Similarities and Differences between the Game of Cops and Robbers and Pursuit Differential Games

BRUNO ANTONIO PANSERA
Department of Law, Economics and Human Sciences & Decisions_Lab
University Mediterranea of Reggio Calabria
Via dell'Università, 25 – 89060 Reggio Calabria
ITALY

Abstract: - This paper examines the conceptual and methodological parallels and divergences between two paradigms of pursuit-evasion: the discrete "cops and robbers" game on graphs and continuous-time pursuit differential games as developed by Isaacs [4]. We first review the foundations of differential games in Isaacs' treatise, with emphasis on the Homicidal Chauffeur model and on geometric and integral constraints on the controls. We then trace the origins of the cops and robbers game as formulated by Quilliot [30] and Nowakowski-Winkler [28], highlighting graph dismantlability and its equivalence to cop-win graphs. Extensions to infinite graphs and to visibility graphs of polygons are surveyed, focusing on results by Lubiw et al. [40] and Bonato-Nowakowski [41]. We also discuss pursuit differential games described by infinite systems of first-order equations in Hilbert spaces (Ibragimov et al. [39, 50]) and games with integral constraints (Mai Umar et al. [48]), as well as Luckraz's work [43] relating cops and robbers to extensive and stochastic games. We then introduce a common reachability framework for both paradigms: we characterize the cop's winning region on a finite graph as the least fixed point of a controllable-predecessor operator, show that folding a dominated vertex delays capture by at most one round, and relate these sets to capture basins and discriminating kernels of differential games. Finally, we compare the two paradigms with respect to information, timing, speed constraints and computational complexity, and identify open problems.

Key-Words: differential games, pursuit-evasion, cops and robbers, graph dismantlability, Hamilton-Jacobi-Isaacs equation, cop-win graphs, integral constraints, Homicidal Chauffeur.

Received: April 28, 2026. Revised: July 6, 2026. Accepted: August 8, 2026. Published: November 4, 2026.

1 Introduction

The methodology of differential games is a combination of optimal control theory and game theory. It is the natural framework for economic analysis with strategic interaction and dynamical optimization [1].

A differential game is a mathematical tool used to analyze strategic interactions between two or more players, whose actions take place in a dynamic system that changes continuously over time. Therefore, differential game theory can be seen as an extension of optimal control theory when multiple decision makers are involved [1].

In this context, the strategies implemented by each player aim to optimize the value function, knowing the strategies used by the other competitors. Each player in a differential game, in fact, has a set of allowed strategies to choose from and an objective function that evaluates the effectiveness of their decisions, based on the current state of the system and the actions of others.

Differential games, understood as games that manifest the evolution of player states over continuous time, form a class of games that contrasts with static or repetitive games. Differential games are typically described using systems of differential equations to better describe the variation of states over time. This model is easily adaptable to the study of complex phenomena in many fields of research, such as duopoly markets or biological processes, in which multiple organisms continuously interact, changing their states over time and interacting with each other in cooperative and noncooperative ways.

In light of the above, we can consider differential games as the mathematical model for studying and analyzing optimization problems defined through the dynamic interaction of so-called rational agents. The importance of this field of research has expanded significantly in recent decades, thanks to the contributions of numerous researchers who have developed this class of games in diverse and interesting contexts.

The theoretical origins of the field can be traced to the 1950s and 1960s, when mathematicians such as Lev Pontryagin [3] established foundational principles of optimal control through the maximum principle, and Rufus Isaacs formally introduced differential game theory in his seminal work [4]. These contributions established the theoretical foundation for analyzing optimal strategies in complex, dynamic multi-agent scenarios, where players' decisions are interdependent and evolve temporally, thereby influencing the system's state over time.

The state of the game at time t is a vector x(t) $\in\mathbb{R}^n$ determined by the system of differential equations:

$$\dot{x}(t) = f\big(t, x(t), u(t), v(t)\big), \qquad x(t_0) = x_0 \quad (1)$$

where $u(t)$ and $v(t)$ are chosen by Player I and Player II at each time t, respectively [2].

Players' choices are not free, but are constrained by conditions imposed on $u(t) \in U$ and $v(t) \in V$ which vary within a set of choices, often represented by Euclidean spaces. The evolution of the game, and therefore the choice of $u(t)$ and $v(t)$, are contingent on the choice of a strategy by Player I and Player II.

The evolution of the game proceeds from the initial point (t_0, x_0) to the final point (t_f, $x(t_f)$), where $x(\cdot)$ is the solution of (1) and $t_f > t_0$ is the terminal time. The payoff is

$$P(t_0, x_0; u, v) = \int_{t_0}^{t_f} f^0\left(s, x(s), u(s), v(s)\right)ds + g\left(t_f, x\left(t_f\right)\right) \quad (2)$$

where g and f^0 are given functions. In the general formulation of [2], Player I chooses the control u to maximize P and Player II chooses v to minimize it; in the pursuit-evasion games considered below, the pursuer minimizes and the evader maximizes the payoff (for instance, the time of capture). Common objectives include:

1. Minimizing the time-to-capture (for the pursuer);
2. Maximizing the time-to-capture (for the evader);
3. Optimizing trade-offs between resource expenditure and strategic gain (in economic contexts).

An important subclass is that of pursuit-evasion games, which model strategic and dynamic interactions between players with diametrically opposed objectives.

This class of games allows us to define multiple strategic interactions between players whose states evolve over time to define strategies that lead one or a group of pursuers to capture one or a group of evaders; the latter define strategies to permanently avoid capture. Pursuit-evasion games combine theory and practice, defining the tools of dynamic decision-making through strategy selection. The nature of the game leads to adversarial reasoning defined on both continuous and discrete spaces. This is the case for pursuit-evasion games defined on $\mathbb{R}^n$ or those defined on graphs, known as "cop-robber games."

These games are inspired by natural phenomena involving a predator and its prey. Their models are now applied wherever a similar pairing arises: in cybersecurity, where a virus must be tracked and contained; in missile defense; and in robotics, for example with self-driving cars. Pursuit-evasion games are therefore a multidisciplinary topic, in which mathematics interacts with physics, computer science and artificial intelligence.

