Heuristic Search Method for Digital IIR Filter Design

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Abstract: - The paper develops innovative methodology for robust and stable design of digital infinite impulse response (IIR) filters using a heuristic search method. The proposed heuristic search method enhances the capability to explore and exploit the search space locally as well globally to obtain optimal filter design parameters. A multicriterion optimization is employed as design criterion to obtain optimal stable IIR filter that satisfies different performance requirements like minimizing L_p-norm approximation error and ripple magnitude. Multicriterion optimization problem has been solved applying weighted sum method and p-norm method. Best weight pattern is searched using evolutionary search method that minimizes the performance criteria simultaneously. The proposed heuristic search method is effectively applied to solve the multicriterion, multiparameter optimization problems of low-pass, high-pass, band-pass, and band-stop digital filters design. The computational experiments show that the proposed heuristic search method is superior or atleast comparable to other algorithms and can be efficiently used for higher filter design.

Key-Words: - Digital infinite impulse response filters, Heuristic search algorithm, Multicriterion optimization, Magnitude Response, Stability, L_p-norm error.

1 Introduction

Digital infinite impulse response (IIR) filter design has attracted growing attention due to their important role in the field of denoising of digital images, artificial vision, biomedical imaging, digital mammography, satellite image processing and many other scientific applications. Digital IIR filter design principally follows two techniques: transformation technique and optimization technique. In transformation technique, first analog IIR filter is designed and is transferred to digital IIR filter. Several well-known filter design approaches, such as Butterworth, Chebyshev and Elliptic function, have been developed using transformation techniques [1]. In accordance, the MATLAB toolbox provides a user-friendly Interface for filter designing [2]. The problem of designing IIR filters has been tackled using various optimizatioqn techniques such as p-error, weighted least square and ripple magnitudes (tolerances) of both pass-band and stop-band [4]-[6]. As compared to finite-impulse-response (FIR) filters digital IIR filters often perform better and have less computation cost. Different iterative design techniques for IIR filters using the Steiglitz-McBride (SM) scheme [3] have been proposed in [4]-[8]. At each iteration, the denominator of an approximation error is replaced by its equivalent obtained at the previous iteration and is combined with a weighting function. Then, the approximation error to be minimized can be described as a quadratic function of filter coefficients. Different stability constraints based on argument Principle [8] and the positive realness based stability constraint [4]–[7] have been employed to guarantee the stability of designed IIR filters. However due to non-linear and multimodal nature of error surface of IIR filters, conventional gradient-based design may easily get stuck in the local minima of error surface. Therefore, researchers have developed design methods based on modern heuristics optimization algorithms such as genetic algorithms [9]-[14], particle swarm optimization [15], seeker-optimization-algorithm [16], simulated annealing [17], tabu search [18], ant colony optimization [19], immune algorithm [21] and many more.

A genetic algorithm is a probabilistic search technique that is based on the principles of genetics. Mainly, genetic algorithms are inspired by evolutionary biology such as inheritance, mutation, natural selection, and crossover. Since its outset, genetic algorithm has been extensively used as a tool in computer programming, artificial intelligence and optimization. The genetic algorithm based optimization methods are not only capable of searching multidimensional and multimodal spaces, but are also able to optimize complex and discontinuous functions that are difficult to analyze mathematically. The use of the genetic algorithm for the digital IIR filter design is practical and attractive [10] as the filter can be constructed in any form, such as cascade, parallel, or lattice and also the low-pass (LP), high-pass (HP), band-pass (BP), and band-stop (BS) filters can be independently designed. Further the classical analog-to-digital transformation is avoided. Multiobjective functions can be simultaneously solved and the obtained model can be of the lowest order. Unfortunately, the performance of genetic algorithm based methods is often compromised by their very slow convergence and they may be trapped in the local optima of the multiobjective functions when the number of the parameters is large and there are numerous local optima [22]**.** Therefore, it is worthy of further developing an efficient heuristic algorithm to solve the problem of designing the optimal digital IIR filters. A hybrid Taguchi genetic algorithm has been applied by Tsai *et al.* [23] to solve the problem of designing optimal digital IIR filters. The hybrid Taguchi genetic algorithm approach is a method of combining the traditional genetic algorithms, which has a powerful global exploration capability, with the Taguchi method, which can exploit the optimum offspring. The Taguchi method is inserted between crossover and mutation operations of traditional genetic algorithms. While genetic algorithm rapidly locates good solutions, even for difficult search spaces, it has some disadvantages associated with it: genetic algorithm may converge towards local optima rather than global optima if the fitness function is not defined properly; it is difficult to operate genetic algorithm on dynamic sets. Conventional optimization algorithms may find better solutions than genetic algorithm in same computation time.

Particle swarm optimization technique attains high quality solutions within shorter calculation time and stable convergence characteristics than other stochastic methods such as genetic algorithm by virtue of being population based optimization algorithm. Based on a variation of particle swarm optimization, namely, quantum-behaved particle swarm optimization, Sun *et al.*[24] have proposed quantum-behaved particle swarm optimization with diversity-guided mutation for the design of 2-D IIR digital filters. The performance of particle swarm optimization or its variants depends on its parameters and may be influenced by premature convergence and stagnation problem [25]. A seeker-optimization-algorithm-based evolutionary method has been proposed by Dai *et al.* [16] for digital IIR filter design. Seeker optimization algorithm is based on the concept of simulating the act of human searching in which the search direction is based on the empirical gradient by evaluating the response to the position changes and the step length is based on uncertainty reasoning by using a simple fuzzy rule. Although seeker-optimization-algorithm is easy to be implemented and good at local convergence, depending upon the initial solution, it might often require too many cost function evaluations to converge to the global minima [16].

Recent heuristic method, namely immune algorithms mimic the antigen-antibody reaction of the immune system in mammals. The antigen and the antibody in the immune algorithms are equivalent to the objective function and the feasible solution for a conventional optimization method. The clonal selection principle of immune algorithms facilitates the efficiency of solving the numerical optimization problems. However, the characteristics of the immune algorithms are similar to those of the traditional genetic algorithms, which are largely based on the stochastic search techniques, have larger standard deviations. The obtained result is more robust if the relevant fitness values have a smaller standard deviation. A robust approach by integrating the immune algorithm and the Taguchi method named as Taguchi-immune algorithm has been purposed by Tsai and Chou [21] . Cooperative co-evolutionary genetic algorithm for digital IIR filter design has been proposed by Yu *et al.* [26]. The magnitude response and the phase response has been considered simultaneously and also tried to find the lowest filter order. The structure and the coefficients of the digital IIR filter have been coded separately, and they evolve coordinately as two different species, i.e., the control species and the coefficient species. The non-dominated sorting genetic algorithm-II has been used for the control species to guide the algorithms toward three objectives simultaneously. The simulated annealing has been used for the coefficient species to keep the diversity but it may require too many function evaluations to arrive at global minima [17]. Various methods exist in the literature that addresses the optimization problem under different conditions. Different optimization methods are classified based on the type of the search space and the objective function. In IIR filter design problems, the evaluation of candidate solutions could be computationally and/or financially expensive since it requires time-consuming computer simulation or expensive physical experiments. Therefore, a method is of great practical interest if it is able to produce reasonably good solutions within a given (often very tight) budget on computational cost/time. This paper aims at developing such a method for approximating the Pareto front of an expensive multiobjective optimization problem exploiting evolutionary search and heuristic approach.

