Adaptive Approach Based on Curve Fitting and Interpolation for Boundary Effects Reduction

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Abstract: - Boundary effects are caused by incomplete data in the boundary regions when the analysis window gets closer to the edge of a signal. Various extension schemes have been developed to handle the boundaries of finite length signals to reduce the boundary effects. Zero padding, periodic extension and symmetric extension are some basic extension methods. However, it is well known that all of these solutions may have drawbacks. In this paper, we consider the problem of handling the boundary effects due to improper extension methods in the wavelet transform. An extension algorithm based on curve fitting with properties that make it more suitable for boundary effects reduction is presented here. This extension algorithm could preserve the time-varying characteristics of the signals and be effective to reduce distortions appearing at the boundary. Then, an interpolation approach is used in the boundary effects region to further alleviate the distortions. Procedures for realization of these two algorithms and relative issues are presented. Several experimental tests conducted on synthetic signals exhibiting linear and nonlinear laws are shown that the proposed algorithms are confirmed to be efficient to alleviate the boundary effects in comparison to the existing extension methods.

1 Introduction

Wavelet transform analysis has been presented as a time-frequency analysis and processing method for over the past two decades [1], [2]. But it has still received increased attention in recent years [3], [6], [7]. Wavelet transform analysis has been widely used for the purpose of denoising, data compression, feature recognition, system nonlinearities detection and so on [4]-[7].

The wavelet transform is calculated as shifting the wavelet function in time along the input signal and calculating the convolution of them. In most practical applications, the signals of interest have finite support. As the wavelet gets closer to the edge of the signal, computing the convolution requires the non-existent values beyond the boundary [8]-[10]. This creates boundary effects caused by incomplete data in the boundary regions. Since the analysis wavelet extends into a region with no available data at both boundaries of the signal. Thus, the results of wavelet transform in these boundary effects regions have questionable accuracy. Actually, the particular impacts of boundary effects become increasingly significant for some systems that may

possess longer period sequence and thus require higher frequency resolutions.

To deal with boundary effects, the boundaries should be treated differently from the other parts of the signal. If not properly made, distortion would appear at the boundaries [3]. Two alternatives to deal with boundary effects can be found. The first one is to accept the loss of data and truncate those unfavorable results at boundaries after convolution between signal and wavelet. But simply neglect these regions in analysis yields to a considerable loss of data which is not allowed in many situations where the edges of the signal contain critical information. The other one is artificial the extension at boundaries before processing signals. In fact, there is another approach that employs the usual wavelet filters for the interior of the signal and constructs different boundary wavelets at the ends of the signal. This method has been shows to be merged into the class of signal extension [10].

Various extension schemes have been developed to deal with the boundaries of finite length signals [11]-[14]. Zero padding, periodic extension and symmetric extension are basic extension methods. It

is well known that each method has its disadvantages [3], [10]. Computing the wavelet transform of an extension signal is equivalent to using the corresponding boundary wavelets. The boundary wavelets corresponding to zero padding and periodic extension have no vanishing moments at the boundaries. Therefore, the transform values behave as if signal were discontinuous at the boundaries. They introduce a singularity in the signal. And boundary wavelets of symmetric extension have one vanishing moment and avoid the discontinuous at the boundaries. So it introduces a singularity in the first derivation. However, if the reflection is symmetric the wavelets must be symmetric to ensure no distortion in the transform values. It is well known that Haar is the only symmetric wavelet with a compact support that has been found so far. One goal of this paper is to seek an extension scheme that preserves the property of vanishing moments.

In addition, these basic extension schemes are usual exploited to the application of data coding which focuses on the procedures of analysis and synthesis using filter banks [15]-[20]. However, when it comes to particular applications that put the emphasis on the ability to recognize coherent structure within a signal, the above mentioned methods don't have the ability to recover those significant features. They only make simple assumptions about the signal's characteristics outside the boundaries. Many signals of interested could not be easily included in the above three categories. So we need a new extension mode appropriate to the requirements of the application of non-stationary signals analysis. In this paper, a new extension mode based on curve fitting technique will be introduced for non-stationary signals analysis. This extension mode extends signal according to the time-varying characteristics of the signals inside of the boundaries so that distortions due to improper extensions could be reduced.

It should be aware that features appearing near the boundaries of transform values will contain information from outside the support of the signal which is synthetic. In other word, the wavelet transform resulting at the boundaries will be affected by the adding data no matter whichever extension mode is employed. Therefore, we will consider the problem from a perspective way that is different from extension method to alleviate these effects. In the paper, we will employ an interpolation processing in the region of the boundary effects to reduce the distortions. We will show that improvement can be obtained by such processing.