The theoretical foundations of pursuit-evasion games can be traced to seminal problems in mathematical literature, such as Richard Rado's classical "The Lion and the Man" problem (posed in 1925) [5], which examines whether a lion can capture a human moving within a circular arena under the constraint of equal speeds. This problem underscores the critical influence of strategy and domain constraints—demonstrating, for example, that under continuous time and specific configurations, perpetual evasion may be possible [5, 44].

Two principal paradigms have emerged. In the discrete setting, the cops and robbers game is played by two adversarial players on the vertices of a graph [28, 30]. The cop attempts to occupy the same vertex as the robber, who moves in response. In the continuous setting, pursuit differential games are played in Euclidean or Riemannian spaces, with dynamics governed by differential equations [4]. Here, the pursuer and evader control their velocities or accelerations subject to integral or geometric constraints. Although both paradigms study the same goal—capture versus evasion—their mathematical formulations differ fundamentally in domain (graphs versus continuous manifolds), time (turn-based versus continuous), and control structure. Rufus Isaacs' classic treatise *Differential Games* [4] laid the groundwork for continuous pursuit-evasion, introducing the Homicidal Chauffeur, minimax principles, and boundary conditions in the state space. Conversely, Quilliot [30] and Nowakowski-Winkler [28] first formalized the cops and robbers game on graphs, discovering that finite cop-win graphs are exactly those that are dismantlable.

In recent decades, researchers have explored many extensions: polygon visibility graphs [40], infinite

cop-win graphs [41], pursuit differential games in Hilbert space [39], and links between cops and robbers and extensive or stochastic games [42, 43]. Yet a systematic comparison of discrete and continuous paradigms—highlighting common mathematical structures and conceptual differences—remains lacking.

This paper surveys key developments, emphasizing how Isaacs' continuous methods and dismantlability in discrete games can inform each other, and outlines open problems at their interface. The remainder of this paper is organized as follows. Section 2 reviews foundational differential game concepts in Isaacs' work. Section 3 examines continuous pursuit-evasion models and infinite-dimensional differential games. Section 4 presents the origins and definitions of the cops and robbers game. Section 5 focuses on cop-win characterizations via dismantlability, visibility graphs and cop-win results in planar regions, and computational complexity. Section 6 contrasts discrete and continuous assumptions on time, information and movement. Section 7 analyzes the elements common to continuous and discrete games and introduces a common reachability framework (Section 7.3). We conclude in Section 8 by identifying open problems and future research directions.

Originality. Sections 2-6 survey known results; all theorems quoted there are due to the cited authors. The graph model of Section 4 and Figures 2-3 follow the author's earlier joint paper [27], where cop-win notions depending on the initial positions were introduced, and the differential-game set-up of Section 3 follows [8, 9, 20, 21]. The new contributions of this paper are: (i) a common state-space formulation of graph pursuit and differential pursuit as a backward-reachability problem (Section 7.3); (ii) Propositions 1 and 2 and Corollary 3, which characterize the cop's winning region and the robber's invariant set as fixed points of one operator and show that folding a dominated vertex delays capture by at most one round, so that dismantlability is exactly the case in which the cop's winning region is the whole state space; (iii) a comparison with capture basins and discriminating kernels of differential games, in which statements without proof are labeled as analogies.

2 Foundations of Differential Games in Isaacs' Works

The pioneer of differential game theory was Rufus Philip Isaacs, who introduced this topic between 1951 and 1955 in his seminal works [4, 6], which have become milestones for those approaching this study (see also [24, 25]). Isaacs's differential game theory gained momentum and recognition when he began his research at the RAND Corporation in Santa Monica. This institute was committed to multidisciplinary research, and the cross-fertilization of knowledge allowed Isaacs to integrate the idea of differential games and their formalism with other sciences to better adapt his model to society's needs.

We had to wait until 1954 to read Isaacs's formalization of specific zero-sum games [6], where players for the first time assumed the roles of evader and pursuer. This dynamically formulated game became a unique starting point for the later development of what we now call pursuit-evasion games. The importance of this introduction lies in the fact that, in a very embryonic form, an attempt was made to define a mathematical model that studied continuous-time evolutionary games. The model underwent modifications over the years, until 1965, when its complete formulation [4] was presented.

Isaacs's differential games received significant impetus from disciplines such as decision theory and optimal control, which were developing at the same time. The continued intersection of these themes was fostered by the work of Isaacs's colleagues at the RAND Corporation, among them Leonard D. Berkovitz and Wendell H. Fleming [25]. In Section 2.1, we study one of the problems Isaacs addressed and solved through the application of differential games, his famous "Homicidal Chauffeur" problem.

This problem has become emblematic of the intersection and close correlations between game theory and optimal control.

Differential games combine elements of the calculus of variations, dynamic programming, and game-theoretic minimax principles to model conflict situations with control-constrained dynamics.

2.1 The Homicidal Chauffeur Model

A classic archetype of differential game theory is the famous Homicidal Chauffeur Problem. Originally formulated by Rufus Isaacs in 1951 during his tenure at the RAND Corporation and later formalized in his monograph *Differential Games* (1965) [4], this problem has become a benchmark for studying pursuit-evasion scenarios with real-world analogues, such as missile interception systems, where similar trade-offs between speed and maneuverability arise.

The problem involves two players with asymmetric abilities. On one side, there is a pursuer (the "driver") who moves at a high speed w but cannot turn more sharply than a minimum turning radius R, resulting in limited maneuverability; on the other, there is an evader (the "pedestrian") who moves at a lower speed

v < w but can change direction instantaneously. Both players can operate indefinitely. The question is whether the pursuer can capture the evader in finite time, or whether the evader can avoid capture indefinitely.