The intent of this paper is to propose a heuristic method that randomly explores the search space for the best solution locally as well globally for the design of IIR filters. The heuristic method proposed is a combination of exploratory move and pattern move which are made iteratively to search optimal design of IIR filter. The unique combination of broad exploration and further exploitation yields a powerful option to solve multimodal optimization problems that designs IIR filters. The values of the filter coefficients are optimized with the heuristic algorithm to achieve L_p-norm error criterion in terms of magnitude response and ripples both in pass band and stop band as objective functions for multiobjective optimization problem. Multicriterion optimization problem is converted into scalar constrained optimization problem employing weighting p-norm method. The weighting technique is used to generate non-inferior solutions, which allow explicit trade-off between conflicting objective levels. The weighting patterns are either presumed on the basis of decision maker's intuition or simulated with suitable step size variation. Further the weightage pattern can also be searched in the non-inferior domain. In the paper, the weightage pattern is searched using Evolutionary search technique. Constraints are taken care of by applying exterior penalty method.

The paper is organized as follows. Section 2 describes the IIR filter design problem statement. The underlying mechanism and details regarding the methodology of the heuristic search algorithm for designing the optimal digital IIR filters is described in Section 3. In Section 4, the performance of the proposed heuristic search method has been evaluated and achieved results are compared with the design results by Tang *et al.* [10], Tsai *et al.* [23] and Tsai and Chou [21] for the low-pass, high-pass, band-pass, and band-stop filters. Finally, the conclusions and discussions are outlined in Section 5.

2 Filter Design Problem

Digital filter design problem involves the determination of a set of filter coefficients which meet performance specifications such as pass-band width and corresponding gain, width of the stop-band and attenuation, band edge frequencies and tolerable peak ripple in the pass band and stop-band. The traditional design of the IIR filter is described by the following difference equation:

$$y(n) = \sum_{k=0}^{M} p_k x(n-k) - \sum_{k=1}^{N} q_k y(n-k) \qquad (1)$$

where p_k and q_k are the coefficient of the filter. $x(n)$ and $y(n)$ are filter input and output. M and N are the number of p_k and q_k filter coefficients, with $N \geq M$.

The transfer function of IIR filter is stated as below:

$$H(z) = \frac{\sum_{k=0}^{U} p_k z^{-k}}{1 + \sum_{k=1}^{V} q_k z^{-k}} \qquad (2)$$

To design digital filter a set of filter coefficients p_k and q_k are determined, which meet the desired performance indices. A popular way of realizing IIR filters is to cascade many first- and second-order sections together [11] to avoid the coefficient quantization problem which causes instability. The structure of cascading type digital IIR filter is:

$$H(\omega, x) = \beta \left(\prod_{i=1}^{M} \frac{1 + p_{1i} e^{-j\omega}}{1 + q_{1i} e^{-j\omega}} \right) \times \left(\prod_{k=1}^{N} \frac{1 + r_{1k} e^{-j\omega} + r_{2k} e^{-2j\omega}}{1 + s_{1k} e^{-j\omega} + s_{2k} e^{-2j\omega}} \right) \qquad (3)$$

where $x = [p_{11}, q_{11}, \ldots, p_{1M}, q_{1M}, r_{11}, r_{21}, s_{11}, s_{21}, \ldots, r_{1N}, r_{2N}, s_{1N}, s_{2N}, \beta]^T$ and Vector x denotes the filter coefficients of first and second order sections of dimension V×1 with $V = 2M + 4N + 1$.

The IIR filter is designed by optimizing the coefficients such that the approximation error function in L_p-norm and using fixed grid approach [20] for magnitude is to be minimized. The magnitude response is specified at K equally spaced discrete frequency points in pass-band and stop-band. $e_1(x)$ denotes the absolute error L_1-norm of magnitude response and $e_2(x)$ denotes the squared error L_2-norm of magnitude response and are defined as given below:

$$e_1(x) = \sum_{i=0}^{K} \left| H_d(\omega_i) - |H(\omega_i, x)| \right| \qquad (4)$$

$$e_2(x) = \sum_{i=0}^{K} \left(H_d(\omega_i) - |H(\omega_i, x)| \right)^2 \qquad (5)$$

Desired magnitude response $H_d(\omega_i)$ of IIR filter is given as:

$$H_d(\omega_i) = \begin{cases} 1, & for \;\; \omega_i \in passband \\ 0, & for \;\; \omega_i \in stopband \end{cases} \quad (6)$$

The ripple magnitudes are to be minimized of pass-band and stop-band, which are denoted by $\delta_1(x)$ and $\delta_2(x)$ respectively. Ripple magnitudes are defined as:

$$\delta_1(x) = \max_{\omega_i}\{|H(\omega_i, x)|\} - \min_{\omega_i}\{|H(\omega_i, x)|\} \quad for \, \omega_i \in passband \quad (7)$$

and

$$\delta_2(x) = \max_{\omega_i}\{|H(\omega_i, x)|\} \;\; for \, \omega_i \in stopband \quad (8)$$

Aggregating all objectives and stability constraints, the multi-criterion constrained optimization problem is stated as:

Minimize $f_1(x) = e_1(x)$

Minimize $f_2(x) = e_2(x)$

Minimize $f_3(x) = \delta_1(x)$

Minimize $f_4(x) = \delta_2(x)$ (9a)

Subject to: the stability constraints:

$1 + q_{1i} \geq 0 \;\; (i = 1, 2, ..., M)$ (9b)

$1 - q_{1i} \geq 0 \;\; (i = 1, 2, ..., M)$ (9c)

$1 - s_{2k} \geq 0 \;\; (k = 1, 2, ..., N)$ (9d)

$1 + s_{1k} + s_{2k} \geq 0 \;\; (k = 1, 2, ..., N)$ (9e)

$1 - s_{1k} + s_{2k} \geq 0 \;\; (k = 1, 2, ..., N)$ (9f)

The multiple-criterion constrained optimization problem for the design of digital IIR filter is converted into a scalar constrained optimization problem by using a weighted sum of the objectives to generate non-inferior solutions.

$$\text{Minimize } f(x) = \left[\sum_{j=1}^{L} \left(w_j f_j(x)\right)^p\right]^{\frac{1}{p}} \quad (10)$$

such that $(1 \leq p \leq \infty)$

Subject to: The satisfaction of stability constraints given by Eq. (9b) to Eq. (9f).

Where $f_j(x)$ is the j[th] objective function, and w_j is non-negative real number called weight assigned to j[th] objective. This approach yields meaningful results when solved many times for different values of w_j, (*j* = *1, 2,..., L*). The p-norm weighting patterns are either presumed on the basis of decision maker's intuition or simulated with suitable step size variation. A problem with the weighted sum technique arises when the lower boundary of function space is not convex [27], because not every non-inferior solution will have a supporting hyper-plane. In case, the non-inferior surface is non-convex the weighted sum method may yield poor designs no matter what weight or optimization technique is used. The paper presents a systematic weight selection heuristics using Evolutionary search technique. The weight selection techniques used in a digital filter design have been discussed by Cortelazzo and Lightner [27]. The design of causal recursive filters requires the inclusion of stability constraints. Therefore, the stability constraints given by Eq. (9b) to Eq. (9f) which are obtained by using the Jury method [28] on the coefficients of the digital IIR filter in Eq. (3) are included in the optimization process. Scalar constrained optimization problem is converted into unconstrained multivariable optimization problem using penalty method. Augmented function is defined as in Eq. 10 with p=1 is defined as:

$$A(x) = \sum_{j=1}^{L} w_j f_j(x) + r\left(c + d\right)$$

where

$$c = \sum_{i=1}^{M}\langle 1 + q_{1i}\rangle^2 + \sum_{i=1}^{M}\langle 1 - q_{1i}\rangle^2 + \sum_{k=1}^{N}\langle 1 - s_{2k}\rangle^2 \quad (11)$$

$$d = \sum_{k=1}^{N}\langle 1 + s_{1k} + s_{2k}\rangle^2 + \sum_{k=1}^{N}\langle 1 - s_{1k} + s_{2k}\rangle^2$$

r is a penalty parameter having large value. Bracket function for constraint given by Eq. (9b) is stated below:

$$\langle 1 + q_{1i}\rangle = \begin{cases} 1 + q_{1i} & if \;\; (1 + q_{1i}) < 0 \\ 0 & if \;\; (1 + q_{1i}) \geq 0 \end{cases} \quad (12)$$

Similarly bracket functions for other constraints given by Eq. (9c) to Eq. (9f) are undertaken.

