A Sparse Signal Reconstruction Perspective for Direction-of-Arrival Estimation with Minimum Redundancy Linear Array

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Abstract: - In this paper, a new direction of arrival (DOA) estimation method based on minimum redundancy linear array (MRLA) from the sparse signal reconstruction perspective is proposed. According to the structure feature of MRLA which is obtaining larger antenna aperture through a smaller number of array sensors, MRLA is combined with $\ell_1-\mathrm{SVD}$ method to estimate signal DOAs. Simulations demonstrate that the proposed method is effective and compared with $\ell_1-\mathrm{SVD}$ method it could estimate more DOAs of signal source, and it is capable of estimating more DOAs with fewer antenna elements.

Key-Words: - Direction of arrival, minimum redundancy linear array, ℓ_1-singular value decomposition, sparse signal reconstruction

1 Introduction

Source location has been a major objective in the signal processing applications. With sensor arrays, this objective may be translated to the Direction-Of-Arrival (DOA) estimation. Among the existing DOA estimation methods, the signal subspace idea has been a dominant technique according to its superresolution performance (e.g., MUSIC [1]). In recent years, there has been an increasing interest in developing CS theory and its applications [2] [3] [4]. CS has provided many methods to solve the sparse recovery problem. There are two major algorithmic approaches to this problem. Basis pursuit (BP) algorithm [5] relies on an optimization problem which can be solved using linear programming, and it is stable and could reconstruct signal accurately, but it needs large number computation, while the greedy algorithm[6][7]takes advantage of the speed, so it has lower complexity and faster speed. By exploiting sparsity, a number of methods have emerged, which can provide even better resolution performance than MUSIC [8] [9] [10]. Among these, the $\ell_1-\mathrm{SVD}$ method [10] is of particular relevance to this work, which formulates the DOA estimation problem into sparse signal reconstruction one and utilizes the Compressive Sensing (CS) approach for its solution. And the $\ell_1-\mathrm{SVD}$ method has increased resolution and improved robustness to noise.

The linear array is one of the most important types of multi-element antennas, and as such it has played an important role both in communications and in radio astronomy. ALAN T. MOFFET proposed minimum redundancy linear array (MRLA) from [11] in1968, which achieve maximum resolution for a given number of elements by reducing the number of redundant spacing present in the array. Many scholars have been making further research for MRLA [12]. These researches make full use of structure features of MRLA to obtain larger antenna aperture through a smaller number of array sensors. Uniform linear array (ULA) is redundant because its covariance matrix is a Toeplitz matrix which is redundant. Also different array sensor distribution could obtain the same function value of conjugate cyclic correlation. The redundancy degree is increased with the number of array sensors. Therefore, we could combine MRLA with $\ell_1-\mathrm{SVD}$ method to estimate signal DOAs based on the structure features.

The paper is organized as follows. In Section 2, the signal model is introduced. This section also reviews the $\ell_1-\mathrm{SVD}$ method and of the MRLA. Section 3 presents the proposed method. To verify the validity of the proposed algorithm, computer simulation results are given in Section 4. Finally, some concluding remarks for this method are given in Section 5.

2 Problem Formulation

2.1 Signal Model

Consider N narrow-band signals with the same center frequency and independent from distinct directions $\theta_i\left(i=1,...,N\right)$ impinge on an ULA including M isotropic antennae, and array element gap is $d=\lambda/2$. The $M\times 1$ array output vector $\mathbf{y}\left(t\right)$ is given by

$$\mathbf{y}(t)=\mathbf{A}\mathbf{s}(t)+\mathbf{n}(t),\qquad t=1,2,...,T \tag{1}$$

where, $\mathbf{y}(t)=[y_0(t)\ \ y_1(t)\ \ \cdots\ \ y_{M-1}(t)]^T$, $\mathbf{s}(t)$ is $N\times 1$ signal vector, which is the t th snapshot of the arriving signals which are assumed to be zero-mean stationary stochastic processes; $\mathbf{A}=[\mathbf{a}(\theta_1)\ \ \mathbf{a}(\theta_2)\ \cdots\ \mathbf{a}(\theta_N)]$ is $M\times N$ array manifold matrix, where $\mathbf{a}(\theta_i)=[1\ \ e^{-j2\pi d\sin\theta_i/\lambda}\ \cdots\ e^{-j2\pi(M-1)d\sin\theta_i/\lambda}\ ]^T$ is the steering vector at direction $\theta_i(i=1,2,...,N)$, and $\mathbf{n}(t)=[n_0(t)\ \ n_1(t)\ \ \cdots\ \ n_{M-1}(t)]^T\in\mathbb{C}^{M\times 1}$ is the $M\times 1$ vector of Additive White Gaussian Noise (AWGN), which, without a loss in generality, is assumed to be of unit variance and uncorrelated with $\mathbf{s}(t)$, where λ is the carrier wavelength of the signal, and $\left(\cdot\right)^T$ denotes the transpose operator.