The dynamics in the plane are

$$\dot{x}_P = w\sin\theta, \dot{y}_P = w\cos\theta,$$
$$\dot{\theta} = \frac{w}{R}\varphi, \ |\varphi| \leq 1,$$
$$\dot{x}_E = v_1, \dot{y}_E = v_2, {v_1}^2 + {v_2}^2 \leq v^2$$

where (x_P, y_P) and (x_E, y_E) are the positions of the pursuer and of the evader, θ is the pursuer's heading and φ is his steering control. Isaacs introduced universal, dispersal and equivocal surfaces to partition the state space and applied backward integration (the retrograde path equations) to compute the isochrones of the value function; within each region, feedback strategies are given by geometric constructions [4].

The pursuer wishes to minimize the time of capture T. Capture occurs when the state enters the capture set, a disc of radius $\ell > 0$ (the capture radius) around the pursuer:

$$\left\|\left(x_P(t), y_P(t)\right) - \left(x_E(t), y_E(t)\right)\right\| \leq \ell.$$

The evader wishes to maximize the time of capture, possibly making it infinite. The value V(x) of the game is the time of capture under optimal play from the state $x = (x_P, y_P, \theta, x_E, y_E)$. Where V is differentiable, it satisfies Isaacs' main equation, the stationary Hamilton-Jacobi-Isaacs equation:

$$\min_{u_P}\max_{u_E}\nabla V(x)\cdot f(x, u_P, u_E) + 1 = 0,$$

with V = 0 on the capture set; here $u_P = \varphi$ and $u_E = (v_1, v_2)$. The optimal feedback strategies $u_P(x)$ and $u_E(x)$ are obtained as the minimizer and the maximizer in this equation, which requires the Isaacs condition (the min and the max in the Hamiltonian can be interchanged). Since V is in general not differentiable across singular surfaces, the equation is understood in the viscosity sense [7].

2.2 Integral and Geometric Control Constraints

The literature on differential games distinguishes two classes of constraints on control functions [4, 18, 19, 22]:

- **Geometric constraints** (e.g., bounded speed $\| u \| \leq \rho$), modeling instantaneous limitations.
- **Integral constraints** (e.g., $\int_0^T \| u(t) \|^2 \ dt \leq \rho^2$), modeling limited energy or resource budgets [39, 49].

Both appear in continuous pursuit-evasion. The Homicidal Chauffeur uses geometric constraints, whereas variants of simple pursuit on Hilbert spaces employ integral constraints [39, 50]. In real-world systems (e.g., robotics), mixed constraints often apply.

3 Continuous Pursuit-Evasion Models

While Isaacs focused on finite-dimensional Euclidean models, subsequent work extended differential game frameworks to infinite-dimensional systems and alternative metrics.

3.1 Pursuit Differential Games via Infinite Systems

In pursuit-evasion games the players are divided into pursuers and evaders; each group may contain one or more players, and the literature has moved from one-on-one games to games with several pursuers and one or several evaders. The pursuers try to bring their positions into coincidence with those of the evaders, and the evaders try to avoid this. Strategies depend on the constraints imposed on the players' controls, which may be geometric (bounds on speed) or integral (bounds on energy or other resources).

The form of the differential equations depends on the problem: in games with simple motion the players control their velocities directly ($\dot{x} = u$), while in linear games the dynamics have the form $\dot{x} = Ax + u$. The class of pursuit-evasion games is thus characterized by the variety of problems it can address, from the number of players to the type of dynamics and constraints.

Classically, a simple pursuit-evasion game involving M pursuers and N evaders is modeled using a system of differential equations, which formalize the dynamic interactions and constraints governing the agents' motion and objectives as the following [8]:

$$\begin{aligned} \dot{x}_i &= u_i, \quad x_i(0) = x_{i0}, \quad i = 1, \dots, M, \\ \dot{y}_j &= v_j, \quad y_j(0) = y_{j0}, \quad j = 1, \dots, N, \end{aligned} \tag{3}$$

where $x_i, y_j, u_i, v_j \in \mathbb{R}^n, n \geq 2,$ and $x_{i0} \neq y_{j0}, i = 1, \dots, M,$ and $j = 1, \dots, N.$

Integral constraints on the controls of the players have the form:

$$\int_0^\infty \| u_i(s) \|^2 \ ds \leq \rho_i^2, \ i = 1, \dots, M, \tag{4}$$

$$\int_0^\infty \| v_j(s) \|^2 \ ds \leq \sigma_j^2, \ j = 1, \dots, N, \tag{5}$$

where ρ_i, σ_j are given positive numbers for all i = 1,...,M, and j = 1,...,N. Control systems with integral constraints on the control functions appear in problems where the available control resources are depleted through consumption, such as in the cases

of energy, finance, or food (see, for instance [10, 11, 22]).
Mai Umar et al. [48] studied linear pursuit and evasion games in ~~Rn~~$\mathbb{R}^n$ with generalized integral constraints, for one pursuer and one evader and for many pursuers and one evader. They obtained sufficient conditions for the completion of pursuit and for evasion, constructed optimal strategies of the players, and estimated the distance that the evader can maintain from the pursuers. Bounded controls in distributed-parameter systems are studied by Chernous'ko [23].
In the case of geometric constraints, the controls satisfy $\|u_i\| \leq \rho_i$ and $\|v_j\| \leq \sigma_j$, i = 1,...,M, j = 1,...,N. In addition, the players' positions may be restricted to a bounded domain Ω, which is a state constraint rather than a constraint on the controls [15, 17].

Definition 1. [8] Borel measurable functions $u_i(t)$, $t \geq 0$, and $v_j(t)$, $t \geq 0$, that satisfy the constraints (4) and (5), respectively, are called controls of the pursuer x_i, $i \in \{1,...,M\}$, and of the evader y_j, $j \in \{1,...,N\}$.
A strategy must respect these constraints. Time plays a central role: capture must occur in finite time, whereas evasion requires avoiding capture for all $t \geq 0$.

