On the k-Distance Graph of Graphs

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Abstract: - For a positive integer k, the k-distance graph $D_k(G)$ of a graph G has vertex set $V(G)$, with two vertices joined whenever their distance in G equals k. We study $D_k(G)$ for an arbitrary k, extending results previously obtained for the fixed case $k = 3$. We show that the dominating sets of $D_k(G)$ coincide with the k-distance dominating sets of G, so that $\gamma(D_k(G)) = \gamma^k(G)$; we characterize when $D_k(G)$ is edgeless in terms of $\mathrm{diam}(G)$; and we give a complete structural description of $D_k(C_n)$ for cycle graphs C_n, showing that it splits into $\gcd(n,k)$ cycle components once n exceeds $2k$. The results hold for all graphs G and all positive integers k, making them applicable across a wide range of graph families.

Key-Words: - k-distance graphs, k-distance domination numbers, isolated graphs, cyclic graphs, dominating sets, circulant graphs, graph diameter, graph isomorphism.

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1 Introduction and Preliminaries

For vertices u, v of a graph G, let $d_G(u,v)$ denote their distance in G, that is, the length of a shortest u-v path in G. Fixing a positive integer k, one can build a new graph on the vertex set $V(G)$ by joining two vertices exactly when their distance in G equals k. This construction, denoted $D_k(G)$ and called the *k-distance graph* of G, was introduced in [1] in order to study its connectivity for arbitrary k. Thus $V(D_k(G)) = V(G)$, and $d_{D_k(G)}(u,v) = 1$ precisely when $d_G(u,v) = k$. Follow-up work has probed further structural properties of $D_k(G)$, including the diameter of $D_2(G)$, [2], and the periodicity of $D_k(G)$ for general k, [3].

A parallel notion is that of domination at a fixed distance. Recall that a set $D \subseteq V(G)$ is a *dominating set* of G if every vertex not in D has a neighbor in D, and the smallest size of such a set is the *domination number* $\gamma(G)$. Standard references on domination in graphs include, [4], [5], the latter surveying its distance-based variants. Domination at a prescribed distance was first studied in [6]. Replacing "neighbor" (distance 1) by "at distance k" gives the notion introduced, [7], for $k = 3$: a set $Q \subseteq V(G)$ is a *k-distance dominating set* if every vertex outside Q is at distance exactly k from some vertex of Q, and the *k-distance domination number* $\gamma^k(G)$ is the smallest cardinality of such a

set. The same parameter has since been computed for other graph families, including circulant graphs, [8]. Domination in graphs is also studied along directions quite different from ours; recent examples include a linear-time algorithm for connected domination on partial k-trees, [9], and the p-strong Roman domination model, [10].

This paper extends the fixed case $k = 3$, studied in [7], [11], [12], to $D_k(G)$ for an arbitrary positive integer k. All three principal contributions are genuinely new for general k: the domination equivalence (Theorems 2.4–Corollary 2.5) was established for $k = 3$ only in [11]; the edgeless characterisation via $\mathrm{diam}(G)$ (Theorem 2.6) is new; and the complete structural description of $D_k(C_n)$ for cycle graphs C_n (cycles on $n \geq 3$ vertices) reveals a number-theoretic pattern invisible at $k = 3$ (Theorem 2.10). A *circulant graph*, [13], is a Cayley graph on a cyclic group $\mathbb{Z}_n$ in which each vertex i is joined to $i \pm s \pmod{n}$ for each generator s in a fixed connection set S; we identify $D_k(C_n)$ as such a graph and use the coset structure of $\langle k \rangle$ in $\mathbb{Z}_n$ to derive a clean formula: writing $d = \gcd(n,k)$, the graph $D_k(C_n)$ is isolated when $n < 2k$, a perfect matching when $n = 2k$, and otherwise splits into exactly d cycle components each of length n/d.

We clarify the relationship between the present paper and closely related work. The *connectivity* of $D_3(G)$ is examined in [12]; connectivity lies outside our scope and there is no overlap. The *periodicity* of $D_k(G)$ for general k is treated in [3]; periodicity is likewise not addressed here. The present paper therefore makes contributions that are complementary to, but distinct from, the above.

