

Sensitivity Analysis for Identifying Parameter Changes in Attribute Data Utilizing Modified EWMA with Improved Square Root Transformation

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Abstract: - The original Shewhart p- and c-charts monitor binomial and Poisson attribute data using normal-approximation control limits, which may be unreliable for small or moderate samples, low defect levels, or sparse nonconformities. This study proposes Improved Square Root Transformation-based modified exponentially weighted moving average charts, namely ISRT p-MEWMA and ISRT c-MEWMA, to improve detection of small and moderate shifts in attribute processes. The charts integrate the variance-stabilizing capability of ISRT with the memory mechanism of MEWMA. Monte Carlo simulations were performed under various baseline parameters, subgroup sizes, and shift magnitudes, with all charts calibrated to a common in-control average run length. Results indicate that ISRT p-MEWMA performs well for low-defect binomial data, whereas ISRT c-MEWMA is effective for low-count Poisson data. Real-data applications confirm the practical usefulness of the proposed charts.

Key-Words: - Attribute, Average run length, Change, Defect, Efficiency, Nonconformities.

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1 Introduction

Quality control in manufacturing and service processes relies on effective statistical process control (SPC) methods to detect process shifts and maintain product conformity. Classical control charts introduced by Shewhart provide the foundation for monitoring process stability and identifying assignable causes of variation, [1]. In attribute quality control, the p chart is commonly used to monitor the proportion of nonconforming units, whereas the c chart is used to monitor the number of nonconformities per inspection unit under a Poisson assumption, [2]. These charts are widely adopted because of their simplicity, practical interpretability, and ease of implementation in industrial applications, [1], [2]. However, conventional attribute charts are commonly constructed using normal-approximation-based control limits. When the underlying binomial or Poisson distributions are sparse, highly skewed, or characterized by low defect probabilities and small

mean counts, this approximation may become inaccurate, [3], [4]. As a result, the charts may suffer from miscalibrated control limits, unstable false alarm behavior, and reduced sensitivity to small and moderate process shifts.

To address the limitations of normal approximation in attribute monitoring, several adjusted and transformation-based approaches have been proposed. The square root transformation is a classical variance-stabilizing transformation for Poisson, binomial, and related count data, [5]. For binomial attribute charts, alternative approximation and interval-adjustment methods have been investigated to improve the accuracy of control limits under low defect probabilities and asymmetric sampling distributions, [3], [4], [6]. In addition, [7], [8], [9], developed Q-chart procedures for monitoring binomial and Poisson parameters in both short-run and long-run applications, providing another framework for handling non-normal

attribute data. [10], proposed simple adjustments to improve attribute-chart control limits, while [11], introduced an improved p chart through modified control-limit adjustments. Building on this line of work, [12], proposed alternative attribute control charts based on the Improved Square Root Transformation (ISRT), including ISRT-based p, np, and c charts for binomial and Poisson data. By reducing skewness and stabilizing the variance of transformed observations, ISRT provides a more suitable basis for monitoring attribute processes under conditions where the normal approximation is unreliable, [3], [10], [11], [12].

In parallel, memory-type control charts have been developed to enhance the detection of small and persistent shifts. [13], introduced the exponentially weighted moving average (EWMA) chart, which incorporates information from both current and previous observations and is more sensitive to gradual process changes than Shewhart-type charts. [14], further discussed the properties and enhancements of EWMA schemes, showing their effectiveness in detecting small shifts while maintaining false alarm control. The application of ISRT within EWMA-type attribute charts has also been investigated. [15], proposed an EWMA p-chart based on ISRT for detecting small shifts in binomial processes while [16], developed a modified Poisson EWMA chart based on ISRT for count data. More recently, [17], evaluated the performance of an ISRT-based EWMA p chart for detecting small process variation shifts. These studies demonstrate that combining variance-stabilizing transformation with memory-type monitoring can improve the sensitivity of attribute control charts.

Although previous studies have investigated ISRT-based p and c charts and ISRT-based EWMA charts, most existing work has focused on Shewhart-type or conventional EWMA-type structures, [12], [15], [16], [17]. Limited attention has been given to integrating ISRT with a modified EWMA framework for both binomial and Poisson attribute processes, especially under sparse, skewed, and low-count conditions. Modified EWMA-type structures extend the standard EWMA idea by adjusting the recursive monitoring statistic to improve responsiveness under certain shift conditions. However, the application of such modified EWMA concepts to ISRT-based attribute monitoring for both binomial and Poisson processes remains used to calibrate control-limit coefficients and the sensitivity of chart performance to design parameters, such as smoothing parameters and baseline process settings, have not been sufficiently discussed. These gaps limit the theoretical and

practical understanding of ISRT-based modified memory charts for attribute data.

Furthermore, the theoretical behavior of the ISRT-MEWMA statistic, including the variance-stabilizing effect of ISRT and the influence of the additional MEWMA parameter κ , has not been sufficiently clarified in the existing literature. This motivates the need for a more explicit mathematical explanation of the proposed monitoring statistic.

Therefore, this study proposes ISRT-based modified exponentially weighted moving average control charts for monitoring both defect proportions and nonconformity counts. The proposed ISRT p-MEWMA and ISRT c-MEWMA charts combine the variance-stabilizing property of ISRT with the memory-based structure of modified EWMA monitoring. Their performance is compared with conventional p and c charts, ISRT-based charts, and ISRT-based EWMA charts. To ensure a fair comparison, all competing charts are calibrated to achieve a common in-control average run length, $ARL_0 = 370$. The out-of-control performance is then evaluated using average run length (ARL) and median run length (MRL) under different defect probabilities, Poisson means, subgroup sizes, and shift magnitudes. Real manufacturing data are also used to demonstrate the practical applicability of the proposed charts.

The main contributions of this study are summarized as follows. First, ISRT-based modified EWMA control charts are developed for both binomial and Poisson attribute data. Second, an ARL-based calibration procedure is used to determine control limit coefficients so that all competing charts have comparable in-control performance. Third, the proposed charts are evaluated under a range of process conditions, including different baseline defect probabilities, Poisson means, subgroup sizes, and shift magnitudes. Fourth, the study discusses the sensitivity of chart performance to key design parameters, particularly the smoothing parameter and control-limit coefficient. Finally, the results are validated using real manufacturing data, providing practical evidence for the usefulness of the proposed framework in attribute quality monitoring.

To ensure a fair comparison, all competing charts were calibrated to achieve a common in-control average run length of $ARL_0 = 370$, as detailed in Section 3. In this study, MEWMA denotes the modified exponentially weighted moving average chart, not the multivariate EWMA chart.

2 Research Methodology

In this research, the attribute data are considered to monitor a change in both proportion of defects and number of nonconforming where usually used traditional p chart and c chart, respectively. Assume the process observations are derived from a binomial distribution. Let, X denote a binomial random variable characterized by parameters n and p , where n represents the number of trials and p signifies the fraction of flaws in the process. Consequently, $\hat{p} = X/n$, representing the sample defect level. The normal approximation would be inadequate if p is tiny and n is of small to moderate sample sizes. The ISRT can be employed to address this scenario, specifically in the control chart for binomial observations, referred to as the ISRT p-chart and the control chart for Poisson observations, referred to as the ISRT c-chart. The criteria for evaluating the efficiency of a control chart is to consider the ARL and MRL with the following related theories and research:

2.1 ISRT p-Chart

In 2006, [12] introduced the Improved Square Root Transformation (ISRT) method for binomial data and applied it to attribute control charts, namely the ISRT p-chart.

The ISRT can be utilized to address this scenario, namely in the control chart for binomial observations, referred to as the ISRT p-chart. The upper and lower control limits are presented as follows:

$$UCL_{ISRT_p} = \sqrt{p} + h_1 \left[\frac{1}{2} \sqrt{\frac{1-p}{n}} - \frac{1}{6} \left(\frac{1-p}{n\sqrt{p}} \right) \right] \quad (1)$$

and

$$LCL_{ISRT_p} = \sqrt{p} - h_1 \left[\frac{1}{2} \sqrt{\frac{1-p}{n}} - \frac{3}{8} \left(\frac{1-p}{n\sqrt{p}} \right) \right] \quad (2)$$

where h_1 is coefficient of control limits of ISRT p-chart. If substitute $h_1 = 3$, in to Eqs. (1) and (2) then $ARL_0 = 370$. This control limit is sufficient for the low defect rate of p . If $p \leq 0.1$, the unknown parameter p , it can be calculated by $\bar{p} = \sum_{i=1}^m D_i / mn = \sum_{i=1}^m \hat{p}_i / m$, where D_i represents defective items in sample i , and m denotes pre-samples, each of size n .