We could formulate the problem of recovering $\mathbf{s}(t)$ from the under-determined linear equation system (1) into a sparse reconstruction problem similar to many treated in the CS application. $\mathbf{s}(t)$ can be almost surely recovered through ℓ_1 minimization:

$$\min_{\mathbf{s}(t)}\left\|\mathbf{s}(t)\right\|_1\quad s.t.\ \ \mathbf{y}(\mathrm{t})=\mathbf{A}\mathbf{s}(\mathrm{t}) \tag{2}$$

With multiple snapshots the model (1) can be rewritten as:

$$\mathbf{Y}=\mathbf{A}\mathbf{S}+\mathbf{N} \tag{3}$$

where $M\times T$ matrix $\mathbf{Y}=[\mathbf{y}(1),...,\mathbf{y}(T)]$ and $M\times T$ matrix $\mathbf{N}=[\mathbf{n}(t),...,\mathbf{n}(T)]$, T is the number of snapshots and $N\times T$ signal snapshots $\mathbf{S}=[\mathbf{s}(1),...,\mathbf{s}(T)]$, which is the expanded snapshots of the arriving signals.

2.2 $\ell_1-\mathrm{SVD}$ method

For narrowband signal source, when the uncorrelated and coherent signals coexist, multiple measurement vectors:

$$y(t)=A(\theta)x(t)+n(t),\quad t=1,...,T \tag{4}$$

Where $A(\theta)=[a(\theta_1),...,a(\theta_N)]$ is unknown, and under the assumption of sparsity, that is, N is small, Reference [10] has formulated the DOA estimation problem into a sparse signal reconstruction one. For this formulation one defines $\mathbf{A}=[\mathbf{a}(\tilde{\theta}_1),...,\mathbf{a}(\tilde{\theta}_K)]$ where $K\triangleq\max(N,M)$ is the number of DOAs at the resolution of interest represented by angles $\tilde{\theta}_1,...,\tilde{\theta}_K$. Assuming $\{\theta_1,...,\theta_N\}\subset\left\{\tilde{\theta}_1,...,\tilde{\theta}_K\right\}$, and let $s_i(t)=\begin{cases}x_k(t), & \tilde{\theta}_i=\theta_k\\ 0, & otherwise\end{cases}$, $\mathbf{Y}=\mathbf{A}\mathbf{S}+\mathbf{N}$, $\mathbf{Y}=[y(1),...,y(T)]$.

Do the singular value decomposition (SVD) on $\mathbf{Y}=\mathbf{U}\Lambda\mathbf{V}^H$, and let $\mathbf{Y}_{SV}=\mathbf{U}\Lambda\mathbf{D}_K=\mathbf{Y}\mathbf{V}\mathbf{D}_K$, where $\mathbf{D}_K=[\mathbf{I}_K\ \ \mathbf{0}]'$, and $\mathbf{I}_K$ is a $K\times K$ unit matrix, and $\mathbf{0}$ is a $K\times(T\text{-}K)$ zero matrix. Let $\mathbf{S}_{SV}=\mathbf{SVD}_K, \mathbf{N}_{SV}=\mathbf{NVD}_K$, in order to get $\mathbf{Y}_{SV}=\mathbf{AS}_{SV}+\mathbf{N}_{SV}$, consider this equation column by column, there is

$$\mathbf{y}^{SV}(k)=\mathbf{As}^{SV}(k)+\mathbf{n}^{SV}(k),\ \ k=1,...,K \tag{5}$$

define $\tilde{s}_i^{(\ell_2)}=\sqrt{\sum_{k=1}^{K}(s_i^{SV}(k))^2}, \forall i$, so the sparsity of $N_\theta\times 1$ vector $\tilde{s}^{(\ell_2)}$ is corresponding to the sparsity of spatial spectrum, and get the spatial spectrum by the minimized formula (6)

$$|\,\mathbf{Y}_{SV}-\mathbf{AS}_{SV}\,\|_f^2+\lambda\,\|\,\tilde{s}^{(\ell_2)}\,\|_1 \tag{6}$$

where Frobenius is defined as $\left\|\mathbf{Y}_{SV}-\mathbf{AS}_{SV}\right\|_f^2=\left\|\mathrm{vec}(\mathbf{Y}_{SV}-\mathbf{AS}_{SV})\right\|_2^2$.

Therefore, the $\ell_1-\mathrm{SVD}$ method is summarized as three steps: 1st do the SVD on $\mathbf{Y}=\mathbf{U\Lambda V}^H$; 2nd take the first N columns of $\mathbf{YV}$, which is denoted by $\mathbf{Y}_{SV}$, $M\times N$; 3rd solve the following optimization problem:

$$\min_{\mathbf{S}_{SV}}\left\|\mathbf{S}_C\right\|_1 \ \text{s.t.}\ \left\|\mathbf{Y}_{SV}-\mathbf{\Phi S}_{SV}\right\|_f^2\le\beta^2 \tag{7}$$

where $\mathbf{S}_{SV}$, $K\times N$, is the first N columns of $\mathbf{SV}$; $\mathbf{S}_C$, $K\times 1$, is the estimated spatial spectrum whose entries are defined to be the 2-norm of the corresponding rows of $\mathbf{S}_{SV}$ and β is a given regulatory parameter.