The paper is organized as follows. In the next section a brief review of the boundary effects in the basic extension methods is given. A general matrix formulation that is common to all signal extension methods is also included. In Section 3, we give depth analysis of the significant importance of smooth extension and present the design method for adaptive smooth extensions with properties that make it more suitable than other extension for non-stationary signals analysis. In Section 4, we develop a new algorithm based on interpolation technique for further boundary effects reduction along with some discuss on the implement of this technique. In Section 5, we present the method of testing and the results concerning the performance of performance of the proposed methods applied on both linear and nonlinear frequency modulation signals. The performance of proposed adaptive extension method based on curve fitting and interpolation is shown to be superior to all of the other methods. Section 6 summarizes the results obtained throughout the paper.

2 Boundary Effects in the Time-frequency Signal Analysis using Wavelet Scalogram

The need for a signal time-frequency analysis comes from the incomplete of either time domain or frequency domain analysis to fully describe the behavior of non-stationary signals. The time-frequency representation of a signal for time-frequency analysis provides information about how the frequency content varies with time, thus providing an ideal approach to examine, analyse and study non-stationary signals. Time-frequency representation is an image of a two dimensional time-frequency representation mapped from one signal. A number of methods have been developed to obtain the energy distribution function with respect to both the time and frequency. Wavelet transform is one of most notably tools. Wavelets have the great advantage of being able to isolate the fine details in a signal. Very small wavelets can be used to identify very fine details in a signal, while very large wavelets can identify coarse details. Wavelet theory is capable of revealing aspects of data that other signal analysis techniques fail to be present the aspects like trends, breakdown points,

and discontinuities in higher derivatives and self-similarity.

But as mentioned in the previous section, wavelet transform suffers from boundary effects like other signal analysis techniques which involve convolution operation. The boundary effects would lead to serious distortion at both boundaries of signal which makes it hard to distract the right information particularly on the start and the end of signals. Therefore, this section will first explore the effect of basic extension methods, which include zero padding, periodic extension and symmetric extension, on the wavelet transform in order to design a suited extension method that is able to minimize boundary effects.

We start with a general formula of various extension modes. We denote vectors by bold lower case letters. Subscript (superscript) l , c and r represent left, central and right respectively. Matrices are denoted by bold upper case letters. We use subscript, such as $M \times N$, to denote the size of a matrix.

A finite signal with length N is $s(n)$, $n \in 0, 1, \cdots N$. Then we can express this signal in another form as

$$\mathbf{s} = [\mathbf{s}_l^{\mathrm{T}}, \mathbf{s}_c^{\mathrm{T}}, \mathbf{s}_r^{\mathrm{T}}]^{\mathrm{T}} \qquad (1)$$

where $\mathbf{s}_l$ and $\mathbf{s}_r$ are vectors consisting of the first and last M components of the signal. $\mathbf{s}_c$ is the central part. Denote the extension vector of $\mathbf{s}(n)$ as

$$\mathbf{s}_e = [\mathbf{s}_{e,l}^{\mathrm{T}}, \mathbf{s}^{\mathrm{T}}, \mathbf{s}_{e,r}^{\mathrm{T}}]^{\mathrm{T}} \qquad (2)$$

where similarly, $\mathbf{s}_{e,l}$ and $\mathbf{s}_{e,r}$ are the left and right extension vectors of length M . We use subscript to denote the size of matrix. Generalized expression for signal extension methods is given by

$$\mathbf{s}_e = \mathbf{H}_{(2M+N) \times N} \mathbf{s} \qquad (3)$$

where $\mathbf{H}$ is the extension matrix. The basic extension methods are all linear extension. Hence (3) can be written in form

$$\mathbf{s}_e = \begin{bmatrix} \mathbf{H}^l \\ \mathbf{I}_N \\ \mathbf{H}^r \end{bmatrix} \mathbf{s} \qquad (4)$$

where $\mathbf{I}_N$ is an $N \times N$ identity matrix; $\mathbf{H}^l$ and $\mathbf{H}^r$ are respective left and right extension matrices.

For the zero padding extension, the extension matrix is

$$\mathbf{H} = \begin{bmatrix} \mathbf{0}_{M \times N} \\ \mathbf{I}_N \\ \mathbf{0}_{M \times N} \end{bmatrix} \qquad (5)$$

where $\mathbf{0}_{M \times N}$ is an $M \times N$ zero matrix. Since for the periodic extension $\mathbf{s}_{e,l} = \mathbf{s}_r$ and $\mathbf{s}_{e,r} = \mathbf{s}_l$ the extension matrices of the periodic extension are

$$\mathbf{H} = \begin{bmatrix} \mathbf{0}_{M \times (N-M)} \mathbf{I}_M \\ \mathbf{I}_N \\ \mathbf{I}_M \mathbf{0}_{M \times (N-M)} \end{bmatrix} \qquad (6)$$

Similar result is available for the extension matrix of the symmetric extension

$$\mathbf{H} = \begin{bmatrix} \mathbf{J}_M \mathbf{0}_{M \times (N-M)} \\ \mathbf{J}_N \\ \mathbf{0}_{M \times (N-M)}, \mathbf{J}_M \end{bmatrix} \qquad (7)$$

where $\mathbf{J}_M$ (or $\mathbf{J}_N$) is an exchange matrix where the 1 elements reside on the counterdiagonal and all other elements are zero.