3 Heuristic Search Approach for the Design of IIR Filter

A heuristic search is used to describe a sequential examination of trial solutions. The procedure of going from a given point to the next improved point is called a 'move'. A move is termed a 'success' if the objective improves; otherwise, it is a 'failure'. The heuristic search routine makes four types of moves. The first move is random initialization. Random initialization has been framed to acquire best starting point. The second move is exploratory move designed to acquire knowledge concerning the behavior of the function. This move is performed in the vicinity of the current point systematically to find the best point around the current point. The third move is a pattern move with random acceleration factor. Weight Pattern search based on evolutionary search method is applied to search the normalized weights, $w_i \;\; (i = 1,2,\ldots,L)$ assigned to participating objectives and are utilized to generate the non-inferior solutions.

3.1 Random Initialization

In random Initialization the starting point is found with the help of random search. Global search is applied to explore the starting point and then the starting point is perturbed in local search space to record the best starting point. The search process is started by initializing the variable x_i^j using Eq. (13) which is used to calculate objective function using Eq. (3).

$$x_i^j = x_i^{\min} + r_1^j (x_i^{\max} - x_i^{\min}) \quad (i = 1, 2, \ldots, V;\ j = 1, 2, \ldots, NV) \tag{13}$$

where r_i is a random generated number, V is number of variables, NV is the population size.

The concept of opposition-based learning has been applied to accelerate reinforcement learning and back-propagation learning in neural networks [29]. The main idea behind opposition-based learning is the simultaneous consideration of an estimate and its corresponding opposite estimate (i.e., guess and opposite guess) in order to achieve a better approximation for the current candidate solution. The opposition based strategy is applied and starting point x_i^j is further explored using Eq. (14) to record the best starting point.

$$x_i^{j+1} = x_i^{\max} - r_2^j (x_i^{\max} - x_i^{\min}) \quad (i = 1, 2, \ldots, V\ ;\ j = 1, 2, \ldots, NV) \tag{14}$$

Out of 2 x NV members, best NV members constitute pool to initiate the process. For the global search best single, member is selected out of local best population. The stepwise flowchart for the initialization of filter coefficients and local search are given in Fig. 1 and Fig. 2.

3.2 Exploratory Move

In the exploratory move, the current point is perturbed in positive and negative directions along each variable one at a time and the best point is recorded. The current point is changed to the best point at the end of each variable perturbation may either be directed or random. If the point found at the end of all variable perturbations is different from the original point, the exploratory move is a success; otherwise, the exploratory move is a failure. In any case, the best point is considered to be the outcome of the exploratory move. The starting point obtained with the help of random initialization is explored iteratively and variable x_i is initialized as follows:

$$x_i^n = x_i^o \pm \Delta_i u_i^j \quad (i = 1, 2, \ldots, V;\ j = 1, 2, \ldots, V) \tag{15}$$

where

$$u_i^j = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \tag{16}$$

where NV denotes number of variables.

The objective function denoted by $f(x_i)$ is calculated as follows

$$x_i^n = \begin{cases} x_i^o + \Delta_i u_i & ;\ f(x_i^o + \Delta_i u_i) < f(x_i^o) \\ x_i^o - \Delta_i u_i & ;\ f(x_i^o - \Delta_i u_i) < f(x_i^o) \\ x_i^o & ;\ Otherwise \end{cases} \tag{17}$$

where $(i = 1, 2, \ldots, V)$ and Δ_i is random for global search and fixed for local search.

The process is repeated till all the variables are explored and overall minimum is selected as new starting point for next iteration. The stepwise flowchart to explore filter coefficients is outlined in Fig. 3.

3.3 Pattern Move

The pattern move utilizes the knowledge acquired in the exploratory move and attains the minimization of the function by moving in the direction of an established 'pattern'. A new point is calculated by leaping from current best point x_i^c along a direction connecting previous best point x_i^o and is executed as given below:

$$x_i^n = x_i^c + \eta (x_i^c - x_i^o) \qquad (i = 1, 2, \ldots, NV). \tag{18}$$

where η is accelerating factor and is a random number varying between 0.5 and 2.0.

Special consideration has been taken while generating accelerating factor which has been made random. In case the pattern move does not move the solution into a better region, the pattern move is not accepted and the extent of the exploratory move is reduced. The stepwise flowchart to perform pattern search for the filter coefficients is outlined in Fig. 4.

3.4 Weight Pattern Search

Evolutionary search method is proposed to search the optimal weight pattern. One weight assigned to an objective is considered dependent/slack weight required to meet the equality constraint required to ensure normalized weight and search is performed on rest of weights. So, in this method, 2^{L-1} feasible solutions are generated for (L-1) weights assigned to participating objectives except weight assigned to dependent objective. A (L-1) dimensional hypercube of side Δ is formed around the point. α_i^C represents weight pattern that is assigned to objectives from the current point in the hyperspace. The better feasible solution is obtained from objective function of the filter design performance index. Another hypercube is formed around the better point, to continue the iterative process. All the corners of the hypercube represented in binary (L-1) bits equivalent code, generated around the current set of assigned weight pattern of units, are explored for the desired solution simultaneously. Table 1 shows the weight pattern for 4-objectives where 3 bits code is considered to represent the corners of the 3-dimensional hypercube (Fig. 6) because one weight is taken as dependent/slack weight.

Serial numbers of hypercube corners in decimal are converted into their binary equivalent code. The deviation from the current centre point is obtained by replacing 0's with -Δ and 1's with +Δ in code associated with hypercube corners. As the number objectives are increased, the number of hypercube corners increases exponentially. The process of exploring the better solution from all corners of the hypercube becomes time consuming, which needs some efficient search technique that should explore all the corners of the hypercube with minimum number of function evaluations and comparisons. The weights are generated as given below:

$$\alpha_i^j = \alpha_i^c + \Delta_i \, (i = 1,2,\ldots,L; i \neq k \;\; ; j = 1,2,\ldots,2^L) \quad (19)$$

$$\Delta_i = \frac{\alpha^{\max} - \alpha^{\min}}{\delta} \qquad \text{where } \alpha_i \ni \{0,1\} \quad (20)$$

Δ is the distance of the corners of the hypercube from the point around which the hypercube is generated.

$$\alpha_k^j = 1 - \sum_{i=1}^{L} \alpha_i^j \qquad (i \neq k) \quad (21)$$

$$\alpha_{\min}^j = \min\{\alpha_k^j \, (k = 1,2,\ldots,L)\} \; (j = 1,2\ldots,2^{L-1}) \quad (22)$$

The weights w_k^j are obtained as

$$w_k^j = \frac{\alpha_k^j}{\alpha_{\min}^j} \quad (k = 1,2,\ldots,L; \; j = 1,2\ldots,2^{L-1}) \quad (23)$$

The stepwise flowchart to search weight pattern by evolutionary search method is given in Fig. 5.