Unless stated otherwise, all graphs considered here are finite and simple. For a graph G, $\mathrm{diam}(G)$ denotes the largest distance occurring between two vertices lying in a common component of G, with the convention $\mathrm{diam}(G) = 0$ when G has no edges; a graph with no edges is called *isolated*.

Remark 1.1. Taking $k = 1$ gives nothing new: since $d_G(u,v) = 1$ exactly when u and v are adjacent, $D_1(G) = G$ for every graph G, and consequently $\gamma^1(G) = \gamma(G)$. The results that follow are therefore of interest mainly for $k \geq 2$, although we state them for arbitrary $k \geq 1$.

2 Main results

A vertex v is a k-distance neighbor of u in G if $d_G(u,v) = k$. The *k-distance open neighbourhood* of u is

$$N_G^k(u) = \{v \in V(G) \mid d_G(v,u) = k\}, \qquad (1)$$

and the *k-distance closed neighbourhood* of u is $N_G^k[u] = N_G^k(u) \cup \{u\}$. For a set $S \subseteq V(G)$, the *k-distance open neighbourhood* of S is defined, using (1), by

$$N_G^k(S) = \bigcup_{u \in S} N_G^k(u), \qquad (2)$$

and the *k-distance closed neighbourhood* of S is $N_G^k[S] = N_G^k(S) \cup S$. By (2), it is easy to see that Q is a k-distance dominating set if and only if $N_G^k[Q] = V(G)$.

We begin by establishing bounds and extremal characterisations for the k-distance domination number $\gamma^k(G)$. The lower bound $\gamma^k(G) \geq 1$ is immediate since the empty set is never a dominating set, and the upper bound $\gamma^k(G) \leq |V(G)|$ holds because $V(G)$ itself always k-distance dominates G. The two extremes are each characterised precisely by the theorem below, giving the first of our main results.

Theorem 2.1. *Let G be a graph and $k \geq 2$. Then $1 \leq \gamma^k(G) \leq |V(G)|$. Moreover, each of the following holds:*

(1) $\gamma^k(G) = 1$ if and only if G is trivial.

(2) $\gamma^k(G) = |V(G)|$ if and only if $\mathrm{diam}(H) \leq k-1$ for each component H of G.

Proof. Let G be a graph. Since $\emptyset$ is not a k-distance dominating set of G, $\gamma^k(G) \geq 1$. Since any k-distance dominating set Q is always a subset of $V(G)$, it follows that $1 \leq \gamma^k(G) \leq |V(G)|$.

(1) Suppose that $\gamma^k(G) = 1$. Then $\{v\}$ is a minimum k-distance dominating set of G for some $v \in V(G)$. This means that $N_G^k[v] = V(G)$. If G is non-trivial, then $|V(G)| \geq 2$. Suppose that G is disconnected, with components $G_1, \ldots, G_t$ where $t \geq 2$, and that $v \in V(G_i)$ for some i. Then $V(G_j) \nsubseteq N_G^k[v]$ for every $j \neq i$, a contradiction. Suppose instead that G is connected. Let $a \in N_G(v)$; this means $d_G(a,v) = 1$, so $a \notin N_G^k[v]$ since $k \geq 2$, showing that $N_G^k[v] \neq V(G)$, a contradiction. Therefore, G is trivial. Conversely, suppose that G is trivial. Then clearly $\gamma^k(G) = 1$.

(2) Suppose that $\gamma^k(G) = |V(G)|$. Then $V(G)$ is the minimum k-distance dominating set of G. Suppose there is a component H of G with $\mathrm{diam}(H) \geq k$. An initial segment of length k of a shortest path realizing $\mathrm{diam}(H)$ is itself a shortest path (so has length exactly k), and its endpoints $u, v \in V(H)$ satisfy $d_G(u,v) = k$. Then $V(G) \setminus \{u\}$ is a k-distance dominating set of G. Thus $\gamma^k(G) \leq |V(G)| - 1$, a contradiction. Therefore, $\mathrm{diam}(H) \leq k-1$ for every component H of G. Conversely, suppose that $\mathrm{diam}(H) \leq k-1$ for each component H of G. Then, by (1), $N_H^k[w] = \{w\}$ for

each $w \in V(H)$. It follows that $N_G^k[a] = \{a\}$ for each $a \in V(G)$. Thus $V(G)$ is the only k-distance dominating set of G. Consequently, $\gamma^k(G) = |V(G)|$. □

Corollary 2.2. *Let $k \geq 3$. The following statements hold.*

(1) If S_n is a star graph with n vertices, then $\gamma^k(S_n) = n$.

