

A Study on Modeling of Road Pavements Based on Laser Scanned Data and a Novel Type of Approximating Hermite Wavelets

BORIS M. SHUMILOV¹, ANDREY N. BAIGULOV²

¹Department of Applied Mathematics
Tomsk State University of Architecture and Building
2, Solyanaya Sq., 634003, Tomsk

²JSC Indor-Most, 6, Shkolny Lane, 634003, Tomsk
RUSSIA

sbm@tsuab.ru, anbaigulov@sibmail.com

Abstract: - The paper presents a new system of Hermite basic spline-wavelets of any odd degree, realizing orthogonal conditions to all polynomials up to the same degree. Construction and inverting of the block of filters are considered according to processing of regular signals and three-dimensional fields. The problems of modeling automobile road pavements using laser scanned data are described. The results of numerical experiments and imposing of the designed road onto the processed laser points are also presented.

Key-Words: - Laser scanning, Data processing, Wavelets, Roads modeling

1 Introduction

Laser scanning is a new direction in high-precision 3D-measurements [1]. As to mobile scanning, scopes of application include positioning of the automobile and railroads, bridges, overpasses, city streets, the coastline etc. (Fig.1). The main advantage of laser scanning is the possibility to work at objects with heavy traffic, at industrial facilities without stopping of production and in hard-to-reach sites, and also at objects having a complex configuration [2].

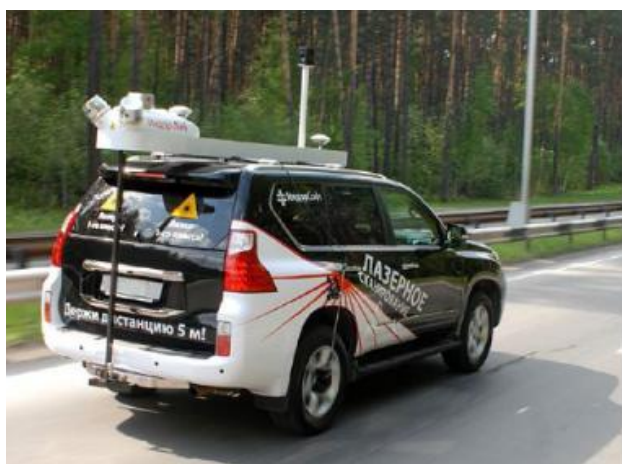


Fig.1: The mobile laser scanner

Cartesian components of the basics of GPS data from laser scanning device are issued in the form of the array ("cloud") of points (Fig.2), in which there is no division into separate cross scans. As the

principle of mobile scanning system allows working on the road without traffic overlapping, the cloud of points contains a roadside landscape and hindrances on the carriageway (reflection from people who are on object, technique, vegetation etc.) (Fig.3).

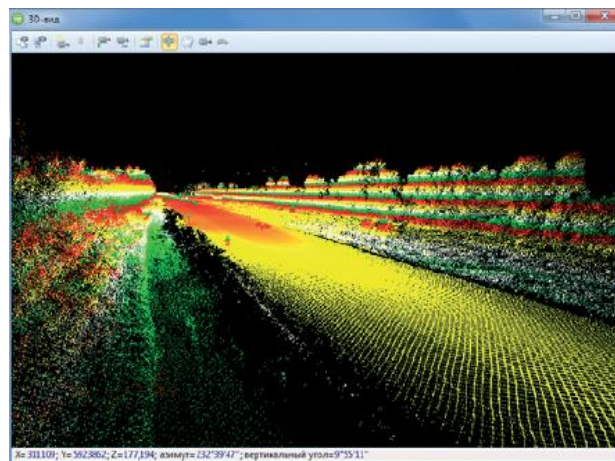


Fig.2: The 3D-view of a cloud of laser points

Main goals of preliminary processing of laser scanned data are removal of a roadside and surrounding landscape, filling of gaps in the scanned data, created by cars passing by on the carriageway, and the elaboration of the planned axial line of the road (Fig.4) [3]. From the mathematical point of view the production of the axis of the road allows to transform bends of the highway to some rectangular area, for which it is possible to apply standard methods of two-dimensional spline interpolation [4]

on a rectangular grid, keeping structural lines of the road (edges, brows), unlike popular method of restoration of a surface of the highway by a triangulation of chaotic points [5].