The merits of the $\ell_1-\mathrm{SVD}$ method lie in its superior resolution performance and robustness to correlation of the signals. The main drawbacks of $\ell_1-\mathrm{SVD}$ are that it requires information about the number of sources N, and its complexity grows proportionally with N. When the distance of the signal source is not too close, $\ell_1-\mathrm{SVD}$ method could resolute $M-1$ signal sources at most, where M is the number of array element.

2.3 Minimum Redundancy Linear Array

For ULA, correlation matrix of $\mathbf{x}(t)$ as follows

$$\mathbf{R}=E\left\{\mathbf{x}(t)\mathbf{x}^H(t)\right\}=\begin{bmatrix} r(0) & r(1) & \cdots & r(M-1)\\ r^*(1) & r(0) & \cdots & r(M-2)\\ \vdots & \vdots & & \vdots\\ r^*(M-1) & r^*(M-2) & \cdots & r(0)\end{bmatrix} \tag{8}$$

where $E(\cdot)$, $(\cdot)^H$ and $(\cdot)^*$ denote expect, transpose and conjugate operator respectively. $r(m),\ m=0,...,M-1$, is correlation function of stochastic processes $x(t)$. From (1) and (8), we have

$$\begin{aligned}\mathbf{R}&=E\left\{[\mathbf{As}(t)+\mathbf{v}(t)][\mathbf{As}(t)+\mathbf{v}(t)]^H\right\}\\ &=\mathbf{AR}_S\mathbf{A}^H+\sigma^2\mathbf{I}_M\end{aligned} \tag{9}$$

where $\mathbf{R}_S=E[\mathbf{s}(t)\mathbf{s}^H(t)]$, obviously, $\mathbf{R}$ is Toeplitz matrix when the signal sources are mutually independent and uncorrelated. According to the structure feature of Toeplitz matrix, we could reconstruct the whole matrix exactly as long as obtain the first row of $\mathbf{R}$.

From (8) it is shown that for ULA with M array elements and its output correlation matrix there are only M independent correlation functions in the M^2 correlation functions. Therefore, correlation matrix of ULA output is a redundant Toeplitz matrix. Using non-uniform linear array is the common method of reducing redundancy of linear array. In fact, different element pairs could obtain the same value of correlation function, which lead to the redundancy of ULA. The redundancy degree is increased with the number of array element M. The principle of designing MRLA is that equating the M array elements of non-uniform linear array with the P array elements of ULA where $M<P$. Table 1 show some optimal MRLA configurations, where $\{d_i\}$ represents the i th sensor locations with respect to a reference point [12].

Table 1 Some MRLA Configurations

M	P	$\{d_i\}$
2	2	{0,1}
3	4	{0,1,3}
4	7	{0,1,4,6};{0,2,5,6}
5	10	{0,1,4,7,9}
6	14	{0,1,6,9,11,13}
7	18	{0,1,4,10,12,15,17}
8	24	{0,1,4,10,16,18,21,23}
9	30	{0,1,4,10,16,22,24,27,29}
10	37	{0,1,3,6,13,20,27,31,35,36}

3 The proposed method

Consider a passive linear array consisting of M isotropic sensors. Let $d_1 < d_2 < \cdots < d_M$ represents the sensor locations with respect to a reference point. It is assumed, hereinafter, that the array belongs to the class of thinned arrays derived from a ULA by the removal of some of its elements. As a result, each sensor location d_i is an integer multiple of a fixed spacing d, which, in the following, will be assumed unitary without loss of generality.

Suppose a non-uniform linear array, for N independent narrow-band signal sources, and M elements non-uniform linear array, the number of sources N is known. Here, we take $M = 4$ for example, so the sensor locations with respect to a reference point $\{d_1, d_2, d_3, d_4\} = \{0, 2d, 5d, 6d\}$, where $d = \lambda/2$ and λ is the carrier wavelength of the signal, the 4×1 array output vector $\mathbf{X}(t) = [x_0(t)\ \ x_2(t)\ \ x_5(t)\ \ x_6(t)]^T$ is then given by

$$\begin{bmatrix} x_0(t) \\ x_2(t) \\ x_5(t) \\ x_6(t) \end{bmatrix} = \left[\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), ..., \mathbf{a}(\theta_N)\right] \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_N(t) \end{bmatrix} + \begin{bmatrix} v_0(t) \\ v_2(t) \\ v_5(t) \\ v_6(t) \end{bmatrix}$$

(10)

where, $t = 1, 2, ..., T$ and $\mathbf{a}(\theta_i) = \begin{bmatrix} 1 & e^{-j2\pi d \sin\theta_i/\lambda} & e^{-j2\pi 2d \sin\theta_1/\lambda} & e^{-j2\pi 5d \sin\theta_1/\lambda} & e^{-j2\pi 6d \sin\theta_1/\lambda} \end{bmatrix}^T$, we could get array output vector correlation matrix

$$\mathbf{R} = E\left\{\mathbf{X}(t)\mathbf{X}^H(t)\right\} = \begin{bmatrix} r(0) & r(2) & r(5) & r(6) \\ r*(2) & r(0) & r(3) & r(4) \\ r*(5) & r*(3) & r(0) & r(1) \\ r*(6) & r*(4) & r*(1) & r(0) \end{bmatrix}$$

