nth chip of the lth satellite spreading code. The chip waveform signal $\check{P}_{T_c}(t)$ is assumed to have a square root raised-cosine pulse shape.

At the receiver, the signal is processed by a matched filter producing $s_l(t)$

$$s_l(t) = \check{s}_l(t) \star \check{P}_{T_c}(t) = d_{l,m}c_l(t - (m-1)T) \tag{3}$$

for $(m-1)T \leq t < mT$, where $\star$ denotes convolution, $c_l(t)$ is defined as

$$c_l(t) = \sum_{n=0}^{N-1} a_{l,n} P_{T_c}(t - nT_c) \quad (4)$$

and $P_{T_c}(t)$ is a raised-cosine pulse.

The filtered received signal is

$$r(t) = \sum_{l=1}^{L} \beta_l s_l \left(t - \delta_l(t)\right) e^{j2\pi f_{d,l} t} + n(t) \quad (5)$$

where β_l and $f_{d,l}$ are the propagation factor and Doppler shift for the lth satellite signal, respectively. Also, $n(t)$ is an additive white Gaussian noise (AWGN) with zero mean and variance of σ_n^2, and $\delta_l(t)$ is the delay of the lth satellite signal. The received signal is then sampled with a sampling frequency of $1/T_s$, where $T_s = T_c/Q$ and Q is an integer that represents the over sampling gain. The sampled received signal takes the form

$$r(iT_s) = \sum_{l=1}^{L} \beta_l s_l \left(iT_s - \delta_l(iT_s)\right) e^{j2\pi f_{d,l} iT_s} + n(iT_s) \quad (6)$$

where $\delta_l(iT_s)$ is the discrete version of the delay $\delta_l(t)$. For simplicity, the variable T_s will be dropped in the term iT_s.

3 Acquisition of Delay/Doppler/ Propagation Factor

The received signal, $r(i)$, is correlated at the receiver with a specific code to retrieve the transmitted bits. So, considering the xth code for the xth satellite, then,

$$\begin{aligned} z(i) &= r(i) * c_x(i) \\ &= \sum_{l=1}^{L} \beta_l d_{l,m} \lambda_{l,x}(i - \delta_l(i)) e^{j2\pi f_{d,l} i} + n_z(i) \end{aligned} \quad (7)$$

where the operator $*$ denotes the correlation operator, $\lambda_{l,x} = c_l(i) * c_x(i)$ is the corsscorrelation function, and $n_z(i)$ is AWGN which results from the convolution of $n(i)$ with $c_x(i)$.

For sake of simplicity, assume that a specific satellite (the xth satellite) is considered as the satellite of interest. Also, assume that other satellite received signals in $z(i)$ are interfering signals, i.e.,

$$\begin{aligned} z(i) &= \beta_x d_{x,m} \lambda_{x,x}(i - \delta_x(i)) e^{j2\pi f_{d,x} i} \\ &+ \sum_{\substack{l=1 \\ l \neq x}}^{L} \beta_l d_{l,m} \lambda_{l,x}(i - \delta_l(i)) e^{j2\pi f_{d,l} i} + n_z(i) \end{aligned} \quad (8)$$

where $\lambda_{x,x}(i)$ is the autocorrelation function (ACF) for the xth satellite.

Since $\lambda_{l,x}(i)$ for $l \neq x$ takes a small value, the second term in (8) can be neglected. Therefore,

$$z(i) = \beta_x d_{x,m} \lambda_{x,x}(i - \delta_x(i)) e^{j2\pi f_{d,x} i} + n_z(i). \quad (9)$$

Considering the fact that $\lambda_{x,x}(i - \delta_x(i))$ is significantly larger than $n_z(i)$ term for sufficiently large N, the noise term can be safely neglected leading to the noise free version of $z(i)$, denoted as $\bar{z}(i)$, expressed as

$$\bar{z}(i) = \beta_x d_{x,m} \lambda_{x,x}(i - \delta_x(i)) e^{j2\pi f_{d,x} i}. \quad (10)$$

Now, to estimate the Doppler shift for the xth satellite received signal, $f_{d,x}$, we first eliminate the effect of $d_{x,m}$ on the phase of $z(i)$ by computing the square of $\bar{z}(i)$, and noting that $d_{x,m}^2 = 1$ we have

$$\bar{z}^2(i) = \beta_x^2 \lambda_{x,x}^2(i - \delta_x(i)) e^{j4\pi f_{d,x} i}. \quad (11)$$

Then to exclude the effect of β_x, defining the ratio

$$\begin{aligned} \mu(i) &= \frac{\bar{z}^2(i)}{\bar{z}^2(i-1)} \\ &= \frac{\lambda_{x,x}^2(i - \delta_x(i)) e^{j4\pi f_{d,x} i}}{\lambda_{x,x}^2(i - 1 - \delta_x(i-1)) e^{j4\pi f_{d,x}(i-1)}}. \end{aligned} \quad (12)$$