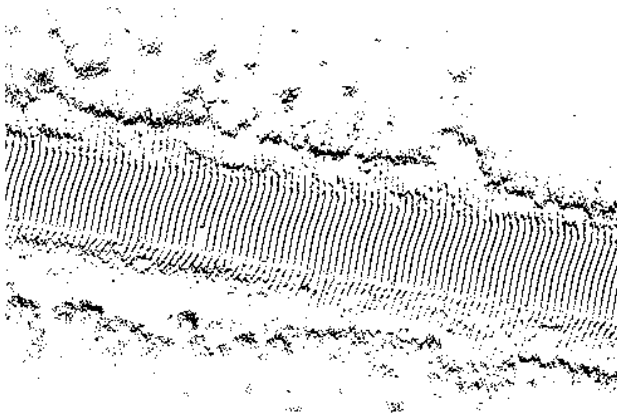


Fig.3: The plane view of points cloud

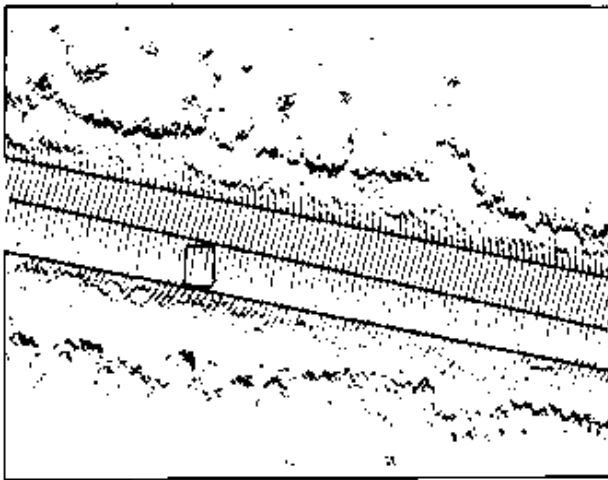


Fig.4: Straightening of laser data

Very important is that at such approach high precision of detection of cracks and damages of a road pavement in the places demanding repair is guaranteed and construction and application of wavelet-transformation of interpolation splines for compression of scanned information in the places of highways not demanding repair is significantly facilitated also [6].

2 Hermite Wavelets and Processing of Regular Signals

Cubic splines of smoothness of C^2 became popular for road engineers as an adequate way of mathematical representation of picket method of tracing of the reconstructing highways. As for Hermite splines they allow in an explicit form, using

the values of spline coefficients, to consider geometrical restrictions on control points (pickets of the automobile route), and on route tangents (the directions of tangent lines at entrance on bridge construction or at adjunction) and radiuses of transition curves (values of curvature on accelerating and brake sections of the route) [7]. Shortcomings of bi-orthogonal Hermite wavelets consist in fractal type of basic functions [8]. So to recalculate onto arbitrary points we need to return to the most dense grid, and then to interpolate the filtered values. These shortcomings semi-orthogonal Hermite spline-wavelets are deprived instead of the lack of mutually orthogonal basic functions [9]. However in this case from the computing point of view the local averaging formulas don't exist and calculation is reduced to:

- a) truncation of the infinite series for coefficients of spline-wavelets accompanied by truncation errors;
- b) or numerical calculation of integrated Fourier coefficients accompanied by integration errors;
- c) or the inverse of the equations systems complicated by sparse structure of the refinement matrix.

