Boundary Effects Reduction in Wavelet Transform for Time-frequency Analysis

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Abstract: - Boundary effects are very common in the processing of finite-length signals. In this paper, we consider the problem of handling the boundary effects that can occur due to improper extension methods. Contrary to traditional methods including zero padding, periodic extension and symmetric extension, we propose an extension algorithm based on Fourier series with properties that make it more suitable for boundary effects reduction in the application of time-frequency signal analysis. This extension algorithm could preserve the time-varying characteristics of the signals and be effective to reduce artificial singularities appearing at the boundary. Procedures for realization of the proposed algorithm and relative issues are presented. Accurate expressions for the extent of boundary effects region are derived and show that the extent of boundary effects region is not equivalent to the width of wavelet under current mean square definition. Then, an interpolation approach is used in the boundary effects region to further alleviate the distortions. Several experimental tests conducted on synthetic signals exhibiting linear and nonlinear laws are shown that the proposed algorithms are confirmed to be efficient to alleviate the boundary effects in comparison to the existing extension methods.

Key-Words: - Finite-length Signals, Convolution, Wavelet Transform, Boundary Effects, Fourier Series Extension, Interpolation

1 Introduction

Wavelet transform analysis has been presented as a time-frequency analysis and processing method for over the past two decades[1, 2]. But it has still received increased attention in recent years [3]. Wavelet transform analysis has been widely used for the purpose of denoising, data compression, feature recognition, system nonlinearities detection and so on [4-7].

The wavelet transform is calculated as shifting the wavelet function in time along the input signal and calculating the convolution of them. In most practical applications, the signals of interest have finite support. As the wavelet gets closer to the edge of the signal, computing the convolution requires the non-existent values beyond the boundary [8-10]. This creates boundary effects caused by incomplete information in the boundary regions. Thus, the results of wavelet transform in these boundary effects regions have questionable accuracy. Actually, the particular impacts of boundary effects become increasingly significant for some systems that may possess longer period sequence and thus require higher frequency resolutions [7].

To deal with boundary effects, the boundaries should be treated differently from the other parts of the signal. If not properly made, distortion would appear at the boundaries [3]. Two alternatives to deal with boundary effects can be found. The first one is to accept the loss of data and truncate those unfavorable results at boundaries after convolution between signal and wavelet. But simply neglect these regions in analysis yields to a considerable loss of data which is not allowed in many situations where the edges of the signal contain critical information. The other one is artificial the extension at boundaries before processing signals. In fact, there is another approach that employs the usual wavelet filters for the interior of the signal and constructs different boundary wavelets at the ends of the signal. This method has been shows to be merged into the class of signal extension [11, 12].

Various extension schemes have been developed to deal with the boundaries of finite length signals [13,14]. Zero padding, periodic extension and symmetric extension are basic extension methods. It is well known that each method has its drawbacks [3,11]. So we should choose one most appropriate to the requirements of the application. Most of existing extension methods are exploited to the application of data compression [15]. In such applications, they have paid much attention on the procedures of analysis and synthesis using filter banks [16]. However, for other applications, such as diseases diagnosis and machine condition monitoring [17], it is desired to analyze the time-frequency content of arbitrary non-stationary signals. And traditional methods are not appropriate for such applications. They only make simple assumption about the signal characteristics outside the boundaries so that they fail to produce satisfactory results [11]. Zero padding assumes signals outside boundary are zero; periodic extension assumes that the signals are periodic and symmetric extension assumes the signals are symmetric about the boundaries. Many signals with time-varying frequency content do not belong to the above three types. Thus, we will present an extension mode that is suited for the application of time-frequency analysis. Our method preserves the time-varying characteristics of the signals while reduces the distortions due to improper extensions at the boundaries.

It should be aware that features appearing near the boundaries of transform values will contain information from outside the support of the signal which is synthetic. In other word, the wavelet transform resulting at the boundaries will be affected by the adding data no matter whichever extension mode is employed. Therefore, we will consider the problem from a perspective way that is different from extension method to alleviate these effects. In the paper, we will employ an interpolation processing in the region of the boundary effects to reduce the distortions. We will show that improvement can be obtained by such processing. In some literatures [7], the extent of these boundary effects regions has been mentioned but not been given an explicit definition. Therefore, we will show that the extent of boundary effects region is not equivalent to the width of wavelets under traditional mean square definition [18].

The paper is organized as follows. In Section 2, there is a brief review of the boundary effects in the wavelets transform and the shortcomings of traditional extension methods in the application of time-frequency signal analysis. In Section 3, we outline the proposed Fourier extension method and present the main properties of the proposed approach. A modified definition of the duration of wavelet, which contains dominating fraction of the signal energy, is introduce to measure the extent of boundary effects region in Section 4. In Section 5, a new algorithm based on interpolation technique for further boundary effects reduction is developed. Numerical examples are given to illustrate the improved performance of the proposed methods applied on both linear and nonlinear frequency modulation signals in Section 6. Section 7 summarizes the results obtained throughout the paper.