Definition 2. [8] If $y_j(t_j) = x_i(t_j)$ at some $j \in \{1,...,N\}$, $i \in \{1,...,M\}$, and $t_j > 0$, then we say that the evader y_j is captured by the pursuer x_i at the time t_j. Evasion is possible if $x_i(t) \neq y_j(t)$ for all $t \geq 0$ and all $i \in \{1,...,M\}$, $j \in \{1,...,N\}$.

Ibragimov et al. [39, 50] studied pursuit differential games in the Hilbert space ~~l2~~ℓ^2, which arise, for example, from distributed-parameter systems and from the reduction of partial differential equations. The dynamics take the form

$$\dot{z}_k(t) + \lambda_k z_k(t) = u_k(t) - v_k(t),\ z_k(0) = z_{k0},\ k = 1,2,\dots,$$

where the controls $u = (u_1, u_2, ...)$ and $v = (v_1, v_2, ...)$ take values in l^2 and are subject to integral or component-wise constraints. Closed-form formulas for the guaranteed pursuit time and explicit pursuer strategies were derived using resolving functions [39, 50, 51].
Indeed, one can discern clear parallels between the development of evasion strategies and the principal findings in their work [8, 9, 12-15, 17, 20, 21].
Specifically, these parallels manifest in the following elements:

1. the formal delineation of temporal intervals;
2. the design of an evader's control policy enabling targeted maneuvers against each individual pursuer;
3. the derivation of lower bounds on the separation between the evader and the pursuers, thereby demonstrating the feasibility of evasion under prescribed conditions.

In [9], the proof proceeds in several stages. First, the problem is reduced to a simpler equivalent formulation. Next, explicit strategies are constructed for both the evader and the pursuers, and their admissibility is rigorously verified. Finally, quantitative estimates of the inter-agent distances are established, confirming that evasion is attainable. In this field of research, many researchers have attempted to introduce models to define evasion and/or capture strategies. The papers that followed had different themes in terms of the number of evaders and, subsequently, geometric or integral constraints, and the space on which to study the game, such as Hilbert space.
Satimov et al. [18] (see also [16]) studied for the first time a linear differential game of many pursuers and one evader (N = 1) in which the control functions of the players are subject to integral constraints, and obtained a result in terms of the players' energies. For the game described by (3)-(5), they showed that if $\rho_1^2 + \dots + \rho_M^2 > \sigma^2$, then pursuit can be completed.

In [19], it was shown that, for the same games, if $\rho_1^2 + \cdots + \rho_M^2 \leq \sigma^2$, then evasion is possible from some initial positions of players. This introduction gave rise to many papers inspired by it that generalized the case to N > 1, such as the paper by Satimov and Ibragimov [17], Ibragimov et al. [8], Salimi and Ferrara [52].

4 Origins and Definitions of the Cops and Robbers Game

In the context of pursuit-evasion games, it is necessary to consider their application within a geometric structure: the graph. In fact, within this structure, the pursuit-evasion game takes the form of a "cop-and-robber game" (Nowakowski and Winkler [28], Quilliot [30]). This game represents the discrete version of differential pursuit-evasion games, such as the lion-and-man problem (Rado, 1925) and the Homicidal Chauffeur game (Isaacs, 1960s), mentioned previously. This class of games allows us to study problems defined in discrete time. We move from a continuous space to a discrete structure, in which the possible positions of the players are the vertices of the graph.

In the standard model, pursuers and evaders sequentially choose their positions on the graph under conditions of perfect information, being able to move only to adjacent vertices at each move [27]. Thus, a player's move consists either of remaining stationary at a node or of moving along an arc of G toward an adjacent vertex. The cop wins if he captures the robber after a finite number of moves.

Since the game has perfect information and finitely many positions, one of the players has a winning strategy (an argument going back to Zermelo [26]; see also Proposition 1 below). If the cop has a winning strategy, G is called cop-win; otherwise G is robber-win [29].
The most important result achieved by Nowakowski and Winkler [28] is the definition of the characteristics a graph must possess to be cop-win. Their focus centers on the property of "dismantlability."
Subsequently, Clarke and MacGillivray [31] extended this result to the case of multiple cops using the categorical product of graphs. It is interesting how these games have attracted the attention of many researchers due to their numerous applications, for example, a computer network subjected to a virus attack. Dismantlable graphs also appear in statistical physics: Brightwell and Winkler [33] related dismantlability to Gibbs measures of graph homomorphisms.
Crucial in this context are some classic elements of graph theory.
A graph G is reflexive if it has a loop at every vertex. Throughout this paper all graphs are reflexive, so that a player may always stay at his current vertex.

The concept of retracted graphs warrants some fundamental consideration. A retract of a graph is a subgraph that preserves, via an appropriate mapping, the structure of the original graph.

Definition 3. Let H be a subgraph of a reflexive graph G. We say that H is a retract of G if there is a homomorphism r: G → H, that is, a map from V(G) to V(H) sending adjacent vertices to adjacent or equal vertices, such that r(v) = v for every v ∈ V(H). The map r is called a retraction.

The minimum number of cops for which the cops have a winning strategy is the cop number c(G) [38]; it is well defined, since |V(G)| cops occupying all vertices win at once. For example, a single vertex is always a retract of a reflexive graph, and so is the 4-cycle in the graph of Figure 1.

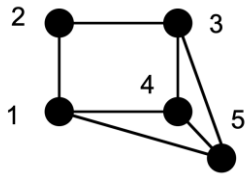


Figure 1: A graph G on the vertices 1-5 with edges 12, 14, 15, 23, 34, 35, 45 (loops omitted). The map r with r(5) = 4 and r(x) = x otherwise is a retraction of G onto the 4-cycle H induced by {1, 2, 3, 4}.
The map in Figure 1 is a retraction because the neighbors 1, 3 and 4 of vertex 5 all lie in N[4]. Since H is a 4-cycle, G is robber-win by Corollary 2 below. In particular, G is not dismantlable: its only dominated vertices are 4 and 5, with N[4] = N[5] = {1, 3, 4, 5}, and deleting either of them leaves a 4-cycle, which has no dominated vertex.

