Unearthing Clues to Reduce the Devastating Effects of Earthquakes: The Hilbert-Huang Transform

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Abstract: - In this paper the use of the energy-time-frequency representation, the Hilbert-Huang Transform HHT, for the decomposition and characterization of seismic ground response signals, is discussed. The HHT, integrated by the Empirical Mode Decomposition (EMD) and the Hilbert Transformation (HT), enables engineers to analyze non-stationary oscillation systems and to obtain more detailed intensity descriptions on time-varying frequency diagrams. The advantage of the HHT over other representations is its sharp intensity-localization properties in the time-frequency plane. This paper first provides the fundamentals of the HHT method, and then uses them to analyze recordings of Mexico City soft-soil deposits, indicating that the HHT method is able to extract some motion characteristics useful in studies of seismology and geotechnical engineering, which might not be exposed effectively and efficiently by other conventional data processing techniques.

Key-Words: - Hilbert-Huang Transform, Seismic recordings, Site effects, Time Series Analysis

1 Introduction

Earthquakes represent the single largest potential cause of damage (from natural hazard) facing many countries in the world. The severity of the impact is a direct function of the performance of structures during strong earthquake. Although damaging earthquakes are infrequent, their consequences can be immense. According to recent studies of Federal Emergency Management Agency (USA), damages and losses from earthquakes have steadily increased in recent years. Population growth, urbanization, and the expansion of the built environment, all contribute to this trend. Decisive action is necessary to counter this trend of increasing losses; improved engineering design must part of that action.

Proper data analysis methods for the extraction of the temporal-frequency-energy distribution of motion recordings (ground acceleration) can help to explain earthquake phenomena, to understand important seismic issues (source mechanism, directivity influence, and soil dynamic nonlinearity) and to improve our knowledge of the underlying physical process the data expose. These facts must be taking into account in statutory regulations in order to develop more secure metropolis. The revised building code for Mexico City, for example, establishes geotechnical zones (similar dynamic properties and responses) within which specific provisions regulate the design of foundations and structures. A geotechnical microzonation map should not preclude careful analysis; it should rather be taken as a means of gaining broad insight into de gross overall properties of a particular site. In seismically active regions, the influence of local site conditions on the response of soil deposits and structures is a crucial aspect in the analysis. Making use of accelerograms recorded in Mexico City soil deposits (Fig. 1), this paper illustrates an alternative and more efficient manner in which this aspect can be dealt with.

A recently developed method, the Hilbert–Huang Transform HHT (Huang et al. 1999) seems to be able to meet some of these challenges. This study seeks to use the Hilbert-Huang transform to analyze dynamic and earthquake motion recordings and to examine the rationale of the HHT for studies of engineering and seismology. The objective of the analysis is to reveal useful information from motion recordings that might be either hidden or distorted by conventional data-analysis approaches. The examples are not exhaustively described but they are listed in the hope of attracting the attention of the geotechnical, seismic and engineering societies to this interesting, challenging and critical research area.

2 The Hilbert-Huang Transform

The HHT was proposed by Huang et al. (1998) and consists of two parts: i) Empirical Mode

Decomposition EMD, and ii) Hilbert Spectral Analysis HAS (Hilbert transformation). With EMD any data set can be decomposed into a finite number of intrinsic mode functions IMFs. An IMF is described as a function satisfying the following conditions: i) the number of extrema and the number of zero-crossings must either equal or differ at most by one; and ii) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero. An IMF admits well-behaved Hilbert transforms (Hongzhi and Chen, 1998). For an arbitrary function $X(t)$ in linear programming-class (Titchmarsh, 1948), its Hilbert transform $Y(t)$, is defined as,

$$Y(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{X(t')}{t-t'} dt' \quad (1)$$

where P indicates the Cauchy principal value. Consequently an analytical signal $Z(t)$, can be produced by,

$$Z(t) = X + iY(t) = a(t)e^{i\theta(t)} \quad (2)$$

where $a(t) = [X^2(t) + Y^2(t)]^{1/2}$, and $\theta(t) = arctan\frac{Y(t)}{X(t)}$ are the instantaneous amplitude and phase of $X(t)$, respectively. Since Hilbert transform $Y(t)$ is defined as the convolution of $X(t)$ and $1/t$ by Eq. (1), it emphasizes the local properties of $X(t)$ even though the transform is global. Using $a(t)$ and $\theta(t)$ expressions, the instantaneous frequency of $X(t)$ is defined as,

$$\varpi(t) = \frac{d\theta(t)}{dt} \quad (3)$$

However, there is still considerable controversy on this definition. A detailed discussion and justification can be found in (Huang et al., 1998).

EMD is a necessary pre-processing of the data before the Hilbert transform is applied. It reduces the data into a collection of IMFs and each IMF, which represents a simple oscillatory mode, is a counterpart to a simple harmonic function, but is much more general. We will not describe EMD algorithm here due to the limitation of the length of the paper. The readers are referred to (Huang et al., 1998) for details.