4 Design Examples and Comparisons

For the purpose of comparison, the lowest order of the digital IIR filter is set exactly the same as that given by Tang *et al.* in [10] for the LP, HP, BP, and BS filters. Therefore, in this paper, the order of the digital IIR filter is a fixed number not a variable in the optimization process. The objective of designing the digital IIR filters is to minimize the objective function given by Eq. (10) with the stability constraints stated by Eq. (9b) to Eq. (9f) under the prescribed design conditions given in Table 2.

The examples of the IIR filters considered by Tang *et al.* [10], Tsai *et al.*[23] and Tsai and Chou [21] are referred to test and compare the proposed heuristic approach. For designing digital IIR filter 200 evenly distributed points are set within the frequency span $[0,\pi]$, such that the number in Eq. (4) and Eq. (5) of discrete frequency points is 182 for the LP and HP filters, or 143 for the BP and BS filters in the union of prescribed pass-band and stop-band shown in Table 2. In the proposed heuristic approach the different design criterion has been adopted as that used by the genetic algorithm based method of [10], but same design criterion as that of hybrid Taguchi genetic algorithm approach [23] and immune algorithm approach [21] to find the optimal digital IIR filters dimensionally. If you must use mixed units, clearly state the units for each quantity in an equation.

In the purposed heuristic approach the combination of four criteria, L_1-norm approximation error, L_2-norm approximation error, and the ripple magnitudes of pass-band and ripple magnitude of stop-band, are considered. These four criteria are contrary to each other in most situations. The filter designer needs to adjust the weights of criteria to design the filter depending on the filter specifications. Weights are adjusted using evolutionary search method. In the purposed heuristic approach larger value of weights w_3 and w_4 are chosen to obtain small ripple magnitude of both pass-band and stop-band. The weights w_1, w_2, w_3 *and* w_4 are set to be 1, 1, 6.6, and 11.4, respectively, for the LP, HP, BP and BS filter. The computational results obtained by the proposed heuristic approach compared to the genetic algorithm based method given by Tang *et al.* [10], Tsai *et al.*[23].and Tsai and Chou [21] respectively, are given in Table 3, 4, 5, 6 and shown in Fig. 7 to

Fig. 10. The designed IIR filter models obtained by the Heuristic approach are given below.

$$H_{LP}(z) = 0.045310\frac{(z+0.993990)(z^2-0.637952z+0.975035)}{(z-0.662967)(z^2-1.397013z+0.751778)} \quad (24)$$

$$H_{HP}(z) = 0.054670\frac{(z-1.117431)(z^2+0.703180z+0.978792)}{(z+0.616401)(z^2+1.348435z+0.729989)} \quad (25)$$

$$H_{BP}(z) = 0.026544\left(\frac{(z^2-0.268206z-1.317565)(z^2+0.228127z-0.670616)}{(z^2-0.637220z+0.762293)(z^2+0.009465z+0.530211)}\right)\times\left(\frac{(z^2-0.076907z-1.094593)}{(z^2+0.640883z+0.764888)}\right) \quad (26)$$

$$H_{BS} = 0.435983\frac{(z^2+0.436461z+.995824)(z^2-0.434855z+0.982156)}{(z^2-0.832325z+0.541472)(z^2+0.831384z+0.541033)} \quad (27)$$

The results obtained by proposed heuristic approach shown in Tables 3 to 6 and Fig. 7 to Fig. 10, clearly depict that, for the LP, HP, BP, and BS filters, the proposed heuristic approach gives the smaller L_1-norm approximation errors, the smaller L_2–norm approximation errors, and the better magnitude performances in both pass-band and stop-band than the genetic algorithm based methods given by Tang *et al.* [10], Tsai *et al.* [23] and Tsai and Chou [21] respectively.

5 Conclusion

This paper proposes a heuristic search method for design of digital IIR filters. As shown through experimental results, the heuristic search method works well with an arbitrary random initialization and it satisfies prescribed amplitude specifications consistently. On the basis of above results obtained for the design of digital IIR filter, we can conclude that: i) the proposed Heuristic approach is superior to the GA-based method presented by Tang *et al.* [10] Tsai *et al.* [23] and Tsai and Chou [21]; ii) the proposed Heuristic approach for the design of digital IIR filers allows each filter, whether it is LP, HP, BP, or BS filter, to be independently designed; and iii) the proposed Heuristic approach is very feasible to design the digital IIR filters, particularly when the complicated constraints, the design requirements, and the multiple criteria are all involved. However, there remain two aspects which need to be addressed: Random exploration and exploitation should be clubbed with local exploration and exploitation at what stage; Refinement of prescribed limits of parameters. Concluding, the results show that the proposed heuristic search method is a competent optimizer providing satisfactory parameters for the design of digital IIR filters.

References:

[1] A.V. Oppenheim, R.W. Schafer, and J.R. Buck, *Discrete-Time Signal Processing*, Second Edition, Englewood Cliffs, NJ: Prentice Hall, 1999.

[2] MATLAB, *MATLAB User's Guide*, The MathWorks, Natick, MA, 1991.

[3] K. E. Steiglitz and L. E. McBride, A technique for the identification of linear systems, *IEEE Trans. Automat. Contr.*, vol. AC-10, Oct. 1965, pp. 461-464.

[4] W.-S. Lu, S. Pei, and C. Tseng, "A weighted least-squares method for the design of stable 1-D and 2-D IIR digital filters," *IEEE Trans. Signal Process.*, vol. 46, no. 1, pp. 1–10, Jan. 1998.

[5] C. Tseng and S. Lee, Minimax design of stable IIR digital filter with prescribed magnitude and phase responses, *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 49, no. 4, Apr. 2002, pp. 547–551.

[6] C. Tseng, Design of stable IIR digital filter based on least p-power error criterion, *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 51, no.9, Sep. 2004, pp. 1879–1888.

[7] S. C. Chan, K. M. Tsui, and K. W. Tse, Design of Constrained Causal Stable IIR Filters Using a New Second-Order-Cone-Programming-Based Model-Reduction Technique, *IEEE*

Trans. Circuits Syst. II, Exp. Briefs, vol. 54, no.2, Feb. 2007,pp. 107-111.

[8] A. Jiang and H. K. Kwan, IIR digital filter design with new stability constraint based on argument principle, *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 56, no. 3, Mar. 2009, pp. 583–593.

[9] S.P. Harris and E.C. Ifeachor, Automatic design of frequency sampling filters by hybrid genetic algorithm techniques, *IEEE Trans. Signal Process.*, vol. 46, no. 12, Dec. 1998, pp. 3304–3314.

[10] K.S. Tang, K.F. Man, S. Kwong and Z.F. Liu, Design and optimization of IIR filter structure using hierarchical genetic algorithms, *IEEE Trans. Ind. Elecron.*, vol. 45, no. 3, June 1998, pp. 481–487.

[11] G. Vanuytsel, P. Boets, L.V. Biesen and S. Temmerman, Efficient hybrid optimization of fixed-point cascaded IIR filter coefficients, in *Proc. IEEE Int. Conf. Instrum. Meas. Technol.*, Anchorage, AK, 2002, pp.793–797.

[12] G.X. Zhang, W.D. Jin, and F. Jin, Multi-criterion satisfactory optimization method for designing IIR digital filters, in *Proc. Int. Conf. Commun. Technol.*, Beijing, China, 2003, pp. 1484–1490.