(2) If K_n is a complete graph with n vertices, then $\gamma^k(K_n) = n$.

(3) If W_n is a wheel graph with n vertices, then $\gamma^k(W_n) = n$.

(4) If F_n is a fan graph with n vertices, then $\gamma^k(F_n) = n$.

(5) If $G+H$ is the join of two graphs G and H, then $\gamma^k(G+H) = |V(G)| + |V(H)|$.

Proof. Immediate from Theorem 2.1(2): S_n, K_n, W_n, F_n, and $G+H$ all have diameter at most $2 \leq k-1$. □

Corollary 2.3. *Let G be a graph and $k \geq 2$. If $\gamma^k(G) = |V(G)|$, then $\gamma^l(G) = |V(G)|$ for all $l \geq k$.*

Proof. By Theorem 2.1(2), $\text{diam}(H) \leq k-1 \leq l-1$ for each component H of G, so $\gamma^l(G) = |V(G)|$ by Theorem 2.1(2) again. □

Theorem 2.4. *Let G be a graph and k a positive integer. The set of all k-distance dominating sets of G and the set of all dominating sets of $D_k(G)$ coincide.*

Proof. Since $V(D_k(G)) = V(G)$, a subset Q of $V(G)$ can equally well be regarded as a subset of $V(D_k(G))$. By the definition of $D_k(G)$, a vertex w is a neighbor of u in $D_k(G)$ if and only if $d_G(u,w) = k$. Consequently, for $Q \subseteq V(G)$ and $u \notin Q$, the statement "u has a $D_k(G)$-neighbor in Q" and the statement "u is at distance k from some vertex of Q" are one and the same. Reading this for every $u \in V(G) \setminus Q$ shows that Q dominates $D_k(G)$ if and only if Q is a k-distance dominating set of G. As Q was arbitrary, the two families of sets coincide. □

Corollary 2.5. *Let G be a graph and k a positive integer. Then $\gamma(D_k(G)) = \gamma^k(G)$.*

Proof. By Theorem 2.4 the two families of sets are literally the same collection of subsets of $V(G)$, so they certainly have the same minimum cardinality. That common minimum is $\gamma(D_k(G))$ when the sets are regarded as dominating sets of $D_k(G)$, and $\gamma^k(G)$ when they are regarded as k-distance dominating sets of G. □

Figure 1 illustrates Theorems 2.4–2.5 on a concrete example. The left panel depicts a spider tree G with centre 0 and three legs of length 2, with $k = 2$. Since $d_G(0,2) = d_G(0,4) = d_G(0,6) = 2$ and $d_G(1,3) = d_G(1,5) = 2$, the set $Q = \{0,1\}$ is a minimum 2-distance dominating set of G, confirming $\gamma^2(G) = 2$. The right panel displays $D_2(G)$, whose edges correspond precisely to the distance-2 pairs in G: namely $\{0,2\}$, $\{0,4\}$, $\{0,6\}$, $\{1,3\}$, $\{1,5\}$, and $\{3,5\}$ (since $d_G(3,5) = 2$ as well). By Theorems 2.4–2.5, the same set $Q = \{0,1\}$ is a minimum dominating set of $D_2(G)$, confirming $\gamma(D_2(G)) = 2$.

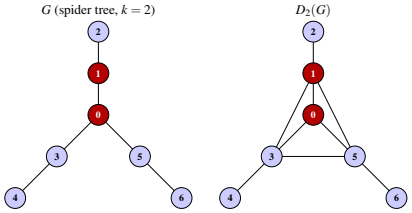

Figure 1: A tree G with $k = 2$: $Q = \{0,1\}$ minimally 2-distance dominates G, so $\gamma^2(G) = 2$; by Theorems 2.4–2.5, Q also minimally dominates $D_2(G)$ (right), so $\gamma(D_2(G)) = 2$.