2.1 Construction of a new system of basic Hermite spline-wavelets

Let the interval $[0, 2^L]$, $L \geq 1$, with a grid Δ^L : $x_i = i$, $i = 0, 1, \dots, 2^L$, be set. Define the system of basic functions $N_{i,k}^L(x) = \varphi_k(x - i)$, $k = 0, 1, \dots, r \forall i$, where [4]

$$\varphi_k(t) = \begin{cases} (-1)^k \omega_k(-t), & -1 \leq t \leq 0, \\ \omega_k(t), & 0 \leq t \leq 1, \\ 0, & t \notin [-1, 1], \end{cases}$$

$$\text{and } \omega_k(t) = (1-t)^{r+1} \sum_{\beta=0}^{r-k} \frac{(r+\beta)!}{k!\beta!r!} t^{k+\beta}, k = 0, 1, \dots, r,$$

with the centers in integers, as $\varphi^L = [N_{0,0}^L, N_{0,1}^L, \dots, N_{0,r}^L, N_{1,0}^L, N_{1,1}^L, \dots, N_{2^L,r}^L]$.

Then interpolating Hermite spline of any odd degree $2r+1$ can be presented as $S^L(x) = \varphi^L(x) C^L$,

$$\text{where } C^L = [C_0^{L,0}, C_0^{L,1}, \dots, C_0^{L,r}, C_1^{L,0}, \dots, C_{2^L}^{L,r}]^T,$$

whereas coefficients of $C_i^{L,k}$, $k = 0, 1, \dots, r \forall i$, denote the values and corresponding derivatives of the approximated function at grid nodes. We will seek for basic wavelets with the centers in even integers as linear combinations of basic Hermite splines on a grid Δ^{L+1} of the form

$$M_{i,k}^L(x) = \sum_{l=0}^r \sum_{j=0}^2 \alpha_l^j \varphi_l(2t-j), t = (x-x_{2i}),$$

$$-1 \leq t \leq 3, \quad (1)$$

meeting orthogonal conditions to all polynomials of $2r+1$ -st order, that is

$$\int_0^{2^{L+1}} M_{i,k}^L(x) x^m dx = 0, k = 0, 1, \dots, r \forall i$$

$$(m = 0, 1, \dots, 2r+1). \quad (2)$$

Theorem. Let the coefficients $\alpha_l^j = \{1, l = k; 0, l \neq k\}$, three matrixes of R^0, R^1, R^2 of dimension $(2r+2) \times (r+1)$ are set by elements, respectively

$$R_{m,l}^j = \int_{j-1}^{j+1} \varphi_l(2t-j) t^m dt, j = 0, 1, 2, l = 0, 1, \dots, r,$$

$$m = 0, 1, \dots, 2r+1.$$

Then each of $r+1$ of columns of block matrix $[A_0^{\text{inner}} / A_2^{\text{inner}}] = -[R^0 | R^2]^{-1} R^1$ gives values of coefficients $\alpha_j^l, j = 0, 2, l = 0, 1, \dots, r$, of the corresponding k -st of the basic wavelets which are completely lying in interval $[0, 2^{L+1}], L \geq 1$. At $L = 0$ elements of matrixes of R^0, R^2 are calculated within interval $[0, 2]$, and the matrix of coefficients α_j^l accepts designation $[A_0^{\text{center}} / A_2^{\text{center}}]$.

At $L > 0$ for extreme at the left basic wavelets the elements of matrix of R^0 are calculated on the interval truncated at the left

$$R_{m,l}^0 = \int_0^1 \varphi_l(2t) t^m dt, l = 0, 1, \dots, r, m = 0, 1, \dots, 2r+1,$$

and coefficients of decomposition (1) are given by values of columns of matrix $[A_1^{\text{left}} / A_2^{\text{left}}] = -[R^1 | R^2]^{-1} R^0$ provided that coefficients $\alpha_0^l = \{1, l = k; 0, l \neq k\}$. For extreme at the right basic wavelets the elements of matrix of R^2 are calculated on the interval truncated at the right

$$R_{m,l}^2 = \int_1^2 \varphi_l(2t-2) t^m dt, l = 0, 1, \dots, r, m = 0, 1, \dots, 2r+1,$$

and coefficients of decomposition (1) are given by values of columns of matrix

$$[A_0^{\text{right}} / A_1^{\text{right}}] = -[R^0 | R^1]^{-1} R^2 \quad \text{provided that}$$

coefficients $\alpha_2^l = \{1, l = k; 0, l \neq k\}$.