By EMD, any signal $X(t)$ can be decomposed into finite IMFs, $imf_j(t)(j = 1, ..., n)$, and a residue $r(t)$, where n is nonnegative integer depending on $X(t)$, i.e.,

$$X(t) = \sum_{j=1}^{n} imf_j(t) + r(t) \quad (4)$$

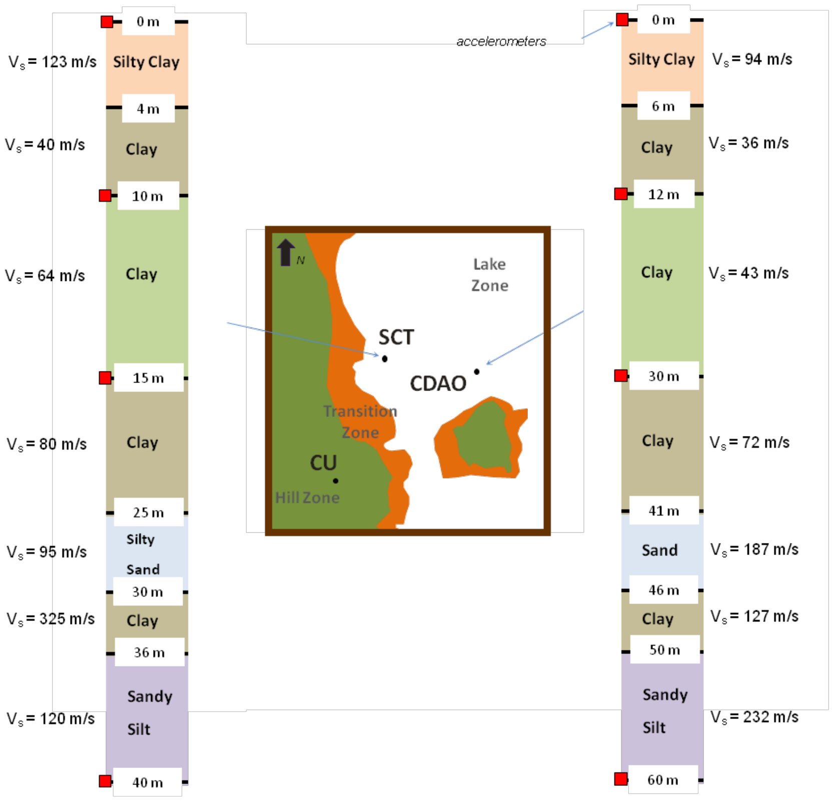


Figure 1 Mexico City Geotechnical Zonation: CDAO and SCT: soft-soils sites, CU: rock-site

For each imf_j, let $X_j(t) = imf_j(t)$, its corresponding instantaneous amplitude, $a_j(t)$, and instantaneous frequency, $\varpi_j(t)$, can be computed with Eqs. (2) and (3). By (3) imf_jcan be expressed as the real part RP in the following form,

$$imf_j(t)RP\left|a_j(t)\exp\left(i\int w_j(t)dt\right)\right| \quad (5)$$

Therefore, by Eqs. (4) and (5), $X(t)$ can be expressed as the IMF expansion as follows,

$$X(t) = RP\sum a_j\,(t)exp\left(i\int \varpi_j(t)dt\right) + r(t)$$

which generalize the following Fourier expansion

$$X(t) = \sum_{j=1}^{\infty} a_j e^{iwjt} \quad (6)$$

by admitting variable amplitudes and frequencies. Consequently, its main advantage over Fourier expansion is that it accommodates nonlinear and non-stationary data perfectly. Eq. (6) enables us to represent the amplitude and the instantaneous frequency as functions of time in a three-dimensional plot, in which the amplitude is contoured on the time-frequency plane. The time-frequency distribution of amplitude is designated as the HAS or simply Hilbert spectrum, denoted by $H\,(\varpi, t)$. For a comprehensive illustration of the HHT-seismic approach, García et al., (2006) is suggested.

3 HHT for studying seismic responses

Earthquake-resistant design is the first line of defense in improving structural safety and reducing losses from earthquakes. Structural engineers must take into account two fundamental characteristics of earthquake shaking: 1) how ground shaking propagates through the Earth (especially near the surface) –site effects–, and 2) how buildings respond to this ground motion. Currently neither characteristic is sufficiently well understood.

Presently recordings of ground motion in urban areas seemed to be insufficient to adequately characterize this phenomenon. Despite of the great efforts to improve the earthquake ground motion data (access, size and quality) there are no significant advances in reducing the dangerous difference between what is measured and predicted.

The intention of this work is point out a possible reason for this situation: the useful information from motion recordings is being either hidden or distorted by conventional data-analysis approaches.