Source: created by the authors.

Theorem 2.6. *Let G be a graph and k a positive integer. Then $\text{diam}(G) \leq k-1$ if and only if $D_k(G)$ is isolated.*

Proof. By definition, an edge uv of $D_k(G)$ corresponds exactly to a pair $u, v \in V(G)$ with $d_G(u,v) = k$; thus $D_k(G)$ is isolated if and only if no two vertices of G are at distance k. If $\text{diam}(G) \leq k-1$, then every pair of vertices lying in a common component of G is at distance at most $k-1 < k$, and vertices in different components are not at distance k either (their distance being undefined); so no pair attains distance k, and $D_k(G)$ is isolated.

For the reverse implication we argue contrapositively: suppose $\text{diam}(G) \geq k$, and pick x, y in a common component with $d_G(x,y) = m \geq k$. Write a shortest x-y path as $x = w_0, w_1, \ldots, w_m = y$. Any initial segment $w_0, w_1, \ldots, w_i$ of a shortest path is itself a shortest w_0-w_i path — otherwise splicing in a shorter w_0-w_i path would shorten the whole x-y walk below m, contradicting minimality of m. Taking $i = k$ gives $d_G(w_0, w_k) = k$, so w_0 and w_k are two vertices of G at distance k, meaning $D_k(G)$ has an edge and is not isolated. Thus whenever $\text{diam}(G) \geq k$, the graph $D_k(G)$ fails to be isolated, which proves the contrapositive of the remaining implication. □

Corollary 2.7. *Let G be a graph with $\operatorname{diam}(G) \leq k-1$ for a positive integer k. Then $D_k(G) = G$ if and only if G is isolated.*

Proof. Theorem 2.6 already guarantees $E(D_k(G)) = \emptyset$. Two graphs on the same vertex set are equal exactly when their edge sets agree, and here $V(G) = V(D_k(G))$ automatically; so $D_k(G) = G$ reduces to the single condition $E(G) = E(D_k(G)) = \emptyset$, i.e., to G being isolated. □

Corollary 2.8. *Let $k \geq 3$. The following statements hold.*

(1) If S_n is a star graph with n vertices, then $D_k(S_n)$ is isolated.

(2) If K_n is a complete graph with n vertices, then $D_k(K_n)$ is isolated.

(3) If W_n is a wheel graph with n vertices, then $D_k(W_n)$ is isolated.

(4) If F_n is a fan graph with n vertices, then $D_k(F_n)$ is isolated.

(5) If $G+H$ is the join of two graphs G and H, then $D_k(G+H)$ is isolated.

Proof. Each of the listed graphs has diameter at most 2: this is clear for S_n, K_n, W_n, and F_n directly, and for a join $G+H$ because every vertex of G is adjacent to every vertex of H, so any two vertices are joined by a path of length at most 2. Since $k \geq 3$ gives $2 \leq k-1$, Theorem 2.6 applies in each case to yield an isolated $D_k(G)$. □

Remark 2.9. Corollary 2.8 is sharp in the sense that it may fail for $k \leq 2$: e.g. $D_2(S_n)$ is not isolated whenever $n \geq 3$, since the $n-1$ leaves of S_n are pairwise at distance 2.

The next theorem gives a complete description of the k-distance graph of a cycle graph for every k, in terms of $d = \gcd(n,k)$.

Theorem 2.10. *Let C_n be a cycle graph with $n \geq 3$ vertices and let k be a positive integer. The following statements hold.*

(1) If $n < 2k$, then $D_k(C_n)$ is isolated.

(2) If $n = 2k$, then $D_k(C_n)$ consists of k disjoint edges (a perfect matching).

(3) If $n > 2k$ and $d = \gcd(n,k)$, then $D_k(C_n)$ has d components, each of which is a cycle graph with $\frac{n}{d}$ vertices.

Proof. Label the vertices of C_n as $v_0, v_1, \ldots, v_{n-1}$ in cyclic order; identifying subscripts modulo n, the graph distance satisfies

$$d_{C_n}(v_i, v_j) = \min\big(|i-j|, n-|i-j|\big) \qquad (3)$$

for representatives $i, j \in \{0, \ldots, n-1\}$.