Theorem 1. [47] If H is a retract of G, then c(H) ≤ c(G).

Corollary 1. If G is cop-win and H is a retract of G, then H is cop-win.

Theorem 2. [47] If H is a retract of G, then c(G) ≤ max{c(H), c(G − H) + 1}, where G − H denotes the subgraph of G induced by V(G) \ V(H).

Theorem 3. [29] If G is a retract of a finite strong product of cop-win graphs, then G is a cop-win graph.

Corollary 2. [29] If G has a retract which is an n-cycle, n ≥ 4, then G is robber-win; if G is a retract of a finite product of paths, it is cop-win.

The closed neighborhood N[v] of a vertex v in G consists of v and the vertices adjacent to v. A vertex v is dominated (irreducible, in the terminology of [27]) if N[v] ⊆ N[u] for some vertex u ≠ v; in that case u is said to dominate (or cover) v. A finite graph G is dismantlable if its vertices can be ordered v_1, ..., v_n so that, for each i < n, the vertex v_i is dominated in the subgraph G_i induced by {v_i, v_{i+1}, ..., v_n}.

Theorem 4. [28] A finite graph is cop-win if and only if it is dismantlable.

If a graph cannot be dismantled, the Nowakowski-Winkler theorem cannot be applied. To solve this problem, the question is how to determine the minimum number of cops to ensure the capture of the evader. The current literature is dominated by the still unsolved Meyniel conjecture [34, 35], which concerns precisely this number. Chiniforooshan [36]

provided an important upper bound on the number of cops, later improved by Lu and Peng [37].

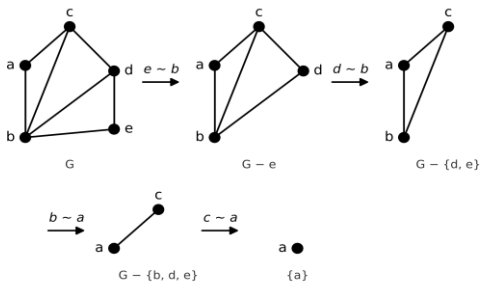


Figure 2: A dismantling of a graph G on the vertices a-e with edges ab, ac, bc, bd, be, cd, de (adapted from [27]). An arrow labeled x ~ y deletes a vertex x that is dominated by y (N[x] ⊆ N[y]) and folds x onto y. The order e, d, b, c leaves the single vertex a, so G is dismantlable and hence cop-win by Theorem 4.

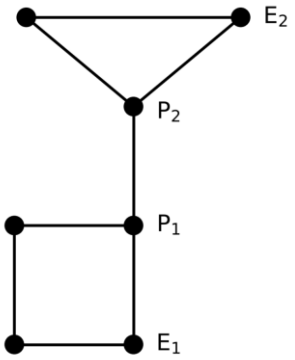


Figure 3: A graph that is not dismantlable, but in which the outcome depends on the initial positions (adapted from [27]). It consists of a 4-cycle through P_1 and E_1, an edge P_1P_2 and a triangle through P_2 and E_2.

Figure 2 shows a dismantling step by step. In G we have N[e] = {b, d, e} ⊆ N[b] = {a, b, c, d, e}, so e can be folded onto b; in G − e, N[d] = {b, c, d} ⊆ N[b]; in the remaining triangle, N[b] = N[a] = {a, b, c}; finally, in the edge ac, N[c] = N[a] = {a, c}. Note that b is adjacent to every vertex of G, so a cop starting at b captures the robber at his first move. Figure 3 illustrates the dependence on the initial positions studied in [27], with the robber moving first. If the cop starts at P_2 and the robber at E_2, every move of the robber stays in $N[P_2]$, so the cop captures him at his first move. If the cop starts at P_1 and the robber at E_1, the robber moves to the vertex of the 4-cycle opposite P_1 and, as on any 4-cycle, can avoid capture forever. The graph is not dismantlable: mapping P_2 and the other two vertices of the triangle onto P_1 is a retraction onto the 4-cycle, so the graph is robber-win by Corollary 2.

Whether a finite graph is cop-win can be decided in polynomial time, by testing whether it is dismantlable [28, 41]. For graphs that are not cop-win, computing the cop number is much harder: Fomin et al. [32] proved that it is NP-hard, and Kinnersley [56] proved that deciding whether c(G) ≤ k, when k is part of the input, is EXPTIME-complete (see Section 5.4).

Following [27], we also consider a version of the game with given initial positions. Let G = (V, E) be a finite connected reflexive graph with |V| ≥ 2. Time is discrete; the cop and the robber start at given vertices $p_0 \neq e_0$, and their positions in round t are denoted p_t and e_t. In each round the robber moves first (odd periods) and the cop moves second (even periods); a move consists in staying at the current vertex or moving to an adjacent one. Both players have complete information about the game at every decision point.

Definition 4. [27] A strategy for the cop is a function s_p: $V \times V \times T^{even} \to V$ with $s_p(p, e, t) \in N[p]$, where T^{even} is the set of even positive integers; a strategy for the robber is a function s_e: $V \times V \times T^{odd} \to V$ with $s_e(p, e, t) \in N[e]$, where T^{odd} is the set of odd positive integers. Let S_p and S_e denote the sets of all strategies of the cop and of the robber.

In the classical game with k cops [38], by contrast, the initial positions are chosen by the players:

1. **Setup**: the k cops choose their initial vertices, then the robber chooses his.
2. **Moves**: Cops move simultaneously (or one per turn in lazy variants) to adjacent vertices or stay. Robber moves similarly.
3. **Capture**: If a cop occupies the robber's vertex, the cops win; otherwise, the robber wins by indefinite evasion.

5 Cop-Win Graphs and Dismantlability

5.1 Dismantlability: Definition and Equivalence

Following [27], dismantlability can be described through one-point retracts. If v is dominated by u in G, the map f: V(G) → V(G − v) with f(v) = u and f(x) = x for x ≠ v is a retraction of G onto G − v (it is a homomorphism because N[v] ⊆ N[u]); we say that v is folded onto u.