Since $\lambda_{x,x}(i - \delta_x(i))$ is an ACF it has real values and so does $\lambda_{x,x}^2(i - \delta_x(i))$. Consequently, $\angle \lambda_{x,x}^2(i - \delta_x(i)) = 0^o$. Thus, it is clear that

$$\angle \mu(i) = 4\pi f_{d,x} \ . \quad (13)$$

Hence, the estimated Doppler shift for the lth satellite signal, $f_{d,x}$, is

$$\hat{f}_{d,x} = \frac{\angle \mu(i)}{4\pi}. \quad (14)$$

Clearly, the Doppler shift is estimated without the need for any searching technique.

Consequently, the Doppler shift effect on $\bar{z}(i)$ can be simply compensated as follows

$$\bar{u}(i) = \bar{z}(i) e^{-j2\pi f_{d,x} i} = \beta_x d_{x,m} \lambda_{x,x}(i - \delta_x(i)). \quad (15)$$

We proceed to estimate the phase of the propagation factor $\beta_x = \alpha_x e^{j\phi_x}$. Starting with ϕ_x estimate, we proceed as before and consider the phase of $\bar{u}^2(i)$, which is

$$\angle\bar{u}^2(i) = \angle\{\beta_x^2 d_{x,m}^2 \lambda_{x,x}^2(i-\delta_x(i))\} = \angle\{\beta_x^2\} \quad . \tag{16}$$

Leading to the simple estimate

$$\hat{\phi}_x = \frac{\angle\bar{u}^2(i)}{2} \quad . \tag{17}$$

To estimate the delay and α_x, we compute the FFT of $\bar{u}(i)$, which is

$$\bar{U}(e^{j\omega}) = \beta_x d_{x,m} \Lambda_{x,x}(e^{j\omega}) e^{-j\omega\delta_x} \tag{18}$$

where $\Lambda_{x,x}(e^{j\omega})$ is the FFT of $\lambda_{x,x}(i)$ (which is known by the receiver, because the receiver knows the code $c_x(i)$ beforehand). Dividing both sides of (18) by $\Lambda_{x,x}(e^{j\omega})$, then

$$V(e^{j\omega}) = \frac{\bar{U}(e^{j\omega})}{\Lambda_{x,x}(e^{j\omega})} = \beta_x d_{x,m} e^{-j\omega\delta_x}. \tag{19}$$

Having done that, α_x is estimated by

$$\hat{\alpha}_x = |V(e^{j\omega})| = |\beta_x d_{x,m}| \quad . \tag{20}$$

Hence, $\hat{\beta}_x = \hat{\alpha}_x e^{j\hat{\phi}}$. To estimate the delay define

$$G(e^{j\omega}) = \frac{V(e^{j\omega})}{\beta_x} = d_{x,m} e^{-j\omega\delta_x} \quad . \tag{21}$$

and take its square (which is the hth sample of $G(e^{j\omega})$) as follows:

$$G_h^2(e^{j\omega_o}) = e^{-j2h\omega_o\delta_x} \quad . \tag{22}$$

Thus,

$$\hat{\delta}_x = \frac{-\angle G_h^2(e^{j\omega})}{2h\omega_o} \quad . \tag{23}$$

Finally, in order to estimate the data bit $d_{x,m}$ for some sample h we write

$$\hat{d}_{x,m} = \text{sign}\left(G_h(e^{j\omega}) e^{jh\omega_o\hat{\delta}_x}\right) \tag{24}$$

where sign(c) is the sign (or parity) of some argument c.

4 Simulation results

Simulations for estimating the delay, Doppler shift, propagation factor magnitude and phase for the desired satellite signal were completed to assess the proposed method performance. There are many CDMA codes discussed in the literature one of which is Gold-code. Gold-codes are generated using shift two polynomials. In this paper Gold-codes are used to model the CDMA code sequence with 31chips length and generated using the polynomials $x^5 + x^2 + 1$ and $x^5 + x^4 + x^3 + x^2 + 1$. The delay, Doppler shift, propagation factor magnitude and phase of the desired satellite signal were $2T_c$, 5 Hz, 10 and $\frac{\pi}{4}$ radians, respectively. Three values of over sampling gain Q were considered ($Q = 10$, 15 and 20). The results for the delay, Doppler shift, propagation factor magnitude and phase root mean square estimation error were obtained by averaging over 1000 independent simulation runs.

Figs. (1), (2), (3) and (4) show the delay, Doppler shift, propagation factor magnitude and phase root mean square estimation error, respectively, versus different signal to noise ratios (SNR)s in dB using the proposed method. The results indicate that the proposed method managed to estimate the required parameters with high accuracy. Also, the figures indicate that the performance is enhanced as the SNR and over sampling gain (Q) are increased.