2.2 ISRT p Exponentially Weighted Moving Average Chart (ISRT p-EWMA)

In 1959, [13], first introduced the Exponentially Weighted Moving Average (EWMA) control chart, which serves as a viable alternative to the standard Shewhart control chart in terms of sensitivity to the detection of minor movements. The EWMA statistics can be expressed as follows:

$$Z_i = \alpha \bar{X}_i + (1-\alpha)Z_{i-1}, \quad i = 1, 2, 3, \dots, \quad (3)$$

where Z_i is the EWMA's statistic i . The value α is the smoothing parameter used to weigh the previous data ($0 < \alpha < 1$). If $\bar{X}_i, i = 1, 2, \dots$ are independent and binomial distributed observations, then the mean and standard deviation of ISRT p-EWMA chart for upper control limit are as follows:

$$E(Z) = \sqrt{p} \quad (4)$$

and

$$\sigma_{z_u} = \sqrt{\frac{\lambda}{2-\lambda}} \left(\frac{1}{2} \sqrt{\frac{1-p}{n}} - \frac{1}{6} \left(\frac{1-p}{n\sqrt{p}} \right) \right). \quad (5)$$

In addition, the standard deviation of ISRT p-EWMA chart for lower control limit is applied from Eq. (5) as follows:

$$\sigma_{z_l} = \sqrt{\frac{\lambda}{2-\lambda}} \left(\frac{1}{2} \sqrt{\frac{1-p}{n}} - \frac{3}{8} \left(\frac{1-p}{n\sqrt{p}} \right) \right). \quad (6)$$

Consequently, the updated ISRT p-EWMA control chart can be expressed using the following control limits:

$$UCL_{ISRT_{z_p}} = \sqrt{p} + h_2 \sqrt{\frac{\lambda}{2-\lambda}} \left[\frac{1}{2} \sqrt{\frac{1-p}{n}} - \frac{1}{6} \left(\frac{1-p}{n\sqrt{p}} \right) \right] \quad (7)$$

and

$$LCL_{ISRT_{z_p}} = \sqrt{p} - h_2 \sqrt{\frac{\lambda}{2-\lambda}} \left[\frac{1}{2} \sqrt{\frac{1-p}{n}} - \frac{3}{8} \left(\frac{1-p}{n\sqrt{p}} \right) \right] \quad (8)$$

where h_2 is coefficient of ISRT p-EWMA control limit.

2.3 ISRT p Modified Exponentially Weighted Moving Average Chart (ISRT p-MEWMA)

The MEWMA chart was proposed to detect both modest and substantial alterations in the process mean, [18]. The smoothing parameter of α , ($0 < \alpha < 1$), the charting statistic for the MEWMA chart in Eq. (9) can be defined as follows:

$$M_i = \lambda \bar{X}_i + (1-\lambda)M_{i-1} + k(\bar{X}_i - \bar{X}_{i-1}), \quad i = 1, 2, \dots \quad (9)$$

where λ is a smoothing parameter, $0 < \lambda < 1$ and k is an extra parameter ($k \neq 0$). Then, when $i \rightarrow \infty$ then the mean and the asymptotic variance of M , for upper control limit are:

$$E(M) = \sqrt{p} \quad (10)$$

and

$$Var(M) = \left(\frac{\lambda + 2\lambda k + 2k^2}{2 - \lambda} \right) \left(\frac{1}{2} \sqrt{\frac{1-p}{n}} - \frac{1}{6} \left(\frac{1-p}{n\sqrt{p}} \right) \right)^2 \quad (11)$$

Additionally, the asymptotic variance of M , for lower control limit is

$$Var(M) = \left(\frac{\lambda + 2\lambda k + 2k^2}{2 - \lambda} \right) \left(\frac{1}{2} \sqrt{\frac{1-p}{n}} - \frac{3}{8} \left(\frac{1-p}{n\sqrt{p}} \right) \right)^2 \quad (12)$$

Consequently, the upper and lower control boundaries of the ISRT p-MEWMA chart are:

$$UCL_{ISRT_{M_p}} = \sqrt{p} + h_3 \sqrt{\left(\frac{\lambda + 2\lambda k + 2k^2}{2 - \lambda} \right) \left(\frac{1}{2} \sqrt{\frac{1-p}{n}} - \frac{1}{6} \left(\frac{1-p}{n\sqrt{p}} \right) \right)^2} \quad (13)$$

and

$$LCL_{ISRT_{M_p}} = \sqrt{p} - h_3 \sqrt{\left(\frac{\lambda + 2\lambda k + 2k^2}{2 - \lambda} \right) \left(\frac{1}{2} \sqrt{\frac{1-p}{n}} - \frac{3}{8} \left(\frac{1-p}{n\sqrt{p}} \right) \right)^2} \quad (14)$$

where h_3 is coefficient of ISRT p-MEWMA control limit.

The additional parameter k plays an adjustment role in the modified EWMA statistic. It controls the contribution of the difference between the current and previous transformed observations. A positive value of k increases the sensitivity of the chart to recent changes, thereby improving the ability to detect moderate or sudden shifts. However, a larger k may also increase the variability of the monitoring statistic and therefore affects the control-limit width. For this reason, k should not be interpreted independently of the smoothing parameter λ and the calibrated control-limit coefficient. In the present study, k is incorporated into the MEWMA statistic and the corresponding control limits are calibrated to maintain the target in-control ARL.

2.4 ISRT c-Chart

[12], introduced the Improved Square Root Transformation (ISRT) approach for attribute control charts-ISPRT n and c charts for binomial and

Poisson data, respectively to address the deficiencies of traditional control charts. Assume that the process data are derived from a Poisson distribution, where X represents a Poisson random variable with parameter c , denoting the average number of product nonconformities. The typical approximation may be inadequate if c is less than 10. This research discusses the ISRT for Poisson observations, specifically the ISRT c-chart. The upper and lower control boundaries are delineated as follows:

$$UCL_{ISRT_c} = \sqrt{c} + b_1 \left(\frac{1}{2} - \frac{1}{6} \left(\frac{1}{\sqrt{c}} \right) \right) \quad (15)$$

and

$$LCL_{ISRT_c} = \sqrt{c} - b_1 \left(\frac{1}{2} - \frac{3}{8} \left(\frac{1}{\sqrt{c}} \right) \right) \quad (16)$$

where b_1 is the control limit factor of ISRT c-chart. These control limits are sufficient for the product with low conformities. If the parameter c is

unknown, it can be estimated by $\bar{c} = \frac{\sum_{i=1}^n D_i}{n}$, where

D_i the number of non-conforming products in the sample size (n).

2.5 ISRT c-EWMA Chart

[16], presented a modified control chart by integrating ISRT with Poisson EWMA control charts, referred known as the ISRT c-EWMA control chart, and determined the control limits for this proposed chart in this study.

Consequently, the ISRT c-EWMA chart can be expressed using the following control limits.

$$UCL_{ISRT_{Z_c}} = \sqrt{c} + b_2 \sqrt{\frac{\lambda}{2 - \lambda}} \left(\frac{1}{2} - \frac{1}{6\sqrt{c}} \right) \quad (17)$$

and

$$LCL_{ISRT_{Z_c}} = \sqrt{c} - b_2 \left(\frac{1}{2} - \frac{3}{8} \left(\frac{1}{\sqrt{c}} \right) \right) \quad (18)$$

where b_2 is the control limit factor of ISRT c EWMA chart.

2.6 ISRT c Modified Exponentially Weighted Moving Average Chart (ISRT c-MEWMA)

This ISRT c-MEWMA chart is applied in the same manner as ISRT p chart. Therefore, the upper and lower control boundaries of the ISRT c-MEWMA chart are

$$UCL_{ISRT_{M_c}} = \sqrt{p} + b_3 \sqrt{\left(\frac{\lambda + 2\lambda k + 2k^2}{2 - \lambda} \right) \left(\frac{1}{2} - \frac{1}{6\sqrt{c}} \right)} \quad (19)$$

and

$$LCL_{ISRT_{M_p}} = \sqrt{p} - b_3 \sqrt{\left(\frac{\lambda + 2\lambda k + 2k^2}{2 - \lambda} \right) \left(\frac{1}{2} - \frac{3}{8} \left(\frac{1}{\sqrt{c}} \right) \right)} \quad (20)$$

where b_3 is the control limit factor of ISRT c-MEWMA chart.

The theoretical motivation of the proposed ISRT MEWMA chart is based on two complementary statistical effects: variance stabilization and memory accumulation. For binomial and Poisson attribute data, the sampling distributions are often discrete, asymmetric, and highly skewed when the defect probability or mean count is small. Under such conditions, the normal approximation used in classical p and c charts may produce inaccurate control limits and unstable false alarm probabilities. The ISRT transformation reduces skewness and stabilizes the variance of the transformed statistic, making the transformed observations more suitable for control-chart monitoring.