Thus a finite graph G is dismantlable if and only if a sequence of such retractions reduces it to a single vertex; equivalently, there is an ordering $v_1, ..., v_n$ of its vertices such that for each i < n there is j > i with

$$N_{G_i}[v_i] \subseteq N_{G_i}[v_j],$$

where G_i is the subgraph induced by $\{v_i, v_{i+1}, ..., v_n\}$ [28]. A dismantling order yields a winning strategy for the cop: on $G - v_1$ the cop plays a winning strategy

against the image f(e) of the robber under the retraction f, and once he reaches this image he needs at most one more move to capture the robber (Proposition 2 in Section 7.3 makes this precise). The capture time of a cop-win graph on n vertices is at most $n - 4$ for $n \geq 7$ [59, 60].

5.2 Extensions: Cop Number $\kappa > 1$ and Infinite Graphs

Aigner and Fromme [38] introduced the cop number c(G). Seymour and Thomas [58] related a different pursuit game, graph searching with "helicopter" cops and a visible robber of unlimited speed, to the treewidth of the graph. For infinite graphs the characterization of Theorem 4 fails in both directions: the infinite ray can be built from a single vertex by repeatedly adding dominated vertices, yet it is robber-win, and there are cop-win graphs that cannot be built in this way [46]; on infinite cop-win graphs see also [41, 45, 53]. Flídr and Ivan [46] constructed an infinite cop-win graph in which every game ends after finitely many moves, but the number of moves needed is unbounded (capture time ω).

5.3 Visibility Graphs and Cop-Win Results

Discrete-geometric pursuit aims to link geometric and graph evasion. That visibility graphs of simple polygons are dismantlable, and hence cop-win, is implicit in the work of Aichholzer et al. [57]; Lubiw et al. [40] proved it explicitly, in the stronger form of 2-dismantlability. In these graphs, vertices represent polygon vertices, and edges connect "visible" pairs. Lubiw et al. [40] also showed that point-visibility graphs inside polygons, which are infinite, are cop-win when each move is along a single segment (distance one in the link metric).

The cop's strategy is to move along the first segment of a shortest path to the robber, thereby progressively chopping off unreachable subregions. In a polygon of n vertices, capture occurs in at most n steps, mirroring the finite dismantlability argument.

5.4 Computational Complexity of Cops and Robbers Games

From a complexity perspective, deciding whether k cops suffice on a given graph is polynomial for fixed k [41], but computing the cop number is NP-hard [32], and the problem is EXPTIME-complete when k is part of the input [56]; for directed graphs, and for given initial positions, EXPTIME-completeness was proved earlier by Goldstein and Reingold [55]. The capture time is computable in polynomial time for one cop [59], while variants of the game with a bounded number of rounds are PSPACE-hard [61]; see also [54]. The computational difficulty of continuous pursuit-evasion games is discussed in Section 7.4.

6 Discrete versus Continuous Pursuit-Evasion Paradigms

The preceding paragraphs show how the definition of a differential game and subsequently the definition of pursuit-evasion games have facilitated the study of mathematical models useful for studying strategic interaction in many fields of research involving mathematics, computer science, and engineering. The need to define similarities and differences between two categories of pursuit games, such as pursuit-evasion and cop-win games, allows us to outline a new perspective that highlights the practical application of these mathematical tools.

1. **Domain**: graph vertices versus Euclidean/Riemannian spaces.
2. **Movement**: discrete turn-taking versus continuous flow.
3. **Capture condition**: coincidence of vertices versus entry into a target set (for example, distance at most a capture radius ℓ).
4. **Information**: both classical models assume perfect information; partial information (for example, line-of-sight visibility) appears in variants of both.
5. **Control constraints**: discrete adjacency moves versus geometric/integral control limits.

These differences show that the two paradigms rest on different mathematical models: continuous dynamics for pursuit differential games and discrete structures for cops and robbers on graphs. Nevertheless, both games can be solved by the same backward-reachability iteration on their state spaces (Section 7.3). In the graph game, a dismantling order certifies that this iteration covers the whole state space; in the differential game, the corresponding object is the capture basin, characterized by the Hamilton-Jacobi-Isaacs equation. The parallel between folding a dominated vertex and reducing the state space of a differential game is an analogy that we do not formalize. These differences are reflected in the objectives of the two theories. In a pursuit differential game, one minimizes or maximizes a payoff such as the time of capture or the energy used, and the solution is typically obtained from the Hamilton-Jacobi-Isaacs equation or from reachable sets. In the cops and robbers game, the main questions are whether the cops can capture the robber

and how many cops are needed, that is, the cop number.

7 Comparative Analysis

The differences highlighted in the previous paragraph show how these structures are apparently different. However, both have a common root in their objective: to capture or escape. In fact, both mathematical models formalize the same issue, namely that of defining and formulating the optimal strategy for achieving capture or escape.

7.1 State-Space Structure

In Cops and Robbers, the state space is a discrete set of cop–robber configurations, where reachability reduces to graph connectivity. In differential games, however, the state evolves in $\mathbb{R}^n$, with continuous trajectories constrained by differential dynamics.

7.2 Movement Constraints

Discrete moves traverse edges in unit time; captures occur at graph vertices. Continuous moves follow vector fields; capture is defined via zero-distance attainment.

7.3 A Common Reachability Framework

In both paradigms the pursuer tries to drive the state of the game into a target set and the evader tries to keep it outside. In this subsection we make this common structure precise for the game on a finite graph and then describe its continuous counterpart. Throughout, G = (V, E) is a finite reflexive graph and N[v] is the closed neighborhood of v. We consider one cop and one robber. The state of the game is the pair (p, e)∈V×V of the positions of the cop and of the robber. As in Section 4, in each round the robber moves first, from e to some e′∈N[e], and then the cop moves from p to some p′∈N[p]; the robber is captured as soon as both players occupy the same vertex. The target set is the diagonal Δ = {(p, e) ∈ V×V: p = e}.