In order to illustrate the boundary effects of various basic methods, we consider a linear frequency modulation signal $s(t)$ with constant amplitude and frequency varied with time from 0.1 to 0.4(normalized frequency). The sampling frequency used is $f_s = 100 \mathrm{Hz}$ s with 300 data points. We perform different extension methods on the test data and extract the instantaneous frequency from wavelet transform of extension data. The estimation error of the instantaneous frequency obtained from three basic extension methods is shown in Fig. 1. The symmetric extension performs better than zero and periodic methods. This is due to symmetric extension have one vanishing moment and zero padding and periodic extension have no vanishing moments. This paper is to seek an extension scheme that preserves the property of vanishing moments.

Define the k moments of wavelet function as

$$m_k = \int_{-\infty}^{\infty} t^k \psi(t) \mathrm{d}t \qquad (8)$$

As a consequence of the Fourier transform properties, we can obtain

$$m_k = (-j)^{-k} \frac{d^k \Psi(\omega)}{d\omega^k}\bigg|_{\omega=0} \qquad (9)$$
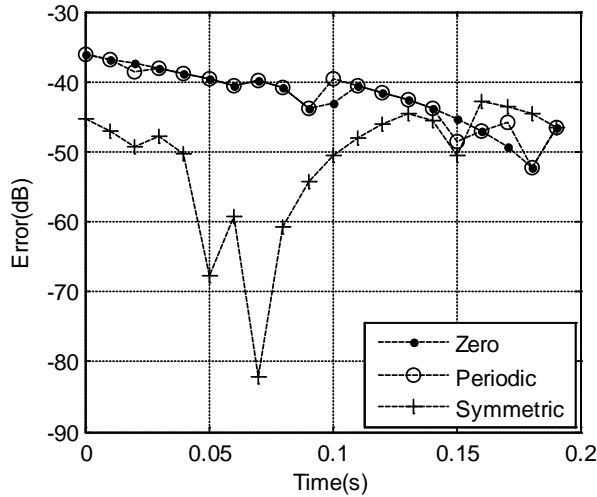

Fig. 1. Comparison of boundary effects for linear FM signal of three basic methods. The values on the vertical axes are normalized to the adopted sample rate.

where $\Psi(\omega)$ is the Fourier transform of $\psi(t)$. If $\Psi(\omega)$ has p order multiple zeros at $\omega = 0$, that is

$$\Psi(\omega) = \omega^p \Psi_0(\omega), \;\; \Psi_0(\omega)\big|_{\omega=0} \neq 0 \qquad (10)$$

then we can find

$$m_k = \int_{-\infty}^{\infty} t^k \psi(t) dt = 0, \;\; k = 0,1,\cdots p-1 \quad (11)$$

If a wavelet function $\psi(t)$ satisfies(11), then we say this wavelet function has p vanishing moments.

Assume signal $s(t)$ is a polynomial of degree $p-1$, which is given by

$$s(t) = \sum_{k=0}^{p-1} \alpha_k t^k \qquad (12)$$

where $\alpha_0, \alpha_1, \cdots, \alpha_{p-1}$ are constant coefficients. Additional, we assume $\psi(t)$ has p vanishing moments. Equation (11) indicates that

$$\langle s(t), \psi(t) \rangle = 0 \qquad (13)$$

In other words, the wavelet transform of $s(t)$ is identical to zero. If $s(t)$ can be expanded into a high-order polynomial of degree N with $N > p$, then the terms of the polynomial with degree lower than p contribute nothing to the wavelet transform which only reflects the terms with degree higher than p (high frequency component). Such a wavelet has the advantages to capture the high frequency component and breakpoints of signals. Therefore, $\psi(t)$ is required to have an as high as possible vanishing moments so that $\Psi(\omega)$ is smooth at $\omega = 0$ to possess a satisfied band-pass property.

3 Boundary Effects Reduction via Adaptive Smooth Extension

It has been shown that every basic extension method has its own drawbacks. We should seek a method representing the feature of signal. Moreover, smooth extension is also critical to the reduction of boundary effects.