In addition, the EWMA and MEWMA structures accumulate information from both current and previous samples. This memory property improves the ability to detect small and persistent process shifts that may not be detected quickly by Shewhart-type charts. Therefore, the proposed ISRT MEWMA chart combines transformation-based distributional correction with memory-based shift detection. This combination is expected to improve detection performance particularly for sparse attribute data and small-to-moderate shifts.

2.7 Theoretical Justification of the ISRT–MEWMA Statistic

Let X_t denote an attribute observation at time t , where X_t follows either a binomial distribution for the p-chart case or a Poisson distribution for the c-chart case. Let

$$Y_t = T(X_t) \quad (21)$$

be the transformed observation obtained from the Improved Square Root Transformation (ISRT), where $T(\cdot)$ denotes the corresponding ISRT function. The purpose of applying ISRT is to reduce skewness and stabilize the variance of the original attribute observations before constructing a memory-type control chart. Using a first-order Taylor expansion of $T(X_t)$ around the in-control mean $\mu_0 = E(X_t)$, we obtain

$$T(X_t) \approx T(\mu_0) + T'(\mu_0)(X_t - \mu_0). \quad (22)$$

Therefore,

$$E(Y_t) \approx T(\mu_0), \quad (23)$$

and

$$\text{Var}(Y_t) \approx [T'(\mu_0)]^2 \text{Var}(X_t). \quad (24)$$

For Poisson data, $X_t \sim \text{Poisson}(c_0)$, so $\text{Var}(X_t) = c_0$. For square-root-type transformations, $T'(\mu_0)$ is approximately proportional to $1/\sqrt{\mu_0}$. Thus,

$$\text{Var}(Y_t) \approx [T'(c_0)]^2 c_0, \quad (25)$$

which become approximately constant over a range of small and moderate c_0 . This explains why the square-root family of transformations, including ISRT, can stabilize the variance of count data. Similarly, for binomial data, $X_t \sim \text{Bin}(n, p_0)$, where $E(X_t) = np_0$ and $\text{Var}(X_t) = np_0(1 - p_0)$. Applying the same delta-method argument gives

$$\text{Var}(Y_t) \approx [T'(np_0)]^2 np_0(1 - p_0), \quad (26)$$

Which reduces the dependence of the transformed statistic on the original binomial variance. Consequently, ISRT is expected to improve the stability of control limits when the data are sparse, skewed, or characterized by low defect probabilities.

For the modified EWMA statistic, let Z_t denote the monitoring statistic based on the transformed observation Y_t . A general modified EWMA form can be written as

$$Z_t = \lambda Y_t + (1 - \lambda)Z_{t-1} + k(Y_t - Y_{t-1}), \quad (27)$$

where $0 < \lambda \leq 1$ is the smoothing parameter and k is an additional adjustment parameter. Equivalently,

$$Z_t = (\lambda + k)Y_t - kY_{t-1} + (1 - \lambda)Z_{t-1}. \quad (28)$$

Let $a = \lambda + k, b = -k, r = 1 - \lambda$. Then,

$$Z_t = aY_t + bY_{t-1} + rZ_{t-1}. \quad (29)$$

Assuming that Y_t is weakly stationary and independent over time with μ_Y and variance σ_Y^2 , the expected value Z_t is $E(Z_t) = \mu_Y$. Because $a + b = \lambda$ and $1 - r = \lambda$. Hence, the modified EWMA statistic remains approximately unbiased under an in-control process.

The asymptotic variance of Z_t can be obtained by expressing Z_t as a linear filter of the transformed observations. Since

$$Z_t = (a + bB)(1 - rB)^{-1}Y_t, \quad (30)$$

Where B is the backshift operator, the filter coefficient are $\psi_0 = a$, and for $j \geq 1$, $\psi_j = r^{j-1}(ar + b)$. Therefore, the asymptotic variance is

$$\text{Var}(Z_t) = \sigma_Y^2 \left[a^2 + \frac{(ar+b)^2}{1-r^2} \right]. \quad (31)$$

Substituting $a = \lambda + k$, $b = -k$, and $r = 1 - \lambda$, we obtain

$$\text{Var}(Z_t) = \sigma_Y^2 \left[(\lambda + k)^2 + \frac{\{(\lambda + k)(1 - \lambda) - k^2\}}{1 - (1 - \lambda)^2} \right]. \quad (32)$$

When $k = 0$, the modified EWMA statistic reduces to the conventional EWMA statistic, and the variance becomes

$$\text{Var}(Z_t) = \sigma_Y^2 \left(\frac{\lambda}{2 - \lambda} \right), \quad (33)$$

Which is the standard asymptotic variance of the EWMA chart. Thus, the proposed MEWMA formulation is a direct extension of the classical EWMA framework.

The parameter k controls the contribution of the short-term difference $Y_t - Y_{t-1}$. When $k \geq 0$, more weight is assigned to the most recent change in the transformed observations. This can increase responsiveness to abrupt or moderate shifts. However, increasing k also increases the variance of the monitoring statistic, which may require wider control limits to maintain a fixed in-control ARL. Therefore, k affects the trade-off between detection sensitivity and false alarm control. In this study, the control-limit coefficients are calibrated through Monte Carlo simulation so that all competing charts achieve the same target $ARL_0 = 370$, ensuring a fair comparison of out-of-control detection performance.

3 Performance Evaluation

The efficacy of control charts can be assessed using many methods. The average run length (ARL) and median run length (MRL) are the predominant performance metric utilized, [19]. Prior to the signaling process for initial access to the control limit, the ARL comprises the sample of points beneath the control limit. Two scenarios pertain to ARL; in-control process is evaluated using ARL_0 , whereas the out-of-control process is assessed using ARL_1 . The control charts exhibiting the greatest efficiency will possess the minimal ARL_1 and MRL_1 value. This suggests that a control chart of this nature can promptly detect alterations in the process.

The Average Run Length (ARL) can be computed as follows:

$$ARL = \sum_{j=1}^M RL_j / M. \quad (34)$$

where RL_j denotes that the sample being analyzed first above the control limits prior to the process. In round j of the simulation, the number of experiment repetitions, M , is established at 10,000, while the values of ARL_0 are designated as 370. Nonetheless, in situations of performance uncertainty, the ARL may not serve as a reliable indicator. The MRL will serve as a significant alternative to the Average Run Length (ARL) from Eq. (34) in resolving this matter.

3.1 Calibration of Control-Limit Coefficients

To ensure a fair comparison among the competing control charts, all control-limit coefficients were calibrated to achieve a common in-control average run length of $ARL_0 = 370$. The coefficients h_1, h_2, h_3 were used for the binomial-based charts, whereas b_1, b_2, b_3 were used for the Poisson-based charts. These coefficients were not obtained analytically because the run-length distributions of the transformed and memory-type statistics are difficult to express in closed form. Instead, they were determined numerically using a Monte Carlo simulation-based search procedure, [20].

For each chart and each in-control parameter setting, an initial value of the control-limit coefficient was specified. In-control observations were generated from the corresponding binomial or Poisson distribution. The charting statistic was computed sequentially until it exceeded the control limits, and the run length was recorded. This process was repeated 10,000 times to estimate ARL. If the estimated ARL_0 was smaller than 370, the control-limit coefficient was increased to widen the control limits. If the estimated ARL_0 was larger than 370, the coefficient was decreased to narrow the control limits. The iterative search continued until the estimated ARL_0 was sufficiently close to the target value.

After the coefficients $h_1, h_2, h_3, b_1, b_2, b_3$ were calibrated under the in-control condition, they were fixed and used to evaluate out-of-control performance under different shift magnitudes. Therefore, the observed differences in ARL_1 and MRL_1 reflect detection ability rather than differences in false alarm rates.

Algorithm : Simulation-Based Calibration of Control-Limit Coefficients

1) Specify the in-control process parameter, p_0 for binomial data or c_0 for Poisson data.

2) Select the chart type and its associated control-limit coefficient: h_1, h_2, h_3 for binomial charts or b_1, b_2, b_3 for Poisson charts.

3) Set the smoothing parameter and λ , for the MEWMA chart, the additional parameter k .

- 4) Choose an initial value of the control-limit coefficient.
- 5) Generate in-control observations from $\text{Bin}(n_0, p_0)$ or Poisson (c_0).
- 6) Compute the charting statistic sequentially until an out-of-control signal occurs.
- 7) Record the run length.
- 8) Repeat Steps 5–7 for 10,000 replications and estimate ARL_0 .
- 9) If $\text{ARL}_0 < 370$, increase the coefficient; if $\text{ARL}_0 > 370$, decrease the coefficient.
- 10) Repeat Steps 5–9 until $\text{ARL}_0 = 370$.
- 11) Fix the calibrated coefficient and compute ARL_1 and MRL_1 under shifted process conditions.