Definition 5. For W⊆V×V, the controllable predecessor of W is

$$\mathrm{Pre}(W) = \{(p,e) \in V \times V\colon\ \forall e' \in N[e]\ \exists p' \in N[p] \text{ with } (p',e') \in W\}.$$

We set $W_0 = \Delta$, $W_{k+1} = \mathrm{Pre}(W_k)$ for $k \geq 0$, and $W^* = \bigcup_{k\geq 0} W_k$. Thus Pre(W) consists of the states from which, whatever the robber does in the next round, the cop can reply so that the new state lies in W.

Proposition 1. Let G be a finite reflexive graph.
(a) $W_0 \subseteq W_1 \subseteq W_2 \subseteq \cdots$.
(b) For every $k \geq 0$, (p, e) ∈ W_k if and only if, starting from the state (p, e) with the robber to move, the cop has a strategy that captures the robber within k rounds, whatever the robber does.
(c) There is $K \leq |V|^2$ such that $W_K = W^*$. Moreover, W* = Pre(W*), and W* is the smallest fixed point of Pre that contains Δ.
(d) Let R = (V×V)\W*. For every (p, e)∈R there is e′∈N[e] such that (p′,e′)∈R for every p′∈N[p]. Hence R is exactly the set of states from which the robber can avoid capture forever.

Proof. (a) The operator Pre is monotone: W ⊆ W′ implies Pre(W) ⊆ Pre(W′). Since G is reflexive, for every (p, p) ∈ Δ and every e′ ∈ N[p] the cop can reply p′ = e′, so Δ ⊆ Pre(Δ), that is, $W_0 \subseteq W_1$. Applying Pre k times gives $W_k \subseteq W_{k+1}$.
(b) We argue by induction on k. For k = 0, capture within zero rounds means p = e, that is, (p, e) ∈ W_0. Assume the claim holds for k. If (p, e) ∈ W_{k+1}= Pre(W_k), then for every move e′ ∈ N[e] of the robber the cop can reply with some p′ ∈ N[p] such that (p′, e′) ∈ W_k, and by the induction hypothesis he can then capture the robber within k further rounds. Conversely, if (p, e) ∉ Pre(W_k), there is e′ ∈ N[e] such that (p′, e′) ∉ W_k for every p′ ∈ N[p]. In particular e′ ∉ N[p], since otherwise the reply p′ = e′ would give (e′, e′) ∈ Δ ⊆ W_k; so the robber is not captured in this round, and by the induction hypothesis, whatever the cop's reply, the robber can avoid capture during the next k rounds. Hence the cop cannot guarantee capture within k + 1 rounds.
(c) By (a), (W_k) is an increasing sequence of subsets of the finite set V × V, so it is constant from some $K \leq |V|^2$ on; then W* = W_K = W_{K+1} = Pre(W*). If W is a fixed point of Pre with Δ ⊆ W, then $W_k \subseteq W$ for every k, by monotonicity and induction, so W* ⊆ W.
(d) Since W* = Pre(W*), a state (p, e) lies in R if and only if there is e′ ∈ N[e] with (p′, e′) ∉ W* for every p′ ∈ N[p]; as in (b), e′ ∉ N[p], so the robber is not captured in that round. Choosing such a move in every round, the robber keeps the state in R forever and is never captured. Conversely, by (b), from every state of W* the cop can force capture. ■

Proposition 2. Let u ≠ v be vertices of G with N[v] ⊆ N[u], let H = G − v be the subgraph induced by V \ {v}, and let f: V → V \ {v} be defined by f(v) = u and f(x) = x for x ≠ v. Then f is a retraction of G onto H, and for every $k \geq 0$, every p ∈ V(H) and every e ∈ V,

$$\big(p, f(e)\big) \in W_k(H) \ \Rightarrow\ (p,e) \in W_{k+1}(G),$$

where W_k(H) and W_k(G) denote the sets of Definition 5 computed in H and in G, respectively.

Proof. Let x and y be adjacent or equal vertices of G. If $v \notin \{x, y\}$, then f(x) = x and f(y) = y are adjacent or equal in H. If x = v, then $y \in N[v] \subseteq N[u]$, so $f(y) \in N[u] = N[f(x)]$ (if y = v, then f(y) = u = f(x)). Hence f maps adjacent vertices to adjacent or equal vertices and fixes every vertex of H, so it is a retraction of G onto H.

Now assume $(p, f(e)) \in W_k(H)$. By Proposition 1(b) applied to H, the cop has a strategy σ in H that captures, within k rounds, a robber who starts at f(e) and moves first. In G, the cop uses σ against the shadow $f(e_t)$ of the robber's position e_t. This is possible: when the robber moves from e_t to $e_{t+1} \in N[e_t]$, the shadow moves from $f(e_t)$ to $f(e_{t+1})$, which is adjacent or equal to $f(e_t)$ because f is a homomorphism, so it is a legal robber move in H; and every move prescribed by σ is a move in H, hence in G. Therefore, within k rounds, the cop and the shadow occupy the same vertex. If at that moment $e_t \neq v$, then $e_t = f(e_t)$ and the robber is captured. Otherwise, the robber is at v and the cop at u. If it is the cop's turn, he moves to $v \in N[u]$ and captures the robber; if it is the robber's turn, the robber moves to some $e' \in N[v] \subseteq N[u]$, and the cop captures him by moving to e′. In all cases capture occurs within k + 1 rounds, so $(p, e) \in W_{k+1}(G)$ by Proposition 1(b). ■

Corollary 3. Let G be a finite reflexive graph. If G is dismantlable, then $W^* = V \times V$, that is, the cop captures the robber from every pair of initial positions. Consequently, G is cop-win if and only if $W^* = V \times V$, equivalently $R = \emptyset$.