3.1 Seismic-Wave Amplification

It is well established in geotechnical engineering that soil response is nonlinear and hysteretic beyond a certain level of deformation (Erdik, 1987, Finn, 1991). Such behavior brings about a reduction of soil strength and stiffness and increased soil damping, which modifies the wave form as compared with that of the linear case (e.g., Hardin and Drnevich 1972a, b; Erdik 1987; Vucetic and Dobry 1991).

With a reduction of soil strength and shear modulus (*G*) for nonlinear soil, the shear wave velocity V_s and thus the fundamental resonant frequency f_n of the soil layer decreases. Therefore, the resonant frequency of a “nonlinear-soil deposit” would be lower than the resonant frequency of the same layer with linear behavior. Accordingly, increased site amplification at the downshifted soil resonant frequency can be regarded as a signature of soil nonlinearity observable in ground motion records (e.g., Field et al. 1997, Beresnev et al. 1998).On the other hand; increased damping due to nonlinear soil behavior will decrease ground motions, thus moderating the site amplification. The effect of nonlinear behavior is likely to be lower at higher frequencies (Joyner 1999). On the basis of these comments, it becomes natural to wonder how the soil nonlinearities would be recognized in ground motion records?

3.1.1 Evidence of Nonlinear Site Effects

There is a bulk of technical information showing that both linear and nonlinear analytical methods are capable of computing with a good degree of accuracy the response of soil deposits under earthquake loading. This sole fact does not elucidate the rather generalized controversy about the significance of nonlinear soil effects (Finn. 1991; Yu et al., 1993; Aki and Irikura, 1991; Aki, 1993). In the author’s opinion the direct way to clarify this question is to look into hard seismological evidence showing the influence of such phenomenon. Thus, it boils down to decode the encrypted information that records of ground motions posses.

In practice, Fourier series expansion is typically used for representing and analyzing the recorded digital data of earthquake ground motion. For specific earthquakes, soil-site amplification is usually obtained by computing the ratio between the ground motion Fourier spectra and the Fourier spectra of the motions recorded on a nearby rock outcrop. Accordingly, comparisons between site-response factors for large- and low-amplitude events yield information on the soil site potential nonlinearity.