(11)

Do the SVD on $\mathbf{R}$, retain its signal subspace $\mathbf{U}_S$, and construct the dictionary $\mathbf{A} \in \mathbb{R}^{4\times N_\theta}$, where N_θ is the number of angular samples, the sparsity of the $N_\theta \times 1$ vector $\tilde{\mathbf{S}}$ is corresponding to the sparsity of the spatial spectrum, estimate the DOA by minimizing the formula (12)

$$\left\|\mathbf{U}_S - \mathbf{A}\tilde{\mathbf{S}}\right\|_f^2 + \lambda\left\|\tilde{\mathbf{S}}\right\|_1$$

(12)

According to the correlation matrix of independent signals with Toeplitz structure and matrix $\mathbf{R}$ includes the $r(0)$, $r(1)$, $r(2)$, $r(3)$, $r(4)$, $r(5)$, $r(6)$, we could obtain an extended correlation matrix $\tilde{\mathbf{R}}$ of some array elements in the ULA by $\mathbf{R}$. So we construct 7×7 matrix

$$\tilde{\mathbf{R}} = \begin{bmatrix} r(0) & r(1) & r(2) & r(3) & r(4) & r(5) & r(6) \\ r(-1) & r(0) & r(1) & r(2) & r(3) & r(4) & r(5) \\ r(-2) & r(-1) & r(0) & r(1) & r(2) & r(3) & r(4) \\ r(-3) & r(-2) & r(-1) & r(0) & r(1) & r(2) & r(3) \\ r(-4) & r(-3) & r(-2) & r(-1) & r(0) & r(1) & r(2) \\ r(-5) & r(-4) & r(-3) & r(-2) & r(-1) & r(0) & r(1) \\ r(-6) & r(-5) & r(-4) & r(-3) & r(-2) & r(-1) & r(0) \end{bmatrix}$$

(13)

According to the correlation matrix $\mathbf{R}$ of independent signals with Toeplitz matrix structure and $r(\text{-}m) = r*(m), (m = 0,1,...,6)$, we could obtain correlation matrix $\tilde{\mathbf{R}}$ of ULA with seven array elements by $\mathbf{R}$.

$$\tilde{\mathbf{R}} = \begin{bmatrix} r(0) & r(1) & r(2) & r(3) & r(4) & r(5) & r(6) \\ r^*(1) & r(0) & r(1) & r(2) & r(3) & r(4) & r(5) \\ r^*(2) & r^*(1) & r(0) & r(1) & r(2) & r(3) & r(4) \\ r^*(3) & r^*(2) & r^*(1) & r(0) & r(1) & r(2) & r(3) \\ r^*(4) & r^*(3) & r^*(2) & r^*(1) & r(0) & r(1) & r(2) \\ r^*(5) & r^*(4) & r^*(3) & r^*(2) & r^*(1) & r(0) & r(1) \\ r^*(6) & r^*(5) & r^*(4) & r^*(3) & r^*(2) & r^*(1) & r(0) \end{bmatrix}$$

(14)

Do the singular value decomposition (SVD) on $\tilde{\mathbf{R}}$, and keep signal subspace $\tilde{\mathbf{U}}_S$. Then use $\ell_1 - \text{SVD}$ method to estimate signal DOAs by solving the following optimization problem:

$$\min_{\tilde{\mathbf{S}}} \left\|\tilde{\mathbf{s}}^{(\ell_2)}\right\| \quad subject\ to \quad \left\|\tilde{\mathbf{U}}_S - \tilde{\mathbf{A}}\tilde{\mathbf{S}}\right\|_f^2 \le \beta^2$$

(15)

where, DOA estimation problem is regarded as subspace block sparse reconstruction, construct an $M\times K$ overcomplete dictionary $\tilde{\mathbf{A}} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_K)]$, K is the number of angular samples and $K >> N$, $[\theta_1, \theta_{2,...,}\theta_K]$ is a sampling grid of all interested signal directions.

$$\left\|\tilde{\mathbf{s}}^{(\ell_2)}\right\|_1 = \sum_{i=1}^{K} \tilde{\mathbf{s}}_i^{(\ell_2)}, \tilde{\mathbf{s}}_i^{(\ell_2)} = \sqrt{\sum_{k=1}^{N} (\tilde{s}_i(k))^2}, \ \forall i, \tilde{s}_i(k)$$

is the element of the ith row of $K\times N$ matrix $\tilde{\mathbf{S}}$, which is the solution of this optimization problem. Frobenius norm is defined as