Proof. We argue by induction on |V|; the case |V| = 1 is trivial. Let $|V| \geq 2$ and let $v_1,...,v_n$ be a dismantling order of G, with $v = v_1$ dominated by u. Then $H = G - v$ is dismantlable, with dismantling order $v_2,...,v_n$, so $W^*(H) = V(H) \times V(H)$ by the induction hypothesis, and Proposition 2 gives $(p, e) \in W^*(G)$ for every $p \neq v$ and every $e \in V$. For the remaining states (v,e), the cop can answer every robber move $e' \in N[e]$ with $p' = u \in N[v]$, and $(u,e') \in W^*(G)$ because $u \neq v$; hence $(v,e) \in \mathrm{Pre}(W^*(G)) = W^*(G)$. For the last statement: if $W^* = V \times V$, the cop wins from any initial positions, so G is cop-win; conversely, if G is cop-win, it is dismantlable by Theorem 4, and $W^* = V \times V$ by the first part. ■

Proposition 1 also describes the position-dependent notions of [27]. In the graph of Figure 3, $(P_2, E_2) \in W_1$ because $N[E_2] \subseteq N[P_2]$, whereas $(P_1, E_1) \in R$. Thus W^* is non-empty but differs from $V \times V$: the graph is robber-win, yet the cop wins from some initial positions.

The same construction underlies the analysis of pursuit differential games. Let the state $x \in \mathbb{R}^n$ evolve according to $\dot{x} = f(x, u, v)$, where $u \in \mathcal{U}$ and $v \in \mathcal{V}$ are the controls of the pursuer and of the evader, and let $T \subset \mathbb{R}^n$ be a closed target set; for the Homicidal Chauffeur, T is the set of states in which the distance between the players is at most ℓ. The set of initial states from which the pursuer can force the state into T in finite time is the capture basin of T, and its complement, from which the evader can keep the state outside T forever, is the evader's victory domain, or discriminating kernel. Under standard assumptions on f, $\mathcal{U}$ and $\mathcal{V}$, these sets can be approximated by iterating discrete-time operators of the same form as Pre, with the neighborhoods $N[\cdot]$ replaced by the sets reachable in one time step [64]; equivalently, the capture basin can be characterized through the value function of a Hamilton-Jacobi-Isaacs equation [63]. In this sense, W^* and R are exact finite counterparts of the capture basin and of the discriminating kernel.

The correspondence can be summarized as follows:

- state space: $V \times V$, versus $\mathbb{R}^n$ or a space of reduced coordinates;
- one step: a robber move $e' \in N[e]$ followed by a cop move $p' \in N[p]$, versus the flow of $\dot{x} = f(x,u,v)$ over a time step;
- target: the diagonal Δ, versus the capture set T;
- pursuer's winning region: W^* (Proposition 1), versus the capture basin [63, 64];
- evader's invariant set: R (Proposition 1(d)), versus the discriminating kernel [64];
- certificate of capture from every state: a dismantling order (Corollary 3), versus a value function solving the Hamilton-Jacobi-Isaacs equation [63];
- reduction: folding a dominated vertex (Proposition 2), versus reducing the state space, as in Isaacs' reduction of the Homicidal Chauffeur to two relative coordinates [4].

7.4 Complexity and Computability

- Computing the cop number is NP-hard [32] and deciding whether $c(G) \leq k$ with k part of the input is EXPTIME-complete [56]; cop-win graphs, however, are recognized in polynomial time via dismantlability.
- No closed-form solution of the Hamilton-Jacobi-Isaacs equation is available in general, and grid-based level-set methods have memory and computational costs that grow exponentially with the dimension of the state space [63]. For pursuit with bounded

velocities among polyhedral obstacles in three dimensions, Reif and Tate [62] showed that deciding the game is hard for exponential time; computability aspects of differential games in Euclidean spaces are discussed in [12].

7.5 Dismantlability vs Invariance Principles

Proposition 1(d) gives a precise form to the invariance arguments used in pursuit-evasion games: R is the largest set of states that the robber can keep invariant against every strategy of the cop, and dismantlability (Corollary 3) is a combinatorial certificate that this set is empty. In differential games the corresponding object is the evader's discriminating kernel, the largest set that the evader can keep invariant, which is characterized through viability theory or through Hamilton-Jacobi-Isaacs equations [63, 64]. Proposition 2 shows that folding a dominated vertex delays capture by at most one round. A continuous analogue would be a reduction of the state space that preserves capture basins up to a bounded delay; we do not formalize this analogy here and list it among the open problems in Section 8.

8 Conclusion and Future Directions

Section 7.3 makes one common structure precise: in both settings the pursuer's winning region is obtained by a backward-reachability iteration, and on finite graphs dismantlability is exactly the case in which this region is the whole state space (Propositions 1-2 and Corollary 3).

Ibragimov's infinite-dimensional pursuit games and Luckraz's stochastic dynamic games underscore rich cross-paradigm connections.

Open problems include:

- Definition of a topological or metric model that can be used for both capture and escape in both discrete contexts, such as graphs, and continuous contexts, such as differential games.
- Simplification of capture-time analysis for infinite graphs and of high-dimensional Hamilton-Jacobi-Isaacs equations for pursuit-evasion games.
- Introduction of new models to characterize the infinite cop-win graphs.
- Definition of mathematical structures for the application and solution of an Isaacs model based on graph dismantling.
- A continuous analogue of Proposition 2: reductions of the state space of a differential game that preserve capture basins up to a bounded delay in the capture time.

Bridging discrete and continuous paradigms promises insights for robotics, surveillance, and cooperative control systems, where agents negotiate both digital networks and physical spaces.

Acknowledgement: The author thanks the editor, Professor Mastorakis, and the anonymous referees for their comments and suggestions, which have improved the quality of this paper.

Declaration of Generative AI and AI-assisted Technologies in the Writing Process

During the preparation of this work, the author(s) utilized artificial intelligence (AI) technologies solely for the purpose of linguistic refinement and grammatical correction. Following the application of these tools, the author(s) comprehensively reviewed, verified, and edited the resulting text. The author(s) maintain full accountability and responsibility for the accuracy, integrity, and final content of the publication.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The author carried out all parts of this work, from the formulation of the problem to the final results.

Conflict of Interest

The author declares that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

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