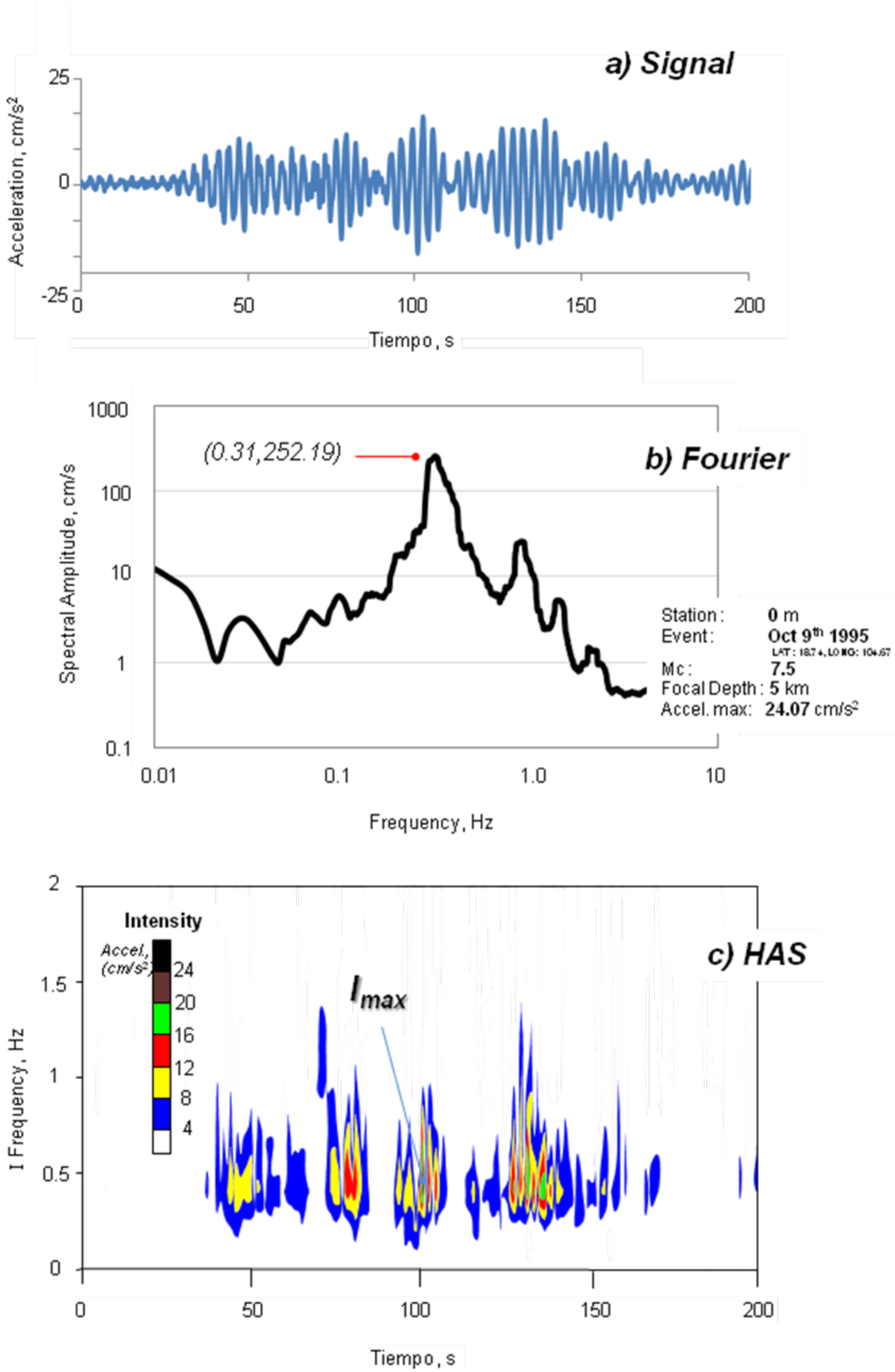


Figure 2 A signal (vector of accelerations), its Fourier spectra and its Hilbert spectra after EMD

The Fourier-based approach or similar methods, widely used, have a number of deficiencies in characterizing the nonstationarity of earthquake ground motion. The Fourier amplitude spectrum defines harmonic components globally (i.e., over the window of the time-data analyzed) and thus yields average characteristics over the entire duration of the earthquake. While the use of windowed (or short-time) Fourier transform may minimize the nonstationarity encoded in the data caused by different types of propagating waves and sources, it also reduces the frequency resolution. More important, the Fourier-based approach cannot accurately resolve issues of nonstationarity rooted in nonlinearity or nonlinear site response. As a result, this approach can lead to a distorted picture of nonlinear site amplification or response (Huang et al., 1999; and Worden and Tomlinson, 2001).

3.2 HHT interpretation of seismic records

In this section, the HHT is used to analyze the recordings from two Lake Zone sites of Mexico City (CDAO and SCT stations in Fig. 1). The strong- to weak-motion amplification ratios for 5 events (Sept 19th 1985, M_c =8.1; Oct 24th 1993, M_c= 6.5; May 23rd 1994, M_c = 5.4; Sept 14th 1995, M_c = 7.3 and Oct 9th 1995, M_c = 7.5) are analyzed to show the effectiveness of the alternative HHT approach in detecting, characterizing, and quantifying site effects.

3.2.1 HHT seismic analysis

The original accelerogram (E-W component) of the October 9th 1995 earthquake recorded in CDAO site is shown in Fig. 2.a and its corresponding Fourier spectrum in Fig. 2.b. The Fourier spectrum indicates that:

a) the frequency content of the waves is spread out with the maximum spectral amplitudes at 0.3 and 0.9 Hz,
b) the Fourier amplitude spectrum of the original data has a well-defined peak amplitude value (~250 cm/s) at the dominant frequency (0.3 Hz).

The HAS (Figure 2c) reveals quantitatively the temporal-frequency distribution of vibration characteristics in the ground-motion showing important features contained in the data:

a) two sets of ground motion in the time intervals 90–110 and 120–140 s
b) the significant accelerations consisting of low-frequency signals between 0.25 to 0.80 Hz,
c) the first waves arrival (90–110s) contains the highest intensity I_{max} (~28 cm/s^2) at 0.4 Hz,
d) I_{max} is surrounded by an important concentration zone (iso-acceleration curves ~ 20 cm/s^2 on a wider frequency band from 0.2 to 0.5 Hz,
e) the second motion arrival (120–140 s) encloses accelerations on the middle of the intensity-scale registered exhibiting frequencies around 1.0 Hz.

Since these quantities (amplitudes and frequencies) are quite often used as an index to measure the seismic energy, the distinction between values based on the two different methods can be critical to the seismic design, retrofit guidelines and codes. It is worth further investigation to see which value (Fourier or HHT-based) is more appropriate for structural design, but the differences between frequency-energy content can be critical when site effects are being analyzed. The full understanding of these characteristics is a subject of continuing study, for now, the potential exists for a useful quantitative measure of a motion's input energy to structural and geotechnical systems.