3.1 Design of Adaptive Extension Method

In the following, we will investigate a new extension mode which could characterize signal better. On the one hand, the signal used in previous section is comprised by many harmonic oscillations, and on the other hand, it is very common to use Fourier series to represent such harmonic oscillations. Thus, Fourier series can be consider as a new mode to extend signal to preserve the harmonic oscillations.

The Fourier series model is given by

$$y(t) = a_0 + \sum_{i=1}^{m} a_i \cos(\omega_i t) + b_i \cos(\omega_i t) \qquad (14)$$

where a_0 is a constant term in the signal, both a_i, b_i and ω_i are parameters that need to be estimated by the fit, m is the number of harmonics in the data.

In summary, the following are the steps of the proposed adaptive algorithm for signal extension:

1) Initialize the number of harmonics, for example, set $m = 3$.

2) Main Iteration: Increment m by 1, and apply these steps:

- Perform data transformations to obtain a linear or simple model.
- Find the above model parameters to minimize the summed square of residual defined as the difference between the real date value s and the

fitted response value y, producing result a_i, b_i and ω_i.

- Update the fitted response value y using a_i, b_i and ω_i.

- If $|s - y|^2$ is smaller than some predetermined threshold, stop. Otherwise, apply iteration.

3) Extend the producing Fourier series to define the data beyond the borders.

3.2 Properties of Smooth Extension

From the perspective of convolution operation, the wavelet transform of a signal could be interpreted as the output of a system whose unit impulse response is the scaled wavelet function

$$\psi_a(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t}{a}\right) \qquad (15)$$

where a is scale. Let's consider a low-pass(smooth) function $\theta(t)$. Set

$$\psi^{(1)}(t) = \mathrm{d}\theta(t)/\mathrm{d}t, \ \psi^{(2)}(t) = \mathrm{d}^2\theta(t)/\mathrm{d}t^2 \quad (16)$$

We use $\psi^{(1)}(t)$ and $\psi^{(2)}(t)$ as the mother wavelets. Then computing the first derivative of a signal after smoothing is equivalent to processing this signal using the first derivative of the smooth function $\psi^{(1)}(t)$. Similarly, computing the second derivative of a signal after smoothing is equivalent to processing this signal using the second derivative of the smooth function $\psi^{(2)}(t)$. This result can be generalized to the higher order. Mathematically, a point of a function with zero first derivative corresponds to extreme value while zero second derivative corresponds to inflection point. Hence, the wavelet transform is able to reflect the extreme and inflection points of a signal if the wavelet is original from a smooth function.

An improper extension maybe results in extra transient component referred to singular points which is defined as points with derivative on the right and the derivative on the left exist with different signs, that is, the points at which its derivative is discontinuous or not defined and finding the amplitudes of the jumps. In other words, singular point represents the extreme and inflection points present in the signal. It is easy to obtain that the singular points of signal are indicated by the amplitudes of its wavelet transform, i.e., zero-

crossing points or maximum points of the transform. In the case of signal extension, an extreme point due to extension would lead to zero point or very small value in the wavelet transform at the corresponding location. More ordinary case is that extension introduces a step at the boundary leading to very large wavelet transform amplitude. For example, the result that wavelet transform of signal $s(n)$ using function $\psi^{(1)}(t)$ is very large indicates the inflection point of $s(n)$.

An unsmooth extension at $s(0)$ or $s(N)$ leads to wavelet transform modulus maximum at the same points which is the reason of distortion. Hence, we should select an extension mode that is as smooth as possible at the boundaries to avoid distortion.

4 Boundary Effects Reduction via Interpolation

Whichever extension method is employed to reduce the boundaries distortion phenomenon, the extension parts would definitely affect the analysis results which are determined by both original and extension signals. If the extension parts do not properly reflect the trend of the original signal, it will fail to produce satisfactory or perfect results. Nevertheless, it is well known that the signals in the application of time-frequency analysis are usually random and it is difficult to estimate the past and future of the signals based on the present data. Hence, this problem should be seen from a perspective that is broader than devising a convenient extension for the signal. Apart from the Fourier series extension method, an additional goal of this paper is to propose an approach to shorten the width of the boundary effect region defined in the above section. This approach is based on interpolation in the boundary effect region to reduce the boundary effects.

Fig. 2 explains the principle of reduction of boundary effects using interpolation method. Without interpolation, the convolution is computed between wavelet and data with length N, from $\mathbf{s}(0)$ to $\mathbf{s}(N-1)$.

After interpolation, the convolution is still computed between wavelet and data. However, the end point of these data has become to $\mathbf{s}(N/2-1)$ if N is odd. As shown in Fig. 2(b), the length of wavelet is the half of that before interpolation and becomes shorter compared to the original signal. Based on the discussion of Section 4, it is easy to show that the boundary effects region which is

decided by the length of wavelet also becomes shorter in consequence of the interpolation procedure. Therefore the boundary effects are alleviated by exploiting interpolation.