2 Boundary Effects in the Time-frequency Signal Analysis using Wavelet Scalogram

The goal of time-frequency analysis has primarily been to characterize and visualize the behavior of non-stationary signals. Wavelet scalogram, which refers to a time-scale energy distribution, is an image of the variation of the frequency content of the signal with respect to time to achieve such purpose. The wavelet scalogram has been widely used for vibration signal analysis, since it is particularly helpful in tackling problems involving signal identification and detection of hidden transients that is hard to detect. But most previously suggested methods have not considered the problems of boundary effects in the wavelet scalogram in many time-frequency analysis applications. Thus this section will investigate the effect of traditional extension methods on the wavelet scalogram in order to seek a suited extension method that can alleviate boundary effects. These traditional methods include zero padding, periodic extension and symmetric extension.

A finite signal with length N is $\mathbf{s}(\mathrm{n})$, $n \in 0,1,\cdots N$. Then we can express this signal in another form as

$$\mathbf{s} = [\mathbf{s}_l^{\mathrm{T}}, \mathbf{s}_c^{\mathrm{T}}, \mathbf{s}_r^{\mathrm{T}}]^{\mathrm{T}} \tag{1}$$

where $\mathbf{s}_l$ and $\mathbf{s}_r$ are vectors consisting of the first and last M components of the signal. $\mathbf{s}_c$ is the central part. Denote the extension vector of $\mathbf{s}(\mathrm{n})$ as

$$\mathbf{s}_e = [\mathbf{s}_{e,l}^{\mathrm{T}}, \mathbf{s}^{\mathrm{T}}, \mathbf{s}_{e,r}^{\mathrm{T}}]^{\mathrm{T}} \tag{2}$$

where similarly, $\mathbf{s}_{e,l}$ and $\mathbf{s}_{e,r}$ are the left and right extension vectors of length M. We use subscript to denote the size of matrix. Generalized expression for signal extension methods is given by

$$\mathbf{s}_e = \mathbf{H}_{(2M+N)\times N}\mathbf{s} \tag{3}$$

where $\mathbf{H}$ is the extension matrix. The traditional extension methods are all linear extension. Hence (3) can be written in form

$$\mathbf{s}_e = \begin{bmatrix} \mathbf{H}^l \\ \mathbf{I}_N \\ \mathbf{H}^r \end{bmatrix} \mathbf{s} \tag{4}$$

where $\mathbf{I}_N$ is an $N \times N$ identity matrix; $\mathbf{H}^l$ and $\mathbf{H}^r$ are respective left and right extension matrices.

For the zero padding extension, the extension matrix is

$$\mathbf{H} = \begin{bmatrix} \mathbf{0}_{M\times N} \\ \mathbf{I}_N \\ \mathbf{0}_{M\times N} \end{bmatrix} \tag{5}$$

where $\mathbf{0}_{M\times N}$ is an $M \times N$ zero matrix. Since for the periodic extension $\mathbf{s}_{e,l} = \mathbf{s}_r$ and $\mathbf{s}_{e,r} = \mathbf{s}_l$ the extension matrices of the periodic extension are

$$\mathbf{H} = \begin{bmatrix} \mathbf{0}_{M\times(N-M)} \mathbf{I}_M \\ \mathbf{I}_N \\ \mathbf{I}_M \mathbf{0}_{M\times(N-M)} \end{bmatrix}. \tag{6}$$

Similar result is available for the extension matrix of the symmetric extension

$$\mathbf{H} = \begin{bmatrix} \mathbf{I}_M \mathbf{0}_{M\times(N-M)} \\ \mathbf{I}_N \\ \mathbf{0}_{M\times(N-M)} \mathbf{I}_M \end{bmatrix}. \tag{7}$$

To show the boundary effects of various traditional methods, consider a linear frequency modulation signal s(t) with constant amplitude and frequency varied with time from 0.1 to 0.4(normalized frequency). The sampling interval used is $T = 0.01$ s with 300 data points. This is a typical non-stationary signal. The analyzing wavelet used to generate the scalogram is a Morlet wavelet since it is very useful and common for the detection of nonlinear characteristics [19]. We extract the ridge of this wavelet scalogram [20] of the test signal that results from extensions by zeros, which amounts to ignoring the need for extensions as shown in Fig. 1. Significant artifacts marked by circled can be observed near the boundaries. It can be see that the middle part of ridge almost perfect coincides with the theoretical result, which is denoted by solid line, while the boundary parts are deviated from that. It should be noted that deviation of the right end is much less than that of the left. This is due to the high frequency in the right side which corresponds to short wavelet length and small boundary effects region that will be discussed later.

Since the three methods only have big different at the boundaries of the ridge, it is more clear to plot one boundary part. In Fig. 2, we show a comparative boundary effects caused by the three different extension schemes. It can be observed that the zero

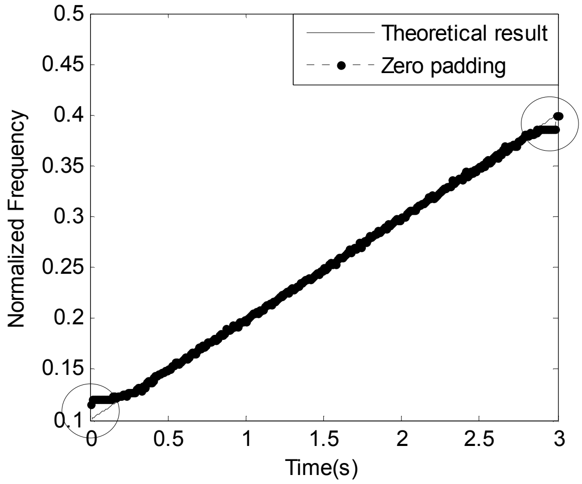


Fig. 1. Wavelet ridge of a linear FM signal using zero padding.