4 HHT and site effects

4.1 Nonstationary data processing

Spectral ratios between lake-bed stations (SCT and CDAO, very soft clay deposits) and a rock-like station (CU) were analyzed. The comparison between the Sept 19th 1985 Michoacan (strong) and the Oct 24th 1993 (weak) events is shown in Fig. 3. Fourier spectra ratios of the weak motions are slightly larger than strong motion ratios over the frequency range of 0.1 to 1.0 Hz but this evidence cannot be interpreted as a clear indication of nonlinear response. The ground motion at SCT is amplified at its natural frequency f_n about 10 times for the strong motion and around 20 times for the weak motion, while CDAO responses show an amplification of approximately 23 times for strong and about 12 times for the weak event, this ambiguous indication of nonlinear clay-behavior could be driven by the analysis tool rather than the actual encrypted in the recorded motions.

The mixed frequency content of the recordings, containing low and high frequencies, is truthfully reflected in the HAS (Fig. 4). The CDAO-HAS show a maximum intensity response (~8 cm/s^2) at 0.2 Hz for weak input (CU) and at 0.4 Hz for strong input (~80 cm/s^2). SCT-HAS, with a broader band of frequencies, depict an intensity of 180 cm/s^2 at 0.85 Hz for the strong event and 18 cm/s^2 at 1.0 Hz for the weak motion. The SCT abnormal peaks, for example at 55 s for the Oct 24th 1993 event and at 60 s for Sept 19th 1985 event, (cusped waveforms and high-frequency spikes) are symptomatic of a nonlinear response at some soil sites (Frankel et al. 2002).

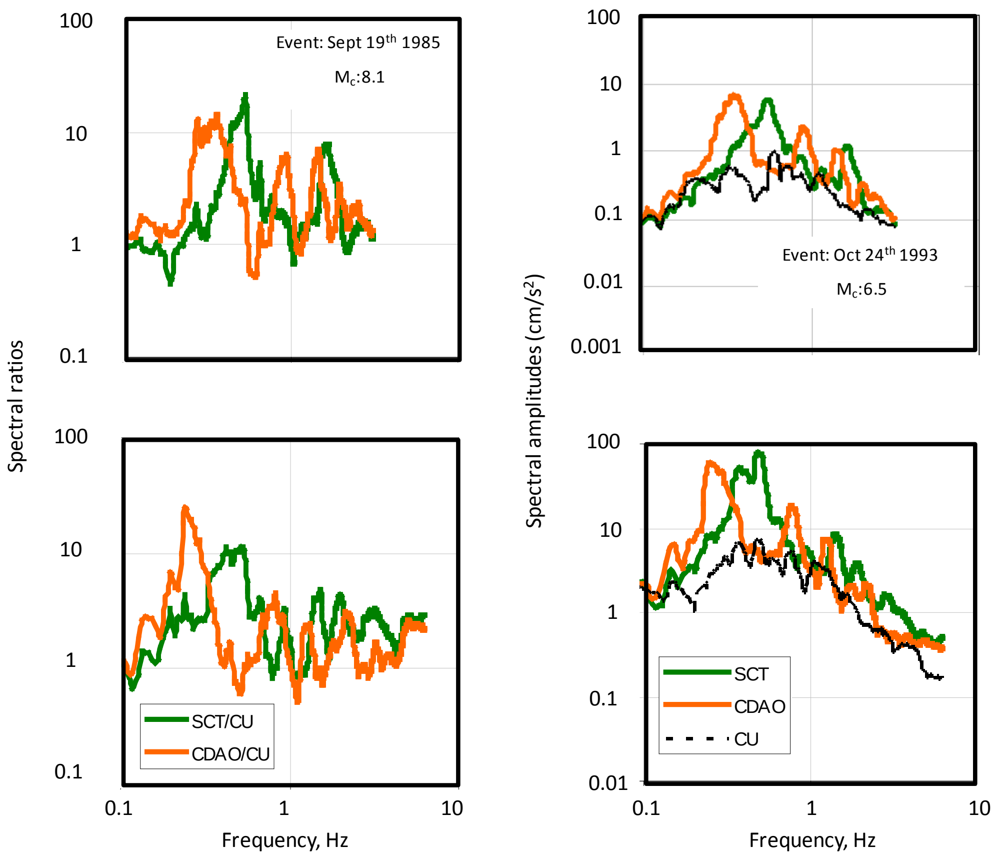


Figure 3 Fourier spectral amplitudes and ratios for two events: a "weak" (M_c: 6.5) and a "strong" (M_c: 8,1)

Contrary to the Fourier results, HHT demonstrates that the responses to the strong movement contain lower frequency levels compared with those generated during the weaker earthquake.

4.2 HHT analysis using down-hole records

A unique opportunity to detect nonlinear amplification effects is provided by borehole vertical arrays. The surface to bedrock spectral ratio provides the transfer function of the soil deposit, nearly isolated from source and path effects. Two earthquakes with different magnitudes, a weak, M_c=6.5 (Fig. 5) and a "strong", M_c=7.5 (Fig. 6), recorded at CDAO site, were used to calculate spectral ratios from bedrock (CU) to 60, 30, 12 and 0 m stations.