In practical applications, we only require to employ interpolation in the boundary effects region to obtain good results without heavy computational burden. Interpolation can be considered as an expansion of the extension method towards the interior of signal. Compare with extension methods, it is easier and more accurate to estimate signal value between two points than predict the data from the view of probability.

4.1 The Range of Interpolation

Performing interpolation processing at the boundaries could further reduce boundary effects. In the implement, we should consider the range of interpolation processing, that is, how long of the
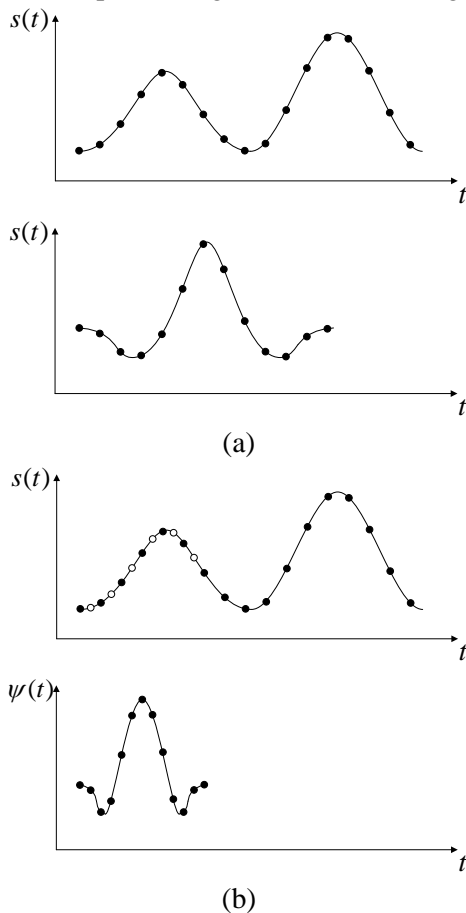
Fig. 2. Convolution between signal and wavelet: (a) before interpolation; (b) after interpolation.

signal should be involved in the interpolation processing. A range which is too long or too short would yield an expensive computation or an inaccurate result. We first discuss the range of

interpolation and then explore how to determine it in practice.

Let us assume signal $s(t)$ has an singular point at $t_0 = 0$. It is obvious that the singular point at $t_0 = 0$ will not impact the whole time-scale plane but only the neighborhood of t_0. We refer it as cone of influence of t_0. The range of interpolation depends on the cone of influence. For the sake of simplicity, suppose the wavelet that we use has a support $[C, C]$. Then the scaled wavelet $\psi_a(t)$ has support $[t - Ca, t + Ca]$. We define the cone of influence as the set of points containing in the support $[t - Ca, t + Ca]$ from the whole time-scale plane. Thus, the cone of influence of t_0 is

$$|t - t_0| \leq Ca \qquad (17)$$

In the cone of influence, the performance of wavelet transform is impacted by the singular point introduced by extension. We refer the cone of influence as the region of boundary effect where the interpolation processing should be performed. It is notice that the range of interpolation is proportional to the scale factor a. Fig. 3. illustrates the length of interpolation required at different scale on the time-scale plane.

4.2 The Implement of Interpolation

Based on the previous discussion, algorithm for the interpolation processing for boundary effect reduction can be summarized as follows:

1) Obtain the wavelet transform of the signal.

2) We find the scale a corresponding to the maximum amplitude of the wavelet transform at $t_0 = 0$.

3) The range of interpolation processing is determined by the scale a from the above step. Fig. 3 shows different range of interpolation processing at different scale.
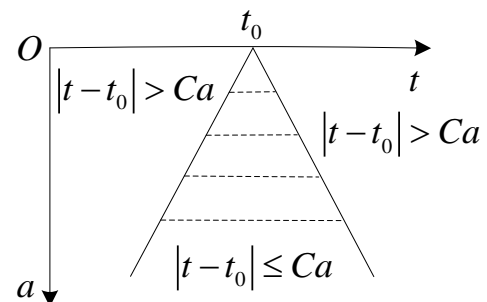

Fig. 3. The range of interpolation processing at different scale.

5 Numerical Examples

In order to validate the results given in Section 3 and Section 5, we present the numerical examples of the proposed algorithms. The performance of the proposed methods has been assessed by means of tests on generic synthetic signals. The purpose of the test is to establish the measurement accuracy of the proposed methods as well as their advantages in boundary effects reduction over the basic methods. The test consists of two parts which involve the proposed extension method and interpolation preprocessing. Two signals exhibiting linear and nonlinear instantaneous frequency laws are used for evaluating the performance of the algorithms.