[13] Y. Liu, S.Q. Shao, H. Zhao, X.F. Liao, and J.B. Yu, An application of genetic algorithms with guiding strategy in the design of digital filters, in *Proc. Int. Conf. Commun., Circuits Syst.*, Chengdu, China, 2004, pp. 1141–1145.

[14] Nikos E. Mastorakis, Ioannis F. Gonos and M. N. S. Swamy, Design of Two-Dimensional Recursive Filters Using Genetic Algorithms *IEEE Trans. Circuits Syst. I, Reg. Papers,* vol. 50, no.5, May 2003, pp. 634-639.

[15] J. Sun, W.B. Xu, and B. Feng,A global search strategy of quantum behaved particle swarm optimization, in *Proc. IEEE Conf. Cybernetics Intell. Syst.*, Singapore, 2004, pp. 111–116.

[16] Chaohua Dai, Weirong Chen and Yunfang Zhu, Seeker optimization algorithm for digital IIR filter design, *IEEE Trans. Ind. Electron.*, vol. 57, no. 5, May 2006, pp. 1710–1718.

[17] S. Chen, R. H. Istepanian, and B. L. Luk, Digital IIR filter design using adaptive simulated annealing, *J. Digital Signal Process.*, vol. 11, no. 3, July 2001,pp. 241–251.

[18] A. Kalinli and N. Karaboga, A new method for adaptive IIR filter design based on Tabu search algorithm, *Int. J. Electron. Commun. (AEÜ)*, vol. 59, no. 2, May 2005, pp. 111–117.

[19] N. Karaboga, A. Kalinli, and D. Karaboga, Designing IIR filters using ant colony optimisation algorithm, *J. Engg. Appl. Artificial Intell.*, vol. 17, no. 3, Apr. 2004, pp. 301–309.

[20] A. G. Deczky, Synthesis of recursive filters using the minimum p-error criterion," IEEE Trans. Audio Electroacoust., vol. AU-20, no. 4, Oct. 1972, pp. 257–263.

[21] J.-T. Tsai and J.-H.Chou, Optimal design of digital IIR filters by using an improved immune algorithm, *IEEE Trans. Signal Process.*, vol. 54, no. 12, Dec. 2006, pp. 4582–4596.

[22] J.M. Renders and S.P. Flasse, Hybrid methods using genetic algorithms for global optimization, *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 26, no. 2, Apr. 1996, pp. 243-258.

[23] J.-T. Tsai, J.-H. Chou, T.-K. Liu, Optimal design of digital IIR filters by using Hybrid Taguchi Genetic Algorithm, *IEEE Trans. Ind. Electron.*, vol. 53, no. 3, June 2006, pp. 867–879.

[24] Jun Sun, Wei Fang and Wenbo Xu, A Quantum-Behaved Particle Swarm Optimization with diversity-guided mutation for the design of two-dimensional IIR digital filters, *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 57, no. 2, Feb. 2010, pp. 141-145.

[25] B. Biswal, P. K. Dash, and B. K. Panigrahi, Power quality disturbance classification using fuzzy C-means algorithm and adaptive particle swarm optimization, *IEEE Trans. Ind. Electron.*, vol. 56, no. 1, Jan. 2009, pp. 212–220.

[26] Y. Yu and Y. Xinjie, Cooperative coevolutionary genetic algorithm for digital IIR filter design, *IEEE Trans. Ind. Electron.*, vol. 54, no. 3, June 2007, pp. 1311–1318.

[27] M. R. Lightner and S. W. Director, Multiple criterion optimization for the design of electronic circuits, *IEEE Trans. Circuits and Syst.*, vol. CAS-28, Mar. 1981, pp 169-179.

[28] G. Cortelazzo and M.R. Lightner, Simultaneous design in both magnitude and group-delay of IIR and FIR filters based on multiple criterion optimization, *IEEE Trans Acoust., Speech, Signal Process.*, vol. 32, 1984, pp. 949–967.

[29] I. Jury, *Theory and Application of the Z-Transform Method*, New York: Wiley, 1964.

[30] Shahryar Rahnamayan, Hamid R. Tizhoosh, and Magdy M. A. Salama, Opposition-Based Differential Evolution, *IEEE Trans. Evol. Comput.*, vol. 12, no. 1, Feb. 2008, pp. 64-78.