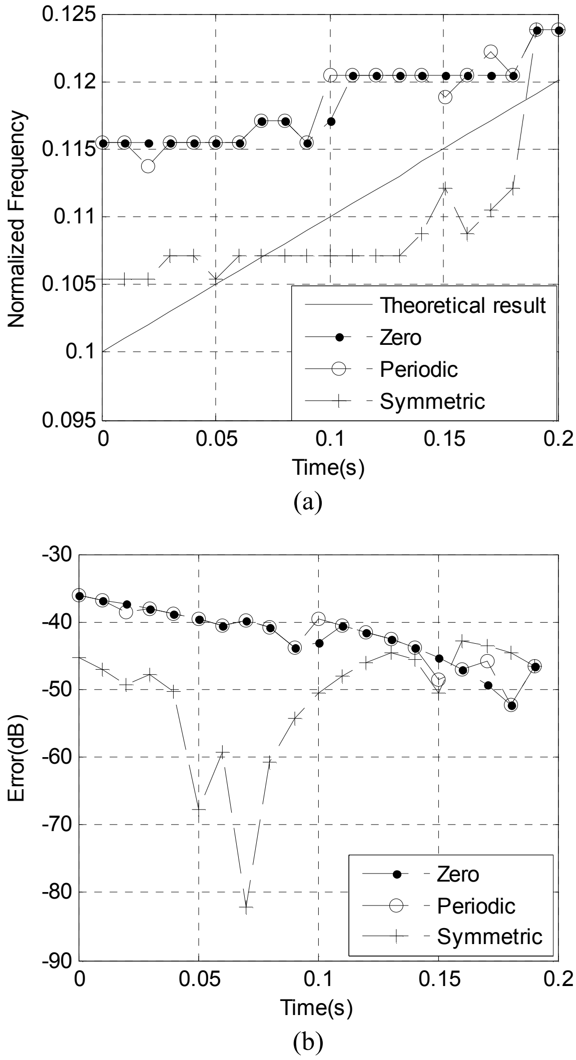


Fig. 2. Comparison of boundary effects for linear FM signal of three traditional methods. (a) The right boundary of wavelet ridge. (b) Estimation error. The values on the vertical axes are normalized to the adopted sample rate.

padding might have similar performance to periodic extension, whereas the symmetric extension presents a slight better result than the other two. This graphic outcome is also confirmed by the error between theoretical result and the results provided by traditional methods shown in Fig. 2(b). All three errors decrease when it gets close to the interior of the signal where the number of adding points participating calculation also decreases.

One of the main problems observed when handling boundaries using periodic extension is that, unless the input sequence is truly periodic and the end-points of the sequence match at the boundaries, artificial singularities can be introduced to wavelet transform near the boundaries. This is due to the discontinuity of the input sequence at the boundaries. Symmetric extension has the advantage, compared with periodic extension, that the extension sequence near the boundaries is continuous. Symmetric extension for handling boundaries is used extensively in many applications. It is the default extension scheme used in some software packages (e.g. MATLAB). However, the first and higher-order derivatives of the extension signal at the boundaries may not be continuous.

3 Boundary Effects Reduction via Fourier Series Extension

As discussed, every traditional method may have drawbacks. We should choose one according to the practical applications. In other words, the extension methods should be consistent with the object to be analyzed. When the object of analysis and processing is a signal with time-varying frequency content, we should choose a method which best matches the signal. The extension part should be consistent with the characteristics of the original part in order to alleviate the impact of artificial extension on features of the original signal. We expect the extension will be smooth and represent the past or future of signal that is nonlinear and non-stationary. In additional, wavelet transform should be computationally efficient depending on the particular application. Real-time applications would need to choose boundary extension methods that provide fast transform implementations.

3.1 Principle of Fourier Extension Method

Let us now consider the same test signal that has been used in Section 2. The signal used in Section 2 contains many harmonic vibrations. The traditional extension schemes cannot reflect such waving feature. It is nature to think of employing Fourier series technical to extension signal. Because Fourier series can be used to represent a signal in terms of the harmonics it is composed of. Although Fourier series is periodic, as we will see later, we only require a small segment of signal to achieve extension process which can be assumed to be periodic. In this paper, we call this novel scheme the Fourier series extension. The main principle of the Fourier series extension scheme is to fit a Fourier series model to the boundary points of the signal and then extrapolate that Fourier series at both ends.

Specifically, the procedure of Fourier series extension can be described as following:

(a)The Fourier series model is given by

$$\mathrm{y}(t) = a_0 + \sum_{i=1}^{m} a_i \cos(\omega_i t) + b_i \cos(\omega_i t) \qquad (8)$$

where a_0 is a constant term in the signal, both a_i, b_i and ω_i are parameters that need to be estimated by the fit, m is the number of harmonics in the data.

Because the above Fourier series model is nonlinear, the first step of Fourier series extension is to perform data transformations to obtain a linear or simple model.