(1) Here $\operatorname{diam}(C_n) = \lfloor n/2 \rfloor$, and $n < 2k$ forces $\lfloor n/2 \rfloor \leq k-1$. Isolation of $D_k(C_n)$ is then immediate from Theorem 2.6.

(2) When $n = 2k$, adding k to an index and subtracting k from it produce the same result modulo n, since $n-k = 2k-k = k$. So, by (3), from any v_i there is only one vertex at distance k, namely v_{i+k}, and this makes $D_k(C_n)$ a 1-regular graph, i.e. a union of disjoint edges. Because the pairing $v_i \leftrightarrow v_{i+k}$ has no fixed points (as $k \neq 0$) and covers all $n = 2k$ vertices, it produces exactly k disjoint edges — a perfect matching.

(3) Suppose now $n > 2k$ and put $d = \gcd(n,k)$. In this range $k < n-k$, so (3) gives $d_{C_n}(v_i, v_{i+k}) = k$ for every i, and these are precisely the edges of $D_k(C_n)$; equivalently, $D_k(C_n)$ is the circulant graph on $\mathbb{Z}_n$ obtained by joining each residue to the ones k above and below it (background on such circulant graphs can be found in [13]). The element $\overline{k} \in \mathbb{Z}_n$ generates a cyclic subgroup H of order $n/\gcd(n,k) = n/d$, and its d cosets partition $\mathbb{Z}_n$ into d blocks of size n/d. Within one such coset, listing its elements in the order obtained by repeatedly adding $\overline{k}$ traces out a cycle of length n/d — and this listing is exactly the sequence of $D_k(C_n)$-edges inside that coset, since consecutive listed elements differ by k. As no edge of $D_k(C_n)$ joins vertices from different cosets, the graph $D_k(C_n)$ splits along the d cosets into d vertex-disjoint cycles, each on n/d vertices. □

Corollary 2.11. *Let G be a graph and k a positive integer. If every component C of G is either isolated or a cycle graph with n vertices for some $n > 2k$ and $\gcd(n,k) = 1$, then $D_k(G) \cong G$.*

Proof. No two vertices lying in different components of G are at distance k (their distance is not even finite), so $D_k(G)$ contributes no edges between components; that is, $D_k(G)$ decomposes as the disjoint union of $D_k(C)$ taken over the components C of G, exactly mirroring the decomposition of G itself. It therefore suffices to check $D_k(C) \cong C$ component by component. When C is isolated, $\operatorname{diam}(C) = 0 \leq k-1$ and Corollary 2.7 gives $D_k(C) = C$ outright. When C is a cycle on $n > 2k$ vertices with $\gcd(n,k) = 1$, part (3) of Theorem 2.10 (with $d = 1$) shows $D_k(C)$ is again a single n-cycle, hence isomorphic to C. Assembling

the isomorphisms $D_k(C) \cong C$ over all components C yields $D_k(G) \cong G$. □

Figure 2 illustrates all four cases of Theorem 2.10 for $k = 3$. Each panel displays C_n and $D_3(C_n)$, with vertices of $D_3(C_n)$ that belong to the same coset of $\langle 3 \rangle$ in $\mathbb{Z}_n$ rendered with identical shading; the cosets are $\{0,3,6,\ldots\}$, $\{1,4,7,\ldots\}$, and $\{2,5,8,\ldots\}$.

Case $n = 5$ ($n < 2k = 6$): every pair of vertices in C_5 is at distance at most $2 < 3$, so $D_3(C_5)$ is isolated (Theorem 2.10(1)).

Case $n = 6$ ($n = 2k$): each v_i has a unique partner v_{i+3} at distance 3, giving the perfect matching $\{v_0v_3, v_1v_4, v_2v_5\}$ with $k = 3$ edges (Theorem 2.10(2)).

Case $n = 9$ ($n > 2k$, $d = \gcd(9,3) = 3$): the three cosets of $\langle 3 \rangle$ in $\mathbb{Z}_9$ are $\{0,3,6\}$, $\{1,4,7\}$, and $\{2,5,8\}$. Within each coset, successive elements differ by 3, tracing a triangle C_3. Hence $D_3(C_9)$ consists of three disjoint triangles (Theorem 2.10(3), $d = 3$, component length $9/3 = 3$).