5.1 Performance Assessment of Fourier Series Extension

First consider the linear FM signal was presented in Section 2. Some results that illustrate the performance of the Fourier series extension in the instantaneous frequency estimation are shown in Fig. 4. For comparison purposes, the extension algorithm is compared with symmetric extension which is superior over the other two basic methods. It is apparent that the results provided by the Fourier series extension method are in better agreement with the theoretical values in Fig. 4(a). As we have done in the previous section, the error between theoretical and estimated wavelet ridge are shown in Fig. 4(b) to illustrate the effect of Fourier series extension. It can be observed that Fourier series extension has less singularity appearing at the boundary than symmetric extension.
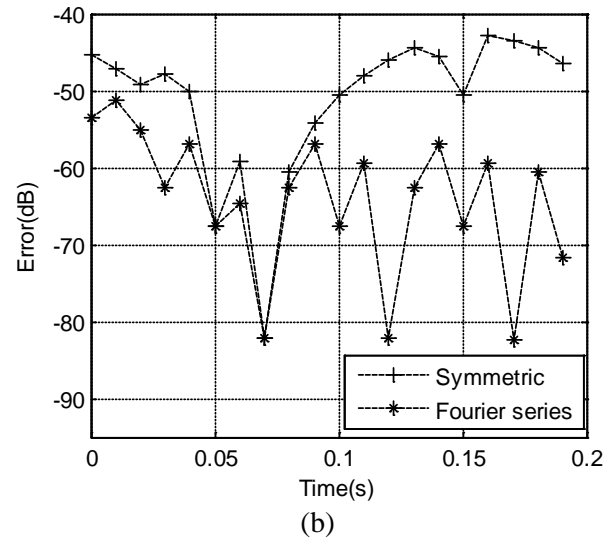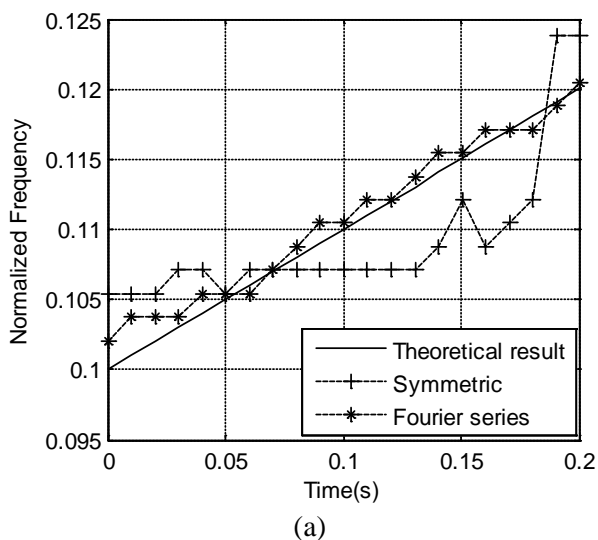

(a)

(b)

Fig. 4. Comparison of boundary effects for linear FM signal of symmetric and Fourier series extension. (a) The right boundary of wavelet ridge. (b) Estimation error. The values on the vertical axes are normalized to the adopted sample rate.

For the nonlinear case, we consider a logarithmic frequency modulated signal with same samples as the linear case and its IF is given by

$$f = f_0 (\frac{f_0}{f_1})^{(\frac{t}{t_1})} \qquad (18)$$

We set $f_0 = 0.1$, $f_1 = 0.4$, $t_1 = 3$. The total signal length and sample period used are $N = 300$, $T = 0.01$ s.

The proposed algorithm is successfully applied on this nonlinear FM signal. Fig. 5 illustrates the results provided by Fourier series extension and symmetric extension applied on the nonlinear FM signal. It can be seen that the results are similar to the results of linear FM signal. Fig. 5(b) shows that the Fourier series extension is indeed efficient to reduce boundary effects for complicated signals with time-varying IF laws.

5.2 Performance Assessment of the Interpolation Method

Several tests have been conducted in order to assess the capability of the proposed method interpolation preprocessing to further reduce the boundary effects. All the basic extension methods and the Fourier series extension will be considered in this section. The performance of the interpolation preprocessing is examined using the same two classes of signals.