The system of the functions $M_{i,k}^L(x), k = 0, 1, \dots, r, i = 1, 2, \dots, 2^L$, with supports no more than two steps of a grid Δ^{L+1} , meets conditions (2) and forms basis.

Proof of theorem follows [10] with using matrix designations.

2.2 Construction and inversion of the block of filters

As the grid $\Delta^{L-1}, L \geq 1$, is received from Δ^L by means of removal of every second knot, then it is possible to write down the corresponding functions φ^{L-1} in the form of linear combinations of functions $\varphi^L: \varphi^{L-1} = \varphi^L P^L$. Here blocks of matrix of P^L are made of coefficients of scale relations [9]

$$\begin{bmatrix} \varphi_0(t) \\ \varphi_1(t) \\ \vdots \\ \varphi_r(t) \end{bmatrix} = \sum_{k=0}^2 H_k \begin{bmatrix} \varphi_0(2t-k) \\ \varphi_1(2t-k) \\ \vdots \\ \varphi_r(2t-k) \end{bmatrix},$$

where $H_2 = U^{-1} \Lambda U, H_1 = \text{diag}(1, 2^{-1}, \dots, 2^{-r}),$
 $H_0 = S H_2 S^{-1}$ and the matrix of U of dimension $(r+1) \times (r+1)$ is set by the elements

$$U_{k,j} = (-1)^{r+1+k-j} \frac{(r+1+k)!}{(r+1+k-j)!}, k, j = 0, 1, \dots, r,$$

while $\Lambda = \text{diag}(2^{-r-1}, \dots, 2^{-2r-1}),$
 $S = \text{diag}(1, -1, \dots, (-1)^{-r}).$ Similarly, we will write

down basic wavelet-functions of $2r+1$ -st degree in the form of matrix line, $\psi^L = [M_{1,0}^L, M_{1,1}^L, \dots, M_{1,r}^L, \dots, M_{2^L,r}^L]$. Then it is possible to express functions ψ^{L-1} as linear combinations of functions $\varphi^L: \psi^{L-1} = \varphi^L Q^L$, where blocks of matrix of Q^L are made of coefficients of decomposition (1). We will collect the corresponding wavelet-coefficients in vector, $D^L = [D_1^{L,0}, D_1^{L,1}, \dots, D_1^{L,r}, \dots, D_{2^L}^{L,r}]^T$. Then, with use of designations for block matrixes, process of receiving C^L from C^{L-1} and D^{L-1} can be written down as [11]:

$$C^L = [P^L | Q^L] \begin{bmatrix} C^{L-1} \\ D^{L-1} \end{bmatrix}. \quad (3)$$

Resolvability of system (3) concerning C^{L-1}, D^{L-1} is guaranteed by linear independence of basic functions. For simplification of calculations at large L the matrix $[P^L | Q^L]$ is offered to be made block three-diagonal [12], having reordered the unknowns so that blocks of matrixes of P^L and Q^L interlaced:

$$\mathbf{K}^L u^{L-1} = C^L,$$

$$\mathbf{K}^1 = \begin{bmatrix} H_1 & A_0^{\text{center}} & O \\ H_2^T & I & H_0^T \\ O & A_2^{\text{center}} & H_1 \end{bmatrix},$$

$$\mathbf{K}^L = \begin{pmatrix} H_1 & I & O & O & \dots & O & O \\ H_2^T & A_1^{\text{left}} & H_0^T & O & \dots & O & O \\ O & A_2^{\text{left}} & H_1 & A_0^{\text{inner}} & \ddots & \vdots & O \\ O & O & H_2^T & I & \ddots & O & \vdots \\ O & O & O & A_2^{\text{inner}} & \ddots & A_0^{\text{right}} & O \\ \vdots & \vdots & \vdots & \ddots & H_2^T & A_1^{\text{right}} & H_0^T \\ O & O & O & \dots & O & I & H_1 \end{pmatrix},$$

$$L > 1,$$

$$u^L = [C_0^{L,0}, C_0^{L,1}, \dots, C_0^{L,r}, D_1^{L,0}, D_1^{L,1}, \dots, D_1^{L,r}, C_1^{L,0}, \dots, D_2^{L,r}, \dots, C_2^{L,r}]^T.$$