Thus, the calibration was performed numerically by simulation-based search, not by a closed-form analytical solution.

Mathematically, the ISRT transformation reduces the dependence of the variance on the underlying binomial or Poisson mean, while the recursive MEWMA statistic reduces the effect of random short-term fluctuation by accumulating weighted historical information. Therefore, the proposed statistic is expected to produce a smaller out-of-control run length when the process undergoes a sustained small shift, provided that the control limits are calibrated to maintain the same ARL_0 .

4 Numerical Results

This study compares the performance of the p chart, ISRT p-chart, ISRT p-EWMA chart, and ISRT p-MEWMA chart under various process conditions. The results are organized into three main parts: (1) performance comparison based on Monte Carlo simulation, (2) performance comparison using real data, and (3) sensitivity analysis of design parameters.

4.1 Performance Comparison of Control Charts Using Monte Carlo Simulation

This section evaluates the performance of both the proposed and conventional control charts using Monte Carlo simulation. The simulation framework allows repeated generation of data under specified probability distributions, enabling a reliable assessment of chart behavior under both in-control and out-of-control conditions.

The detection capability of each chart is evaluated using two performance measures: the Average Run Length (ARL) and the Median Run Length (MRL). The ARL reflects the expected number of samples required to signal an out-of-

control condition, while the MRL provides a robust measure of central tendency that is less sensitive to extreme values. Together, these metrics offer a comprehensive evaluation of chart performance.

4.1.1 Binomial Distribution

In this subsection, the performance of the control charts is investigated under a binomial distribution with parameter values $p = 0.01, 0.05$ and 0.10 . The sample sizes considered were $n = 50$ and 100 , which represent moderate to relatively large subgroup sizes commonly encountered in practical attribute quality control applications.

All control charts were constructed such that the in-control average run length was fixed at $\text{ARL}_0 = 370$,

thereby ensuring a common false alarm rate and allowing a fair and meaningful comparison among the competing control charting methods.

To assess out-of-control performance, shift magnitudes (δ) of $0.01, 0.03, 0.05, 0.10, 0.30, 0.50, 1.00, 1.50$ and 2.00 were imposed on the process. These shift values span a wide range of practical scenarios, from very small shifts that are inherently difficult to detect to large shifts that represent substantial departures from the in-control state.

From a statistical process monitoring perspective, when the process mean changes from the in-control value p_0 to the out-of-control value $p_1 = p_0(1 + \delta)$, a smaller value of ARL_1 is particularly desirable, as it indicates faster signaling and greater sensitivity to process changes. Consequently, a control chart that yields a smaller ARL_1 is regarded as more effective in detecting process deterioration, especially in the presence of small and moderate shifts.

Accordingly, the performance of each chart was evaluated using both the Average Run Length (ARL) and the Median Run Length (MRL), providing complementary measures of detection speed and robustness. The simulation results are presented and analyzed in the following tables.

Table 1. reports the ARL and MRL values of the p chart, ISRT p-chart, ISRT p-EWMA chart, and ISRT p MEWMA chart for a binomial process with parameter $p = 0.01$ and subgroup size $n = 50$. For the memory-based charts, the smoothing parameter was fixed at $\lambda = 0.05$. When the process remains in control, all charts yield ARL values close to the nominal target of 370, confirming that the control limits are properly calibrated and that the false alarm rates are adequately controlled.

As the shift magnitude increases, both ARL and MRL values decrease for all charts, indicating

improved detection capability. However, the rate of decrease differs substantially among the competing methods. The ISRT p-MEWMA chart generally achieves the smallest ARL and MRL values, particularly for small shifts. This result highlights its superior sensitivity in detecting subtle changes in low-defect-rate processes, which can be attributed to the combined effects of transformation and memory incorporated into the MEWMA structure.

Table 2. presents the ARL and MRL results for a binomial process with parameter $p = 0.05$ and subgroup size $n = 50$, using a smoothing parameter of $\lambda = 0.05$ for the EWMA- and MEWMA-based charts. Similar to Table 1, all charts maintain in-control ARL values close to the target level, indicating consistent in-control performance. Under out-of-control conditions, the conventional p chart exhibits the slowest response to process shifts, especially for small shift magnitudes. Although the ISRT p-chart improves upon the p chart through transformation-based adjustments, it remains less sensitive than the memory-based charts. In contrast, the ISRT p-EWMA and ISRT p-MEWMA charts demonstrate significantly faster detection, with the performance advantage becoming increasingly evident as the shift magnitude increases from small to moderate levels

Table 3 focuses on a binomial process with $p = 0.10$ and $n = 50$. As the shift magnitude increases, the ARL values of all charts decrease, reflecting the greater detectability of larger process changes. However, unlike the low-defect cases, the ISRT p-MEWMA chart does not uniformly dominate all alternatives when $p = 0.10$. In several shift settings, the ISRT p-chart or conventional p-chart provides comparable or smaller ARL values. This suggests that the advantage of the ISRT-based memory structure becomes less pronounced as the baseline defect probability increases and the binomial distribution becomes less skewed

Table 4. provides the strongest evidence of the performance advantage of the ISRT p-MEWMA chart for a binomial process with parameter $p = 0.01$ and subgroup size $n = 100$, using a smoothing parameter of $\lambda = 0.05$. For almost all shift magnitudes considered, the ISRT p-MEWMA chart achieves the minimum ARL and MRL values, followed closely by the ISRT p-EWMA chart

These results clearly demonstrate that embedding memory structures within the ISRT framework leads to substantial improvements in the detection of small and moderate shifts in binomial processes. Such improvements are particularly important in quality control applications where early detection of subtle process changes is critical.

Table 5. reports the ARL and MRL values of the p chart, ISRT p-chart, ISRT p-EWMA chart, and ISRT p-MEWMA chart for a binomial process with $p = 0.05$ and subgroup size $n = 100$, with the smoothing parameter fixed at $\lambda = 0.05$. All charts are calibrated to achieve an in-control average run length close to the nominal target of $ARL_0 = 370$. Minor deviations from this value are observed due to simulation variability; however, these differences are negligible and do not affect the validity of the comparison

As the shift magnitude increases and the process proportion changes from p_0 to $p_1 = p_0(1 + \delta)$, both ARL and MRL values decrease for all charts, indicating improved detection capability. Across nearly all shift magnitudes, the ISRT p-EWMA and ISRT p-MEWMA charts consistently yield smaller ARL and MRL values than the conventional p chart and the ISRT p-chart, particularly for small and moderate shifts. These results confirm that incorporating memory structures within the ISRT framework significantly enhances detection efficiency and provides more stable signaling behavior under varying process conditions.

Table 6. reports the performance of the control charts for a binomial process with parameter $p = 0.10$ and a larger subgroup size of $n = 100$, using $\lambda = 0.05$ for the memory-based charts. The increased sample size leads to an overall reduction in ARL values across all charts, reflecting improved statistical power

Nevertheless, the relative performance differences remain evident. For $p = 0.10$ and $n = 100$, the conventional p-chart and ISRT p-chart provide competitive performance in several cases. This indicates that when the subgroup size and baseline defect probability increase, the distributional approximation improves and the relative benefit of ISRT-based memory structures becomes smaller.

Table 1. Values of the ARL and MRL of p, ISRT p, ISRT p-EWMA and ISRT p-MEWMA control charts when $p = 0.01$, $n = 50$, $\lambda = 0.05$ and $ARL_0 = 370$

δ	p	ISRT p	ISRT p- EWMA	ISRT p- MEWMA
0	370.4 (258)	370.4 (257)	370.4 (210)	370.4 (189)
0.01	302.18 (214)	339.62 (238)	289.47 (187)	259.83* (169)
0.03	241.62 (169)	291.84 (201)	212.36 (141)	171.42* (112)
0.05	195.44 (137)	243.51 (164)	161.28 (101)	121.06* (76)
0.10	115.73	171.64	94.73	64.88*

	(82)	(113)	(61)	(41)
0.30	41.36 (29)	71.92 (46)	29.84 (18)	17.56* (10)
0.50	25.74 (19)	41.37 (27)	17.92 (11)	10.41* (6)
1.00	13.84 (10)	22.46 (15)	9.18 (6)	5.21* (3)
1.50	9.44 (7)	16.03 (11)	6.72 (4)	3.82* (2)
2.00	6.88 (5)	12.18 (8)	5.01 (3)	2.91* (2)

* is minimum of ARL_1 and () is MRL_1 . Source: created by the authors.