$\left\|\tilde{\mathbf{U}}_S - \tilde{\mathbf{A}}\tilde{\mathbf{S}}\right\|_f^2 = \left\|\mathrm{vec}(\tilde{\mathbf{U}}_S - \tilde{\mathbf{A}}\tilde{\mathbf{S}})\right\|_2^2$, and β is regularization parameter. We choose β high enough so that the probability that $\left\|\tilde{\mathbf{v}}_{\mathbf{s}}\right\|_2^2 \geq \beta^2$ is small and $\tilde{\mathbf{v}}_{\mathbf{s}} = \mathrm{vec}(\tilde{\mathbf{N}}_{\mathbf{S}})$, $\tilde{\mathbf{U}}_S = \tilde{\mathbf{A}}\tilde{\mathbf{S}} + \tilde{\mathbf{N}}_{\mathbf{S}}$, $\tilde{\mathbf{N}}_{\mathbf{S}}$ is a $M \times N$ matrix. The sparsity of the resulting $K \times 1$ vector $\tilde{\mathbf{s}}^{(\ell_2)}$ corresponds to the sparsity of the spatial spectrum. We can find the spatial spectrum of $\tilde{\mathbf{S}}$ by the formula (15). Our method also requires information about the number of signal sources N .

Similarly, when we consider a $M = 5$ non-uniform linear array, so the sensor locations with respect to a reference point $\{d_1, d_2, d_5, d_8, d_{10}\} = \{0, d, 4d, 7d, 9d\}$, where $d = \lambda/2$ and λ is the carrier wavelength of the signal, the 5×1 array output vector $\mathbf{X}_1(t) = [x_0(t)\ \ x_1(t)\ \ x_4(t)\ \ x_7(t)\ \ x_9(t)]^T$, we could get array output vector correlation matrix

$$\mathbf{R}_1 = E\left\{\mathbf{X}_1(t)\mathbf{X}_1^H(t)\right\} = \begin{bmatrix} r(0) & r(1) & r(4) & r(7) & r(9) \\ r*(1) & r(0) & r(3) & r(6) & r(8) \\ r*(4) & r*(3) & r(0) & r(3) & r(5) \\ r*(7) & r*(6) & r*(4) & r(0) & r(2) \\ r*(9) & r*(8) & r*(5) & r*(2) & r(0) \end{bmatrix} \quad (16)$$

According to the correlation matrix of independent signals with Toeplitz structure and matrix $\mathbf{R}_1$ includes the $r(0)$, $r(1)$, $r(2)$, $r(3)$, $r(4)$, $r(5)$, $r(6)$, $r(7)$, $r(8)$, $r(9)$, we could obtain an extended correlation matrix $\tilde{\mathbf{R}}_1$ of some array elements in the ULA by $\mathbf{R}_1$. So we construct 10×10 matrix

$$\tilde{\mathbf{R}}_1 = \begin{bmatrix} r(0) & r(1) & r(2) & r(3) & r(4) & r(5) & r(6) & r(7) & r(8) & r(9) \\ r(-1) & r(0) & r(1) & r(2) & r(3) & r(4) & r(5) & r(6) & r(7) & r(8) \\ r(-2) & r(-1) & r(0) & r(1) & r(2) & r(3) & r(4) & r(5) & r(6) & r(7) \\ r(-3) & r(-2) & r(-1) & r(0) & r(1) & r(2) & r(3) & r(4) & r(5) & r(6) \\ r(-4) & r(-3) & r(-2) & r(-1) & r(0) & r(1) & r(2) & r(3) & r(4) & r(5) \\ r(-5) & r(-4) & r(-3) & r(-2) & r(-1) & r(0) & r(1) & r(2) & r(3) & r(4) \\ r(-6) & r(-5) & r(-4) & r(-3) & r(-2) & r(-1) & r(0) & r(1) & r(2) & r(3) \\ r(-7) & r(-6) & r(-5) & r(-4) & r(-3) & r(-2) & r(-1) & r(0) & r(1) & r(2) \\ r(-8) & r(-7) & r(-6) & r(-5) & r(-4) & r(-3) & r(-2) & r(-1) & r(0) & r(1) \\ r(-9) & r(-8) & r(-7) & r(-6) & r(-5) & r(-4) & r(-3) & r(-2) & r(-1) & r(0) \end{bmatrix}$$

(17)

According to the correlation matrix $\mathbf{R}_1$ of independent signals with Toeplitz matrix structure and $r(\text{-}m) = r*(m), (m = 0,1,...,9)$, we could obtain correlation matrix $\tilde{\mathbf{R}}_1$ of ULA with seven array elements by $\mathbf{R}_1$.