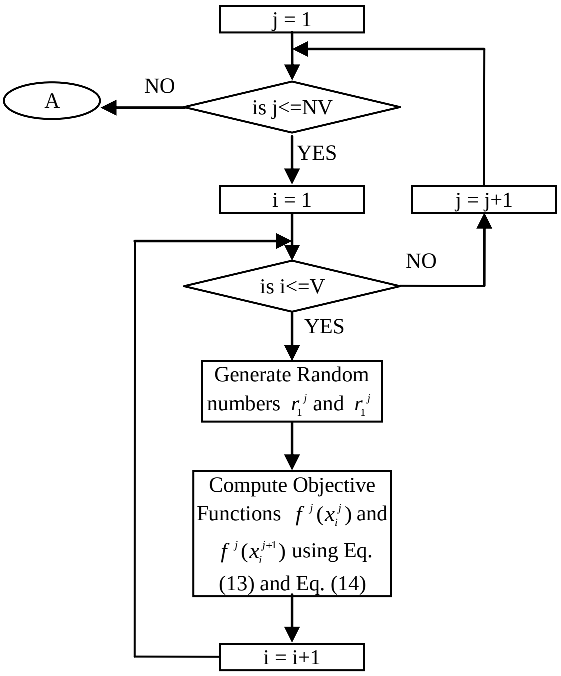

Fig. 1 Flowchart of procedure Random Initialization

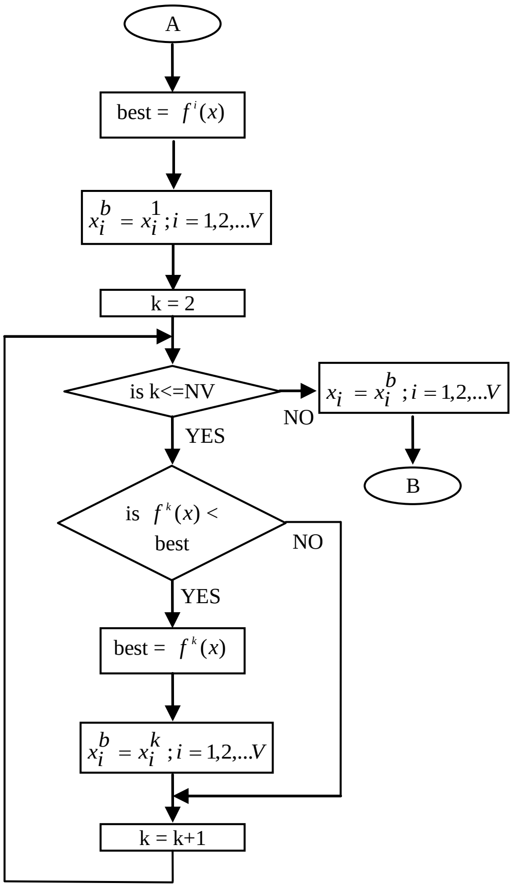

Fig. 2 Flowchart of procedure Local Search

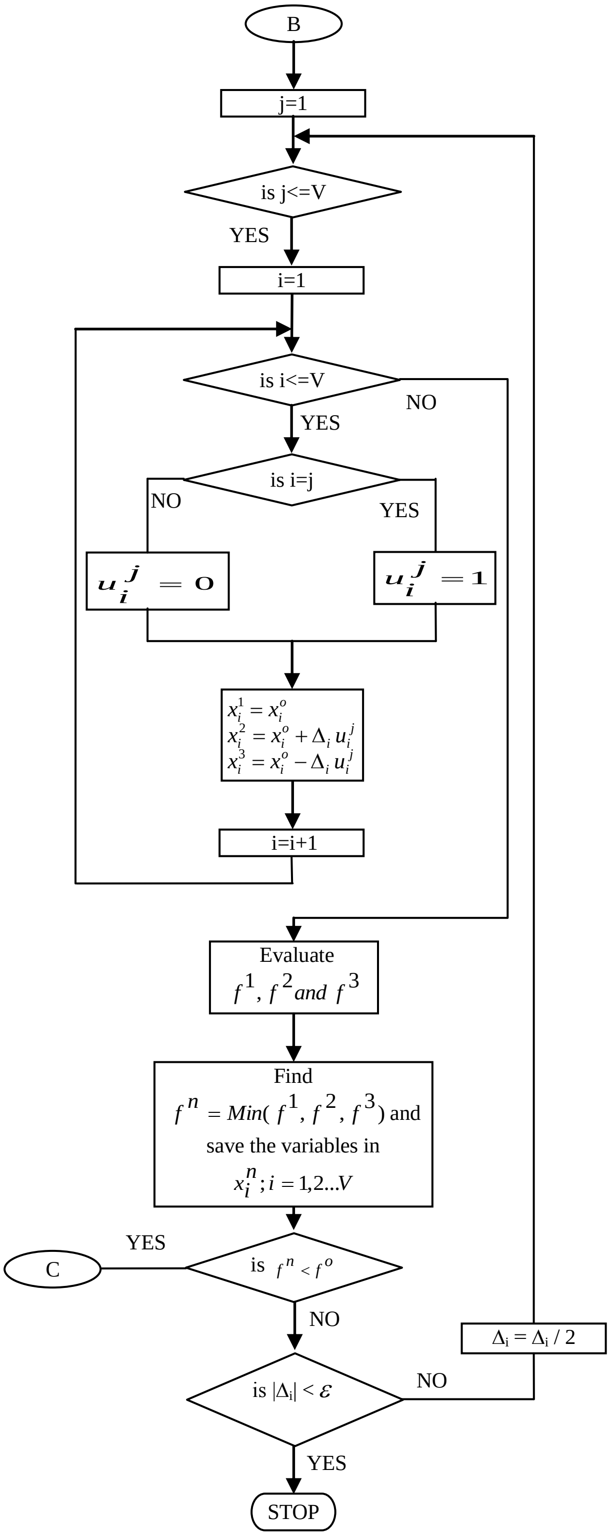

Fig. 3 Flowchart of procedure Exploratory Move

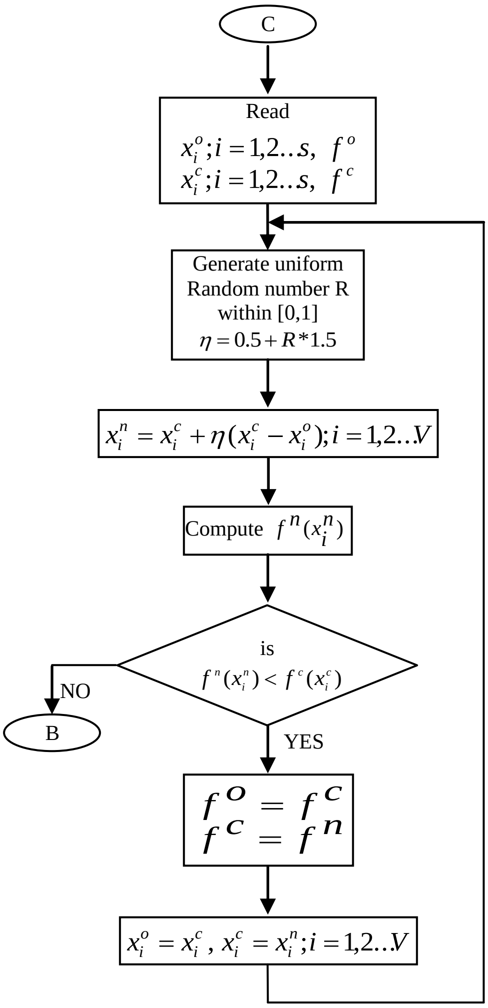

Fig. 4 Flowchart of procedure Pattern Search

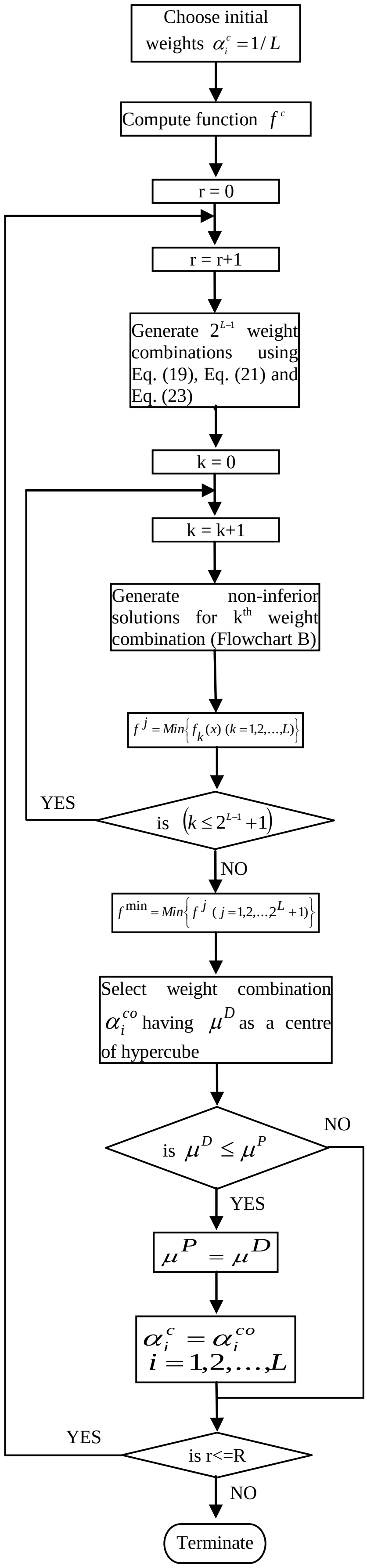

Fig. 5 Flowchart of procedure Weight Pattern Search

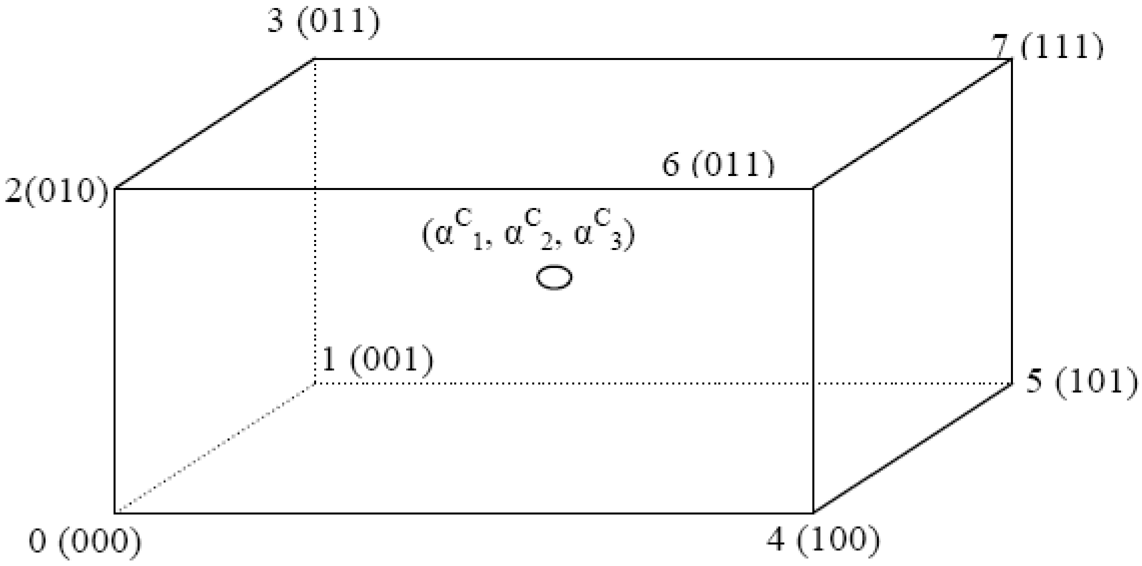

Fig. 6 Three dimensional hypercube representing corners in decimal

Table-1
Weight Pattern Generation Vector at Hypercube Corners for 4-Objectives

Hyper cube Corners	Possible combinations of 3-bits	Distance of hypercube corners from centre point	Possible power generation pattern of units at the hypercube corners		
	$C_2 C_1 C_0$	$\alpha_1^c,\ \alpha_2^c,\ \alpha_3^c$			
0	0 0 0	$-\Delta_1 - \Delta_2 - \Delta_3$	$\alpha_1^c - \Delta_1$	$\alpha_2^c - \Delta_2$	$\alpha_3^c - \Delta_3$
1	0 0 1	$-\Delta_1 - \Delta_2 + \Delta_3$	$\alpha_1^c - \Delta_1$	$\alpha_2^c - \Delta_2$	$\alpha_3^c + \Delta_3$
2	0 1 0	$-\Delta_1 + \Delta_2 - \Delta_3$	$\alpha_1^c - \Delta_1$	$\alpha_2^c + \Delta_2$	$\alpha_3^c - \Delta_3$
3	0 1 1	$-\Delta_1 + \Delta_2 + \Delta_3$	$\alpha_1^c - \Delta_1$	$\alpha_2^c + \Delta_2$	$\alpha_3^c + \Delta_3$
4	1 0 0	$+\Delta_1 - \Delta_2 - \Delta_3$	$\alpha_1^c + \Delta_1$	$\alpha_2^c - \Delta_2$	$\alpha_3^c - \Delta_3$
5	1 0 1	$+\Delta_1 - \Delta_2 + \Delta_3$	$\alpha_1^c + \Delta_1$	$\alpha_2^c - \Delta_2$	$\alpha_3^c + \Delta_3$
6	1 1 0	$+\Delta_1 + \Delta_2 - \Delta_3$	$\alpha_1^c + \Delta_1$	$\alpha_2^c + \Delta_2$	$\alpha_3^c - \Delta_3$