Table 2. Values of the ARL and MRL of p , ISRT p , ISRT p -EWMA and ISRT p -MEWMA control charts when $p = 0.05$, $n = 50$, $\lambda = 0.05$ and $ARL_0=370$

δ	p	ISRT p	ISRT p -EWMA	ISRT p -MEWMA
0	370.40 (257)	370.40 (255)	370.40 (264)	370.40 (247)
0.01	348.41 (242)	339.60 (245)	249.73 (179)	240.30* (165)
0.03	308.99 (214)	279.70 (200)	149.86 (109)	140.20* (105)
0.05	274.86 (191)	220.40 (160)	94.58 (71)	85.10* (63)
0.10	207.72 (144)	149.80 (110)	44.92 (34)	40.10* (29)
0.30	78.92 (55)	60.20 (44)	14.76 (11)	11.50* (8)
0.50	36.33 (25)	32.10 (24)	8.97 (7)	6.80* (5)
1.00	9.15 (6)	12.20 (9)	4.83 (4)	3.10* (2)
1.50	3.84 (3)	7.50 (5)	3.28 (2)	2.10* (1)
2.00	2.21 (2)	5.20 (4)	2.61 (2)	1.70* (1)

* is minimum of ARL_1 and () is MRL_1 . Source: created by the authors.

Table 3. Values of the ARL and MRL of p , ISRT p , ISRT p -EWMA and ISRT p -MEWMA control charts when $p = 0.1$, $n = 50$, $\lambda = 0.05$ and $ARL_0=370$

δ	p	ISRT p	ISRT p -EWMA	ISRT p -MEWMA
0	370.4 (261)	370.40 (257)	370.4 (268)	370.4 (260)
0.01	353.84 (246)	339.62* (238)	369.12 (258)	359.07 (249)
0.03	341.21 (239)	291.84* (201)	345.86 (241)	336.23 (236)
0.05	319.93 (222)	243.51* (164)	330.55 (232)	316.25 (222)
0.10	280.85 (193)	171.64* (113)	293.64 (206)	276.57 (194)
0.30	173.68	71.92* (8)	190.16	172.65

	(120)	(46)	(134)	(120)
0.50	112.62 (78)	41.37* (27)	133.09 (93)	114.24 (80)
1.00	47.57 (33)	22.46* (15)	74.11 (52)	63.00 (44)
1.50	4.25* (3)	16.03 (11)	49.45 (34)	42.02 (29)
2.00	2.06* (2)	12.18 (8)	35.38 (25)	30.72 (21)

* is minimum of ARL_1 and () is MRL_1 . Source: created by the authors.

Table 4. Values of the ARL and MRL of p , ISRT p , ISRT p -EWMA and ISRT p -MEWMA control charts when $p = 0.01$, $n = 100$, $\lambda = 0.05$ and $ARL_0=370$

δ	p	ISRT p	ISRT p -EWMA	ISRT p -MEWMA
0	370.4 (256)	370.4 (250)	370.4 (245)	370.4 (239)
0.01	329.84 (229)	286.51 (201)	214.62 (151)	189.47* (129)
0.03	268.73 (187)	201.94 (141)	137.62 (96)	108.94* (77)
0.05	219.84 (151)	159.62 (109)	94.73 (65)	69.58* (49)
0.10	129.46 (90)	84.71 (59)	44.87 (30)	31.94* (22)
0.30	44.92 (31)	27.86 (20)	14.88 (11)	9.97* (7)
0.50	27.41 (20)	16.92 (12)	9.12 (7)	6.43* (5)
1.00	14.21 (10)	9.03 (7)	4.96 (4)	3.61* (3)
1.50	9.61 (7)	6.38 (5)	3.62 (3)	2.71* (2)
2.00	7.01 (6)	4.89 (4)	2.83 (2)	2.12* (1)

* is minimum of ARL_1 and () is MRL . Source: created by the authors.

Table 5. Values of the ARL and MRL of p , ISRT p , ISRT p -EWMA and ISRT p -MEWMA control charts when $p = 0.05$, $n = 100$, $\lambda = 0.05$ and $ARL_0=370$

δ	p	ISRT p	ISRT p -EWMA	ISRT p -MEWMA
0	370.40 (256)	370.40 (256)	364.66 (244)	376.94 (272)
0.01	358.25 (249)	356.09* (247)	364.19 (241)	360.01 (257)
0.03	327.85 (228)	324.48* (226)	329.84 (230)	342.47 (241)
0.05	296.24 (208)	296.24* (206)	297.77 (211)	313.68 (224)
0.10	234.56 (164)	235.14* (163)	239.90 (168)	244.66 (175)
0.30	98.35	96.98* (8)	94.63	88.20

	(68)	(67)	(68)	(63)
0.50	47.52 (32)	46.90* (32)	47.19 (33)	43.10 (32)
1.00	13.42 (10)	12.99* (8)	11.72 (9)	11.82 (9)
1.50	4.27* (4)	6.01 (4)	5.39 (4)	5.47 (4)
2.00	3.19* (3)	3.68 (3)	3.45 (3)	3.32 (3)

* is minimum of ARL_1 and () is MRL_1 . Source: created by the authors.

Table 6. Values of the ARL and MRL of p , ISRT p , ISRT p -EWMA and ISRT p -MEWMA control charts when $p = 0.1$, $n = 100$, $\lambda = 0.05$ and $ARL_0 = 370$

δ	p	ISRT p	ISRT p -EWMA	ISRT p -MEWMA
0	370.4 (260)	370.4 (269)	370.4 (249)	370.4 (270)
0.01	340.44* (218)	360.87 (238)	342.10 (234)	362.60 (229)
0.03	272.41* (187)	292.42 (201)	276.46 (193)	285.61 (195)
0.05	203.09* (134)	211.39 (142)	216.94 (143)	214.54 (148)
0.10	128.82* (82)	134.23 (86)	139.43 (88)	132.96 (88)
0.30	26.69* (19)	27.60 (20)	30.20 (22)	26.87 (19)
0.50	9.06* (8)	10.09 (8)	11.72 (9)	9.94 (8)
1.00	2.22* (2)	2.25 (2)	2.91 (3)	2.24 (2)
1.50	1.17* (1)	1.18 (1)	1.50 (1)	1.17 (1)
2.00	1.01* (1)	1.01 (1)	1.12 (1)	1.01 (1)

* is minimum of ARL_1 and () is MRL_1 . Source: created by the authors.

4.1.2 Poisson Distribution

This subsection reports the Monte Carlo simulation results for data generated from a Poisson distribution, which is widely used to model count-based processes in quality control applications. The sample sizes considered were $c = 5, 10$ and 15 , representing small to moderate inspection frequencies commonly encountered in practice.

All control charts were constructed such that the in-control average run length was fixed at $ARL_0 = 370$, thereby ensuring a common false alarm rate and enabling a fair comparison among the competing charting methods.

To evaluate out-of-control performance, shift magnitudes (δ) of $0.01, 0.03, 0.05, 0.10, 0.30, 0.50$,

$1.00, 1.50$ and 2.00 were imposed on the process mean. These shifts cover a broad range of scenarios, from subtle changes that are difficult to detect to large departures that are expected to be identified rapidly.

From a statistical process monitoring perspective, when the process mean changes from the in-control value c_0 to the out-of-control value $c_1 = c_0(1 + \delta)$, a smaller value of ARL_1 is desirable, as it indicates faster signaling and improved sensitivity to process changes. Accordingly, the performance of each chart was evaluated using both the Average Run Length (ARL) and the Median Run Length (MRL). The simulation results are summarized and interpreted as follows.

Accordingly, particular attention is given to the behavior of ARL_1 and MRL_1 across different shift magnitudes. Tables 7–9 summarize the comparative performance of the competing control charts under Poisson-distributed data, with Table 7 focusing on a low-count process ($c_0 = 5$), where early detection of small shifts is especially challenging and practically important.

Table 7. reports the ARL and MRL values of the c chart, ISRT c chart, ISRT c -EWMA chart, and ISRT c -MEWMA chart for a Poisson process with in-control mean $c_0 = 5$ and smoothing parameter $\lambda = 0.05$. When the process is in control, all charts yield ARL values close to the nominal target of 370 , confirming appropriate calibration and a common false alarm rate λ .

As the process shifts from c_0 to $c_1 = c_0(1 + \delta)$, both ARL and MRL values decrease for all charts. However, the ISRT c -EWMA and ISRT c -MEWMA charts consistently achieve smaller ARL_1 and MRL_1 values than the conventional C chart and ISRT c chart, particularly for small shift magnitudes. This result demonstrates the clear advantage of incorporating memory structures when monitoring low-count Poisson processes, where subtle changes are otherwise difficult to detect.