Here O designates matrix of $r+1$ -st order with zero coefficients whereas I – the single matrix of $r+1$ -st order, dots designate the repeating K^L matrix blocks. In a case with arbitrary interval $[a, b]$ for the compensation of the single step of a grid in the equations (3) we need to use as initial C^L the values of function and derivatives multiplied by $h = (b - a) / 2^L$ in the corresponding degree: $\{f^{(k)}(i \cdot h) \cdot h^k, k = 0, 1, \dots, r, i = 0, 1, \dots, 2^L\}$, $(r+1) \cdot (2^L+1)$ of values in total.

2.3 Numerical examples for function with two ruptures and a break

For $x \in [0, 1]$ we will consider as an example the approximation of function of Harten [13]

$$f(x) = \begin{cases} \frac{1}{2} \sin(3\pi x), & x \leq \frac{1}{3}, \\ |\sin(4\pi x)|, & \frac{1}{3} < x \leq \frac{2}{3}, \\ -\frac{1}{2} \sin(3\pi x), & x > \frac{2}{3}. \end{cases}$$

If to neglect all wavelet-coefficients, having left only C^0 spline coefficients, some smoothing

polynomial of $(2r+1)$ -th degree very slightly different from the least-square solution will turn out (see Fig.5, 6).

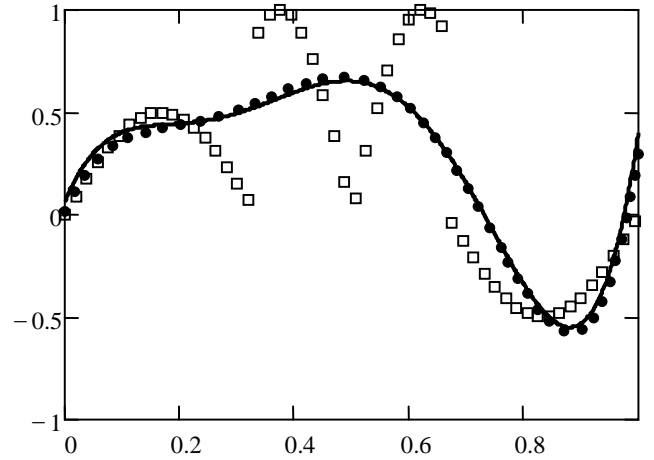


Fig.5: Comparison of values of the smoothing polynomial of 5-th degree (points) and the least-square polynomial of the same degree (the continuous line)

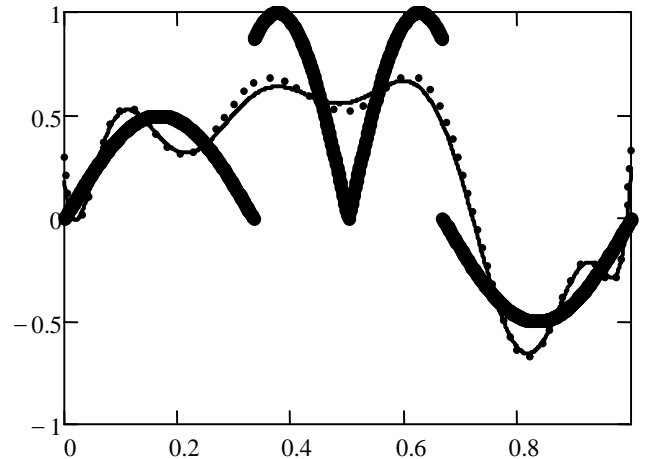


Fig.6: Comparison of values of the smoothing polynomial of 11-th degree (points) and the least-square polynomial of the same degree (thin continuous line)

At the same number of basic functions 12 for a case of $r=3$ the smoothing spline of 7-th degree with one knot in the middle of the interval turns out. As a result the difference from least-square solution looks more considerable after noticeable improvement of adjustment in the vicinity of break of function (Fig.7).