$$\tilde{\mathbf{R}}_1 = \begin{bmatrix} r(0) & r(1) & r(2) & r(3) & r(4) & r(5) & r(6) & r(7) & r(8) & r(9) \\ r^*(1) & r(0) & r(1) & r(2) & r(3) & r(4) & r(5) & r(6) & r(7) & r(8) \\ r^*(2) & r^*(1) & r(0) & r(1) & r(2) & r(3) & r(4) & r(5) & r(6) & r(7) \\ r^*(3) & r^*(2) & r^*(1) & r(0) & r(1) & r(2) & r(3) & r(4) & r(5) & r(6) \\ r^*(4) & r^*(3) & r^*(2) & r^*(1) & r(0) & r(1) & r(2) & r(3) & r(4) & r(5) \\ r^*(5) & r^*(4) & r^*(3) & r^*(2) & r^*(1) & r(0) & r(1) & r(2) & r(3) & r(4) \\ r^*(6) & r^*(5) & r^*(4) & r^*(3) & r^*(2) & r^*(1) & r(0) & r(1) & r(2) & r(3) \\ r^*(7) & r^*(6) & r^*(5) & r^*(4) & r^*(3) & r^*(2) & r^*(1) & r(0) & r(1) & r(2) \\ r^*(8) & r^*(7) & r^*(6) & r^*(5) & r^*(4) & r^*(3) & r^*(2) & r^*(1) & r(0) & r(1) \\ r^*(9) & r^*(8) & r^*(7) & r^*(6) & r^*(5) & r^*(4) & r^*(3) & r^*(2) & r^*(1) & r(0) \end{bmatrix} \quad (18)$$

Do the singular value decomposition (SVD) on $\tilde{\mathbf{R}}_1$, and keep signal subspace. Then use $\ell_1-\mathrm{SVD}$ method to estimate signal DOAs by solving the following optimization problem according to the (15).

4 Simulation results and discussions

In this section, to demonstrate the superiority of the proposed method, we compared with the $\ell_1-\mathrm{SVD}$ method by plots of spectra and the root-mean-square error (RMSE).

Consider the direction of the overcomplete dictionary $\tilde{\mathbf{A}}$ is from 1° to $K = 180°$ with 0.5° uniform sampling, and signal covariance matrix $\mathbf{R}$ can be estimated by these snapshots as

$$\mathbf{R} = \frac{1}{T}\sum_{t=1}^{T}\mathbf{X}(t)\mathbf{X}^H(t) \quad (19)$$

where T is the total number of snapshots.

For simplicity, we suppose that all sources are of equal power σ_s^2 and the input SNR is defined as $10\log10(\sigma_s^2/\sigma_n^2)$, where σ_n^2 is the power of the noise. The RMSE of the DOA estimates from 500 Monte Carlo trials is defined as

$$\mathrm{RMSE} = \sqrt{\sum_{n=1}^{500}\sum_{k=1}^{N_s}(\hat{\theta}_k(n) - \theta_k)^2 \Big/ (500N_s)} \quad (20)$$

where $\hat{\theta}_k(n)$ is the estimate of θ_k for the n th Monte Carlo trial, and N_s is the number of signals. We select $T = 1000$ in the following simulations.

In the simulation, when $M = 4$, consider 3 and 4 independent signals from $[-22°, 3°, 41°]$, $[-22°, 3°, 24°, 41°]$ respectively. Fig.1, Fig.2 show that the signal DOAs estimation based on $\ell_1-\text{SVD}$ method and the proposed method in the two conditions respectively. Obviously, we can't estimate the 3 and 4 signal DOAs effectively by $\ell_1-\text{SVD}$ method. The RMSE of the 3 signals DOA estimates based on the proposed method is shown in Fig.3. In Fig.4 we plot the RMSE of the 4 signals DOA estimates based on the proposed method and Crame- Rao Lower bound (CRLB) under different input signal-to-noise ratio (SNR). Consider 5 independent signals from $[-45°,-22°,3°, 24°, 41°]$, the Fig.5 and Fig.6 show the signal DOAs estimation and its RMSE based on the proposed method.

When $M=5$, consider 6 independent signals from $[-45°,-22°, -12°, 10°,25°,48°]$ and 7 independent signals from $[-44°,-22°, -12°, 12°,26°,48°,63°]$ respectively. The Fig.7, Fig.8, Fig.9 and Fig.10 show the signal DOAs estimation and its RMSE based on the proposed method respectively.

Simulation results show that for four-element non-uniform linear array we could estimate 5 independent source DOAs at most using the proposed method, which is more than the number by $\ell_1-\text{SVD}$ method, and for five-element non-uniform linear array we could estimate 7 independent source DOAs at most. Note that the proposed method is suitable for the independent signals. In the similar way, the number of element M could be other value, we could estimate more independent source DOAs by the proposed method.