From the Fourier spectra ratios it is noted that the strong-motion amplification was significantly larger than weak-motion amplification in a wide frequency band (0.2 to 1 Hz) with the most pronounced resonance slightly shifted to the left. As shown in Figs. 5 and 6, noticeable differences in amplification are detected in 60 m and 30 m responses, while the spectrum at 12 m is equivalent to the surface spectrum implying that this upper layer does not amplify the motions (under strong and weak shakings).

The HAS (Figs.5 and 6) permit to outline the differences between responses and to mark the effects of the input characteristics on the amplification ratios. Variations between the 60 m-HAS for both events, in intensities and frequency content, are consequent with the different responses recorded at 30, 12 and 0 m while the Fourier analysis generates spectra with shapes very similar to the rock-spectra and the relation with the dissimilar soil responses results incongruent. Comparisons between 60 m- and CU-spectra deamplification takes place, alerting about the impact of the reference site-selection on the amplification ratios and soil-nonlinearity conclusions.

Using the spectral ratios in Figs. 5 and 6 three behavior-bands can be observed: the lowest frequency-band where amplification is not affected by nonlinearity (higher ratios for the stronger event), a central band (where f_n is localized) with higher amplification ratios for the strong event (three times larger than those calculated for the weak event) and the high frequency band where amplification is higher for weak event (nonlinear response).