(b)Fourier series fitting process involves finding the above model parameters to minimize the summed square of residual defined as the difference between the real date value s and the fitted response value y. This approach is referred as least-squares method.

(c) After completing the parameters estimate, the resulting Fourier series is extended to define the data beyond the borders so that the convolution can be calculated.

3.2 Design of Fourier Extension

When performing the Fourier series extension, several important issues must be considered.

3.2.1 The Choice of Fitting Number

The number of the boundary samples M to be fitted should be chosen carefully. If M is too small, the fitting result could not give a good represent of the characteristics of the original signal. On the other hand, a large M will lead to extra computation. In some applications, e.g., subband coding, M is equivalent to the length of the filters. As we will discuss later, in the application of time-frequency analysis using wavelet with an infinite support, M is determined by the extent of the boundary effects region.

3.2.2 Periodic Problem
Fourier series extension does not require the signal to represent a periodic function because we only choose boundary parts of the signal to perform extension which could be considered as a segment of a periodic function. However, if the data presented are assumed to represent a full cycle of periodic function, then many terms of a Fourier series are needed to achieve fitting. Therefore it is necessary to think in terms of the data representing only a partial segment of one complete periodic cycle so that only a few terms can give a good fit.

3.2.3 Properties of Fourier Extension
Unlike the traditional extension methods, the advantage of Fourier series extension is that it avoids the artificial discontinuities at the boundaries neither in the extension signal nor in its derivatives. So it prevents the appearance of large wavelet transform values at the ends. Therefore, Fourier series extension is a *smooth* extension. Moreover, using Fourier series to represent signal could well fit the fluctuation features of some very common signals in the areas of diagnosis and monitoring.

It should be noted that there is another *smooth* extension named polynomial extension. But it needs a higher degree polynomial to fit the fluctuation features compared with the Fourier series extension which has much less computational complexity and provides fast transform implementations.

4 Extent of Boundary Effects Region
It is important to examine the extent of the boundary effects region with appearance of the artificial components. The boundary effects region consists of a segment of transform results where the wavelet coefficients are calculated from the part of signal which contains the extension data.

4.1 Mean Square Definition of the Duration of Wavelet
As mentioned in the introduction, border effects root from wavelets analysis window extending beyond the data. Thus, the extent of the border effects region is relative to the width of wavelets analysis window, i.e. the duration of the wavelet.

For the solution of this problem, let take Morlet wavelet for example. Morlet wavelet is a complex sine wave localized with a Gaussian envelope given by

$$\psi(t) = \frac{1}{\sqrt{\pi\gamma_b}} \bullet e^{j2\pi\gamma_c - (t^2/\gamma_b)} \tag{9}$$

where γ_b is a bandwidth parameter defined as the variance of the Fourier transform of the Morlet wavelet and γ_c denotes the wavelet center frequency.

In fact, the strict duration of the Morlet wavelet is not a compact interval but the entire time axis. Hence, a duration where most of the signal energy is contained might be accepted as a practical measure of the signal duration. A widely used definition of signal duration Δt, proposed by Gabor [18] is

$$\Delta t = \sqrt{\frac{\int_{-\infty}^{\infty} t^2 |s(t)|^2 dt}{\int_{-\infty}^{\infty} |s(t)|^2 dt}} \tag{10}$$

which is a mean square definition. Based on this definition, the duration for a scaled Morlet wavelet at scale a can be written as

$$\Delta t = \frac{\sqrt{\gamma_b}}{2} a \,. \tag{11}$$

To examine the relationship between the extent of the border effects region and wavelet duration, we consider a simple sine signal with frequency $f = 8Hz$. The transform result of this signal using the wavelet in Fig. at scale $a = 1$ is plotted in Fig.. Due to the impact of border effects, the transform result which should be constant with time represents a curve at both ends. The length of the curve is the actual extent of border effects. It is much longer, not equivalent, than the duration of the wavelet defined in (9).This is due to the fact that the interval $[-\Delta t, \Delta t]$ just contains roughly 53% of the entire signal energy in this case of Morlet wavelet as shown in the Fig.. Hence, the definition of duration should be modified in order to be applied to the measure of the extent of border effects.

4.2 Modified Definition of the Duration of Wavelet
In this paper, we define a new duration Δt_e where a dominating fraction of the signal energy occurs.

Denote the ratio of integral of the modulus of Morlet wavelet over the interval $[-\Delta t_e, \Delta t_e]$ to over the entire time axis as η, then

$$\eta = \frac{\int_{-\Delta t_e}^{\Delta t_e} |\psi(a,t)| dt}{\int_{-\infty}^{\infty} |\psi(a,t)| dt} \,. \tag{12}$$

The numerator of the (12) is given by

$$\int_{-\Delta t_e}^{\Delta t_e} |\psi(a,t)| dt = \int_{-\Delta t_e}^{\Delta t_e} \frac{1}{\sqrt{\pi \gamma_b a}} e^{\frac{-t^2}{\gamma_b a^2}} dt$$
$$= \sqrt{a} erf(\frac{\Delta t_e}{a\sqrt{\gamma_b}}) \quad (13)$$