For a case of $r=2$ we will put

$$\psi_1^0(x) = 0.922 M_{1,0}^0(x), \psi_2^0(x) = 13.125 M_{1,1}^0(x),$$

$$\psi_3^0(x) = 147.537 M_{1,2}^0(x),$$

$$\psi_{i-2}^L(x) = 0.236 \cdot 2^{L/2} M_{i/3,0}^L(x),$$

$$\psi_{i-1}^L(x) = 2.476 \cdot 2^{L/2} M_{i/3,1}^L(x),$$

$$\psi_i^L(x) = 49.321 \cdot 2^{L/2} M_{i/3,2}^L(x),$$

$$i = 3, 3 \cdot 2^L, L \geq 1;$$

$$\psi_{i-2}^L(x) = 2.226 \cdot 2^{L/2} M_{i/3,0}^L(x),$$

$$\psi_{i-1}^L(x) = 10.999 \cdot 2^{L/2} M_{i/3,1}^L(x),$$

$$\psi_i^L(x) = 233.334 \cdot 2^{L/2} M_{i/3,2}^L(x),$$

$$i = 6, 9, \dots, 3 \cdot 2^L - 3, L \geq 2.$$

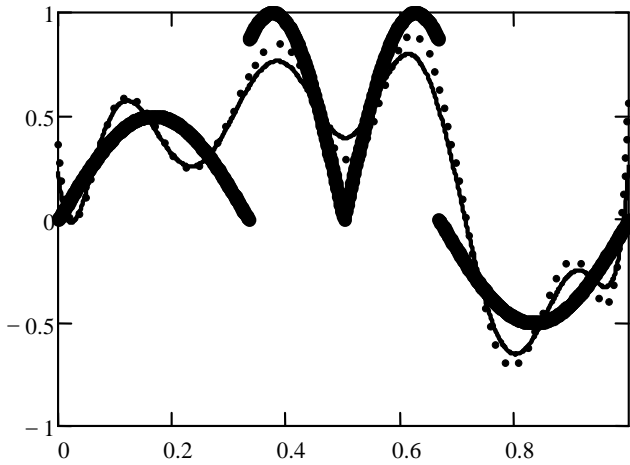


Fig.7: Comparison of values of the smoothing spline of 7-th degree (points) and the least-square spline of 7-th degree (the continuous line)

We will notice that $\psi_i^L(x)$ are normalized so that $\|\psi_i^L(x)\|_{L_2(0,1)} = 1$ for $i = 1, 2, \dots, 3 \cdot 2^L$. Results of synthesis of coefficients of approximating Hermite spline of 5-th degree $S^5(x)$ on condition of zeroing of wavelet-coefficients, on the module smaller then value $0.61 \cdot h^{L/2}$, $h = 2^{-5} = 1/32$, are presented in Fig.8. Thus the coefficient of compression of $K = 99/45 \approx 2$ is reached.

3 Modeling of surfaces

We chose for application the tensor decomposition technique [14] as the array of cross scans of the highway has the prevailing length in one of the directions. As usual we will write down basic spline

functions in the form of two matrix lines for the directions u and v respectively,

$$\phi_u^{L_1} = \left[N_{0,0}^{L_1}, N_{0,1}^{L_1}, \dots, N_{0,r_1}^{L_1}, N_{1,0}^{L_1}, N_{1,1}^{L_1}, \dots, N_{2^{L_1},r_1}^{L_1} \right]$$

and

$$\phi_v^{L_2} = \left[N_{0,0}^{L_2}, N_{0,1}^{L_2}, \dots, N_{0,r_2}^{L_2}, N_{1,0}^{L_2}, N_{1,1}^{L_2}, \dots, N_{2^{L_2},r_2}^{L_2} \right].$$