Case $n = 8$ ($n > 2k$, $d = \gcd(8,3) = 1$): the unique coset is all of $\mathbb{Z}_8$, so $D_3(C_8)$ is connected. The sequence $0 \to 3 \to 6 \to 1 \to 4 \to 7 \to 2 \to 5 \to 0$ (each step adding 3 modulo 8) is a Hamiltonian cycle, confirming $D_3(C_8) \cong C_8$ (Theorem 2.10(3), $d = 1$, component length 8).

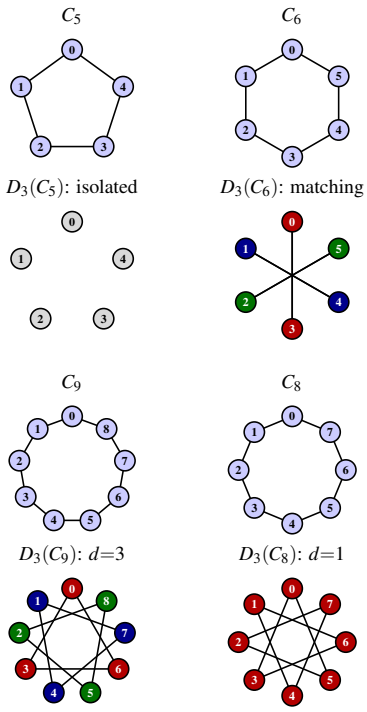

Figure 2: $D_3(C_n)$ (Theorem 2.10): isolated, a perfect matching, d=3 triangles, and a single 8-cycle, for $n = 5,6,9,8$.

Source: created by the authors.

3 Conclusion

We set out to understand the k-distance graph $D_k(G)$ for arbitrary k, a question that had only been settled for the special case $k = 3$. The dominating sets of $D_k(G)$ turn out to coincide exactly with the k-distance dominating sets of G, so $\gamma(D_k(G)) = \gamma^k(G)$ (Theorem 2.4, Corollary 2.5), and along the way we pinned down bounds and extremal cases for $\gamma^k(G)$ itself (Theorem 2.1). We also worked out exactly when $D_k(G)$ has no edges at all, in terms of $\mathrm{diam}(G)$ (Theorem 2.6), which settles the question for several familiar graph families as a corollary (Corollaries 2.7–2.8).

The main result of the paper, though, is the full description of $D_k(C_n)$ for cycle graphs: writing $d = \gcd(n,k)$, the graph $D_k(C_n)$ is isolated when $n < 2k$, a perfect matching when $n = 2k$, and otherwise breaks apart into d disjoint cycles of length n/d (Theorem 2.10). This pins down exactly when $D_k(C_n)$ is isomorphic to C_n itself (Corollary 2.11). Three directions stand out for future work. First, **circulant families**: for which connection sets S does the circulant graph $\mathrm{Circ}(n,S)$ admit a structural description of D_k analogous to Theorem 2.10? The simplest open case is $S = \{1,2\}$, i.e. the cycle with additional chords. Second, **connectivity**: determine $\kappa(D_k(C_n))$ as a function of n and k; by Theorem 2.10(3) the graph $D_k(C_n)$ consists of $d = \gcd(n,k)$ disjoint cycles, each 2-connected, so $\kappa(D_k(C_n)) = 2$ whenever $n > 2k$. It would be interesting to determine $\kappa(D_k(G))$ for more general graph families. Third, **chromatic number**: since each component of $D_k(C_n)$ is a cycle of length n/d, we have $\chi(D_k(C_n)) = 3$ if n/d is odd and $\chi(D_k(C_n)) = 2$ if n/d is even; characterising $\chi(D_k(G))$ for trees, bipartite graphs, and joins remains open.

Declaration of Generative AI and AI-assisted Technologies in the Writing Process

During the preparation of this work, the author(s) utilized artificial intelligence (AI) technologies solely for the purpose of linguistic refinement and grammatical correction. Following the application of these tools, the author(s) comprehensively reviewed, verified, and edited the resulting text. The author(s) maintain full accountability and responsibility for the accuracy, integrity, and final content of the publication.

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