The denominator can be written as

$$\int_{-\infty}^{\infty} \psi(a,t) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi \gamma_b a}} e^{\frac{-t^2}{\gamma_b a^2}} dt = \sqrt{a} \quad (14)$$

Substituting (13) and (14) into (12) yields

$$\eta = erf(\frac{\Delta t_e}{a\sqrt{\gamma_b}}) \quad (15)$$

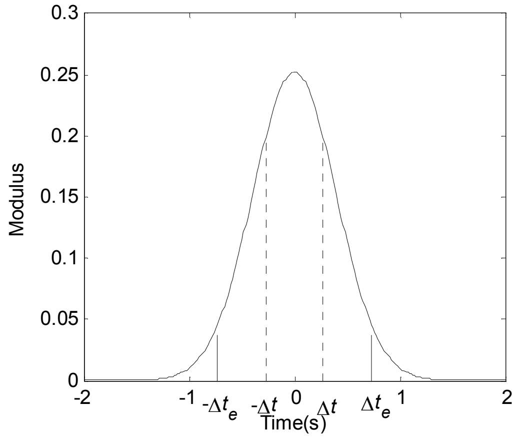
where $erf(x)$ is the error function defined as

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad (16)$$

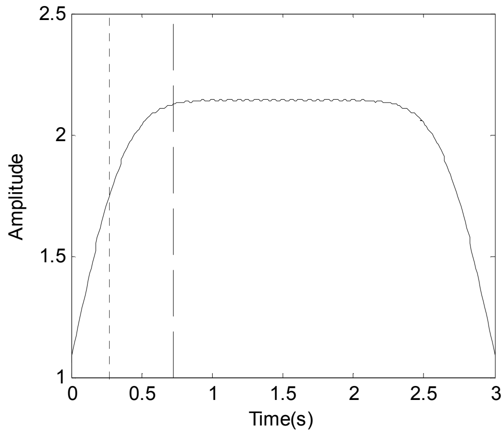
Then, the modified duration is defined as Δt_e that satisfies the formula in(15).

If we set the parameter $\eta = 0.9$, which means the new duration containing majority effective part of the wavelet, the duration of the same Morlet wavelet becomes $[-0.73, 0.73]$ (s) which approaches to the extent of border effects region. Since such a definition yields an interval containing dominating fraction of the wavelet energy. Fig. 3(a) shows the duration of Morlet wavelet defined by (10) and (12). In Fig. 3(b), we can observe that the new duration of wavelet is much more reasonable since the extent of border effects region obtained from it covers the whole distortion part in the wavelet transform.

Note that according to (15), the extent of border effects region increases linearly with scale parameter a. Furthermore, the degree of the border effects will become less as closing to the interior of the signal where the calculation of the wavelet coefficients involves less artificial data.



(a)



(b)

Fig. 3. (a) Duration defined by (10) (dotted) and modified duration (dashed) in the modulus of a Morlet wavelet and (b) related extent of boundary effects region in the wavelet transform of a sine signal with $f = 8$ Hz. The left side of dotted and dashed line in (b) represents the extent of boundary effects region.

5 Boundary Effects Reduction via Interpolation

Whichever extension method is employed to reduce the boundaries distortion phenomenon, the extension parts would definitely affect the analysis results which are determined by both original and extension signals. If the extension parts do not properly reflect the trend of the original signal, it will fail to produce satisfactory or perfect results. Nevertheless, it is well known that the signals in the application of time-frequency analysis are usually random and it is difficult to estimate the past and future of the signals based on the present data. Hence, this problem should be seen from a perspective that is broader than devising a convenient extension for the signal. Apart from the Fourier series extension method, an additional goal of this paper is to propose an approach to shorten the width of the boundary effect region defined in the above section. This approach is based on interpolation in the boundary effect region to reduce the boundary effects.

Fig. 4 explains the principle of reduction of boundary effects using interpolation method. Without interpolation, the convolution is computed between wavelet and data with length N, from $\mathbf{s}(0)$ to $\mathbf{s}(N-1)$.

After interpolation, the convolution is still computed between wavelet and data. However, the end point of these data has become to $\mathbf{s}(N/2-1)$ if N is odd. As shown in Fig. 4(b), the length of

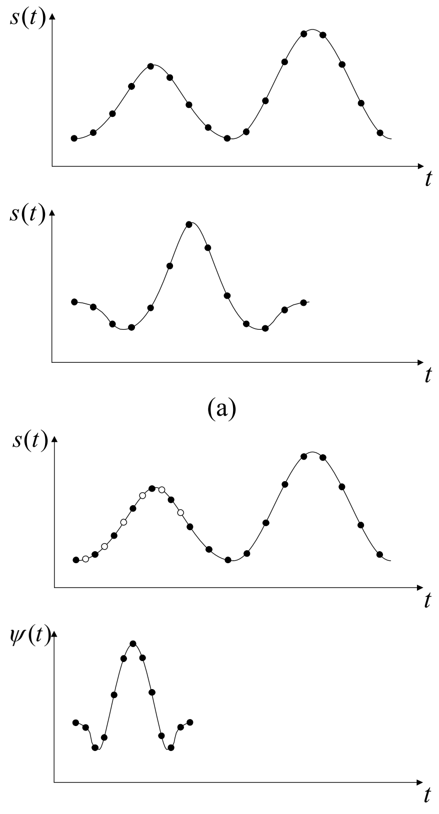


Fig. 4. Convolution between signal and wavelet: (a) before interpolation; (b) after interpolation.

wavelet is the half of that before interpolation and becomes shorter compared to the original signal. Based on the discussion of Section 4, it is easy to show that the boundary effects region which is decided by the length of wavelet also becomes shorter in consequence of the interpolation procedure. Therefore the boundary effects are alleviated by exploiting interpolation.