Table 7. Values of the ARL and MRL of c , ISRT c , ISRT c -EWMA and ISRT c -MEWMA control charts when $c = 5$, $\lambda = 0.05$ and $ARL_0 = 370$

δ	c	ISRT c	ISRT c -EWMA	ISRT c -MEWMA
0	370.4 (257)	370.4 (255)	370.4 (268)	370.4 (272)
0.01	330.25 (229)	315.42 (221)	260.34 (180)	245.18* (170)
0.03	270.46 (188)	245.46 (170)	180.58 (126)	165.27* (118)
0.05	225.13 (158)	205.22 (144)	130.43 (92)	118.37* (85)
0.10	155.84	138.17	72.76	64.45*

	(110)	(99)	(51)	(47)
0.30	62.52 (44)	55.95 (39)	18.43 (13)	15.60* (11)
0.50	34.17 (24)	30.45 (22)	9.24 (7)	7.88* (6)
1.00	13.33 (9)	11.69 (8)	3.6 (3)	3.1* (3)
1.50	8.29 (6)	7.20 (5)	2.3 (2)	2.0* (2)
2.00	5.46 (4)	4.5 (4)	1.6 (1)	1.4* (1)

* is minimum of ARL_1 and () is MRL_1 . Source: created by the authors.

Table 8. Values of the ARL and MRL of c , ISRT c , ISRT c -EWMA and ISRT c -MEWMA control charts when $c = 10$, $\lambda = 0.05$ and $ARL_0=370$

δ	c	ISRT c	ISRT c -EWMA	ISRT c -MEWMA
0	370.4 (253)	370.4 (250)	370.4 (273)	370.4 (260)
0.01	344.32 (244)	257.62* (183)	277.4 (196)	327.62 (231)
0.03	260.03 (186)	136.07* (103)	144.58 (108)	265.69 (192)
0.05	214.15 (154)	82.01* (31)	87.08 (64)	213.53 (155)
0.10	132.76 (93)	33.65* (25)	35.89 (26)	132.32 (92)
0.30	27.55 (19)	8.59* (7)	8.66 (7)	28.57 (20)
0.50	9.5 (7)	5.03* (4)	5.06 (4)	8.89 (6)
1.00	2.02 (1)	2.72 (2)	2.63 (2)	2.01* (1)
1.50	1.19 (1)	2.05 (1)	1.9 (1)	1.17* (1)
2.00	1.03 (1)	1.75 (1)	1.46 (1)	1.02* (1)

* is minimum of ARL_1 and () is MRL_1 . Source: created by the authors.

Table 9. Values of the ARL and MRL of c , ISRT c , ISRT c -EWMA and ISRT c -MEWMA control charts when $c = 15$, $\lambda = 0.05$ and $ARL_0=370$

δ	c	ISRT c	ISRT c -EWMA	ISRT c -MEWMA
0	370.4 (261)	370.4 (260)	370.4 (266)	370.4 (259)
0.01	369.05 (255)	369.68 (258)	354.30 (257)	347.74* (253)
0.03	334.61 (240)	337.35 (241)	300.35* (223)	305.67 (224)
0.05	305.53 (220)	304.61 (219)	263.49* (195)	274.92 (199)
0.10	248.87 (180)	249.82 (180)	203.31* (154)	220.16 (164)
0.30	102.74 (74)	102.44 (74)	73.42* (56)	82.71 (62)
0.50	45.57 (33)	45.96 (34)	32.04* (25)	36.86 (28)
1.00	9.09	9.06	7.11*	7.92

	(7)	(7)	(6)	(6)
1.50	3.97 (3)	3.97 (3)	3.46* (3)	3.67 (3)
2.00	2.40 (2)	2.40 (2)	2.23* (2)	2.34 (2)

* is minimum of ARL_1 and () is MRL_1 . Source: created by the authors.

Table 8. reports the ARL and MRL values of the c chart, ISRT c -chart, ISRT c -EWMA chart, and ISRT c -MEWMA chart for a Poisson process using a smoothing parameter of $\lambda = 0.05$. For $c_0 = 10$, the memory-based charts generally improve detection performance, but the best-performing chart varies across shift magnitudes. In several small and moderate shift cases, the ISRT c -EWMA chart provides the smallest ARL_1 , whereas the ISRT c -MEWMA chart remains competitive and performs well for some larger shifts. These results indicate that the benefit of the proposed MEWMA structure depends on the baseline mean and shift magnitude

As the process mean shifts from c_0 to $c_1 = c_0(1+\delta)$, both ARL and MRL values decrease for all charts, indicating improved detection performance. Compared with the conventional c -chart and ISRT c -chart, the memory-based ISRT charts generally improve detection performance for several shift magnitudes. However, the best-performing chart varies depending on the in-control mean and the shift magnitude. In particular, the ISRT c -EWMA chart provides the smallest ARL values in several moderate-count cases, while the ISRT c -MEWMA chart remains competitive and performs particularly well in low-count settings. This performance gain demonstrates that the advantages of incorporating memory structures remain substantial for moderate-count Poisson processes, where detection efficiency can still be enhanced despite the increased baseline mean.

Table 9. presents the ARL and MRL results for a Poisson process with a smoothing parameter of $\lambda = 0.05$ for the EWMA and MEWMA based charts. For $c_0 = 15$, the ISRT c -EWMA chart provides the lowest ARL_1 for most small-to-large shift magnitudes, whereas the ISRT c -MEWMA chart yields the lowest ARL_1 only for the smallest shift considered. This suggests that, as the Poisson mean increases, the advantage of the MEWMA extension becomes less uniform, and the relative performance depends on both the baseline mean and the shift size

As the shift magnitude increases and the process mean changes to $c_1 = c_0(1+\delta)$, the ARL and MRL values decrease rapidly for all charts, reflecting the greater detectability of larger process shifts. For

small-to-moderate shift magnitudes, the ISRT c-EWMA and ISRT c-MEWMA charts generally provide lower ARL_1 and MRL_1 values than the conventional c-chart and ISRT c-chart in several cases. However, the relative performance varies depending on the in-control mean and the shift magnitude.

Overall, the results presented in Tables 7–9 demonstrate that incorporating memory structures into the ISRT framework substantially enhances the detection performance of control charts for Poisson-distributed data. Across the considered Poisson settings, the memory-based ISRT charts generally provide faster detection than conventional c-type charts, particularly in low-count cases. However, the relative advantage of ISRT c-EWMA and ISRT c-MEWMA varies with the in-control mean and shift magnitude. The ISRT c-MEWMA chart shows a clear advantage when $c_0 = 5$, whereas the ISRT c-EWMA chart is more competitive for several cases with $c_0 = 10$ and $c_0 = 15$. These findings suggest that the proposed MEWMA structure is especially useful for sparse and low-count Poisson processes, while parameter selection remains important for moderate-count processes.

4.1.3 Interpretation of Non-Uniform Performance

The simulation results indicate that the proposed ISRT-based MEWMA charts are not uniformly dominant under all parameter settings. Their advantage is most evident when the underlying attribute distribution is sparse, highly skewed, or characterized by a low defect probability or small Poisson mean. Under these conditions, the ISRT transformation improves the approximation of the transformed statistic, and the MEWMA structure accumulates weak signals over time, leading to faster detection of small and moderate shifts.

However, when the baseline defect probability p_0 or Poisson mean c_0 increases, the original binomial or Poisson distribution becomes less skewed and closer to normality. In such cases, the benefit of the variance-stabilizing transformation becomes less pronounced. As a result, simpler charts such as the ISRT chart, the conventional chart, or the ISRT-EWMA chart may provide comparable or even smaller ARL_1 values in some settings. This explains why the ISRT p-MEWMA and ISRT c-MEWMA charts do not uniformly dominate all alternatives in Tables 3, 5, 6, 8, and 9.

In addition, the MEWMA statistic contains a memory component and an additional adjustment parameter k . While this structure can improve sensitivity to persistent small changes, it may also

increase the variability of the monitoring statistic or introduce a slower response under some moderate-count or higher-defect settings after calibration to the same ARL_0 . Therefore, the proposed method should be regarded as particularly useful for low-rate and low-count monitoring rather than as a universally superior chart for all attribute processes.

The benefit of ISRT is expected to be strongest when the original binomial or Poisson distribution is highly skewed. As p_0 or c_0 increases, the distribution becomes more symmetric and the normal approximation becomes more reliable. Consequently, the marginal gain obtained from variance stabilization decreases. This theoretical behavior is consistent with the simulation results, where the proposed ISRT-MEWMA charts show the clearest advantage in low-defect and low-count settings but lose some advantage when p_0 or c_0 becomes larger.