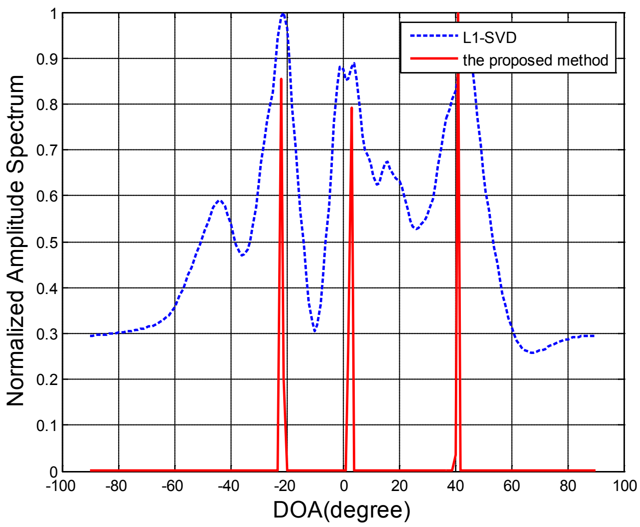

Fig.1. The normalized amplitude spectrum of 3 signals

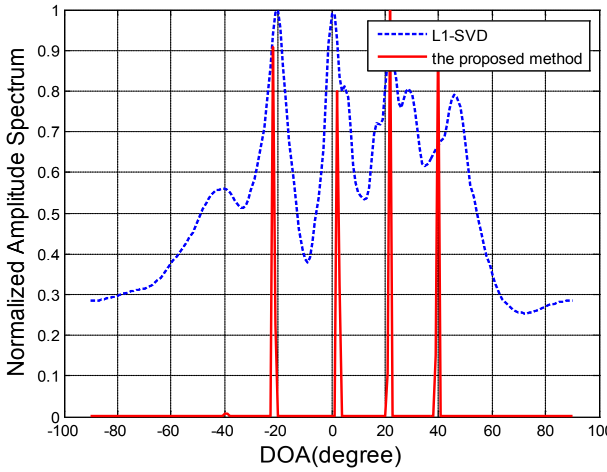

Fig.2. The normalized amplitude spectrum of 4 signals

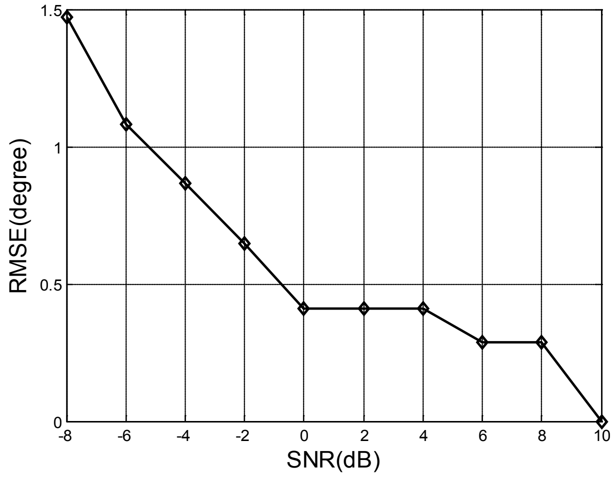

Fig.3. RMSE of the 3 signal DOA estimates versus input SNR

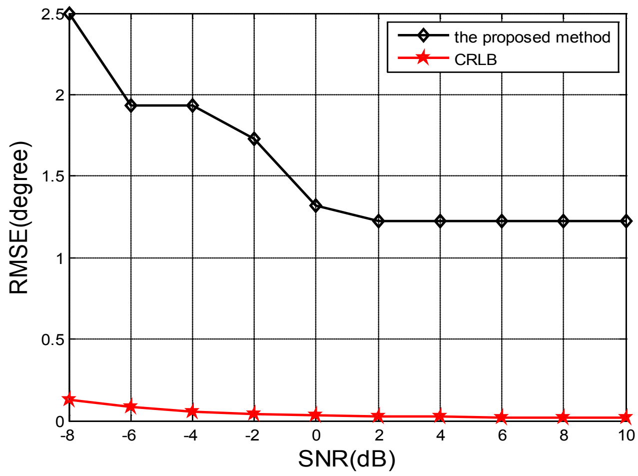

Fig.4. RMSE of the 4 signal DOA estimates versus input SNR

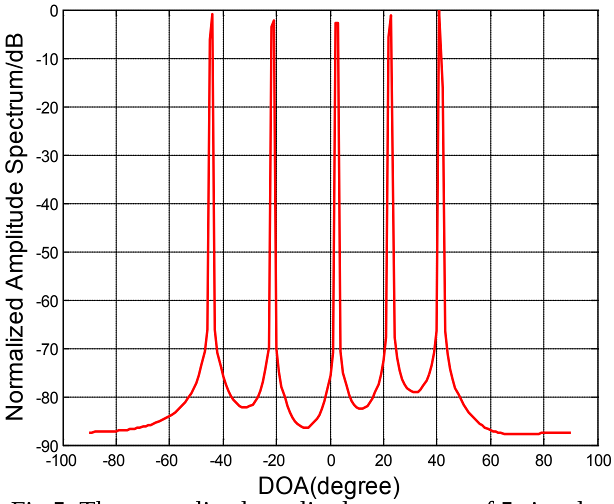

Fig.5. The normalized amplitude spectrum of 5 signals

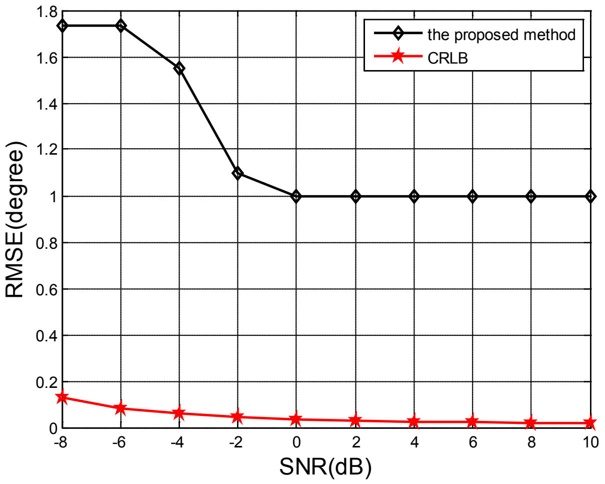

Fig.6. RMSE of the 5 signal DOA estimates versus input SNR

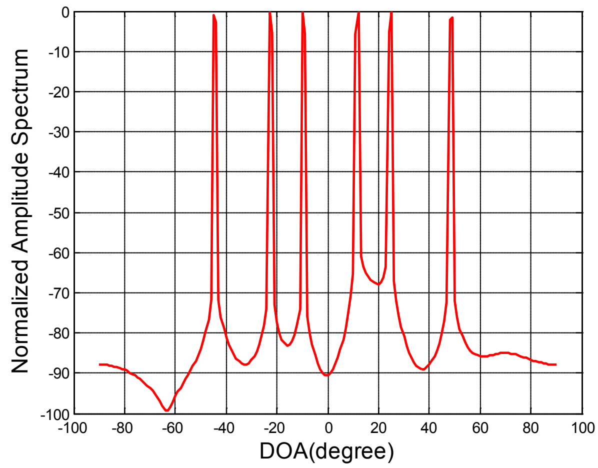

Fig.7. The normalized amplitude spectrum of 6 signals

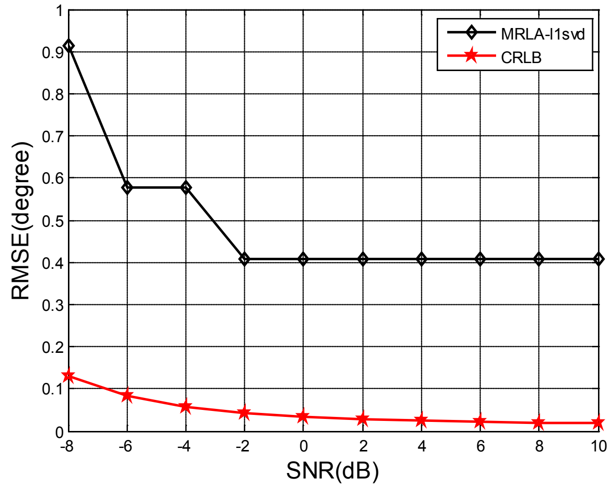