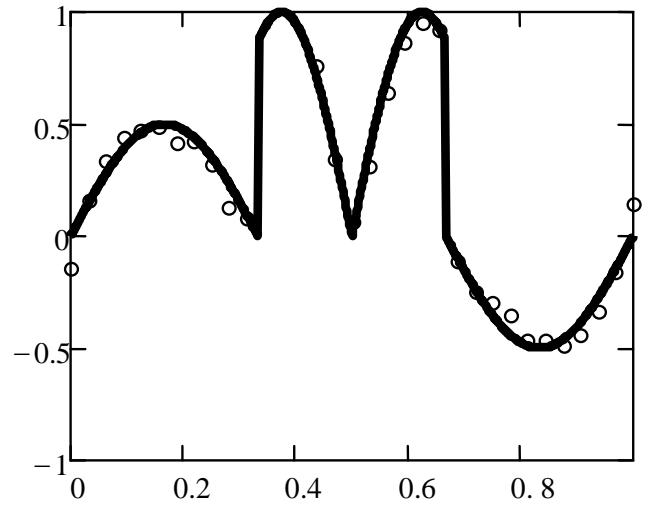


Fig.8: Results of synthesis of nodal values (circles) of the smoothing spline of 5-th degree; the continuous line designates approximated function

Then one-dimensional wavelet-transformations can be written down as $C^{L_1} = P^{L_1} \cdot C^{L_1-1} + Q^{L_1} \cdot D^{L_1-1}$ for variable u and $C^{L_2} = C^{L_2-1} \cdot (P^{L_2})^T + D^{L_2-1} \cdot (Q^{L_2})^T$ for variable v , where vectors D^{L_1-1} and D^{L_2-1} are coefficients of one-dimensional wavelet-decompositions. Let coefficients $C_{i,j}^{k_1,k_2}$ of two-dimensional spline are collected into matrix:

$$C^{L_1,L_2} = \begin{bmatrix} C_{0,0}^{0,0}, & C_{0,0}^{0,1}, & C_{0,0}^{0,2}, & \dots, & C_{0,2^{L_2}}^{0,r_2} \\ C_{0,0}^{1,0}, & C_{0,0}^{1,1}, & C_{0,0}^{1,2}, & \dots, & C_{0,2^{L_2}}^{1,r_2} \\ C_{0,0}^{2,0}, & C_{0,0}^{2,1}, & C_{0,0}^{2,2}, & \dots, & C_{0,2^{L_2}}^{2,r_2} \\ \dots & \dots & \dots & \dots & \dots \\ C_{2^{L_1},0}^{r_1-1,0}, & C_{2^{L_1},0}^{r_1-1,1}, & C_{2^{L_1},0}^{r_1-1,2}, & \dots, & C_{2^{L_1},2^{L_2}}^{r_1-1,r_2} \\ C_{2^{L_1},0}^{r_1,0}, & C_{2^{L_1},0}^{r_1,1}, & C_{2^{L_1},0}^{r_1,2}, & \dots, & C_{2^{L_1},2^{L_2}}^{r_1,r_2} \end{bmatrix}.$$

With such designations the formula for approximation of surface will correspond to:
 $S(u, v) = \varphi_u^{L_1} \cdot C^{L_1, L_2} \cdot (\varphi_v^{L_2})^T$. Then the formula

$$C^{L_1, L_2} = P^{L_1} \cdot \left[C^{L_1-1, L_2-1} \cdot (P^{L_2})^T + E^{L_1-1, L_2-1} \cdot (Q^{L_2})^T \right] + Q^{L_1} \cdot \left[F^{L_1-1, L_2-1} \cdot (P^{L_2})^T + D^{L_1-1, L_2-1} \cdot (Q^{L_2})^T \right],$$

takes place, where

$$C^{L_1-1, L_2-1}, E^{L_1-1, L_2-1}, F^{L_1-1, L_2-1}, D^{L_1-1, L_2-1} -$$

matrixes of coefficients of two-dimensional wavelet-decomposition. The last formula shows that two-dimensional wavelet-decomposition is reduced to one-dimensional wavelet-decomposition for each column of initial matrix of data of C^{L_1, L_2} and to receiving thus two matrixes of intermediate data. Then lines of these intermediate matrixes are also exposed to one-dimensional wavelet-transformation – thus we receive four resulting matrixes etc.