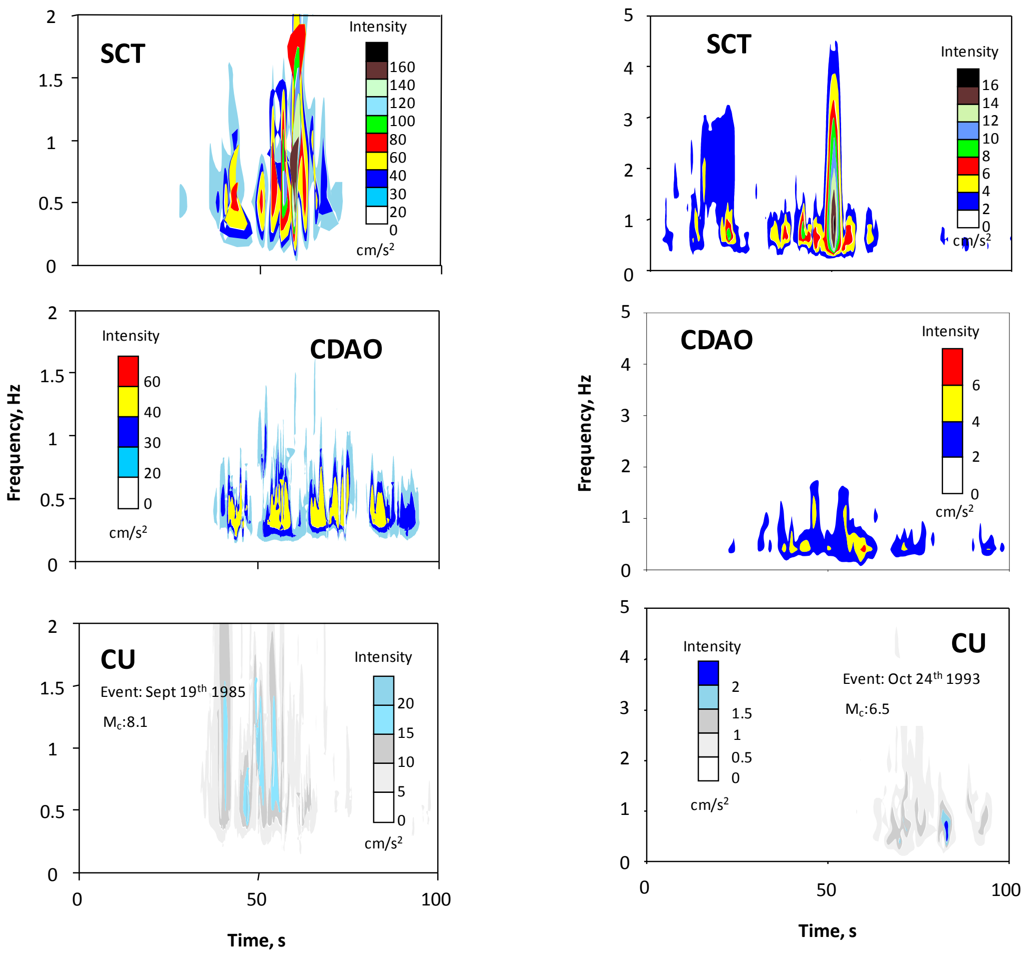


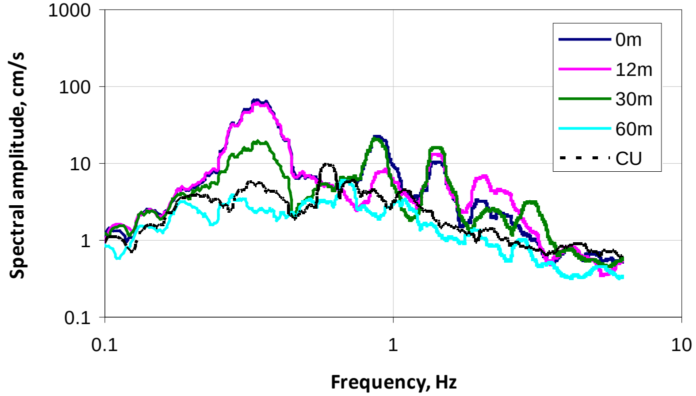
Figure 4 Hilbert spectral after EMD for two events:
a "weak" (M_c: 6.5) and a "strong" (M_c: 8,1)

On the basis of these results, it may be argued that the existence of specific frequency ranges where nonlinear and linear responses develop indicates that nonlinearities cannot be predicted from simple quantitative reasoning through Fourier spectra.

CU- and 60-HAS for weak event result more intense than that calculated for strong shaking but the duration of the intense accelerations differs. Despite of the similar intensity developed in both events, the time interval where maximum accelerations are developed has a profound influence on the minimum response (8 cm/s^2 ,weak). This characteristic cannot be deduced from Fourier analysis.

HHT analysis depicts a deamplification from 12 to 0 m (more energy is dissipated during the strong event) and because of the modification of the frequency content this superior layer can be considered as a low-pass filter. In the frequency range containing most of the radiated energy (around f_n), it is expected that the attenuation of the strong motion by hysteretic damping be larger than for weak motions. This is not observed in the Fourier spectra, only the HAS analysis permits seeing that weak response is due to a collection of acceleration spikes of low frequency from bedrock to the surface while the strong shaking contains a broader intensity-frequency range where higher harmonics are generated increasing strong-motion amplitudes. It seems inappropriate to relate coordinates (frequency, spectral amplitude) for describing a phenomenon that is clearly time dependent. The response summary (red tables inside the Figs.) using HHT permits to analyze the effect that time has on the earthquake phenomena manifestations: displacements. More analysis are being conducted to exploit these important findings, but it is a clear advantage on the results of conventional frequency-domain analyses.

The above arguments indicate that the HHT allows a more precise characterization of the layers behavior as linear or nonlinear and demonstrates that Fourier analysis is not able to reveal effectively the responses characteristics dealing with nonstationary time histories.



Station	Amplitude	Frequency	Time
0 m	57	0.32	
12 m	57	0.32	
30 m	18	0.32	
60 m	7	0.60	
CU	9	0.57	

Station	Intensity	Frequency	Time
0 m	8/6	0.1/0.8	12
12 m	8/6	0.2/1.1	23
30 m	4/6	0.8/1.2	5
60 m	1/1.5	0.6/1.0	3
CU	2/2.5	0.2/0.7	3

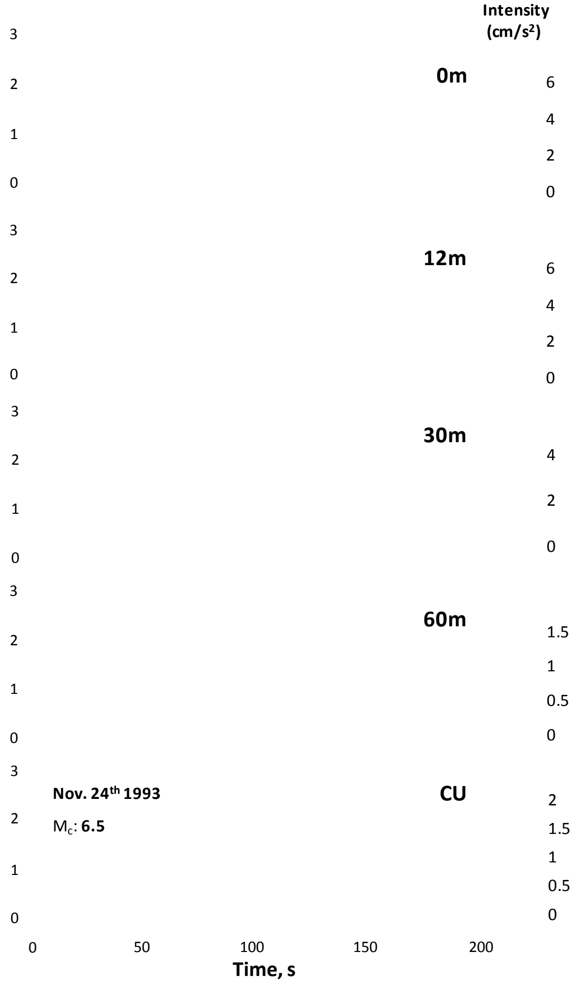
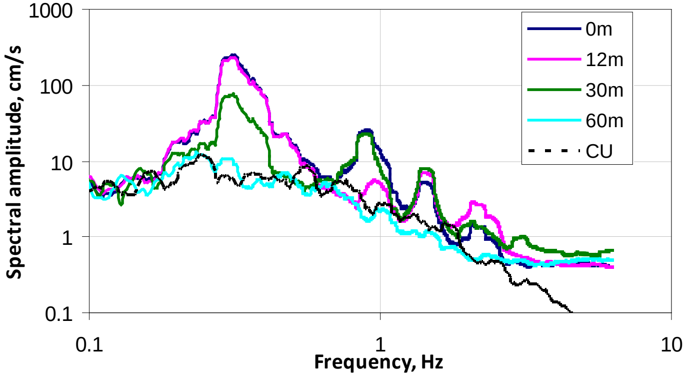