Taken together, the Monte Carlo simulation results for both binomial and Poisson distributions generally indicate the advantage of memory-based ISRT control charts, particularly under sparse and low-count conditions over their conventional and transformation-only counterparts. In both distributional settings, fixing the in-control average run length at $ARL_0 = 370$ ensures a fair comparison, while introducing EWMA and MEWMA structures results in substantial reductions in ARL_1 and MRL_1 , particularly for small and moderate shifts that are of primary concern in statistical process monitoring. Although the magnitude of improvement varies with the underlying distribution and parameter values, the overall pattern remains robust: incorporating memory within the ISRT framework can enhance detection efficiency in many low-rate and low-count scenarios, although the magnitude of improvement depends on the baseline parameter and shift size. These results provide strong empirical support for the effectiveness and general applicability of the proposed ISRT based EWMA and MEWMA control charts.

While the Monte Carlo simulation results in Section 4.1 provide a systematic and controlled evaluation of the detection performance of the competing control charts under predefined distributional assumptions, practical quality monitoring often involves data collected under complex and less ideal conditions. Therefore, an empirical analysis based on real-world data is essential to further assess the robustness and practical usefulness of the proposed control charts.

Accordingly, this section extends the comparative analysis to real data in order to validate the findings obtained from the simulation study and

to examine whether the observed advantages of the ISRT based EWMA and MEWMA charts persist under actual operating conditions. By applying the control charts to real binomial and Poisson data sets, this analysis aims to demonstrate the practical applicability, reliability, and effectiveness of the proposed methods in detecting process shifts beyond controlled simulation environments.

4.2 Real Application

This section provides a comparative analysis of the control charts using real-world data. The objective is to validate the findings obtained from the Monte Carlo simulation and to demonstrate the practical applicability and effectiveness of the proposed charts in detecting process shifts under actual operating conditions.

4.2.1 Bearing and Seal Assembly Nonconformity Data

This subsection examines the performance of the p chart, ISRT p-chart, ISRT p-EWMA chart, and ISRT p-MEWMA chart using real-world data that follow a binomial distribution. The data considered in this analysis report is the number of nonconforming bearing and seal assemblies observed in samples of size 100, which represent a typical attribute quality control scenario in manufacturing processes, [1].

Based on these data, a fraction nonconforming (p) control chart was constructed to monitor the proportion of nonconforming units over time. When sample points were found to plot outside the control limits, it was assumed that assignable causes could be identified and eliminated. The control limits were then recalculated using the remaining in-control observations in order to obtain revised limits that more accurately reflect the stable process behavior.

Figure 1, Figure 2, Figure 3 and Figure 4 illustrate the monitoring results obtained from the competing control charts. Figure 1 presents the conventional p chart, which identifies several out-of-control points, indicating potential process instability. However, due to its reliance solely on current sample information, the p chart exhibits limited sensitivity to subtle or gradually developing shifts. Figure 2 shows the ISRT p-chart, which provides a more stabilized charting statistic as a result of the transformation embedded in the ISRT framework. Although the signaling behavior is more structured than that of the p chart, the absence of a memory mechanism still limits its ability to detect small and persistent changes at an early stage.

In contrast, Figure 3 demonstrates the ISRTp-EWMA chart, where the incorporation of an

exponential weighting structure allows historical information to influence the current monitoring statistic. As a result, out-of-control signals appear earlier and more consistently, and the chart exhibits smoother trajectories with reduced random fluctuation. Figure 4 displays the ISRTp-MEWMA chart, which combines the ISRT transformation with a modified EWMA structure. Among all charts considered, this chart shows the strongest and most persistent signaling behavior, reflecting enhanced sensitivity to subtle and sustained deviations from the in-control state.

Overall, the comparative results presented in Figure 1, Figure 2, Figure 3 and Figure 4 confirm that the advantages of memory-based ISRT charts observed in the Monte Carlo simulation are preserved when applied to real manufacturing data. In particular, the ISRTp-EWMA and ISRT p-MEWMA charts provide superior detection performance and greater stability under realistic operating conditions, thereby offering a more effective framework for monitoring binomial quality characteristics in practice.

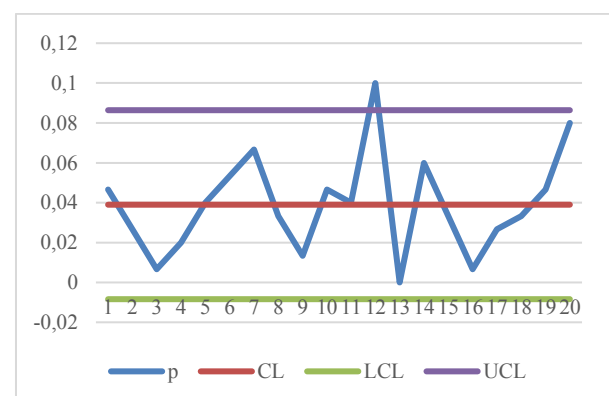


Fig. 1: Efficiency in detecting a change with p chart
Source: created by the authors.

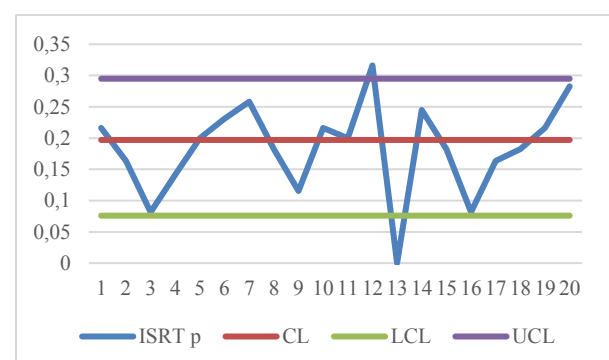


Fig. 2: Efficiency in detecting a change with ISRT p-chart. Source: created by the authors.

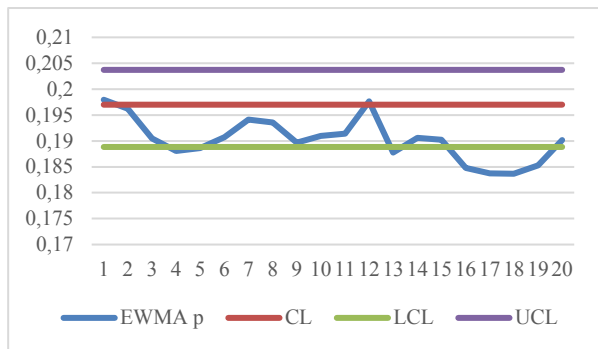


Fig. 3: Efficiency in detecting a change with ISRT p-EWMA chart.

Source: created by the authors.

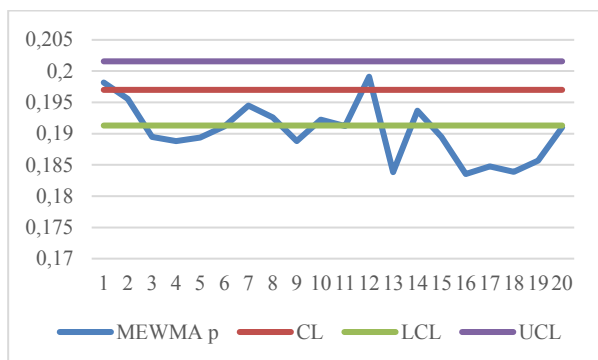


Fig. 4: Efficiency in detecting a change with ISRT p-MEWMA chart.

Source: created by the authors.

4.2.2 Surface defects on steel plates

This subsection evaluates the performance of the c-chart, ISRT c-chart, ISRT c-EWMA chart, and ISRT c-MEWMA chart using real-world data that follow a Poisson distribution. The data considered in this analysis consist of counts of surface defects observed on twenty-five rectangular steel plates, which represent a typical nonconformity-based quality characteristic encountered in manufacturing and materials processing applications, [1].

Based on these data, a control chart for nonconformities was constructed to monitor the number of defects per inspection unit over time. When observations were found to plot outside the control limits, it was assumed that assignable causes could be identified and eliminated. The control limits were subsequently recalculated using the remaining in-control observations in order to obtain revised limits that more accurately represent the stable process behavior.

Figure 5, Figure 6, Figure 7 and Figure 8 illustrate the monitoring results obtained from the competing control charts. Figure 5 presents the conventional C chart, which identifies several out-of-control points associated with pronounced increases in defect counts, indicating potential process instability. However, due to its reliance

solely on current observations, the C chart exhibits limited sensitivity to subtle or gradually developing changes in the underlying defect rate. Figure 6 shows the ISRT c-chart, which provides improved stabilization of the charting statistic through transformation. Although the signaling behavior is more structured than that of the C chart, the absence of a memory component still constrains its ability to detect small and persistent shifts at an early stage.