Note that when using the wavelet-transformations based on Hermite splines it is necessary to calculate approximate values of derivatives in nodes of the most dense grid with suitable accuracy (for example via ENO-scheme [13]) to apply algorithms of wavelet-transformations. From the point of view of compression the quantity of wavelet-coefficients given thus in comparison with the methods, based on B-spline-wavelet-transformations, will multiply increase. However as a result, taking into account neglecting of the less significant wavelet-coefficients, this approach can still be competitive. The stated above algorithms were the basis for the software package [15] for processing of laser scanned data. Results of visualization of the data processed are given in Fig.9-11.

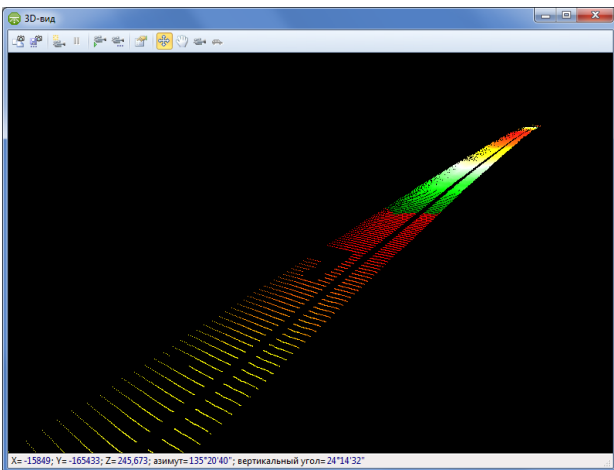


Fig.9: Laser data after preliminary filtering

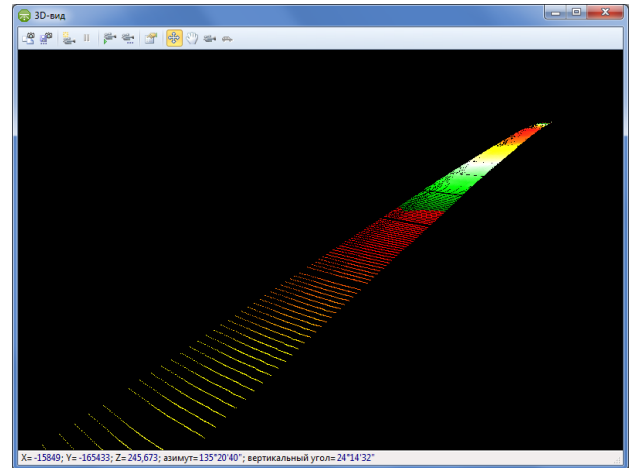


Fig.10: Filling data gaps and forming regular points

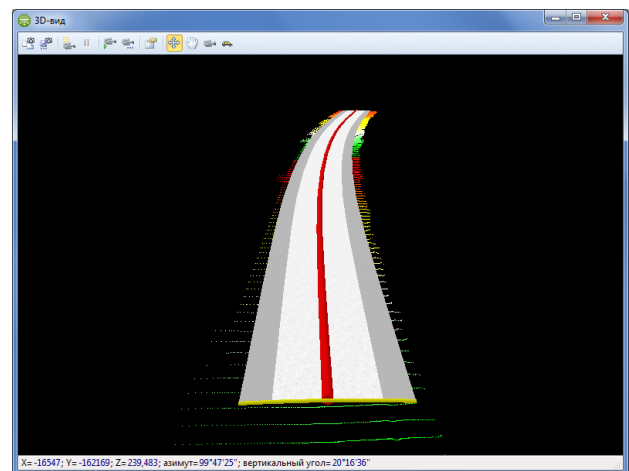


Fig.11: Imposing of the designed route on the processed laser points

4 Conclusion

In this paper, we propose an algorithm based on Hermite wavelet transform to detect cracks and damages of a road pavement. Besides, we design a derivatives filter needed in the transform and make the computation much more efficient by block matrix identities. The figures and data resulting from experiments show that the Hermite wavelet transform is a powerful tool in cracks and damages of road pavement detection.

We believe that the flexible multiscale decomposition of the Hermite wavelet transform have the potential to be used more and more in image and video processing. And we expect a combination of cracks and damages of road pavement detection and machine learning in the future to develop more intelligent road repairing system.

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