7	1 1 1	$+\Delta_1 + \Delta_2 + \Delta_2$	$\alpha_1^c + \Delta_1$	$\alpha_2^c + \Delta_2$	$\alpha_3^c + \Delta_3$

Table 2
Prescribed Design Conditions on LP, HP, BP and BS Filters.

Filter type	Pass-band	Stop-band	Maximum Value of $\lvert H(\omega, x)\rvert$
Low-Pass	$0 \le \omega \le 0.2\pi$	$0.3\pi \le \omega \le \pi$	1
High-Pass	$0.8\pi \le \omega \le \pi$	$0 \le \omega \le 0.7\pi$	1
Band-Pass	$0.4\pi \le \omega \le 0.6\pi$	$0 \le \omega \le 0.25\pi$ $0.75 \le \omega \le \pi$	1
Band-Stop	$0 \le \omega \le 0.25\pi$ $0.75 \le \omega \le \pi$	$0.4\pi \le \omega \le 0.6\pi$	1

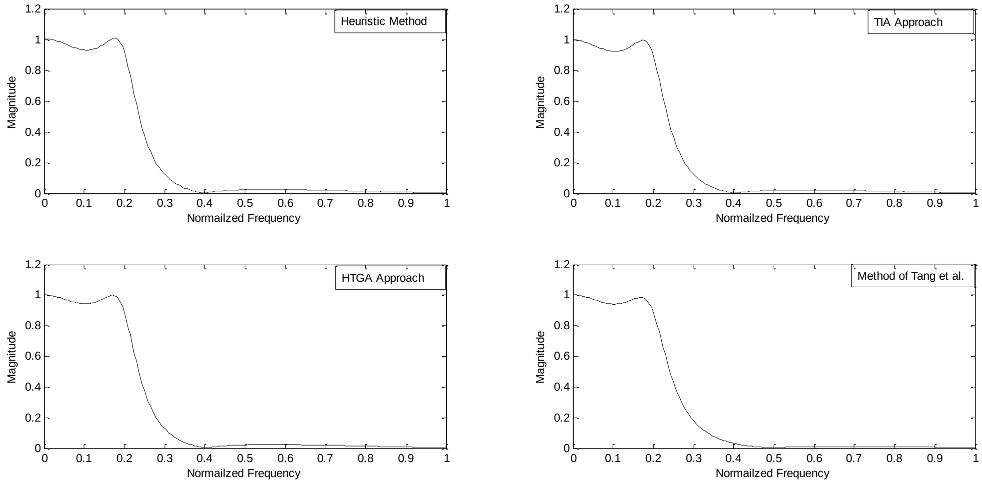

Fig. 7 Frequency responses of low pass filter using the Heuristic approach and the method given in [21], [23] and [10] respectively.

Table 3
Design Results for Low Pass Filter.

Method	L1-norm error	L2-norm error	Pass-band performance (Ripple magnitude)	Stop-band performance (Ripple magnitude)
Heuristic Approach	4.1145	0.4107	$0.9246 \le \lvert H(e^{j\omega})\rvert \le 1.011$ (0.0871)	$\lvert H(e^{j\omega})\rvert \le 0.1238$ (0.1238)
TIA Approach [21]	4.2162	0.4380	$0.9012 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.0988)	$\lvert H(e^{j\omega})\rvert \le 0.1243$ (0.1243)
HTGA Approach [23]	4.2511	0.4213	$0.9004 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.0996)	$\lvert H(e^{j\omega})\rvert \le 0.1247$ (0.1247)
Method of Tang et al.[10]	4.3395	0.5389	$0.8870 \le \lvert H(e^{j\omega})\rvert \le 1.009$ (0.1139)	$\lvert H(e^{j\omega})\rvert \le 0.1802$ (0.1802)

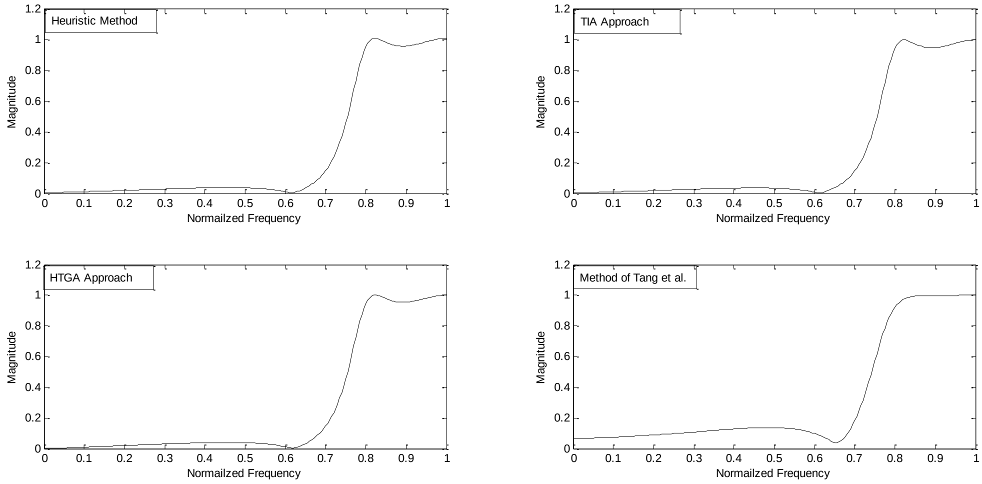

Fig. 8 Frequency responses of high pass filter by using the Heuristic approach and the methods given in [21], [23] and [10] respectively.