5 Conclusion

In this article, a delay/Doppler/propagation factor acquisition method for global positioning system (GPS) signal is proposed. The method provides a closed form solution for the estimation of the required parameters. The method is based upon algebraic manipulation of FFT of the ACF for the received signal. Thus, the proposed method does not requiring any searching procedure to perform the estimation. Simulation results for the proposed method are presented in this article to assess its performance.

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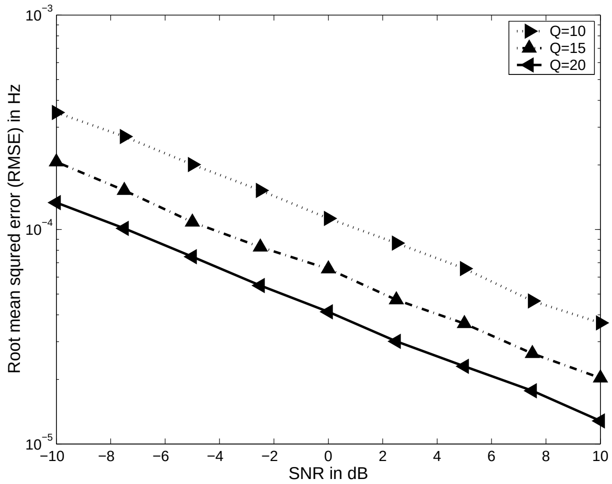

Figure 1: **Root mean square error (RMSE) for the Doppler shift estimation in Hz vs. SNR in dB**.

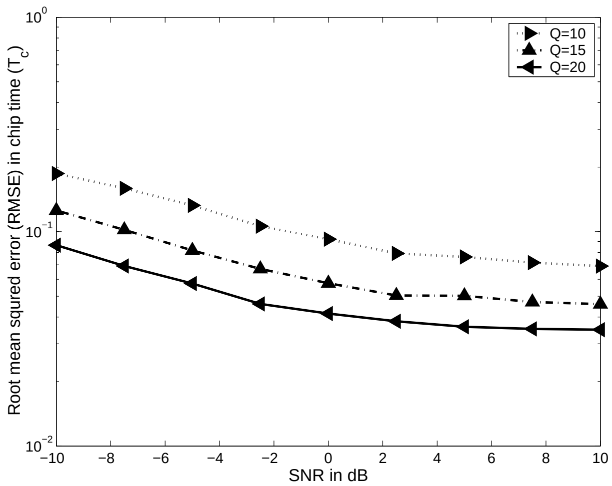

Figure 2: **Root mean square error (RMSE) for the delay estimation in chip time (T_c) vs. SNR in dB**.

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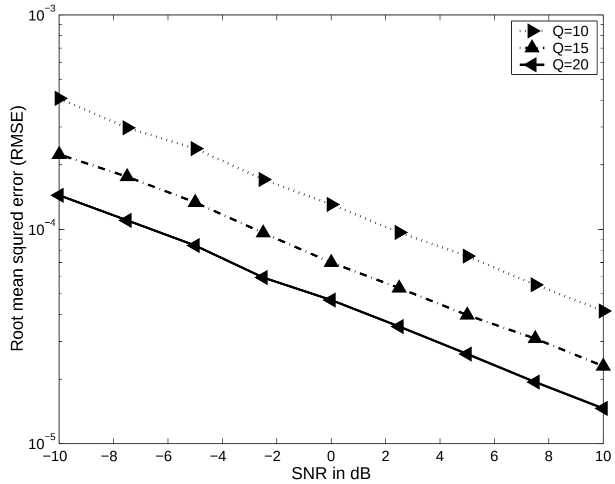

Figure 3: **Root mean square error (RMSE) for the propagation factor magnitude estimation vs. SNR in dB**.

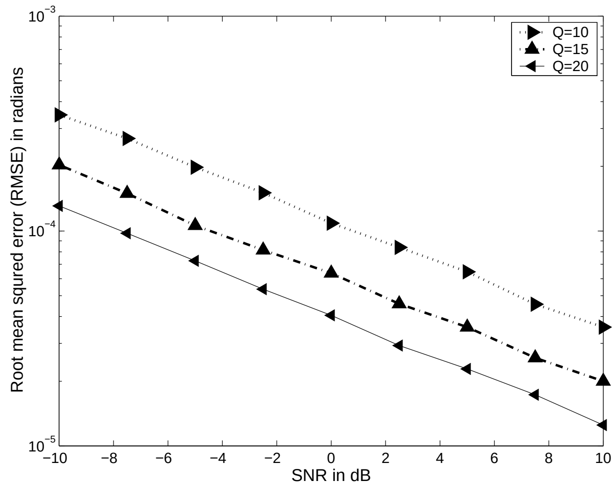

Figure 4: **Root mean square error (RMSE) for the propagation factor phase estimation in radians vs. SNR in dB**.