In practical applications, we only require to employ interpolation in the boundary effects region to obtain good results without heavy computational burden. Interpolation can be considered as an expansion of the extension method towards the interior of signal. Compare with extension methods, it is easier and more accurate to estimate signal value between two points than predict the data from the view of probability.

6 Numerical Examples

In order to validate the results given in Section 3 and Section 5, we present the numerical examples of the proposed algorithms. The performance of the proposed methods has been assessed by means of tests on generic synthetic signals. The purpose of the test is to establish the measurement accuracy of the proposed methods as well as their advantages in boundary effects reduction over the traditional methods. The test consists of two parts which involve the proposed extension method and interpolation preprocessing. Two signals exhibiting linear and nonlinear instantaneous frequency laws are used for evaluating the performance of the algorithms.

6.1 Performance Assessment of Fourier Series Extension

First consider the linear FM signal was presented in Section 2. Fig. 5 depicts the extension result (only the left boundary) using Fourier series extension applied on this signal. Fourier series extension displays both smooth and a good 'explanation' of signal itself.

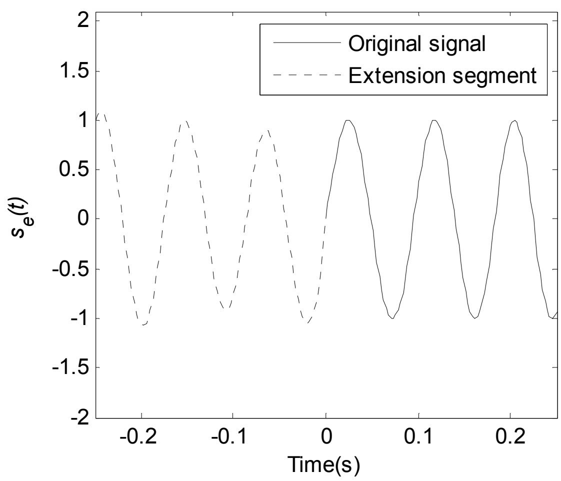
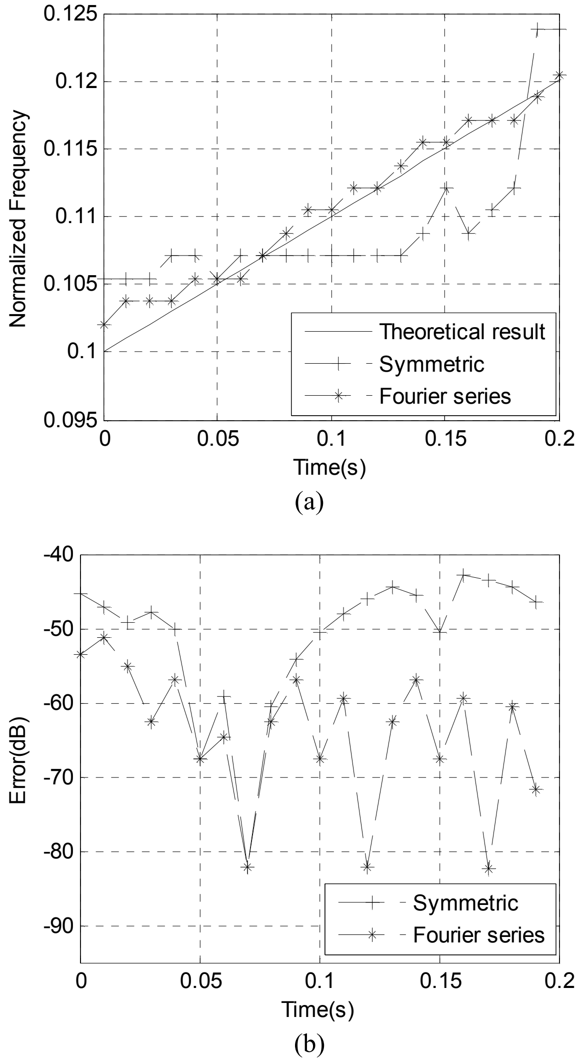


Fig. 5. Fourier series extension of the left boundary of the linear FM signal.

Some results that illustrate the performance of the Fourier series extension in the time-frequency analysis are shown in Fig. 6. For comparison purposes, the extension algorithm is compared with symmetric extension which is superior over the other two traditional methods. It is apparent that the results provided by the Fourier series extension method are in better agreement with the theoretical values in Fig. 6(a). As we have done in the previous section, the error between theoretical and estimated wavelet ridge are shown in Fig. 6(b) to illustrate the effect of Fourier series extension. It can be observed that Fourier series extension has less singularity appearing at the boundary than symmetric extension.