Table 4
Design Results for High Pass Filter

Method	L1-norm error	L2-norm error	Pass-band performance (Ripple magnitude)	Stop-band performance (Ripple magnitude)
Heuristic Approach	4.6635	0.4439	$0.9584 \le \left\|H(e^{j\omega})\right\| \le 1.008$ (0.0504)	$\left\|H(e^{j\omega})\right\| \le 0.1477$ (0.1477)
TIA Approach [21]	4.7144	0.4509	$0.9467 \le \left\|H(e^{j\omega})\right\| \le 1.000$ (0.0533)	$\left\|H(e^{j\omega})\right\| \le 0.1457$ (0.1457)
HTGA Approach [23]	4.8372	0.4558	$0.9460 \le \left\|H(e^{j\omega})\right\| \le 1.000$ (0.0540)	$\left\|H(e^{j\omega})\right\| \le 0.1457$ (0.1457)
Method of Tang et al. [10]	14.5078	1.2394	$0.9224 \le \left\|H(e^{j\omega})\right\| \le 1.003$ (0.0779)	$\left\|H(e^{j\omega})\right\| \le 0.1819$ (0.1819)

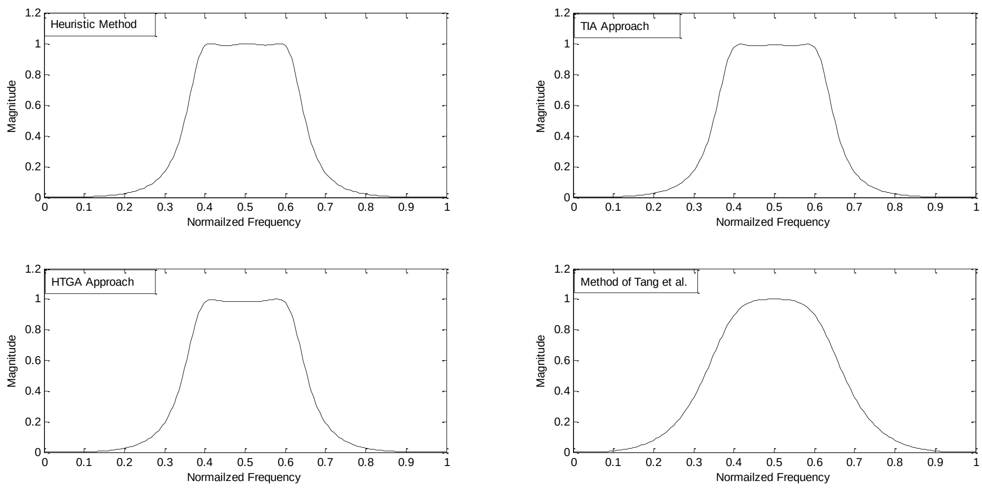

Fig. 9 Frequency responses of band pass filter by using the Heuristic approach and the methods given in [21], [23] and [10] respectively.

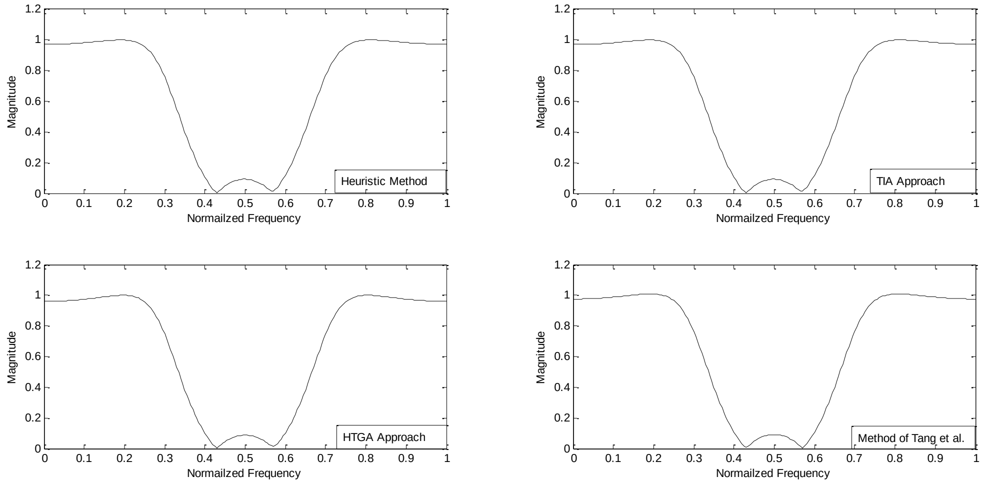

Fig. 10 Frequency responses of band stop filter by using the Heuristic approach and the methods given in [21], [23] and [10] respectively.

Table 5
Design Results for Band Pass Filter

Method	L1-norm error	L2-norm error	Pass-band performance (Ripple magnitude)	Stop-band performance (Ripple magnitude)
Heuristic Approach	1.4360	0.2052	$0.9896 \le \lvert H(e^{j\omega})\rvert \le 1.004$ (0.0147)	$\lvert H(e^{j\omega})\rvert \le 0.0627$ (0.0627)
TIA Approach [21]	1.6119	0.2191	$0.9806 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.0194)	$\lvert H(e^{j\omega})\rvert \le 0.0658$ (0.0658)
HTGA Approach [23]	1.9418	0.2350	$0.9760 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.0234)	$\lvert H(e^{j\omega})\rvert \le 0.0711$ (0.0711)
Method of Tang et al. [10]	5.2165	0.6949	$0.8956 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.1044)	$\lvert H(e^{j\omega})\rvert \le 0.1772$ (0.1772)

Table 6
Design Results for Band Stop Filter

Method	L1-norm error	L2-norm error	Pass-band performance (Ripple magnitude)	Stop-band performance (Ripple magnitude)
Heuristic Approach	3.7699	0.4532	$0.9652 \le \lvert H(e^{j\omega})\rvert \le 1.008$ (0.0434)	$\lvert H(e^{j\omega})\rvert \le 0.1060$ (0.1060)
TIA Approach [21]	4.1275	0.4752	$0.9560 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.0440)	$\lvert H(e^{j\omega})\rvert \le 0.1171$ (0.1171)
HTGA Approach [23]	4.5504	0.4824	$0.9563 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.0437)	$\lvert H(e^{j\omega})\rvert \le 0.1013$ (0.1013)
Method of Tang et al. [10]	6.6072	0.7903	$0.8920 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.1080)	$\lvert H(e^{j\omega})\rvert \le 0.1726$ (0.1726)