data from the boundary part. The number of data participating to the calculation is 50 points.
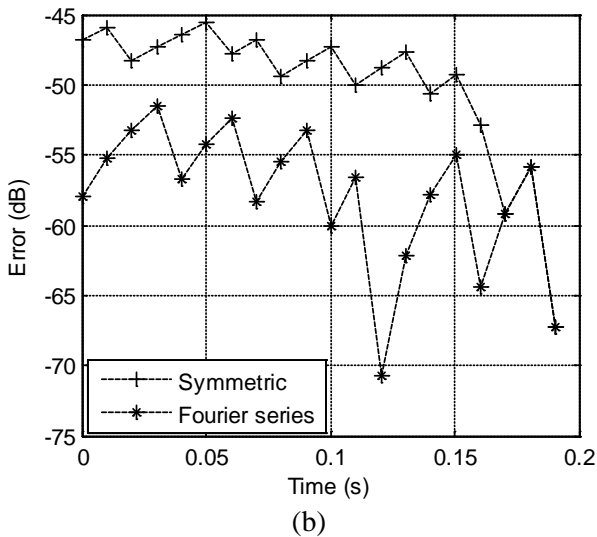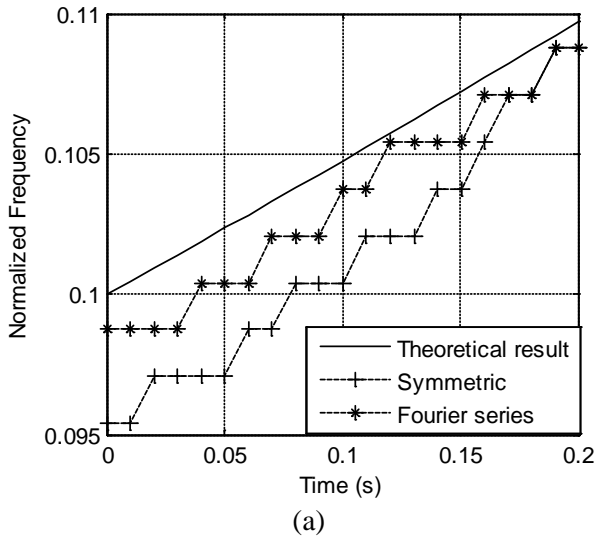

(a)

(b)

Fig. 5. Comparison of boundary effects for logarithmic FM signal of symmetric and Fourier series extension. (a) The right boundary of wavelet ridge. (b) Estimation error. The values on the vertical axes are normalized to the adopted sample rate.

We plot the comparison results of symmetric extension and Fourier extension with and without interpolation processing on the linear FM signal and logarithmic FM signal in Fig. 6 and Fig. 7 respectively. It can be clearly observed that the interpolation processing is indeed able to further reduce the distortion resulted from the boundary effects for both signals.

To show the effect of the interpolation preprocessing, compression test results of the linear FM signal for different extension methods with and without interpolation processing are provided in Table 1. All of the values are calculated from true normalized frequency and estimating normalized frequency. In order to display the performance of boundary effects reduction, we only calculate the
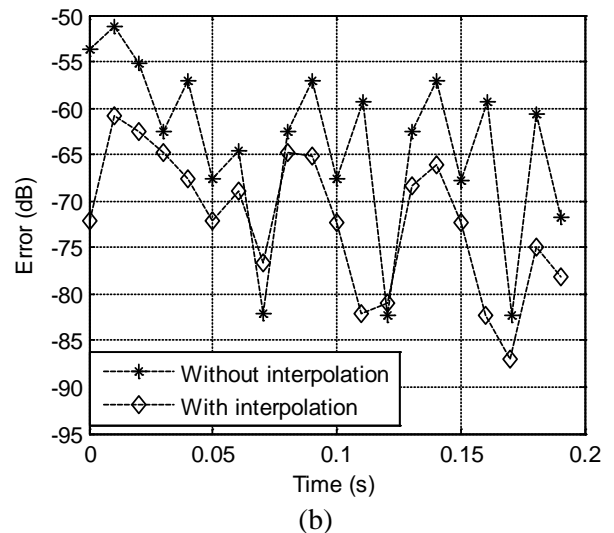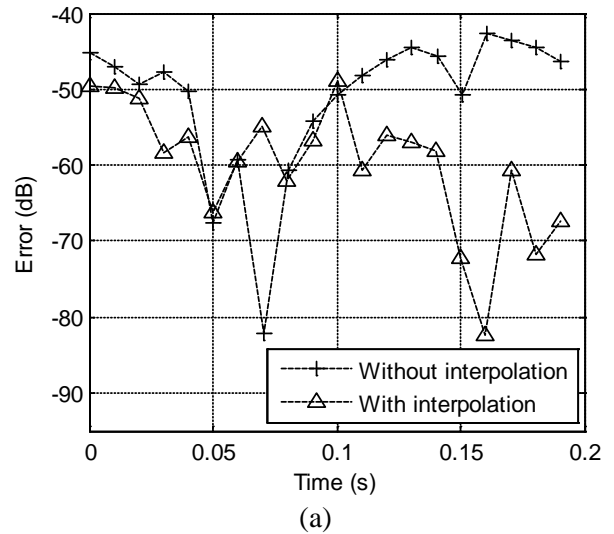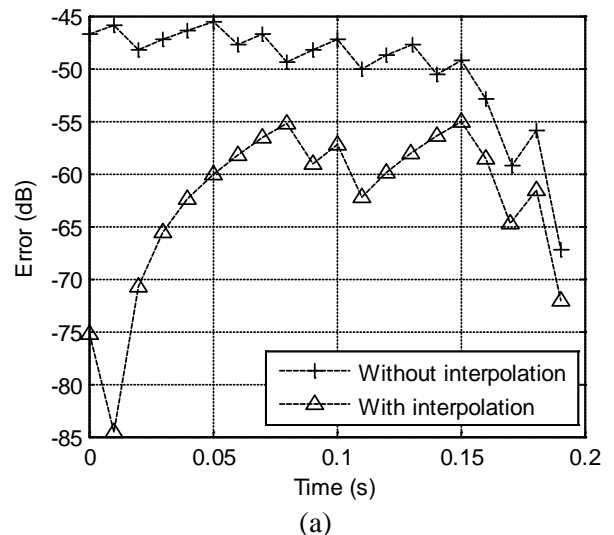