(a)

(b)

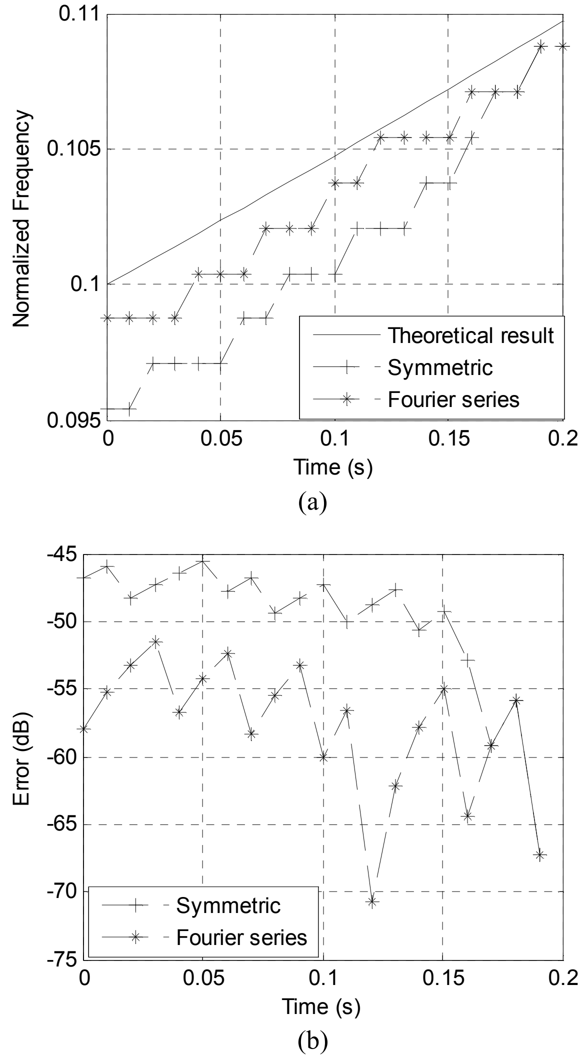
Fig. 6. Comparison of boundary effects for linear FM signal of symmetric and Fourier series extension. (a) The right boundary of wavelet ridge. (b) Estimation error. The values on the vertical axes are normalized to the adopted sample rate.

For the nonlinear case, we consider a logarithmic frequency modulated signal with same samples as the linear case and its IF is given by

$$\mathrm{f} = f_0 (\frac{f_0}{f_1})^{(\frac{t}{t_1})} \tag{17}$$

We set $f_0 = 0.1$, $f_1 = 0.4$, $t_1 = 3$. The total signal length and sample period used are $N = 300$, $T = 0.01$ s.

The proposed algorithm is successfully applied on this nonlinear FM signal. Fig. 7 illustrates the results provided by Fourier series extension and symmetric extension applied on the nonlinear FM signal. It can be seen that the results are similar to the results of linear FM signal. Fig. 7(b) shows that the Fourier series extension is indeed efficient to reduce boundary effects for complicated signals with time-varying IF laws.



(a)

(b)

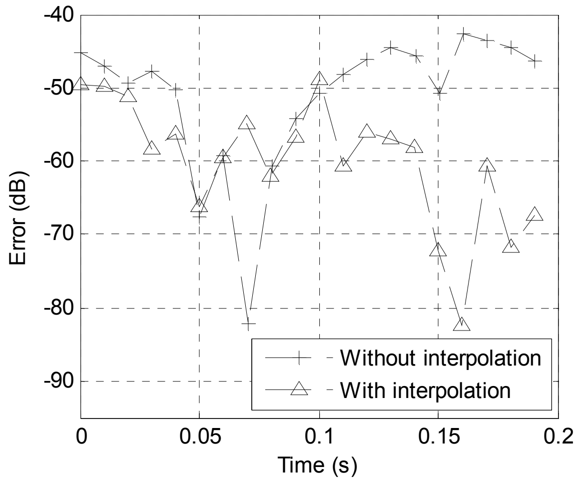
Fig. 7. Comparison of boundary effects for logarithmic FM signal of symmetric and Fourier series extension. (a) The right boundary of wavelet ridge. (b) Estimation error. The values on the vertical axes are normalized to the adopted sample rate.

6.2 Performance Assessment of the Interpolation Method

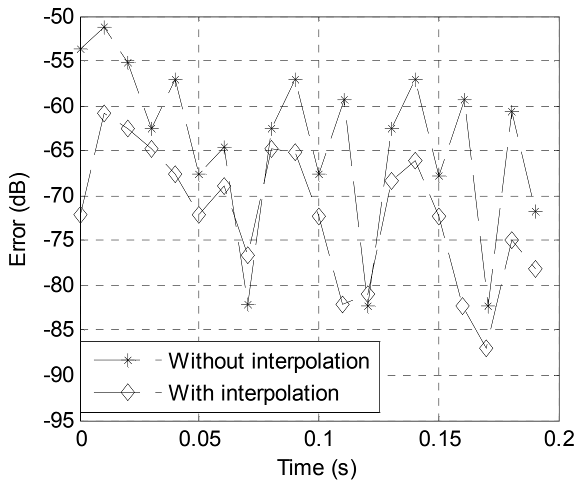
Several tests have been conducted in order to assess the capability of the proposed method interpolation preprocessing to further reduce the boundary effects. All the traditional extension methods and the Fourier series extension will be considered in this section. The performance of the interpolation preprocessing is examined using the same two classes of signals.