Figure 5 Comparison between Fourier and HHT descriptions:
a “weak” event (Nov. 24th 1993, M_c: 6.5)



Station	Amplitude	Frequency	Time
0 m	213	0.30	—
12 m	213	0.30	—
30 m	63	0.29	—
60 m	10	0.23	—
CU	9	0.22	—

Station	Intensity	Frequency	Time
0 m	16/20 12/16	0.25/0.6 0.25/0.8	20 13
12 m	24/28 16/20	0.2/1.3 0.2/1.1	15 8
30 m	4/6	0.2/2.0	30
60 m	0.5/1.0	0.2/0.5	10
CU	1.5/2.0	0.2/1.5	12

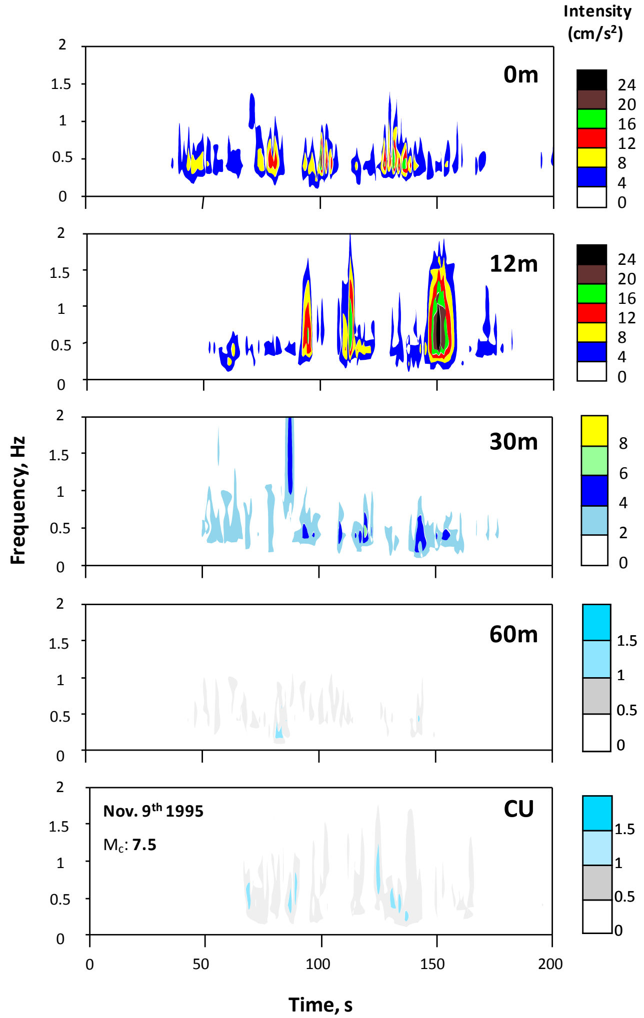


Figure 6 Comparison between Fourier and HHT descriptions: a “strong” event (Nov. 9th 1995 , M_c: 7.5)

5 Conclusion

HHT offers an important next step in a long-term effort to reduce earthquake losses through improved structural and ground-based earthquake strong motion recordings. The dangers of unsafe “underbuilding” and costly “overbuilding” for earthquakes can only be dealt with by the capture of vital information when earthquakes occur.

For seismic processes (nonstationary) the Fourier-based spectral analyses are not adequate because they are based on linear and stationary assumptions. In this short paper we have addressed the possibility of characterizing natural systems (soil deposits) by the time-frequency variations of system signals (accelerograms that represent the systems dynamics). The objective of describing data from engineering perspectives (time-frequency-intensity domain) for finding specific and indicative behavior patterns reducing the error mechanisms is addressed applying the HHT to the seismic information. Our analysis is of a preliminary nature and many issues have to be investigated rigorously but the HHT seems to have much potential for this approach. If new alternatives for analyze the sets of strong motion recordings are not applied on engineer fields by the time of the next damaging earthquake, a great opportunity to understand and to mitigate the effects of such natural disasters will be lost.

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