(a)

(b)

Fig. 6. Estimation error of wavelet ridge of linear FM signal with and without interpolation processing for (a) symmetric extension and (b) Fourier series extension.
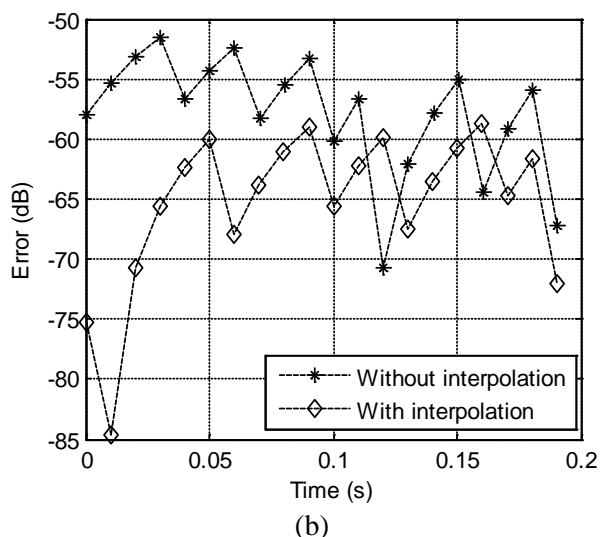

(a)

(b)

Fig. 7. Estimation error of wavelet ridge of logarithmic FM signal with and without interpolation processing for (a) symmetric extension and (b) Fourier series extension.

It can be concluded that the Fourier series extension with the interpolation processing provides the best performance among the four methods mentioned in this study. Furthermore, the interpolation processing is able to produce a better accuracy of the time-frequency characteristics estimate no matter which extension method is applied.

Table 1. Performance comparison of extension methods with and without interpolation preprocessing.

Method	Bias	Variance	MSE[1]
Zero	8.60×10^{-3}	1.28×10^{-5}	8.68×10^{-5}
Zero (interpolation)	3.14×10^{-3}	9.05×10^{-6}	2.07×10^{-5}
Periodic	8.68×10^{-3}	1.21×10^{-5}	8.76×10^{-5}
Periodic (interpolation)	3.53×10^{-3}	7.94×10^{-6}	2.04×10^{-5}
Symmetric	3.77×10^{-3}	4.17×10^{-6}	1.84×10^{-5}
Symmetric (interpolation)	1.41×10^{-3}	1.05×10^{-6}	3.04×10^{-6}
Fourier	9.24×10^{-4}	4.87×10^{-7}	1.34×10^{-6}
Fourier (interpolation)	3.37×10^{-4}	5.61×10^{-8}	1.69×10^{-7}

[1] MSE is the mean square error.

6 Conclusion

In this paper, we have investigated the problem of dealing with the boundary effects that would arise in the application of time-frequency analysis. Basic methods including zero padding, periodic extension and symmetric extension were shown to provide unsatisfied performance to reduce the boundary effects. We derived a generalized expression for various extension methods. The relationship between smooth and the boundaries effect has been stressed. A smooth extension scheme using Fourier series to avoid distortion appearing at the boundaries was proposed. This extension technique possesses the property of preserving the harmonic oscillations of the time-vary signal that makes it more suitable than the other methods for the time-frequency analysis application. A new algorithm based on interpolation technique was proposed from new perspectives to further reduce the boundary effects. It has been shown that the range of interpolation is determined by the scale factor maximized the amplitude of the wavelet transform at the boundaries. Some details on the procedures for implement of the proposed technique have been presented. By comparing the results of the analysis, it has been shown that the adaptive smooth extension with the interpolation processing provided the best performance in the study. Although we have restricted the analysis to the wavelet transform, the proposed methods can be applied on any time-frequency distributions that involve convolution operation.

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