We plot the comparison results of symmetric extension and Fourier extension with and without interpolation processing on the linear FM signal and logarithmic FM signal in Fig. 8 and Fig. 9

respectively. It can be clearly observed that the interpolation processing is indeed able to further reduce the artificial singularities resulted from the boundary effects for both signals.

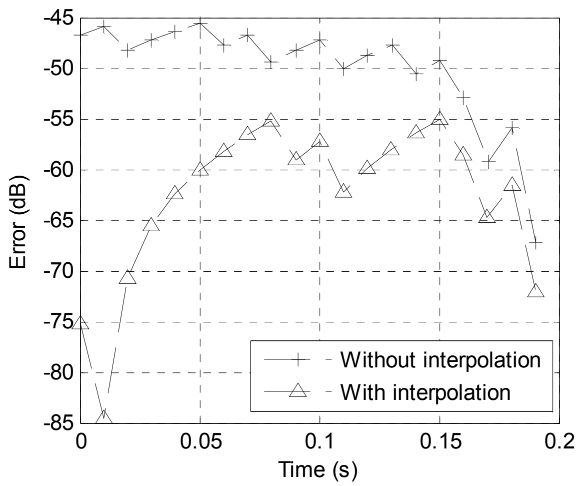


(a)

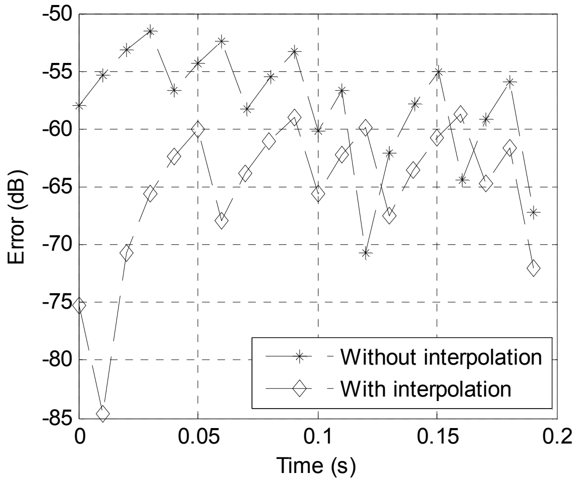


(b)

Fig. 8. Estimation error of wavelet ridge of linear FM signal with and without interpolation processing for (a) symmetric extension and (b) Fourier series extension.



(a)



(b)

Fig. 9. Estimation error of wavelet ridge of logarithmic FM signal with and without interpolation processing for (a) symmetric extension and (b) Fourier series extension.

To show the effect of the interpolation preprocessing, compression test results of the linear FM signal for different extension methods with and without interpolation processing are provided in Table 1. All of the values are calculated from true normalized frequency and estimating normalized frequency. In order to display the performance of boundary effects reduction, we only calculate the data from the boundary part. The number of data participating to the calculation is 50 points.

It can be concluded that the Fourier series extension with the interpolation processing provides the best performance among the four methods mentioned in this study. Furthermore, the interpolation processing is able to produce a better accuracy of the time-frequency characteristics estimate no matter which extension method is applied.

Table 1 Performance comparison of extension methods with and without interpolation preprocessing.

Method	**Bias**	**Variance**	**MSE**[a]
Zero	8.60×10^{-3}	1.28×10^{-5}	8.68×10^{-5}
Zero (interpolation)	3.14×10^{-3}	9.05×10^{-6}	2.07×10^{-5}
Periodic	8.68×10^{-3}	1.21×10^{-5}	8.76×10^{-5}
Periodic (interpolation)	3.53×10^{-3}	7.94×10^{-6}	2.04×10^{-5}
Symmetric	3.77×10^{-3}	4.17×10^{-6}	1.84×10^{-5}
Symmetric (interpolation)	1.41×10^{-3}	1.05×10^{-6}	3.04×10^{-6}
Fourier	9.24×10^{-4}	4.87×10^{-7}	1.34×10^{-6}
Fourier (interpolation)	3.37×10^{-4}	5.61×10^{-8}	1.69×10^{-7}

[a] MSE is the mean square error.

7 Conclusion

In this paper, we have discussed the problem of dealing with the boundary effects that would arise in the application of time-frequency analysis. Traditional methods including zero padding, periodic extension and symmetric extension were shown to provide unsatisfied performance to reduce the boundary effects. A smooth extension scheme using Fourier series to avoid distortion appearing at the boundaries was proposed. This extension technique possesses the property of preserving the waving characteristics of the time-vary signal that makes it more suitable than the other methods for the time-frequency analysis application. Some details on the procedures for realization of the proposed technique have been presented. We modified the current definition of duration of wavelet so that the new strict definition was able to measure the extent of the boundary effects region which would be applied to the procedure of Fourier extension and the interpolation. An example based on Morlet wavelet was used to validate the theoretical derivations. A new algorithm based on interpolation technique was proposed from new perspectives to further reduce the boundary effects. By comparing the results of the analysis, it has been shown that the Fourier series extension with the interpolation processing provided the best performance in the study. Although we have restricted the analysis to the wavelet transform, the proposed methods can be applied on any time-frequency distributions that involve convolution operation.

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