In contrast, Figure 7 demonstrates the ISRTc-EWMA chart, where the incorporation of an exponential weighting structure allows historical information to influence the current monitoring statistic. As a result, out-of-control signals appear earlier and more consistently, and the chart exhibits smoother trajectories with reduced random fluctuation. Figure 8 displays the ISRT c-MEWMA chart, which combines the ISRT transformation with a modified EWMA structure. Among all charts considered, this chart shows the strongest and most persistent signaling behavior, reflecting enhanced sensitivity to subtle and sustained deviations from the in-control state.

Overall, the comparative results presented in Figure 5, Figure 6, Figure 7 and Figure 8 confirm that the advantages of memory-based ISRT charts observed in the Monte Carlo simulation are preserved when applied to real Poisson-distributed defect data. In particular, the ISRTc-EWMA and ISRT c-MEWMA charts provide superior detection performance and greater stability under realistic operating conditions, thereby offering a more effective framework for monitoring count-based quality characteristics in practice.

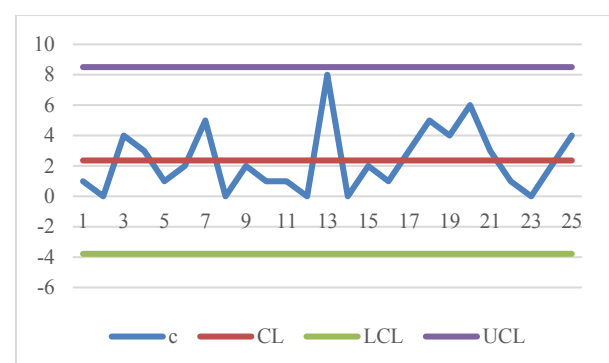


Fig. 5: Efficiency in detecting a change with c chart.

Source: created by the authors.

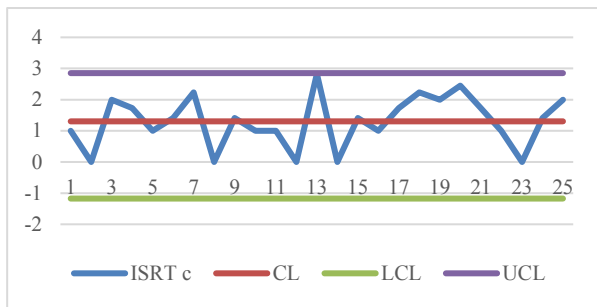


Fig. 6: Efficiency in detecting a change with ISRT c-chart.

Source: created by the authors.

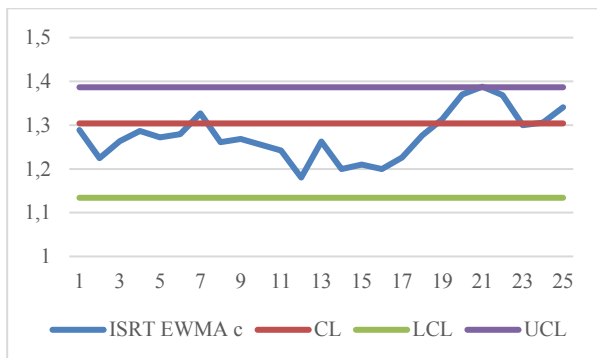


Fig. 7: Efficiency in detecting a change with ISRT c-EWMA chart.

Source: created by the authors.

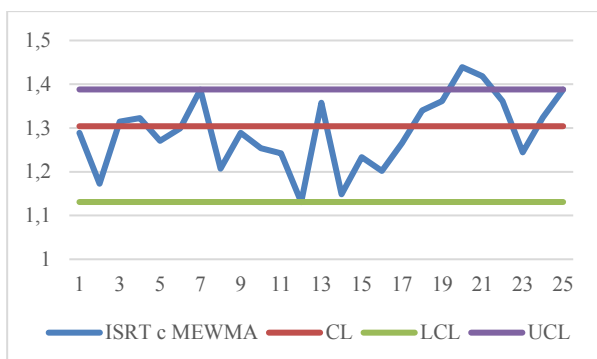


Fig. 8: Efficiency in detecting a change with ISRT c-MEWMA chart.

Source: created by the authors.

4.3 Sensitivity Analysis of Smoothing Parameters

To examine the robustness of the proposed charts, a sensitivity analysis was conducted with respect to the smoothing parameter λ , the control-limit coefficient, and the baseline process parameters. Smaller values of λ , such as 0.03 and 0.05, assign greater weight to historical observations and are therefore more suitable for detecting small and persistent shifts. Larger values of λ , such as 0.10 and 0.20, place greater emphasis on recent observations and may respond more quickly to moderate or large shifts.

The simulation results indicate that the performance of the ISRT based memory charts is sensitive to the choice of λ . In low-defect and low-count settings, smaller smoothing parameters generally provide better detection performance because they accumulate weak signals over time. However, when the baseline defect probability or Poisson mean increases, the advantage of the MEWMA structure becomes less uniform, and the ISRT EWMA or even the conventional chart may provide comparable ARL_1 values in some cases. These findings confirm the importance of ARL based calibration and parameter tuning when applying the proposed charts in practice. A summary of the sensitivity analysis is presented in Table 10.

Table 10. Sensitivity Analysis of Smoothing Parameters

Design parameter	Values considered	Main observation
Smoothing parameter λ	0.03, 0.05, 0.10, 0.20	Smaller values are more suitable for small persistent shifts
Additional MEWMA parameter k	Fixed / varied according to the proposed design	Larger k increases sensitivity to recent changes but may increase the variance of the monitoring statistic.
Binomial control-limit coefficients	h_1, h_2, h_3 , calibrated by simulation-based search	Coefficients were adjusted until $ARL_0 = 370$.
Poisson control-limit coefficients	b_1, b_2, b_3 , calibrated by simulation-based search	Coefficients were adjusted until $ARL_0 = 370$.
Baseline defect probability p_0	0.01, 0.05, 0.10	MEWMA advantage is strongest when (p_0) is low
Poisson mean c_0	5, 10, 15	MEWMA advantage is more evident when (c_0) is low
Subgroup size n	50, 100	Larger (n) improves detectability, but the best chart may vary

Source: created by the authors.

5 Conclusion and Future Work

This study developed and evaluated ISRT-based modified EWMA control charts for monitoring binomial and Poisson attribute data. The proposed ISRT p-MEWMA and ISRT c-MEWMA charts combine the variance-stabilizing property of ISRT with the memory-based structure of MEWMA monitoring. The performance of the proposed charts was compared with conventional p and c-charts, ISRT-based charts, and ISRT-based EWMA charts using Monte Carlo simulation and real manufacturing data.

In the simulation study, all charts were calibrated to achieve a common in-control average run length of $ARL_0=370$, allowing a fair comparison of out-of-control detection performance. The results show that ISRT-based memory charts generally reduce ARL_1 and MRL_1 for small and moderate shifts, particularly under sparse, skewed, low-defect, and low-count conditions. The ISRT p/c-MEWMA charts provide strong performance in several low-defect and low-count scenarios; however, the best-performing chart may vary depending on the baseline parameter, subgroup size, smoothing parameter, and shift magnitude. Therefore, the proposed MEWMA charts should be regarded as robust alternatives rather than uniformly dominant methods.

The real-data applications further support the practical usefulness of the proposed framework. For binomial nonconforming data and Poisson defect-count data, the ISRT-based EWMA and MEWMA charts produced earlier and more stable signaling behavior than conventional charts in several cases. These findings indicate that combining transformation-based correction with memory-based monitoring can improve early shift detection in attribute quality-control applications.

Several directions for future research merit further investigation. First, the current study focuses on binomial and Poisson distributions; future work may extend the proposed framework to other discrete distributions, such as the negative binomial distribution, to accommodate overdispersed count data commonly encountered in practice.

Second, adaptive or data-driven selection of the smoothing parameter λ could be explored to further enhance detection performance under varying process dynamics. Additionally, incorporating robust estimation techniques may improve chart performance in the presence of outliers or model misspecification.

Third, future studies may consider extending the ISRT based memory charts to multivariate attribute processes and high-dimensional monitoring problems, where complex dependence structures are present. Finally, real-time implementation and integration with Industry 4.0 or smart manufacturing systems represent promising avenues for applying the proposed methods in automated quality monitoring environments.

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Declaration of Generative AI and AI-assisted Technologies in the Writing Process

During the preparation of this work, the author(s) utilized artificial intelligence (AI) technologies solely for the purpose of linguistic refinement and grammatical correction. Following the application of these tools, the author(s) comprehensively reviewed, verified, and edited the resulting text. The author(s) maintain full accountability and responsibility for the accuracy, integrity, and final content of the publication.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

-S.P.: Data collection, software, data analysis, original draft writing, methodology.

-Y.A.: Data curation, validate, conclusion.

-S.S.: Conceptualization, investigation, funding acquisition, project administration, original draft writing, reviewing, and editing.

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Conflict of Interest

The authors have no.

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