Fig.8. RMSE of the 6 signal DOA estimates versus input SNR

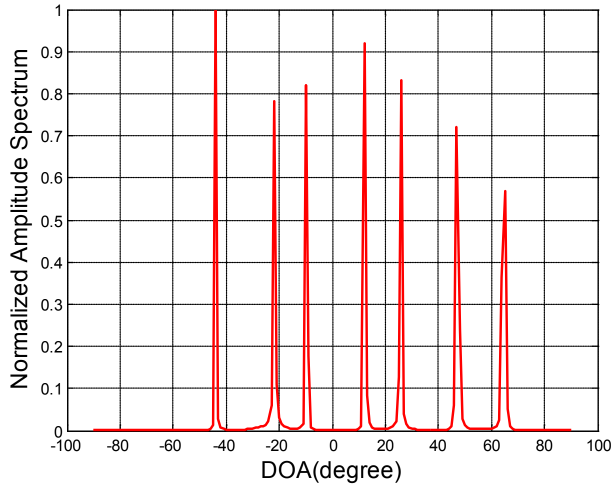

Fig.9. The normalized amplitude spectrum of 7 signals

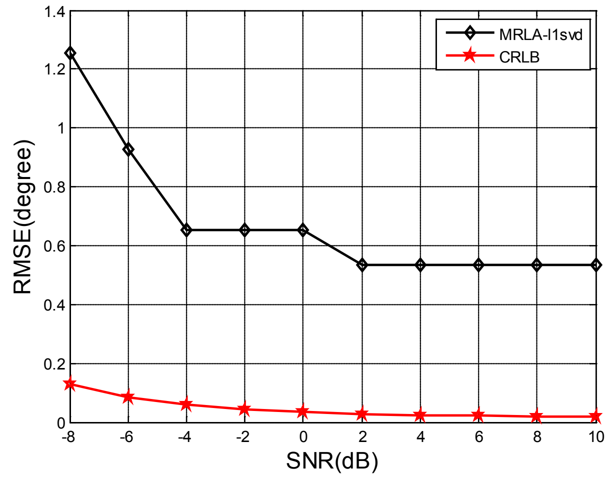

Fig.10. RMSE of the 7 signal DOA estimates versus input SNR

5 Conclusions

In this paper, we propose a new DOA estimation method which combined MRLA with

$\ell_1 - \mathrm{SVD}$ method. Simulation results validate the effectiveness of the proposed method and illustrate that it outperforms $\ell_1 - \mathrm{SVD}$ method in the aspect of estimating more signal sources with the same element, and it is capable of estimating more DOAs with fewer antenna elements.

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